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1 **Effects of discrete-electrode configuration on traveling-** 2 **wave electrohydrodynamic pumping**

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16 17 **ABSTRACT**

18 Traveling-wave electrohydrodynamic (EHD) micropumps can be incorporated
19 into the package of an integrated circuit chip to provide active cooling. They can also be
20 used for fluid delivery in microdevices. The pump operates in the presence of a thermal
21 gradient through the fluid layer such that a gradient in electrical conductivity is
22 established allowing ions to be induced. These ions are driven by a traveling electric field.
23 Such a traveling electric field can be realized in practice only via discrete electrodes upon
24 which the required voltages are imposed. The impact of using discrete electrodes to create
25 the traveling wave on the flow rates generated is explored through numerical modeling.
26 The change in performance from an ideal sinusoidal voltage boundary condition is
27 quantified. The model is used to explore the widths of electrodes and the intervening
28 isolation regions that lead to optimized pumping. The influence of the choice of working
29 fluid on the performance of the pump is determined using an analytical model.

30 31 **KEYWORDS:**

32 electrohydrodynamics, micropump, microfluidics, fluid delivery, electronics cooling

33

1 **1. INTRODUCTION**

2 Thermal management of electronic components is of increasing concern in the
3 development of reliable and portable electronic devices. The need to reduce package
4 weight and volume while increasing the functionality has been widely discussed in recent
5 years. The reduction in transistor size and the increase in power density necessitate
6 alternative cooling techniques to replace conventional air-cooled heat sinks. Among the
7 alternative strategies for improved thermal management of electronic systems, liquid
8 cooling using microchannels offers the ability to increase power dissipation while also
9 maintaining a small form factor. Contact and spreading resistances can be reduced or
10 even eliminated by integrating the channels directly on the back side of common flip-chip
11 designs. Further, by using liquid cooling, the heat-generation and heat-dissipation
12 components can be separated, allowing the convective surface area to be unconstrained by
13 the microprocessor area (Mahajan et al. 2006). Thus, the heat exchanger in the cooling
14 loop can be placed at any convenient location in the system. Typically, desirable
15 attributes of a pump for electronics cooling are that they provide high flow rates with low
16 power consumption. However, high flow rates usually translate to large pumps to drive
17 the liquid flow. The prohibitive pumping requirements have limited the application of
18 microchannel heat sinks in space-constrained electronics (Garimella et al. 2006).
19 Solutions which integrate micropumps directly into the microchannels thus represent an
20 important research area to facilitate broader use of liquid cooling in electronic systems.

21 Additionally, the development of cell analysis tools has recently targeted
22 microfluidic devices since they can be used to sample, trap, separate, sort, treat and
23 analyze cells (Andersson and Van den Berg 2003). Microfluidic devices offer many
24 attractive benefits for biological handling and analysis. For example, reducing device
25 size also reduces sample requirements and reagent volumes, which can reduce overall
26 cost. Test chips are often disposable which is important for sterility. Using microfluidic
27 chips also allows for a closed system, thus protecting the operator from chemical
28 exposure. The small size accommodates parallel operations and thereby reduces cell
29 sorting, analysis and treatment times. Combining different functions on a single
30 microchip is another step toward achieving a completely closed system that can be fully

1 automated, reduce contamination, and eliminate human intervention and error (Wolff et
2 al. 2003). However, for microfluidic devices to capitalize on all of the above benefits,
3 integration of the fluid pumping mechanism is imperative. Early microfabricated cell-
4 sorting devices used electroosmotic flow as the driving mechanism, which resulted in low
5 sample throughput (Fu et al. 1999). An increased throughput has been achieved using
6 pressure-driven alternatives, but at the expense of removing the pumping mechanism
7 from the chip level (Fu et al. 2002, Wolff et al. 2003). Regardless, the target flow rates
8 for pumping biological materials are typically much lower than for convective cooling,
9 allowing reduced demands on pump operating conditions (such as voltage, power, etc.) to
10 obtain the desired range of fluid flow.

11 Recent reviews of possible micropumping mechanisms are available in (Iverson and
12 Garimella 2008, Laser and Santiago 2004, Singhal et al. 2004). Although
13 electrohydrodynamics (EHD) has been studied for many years (Melcher 1966, Melcher
14 and Firebaugh 1967, Seyed-Yagoobi 2005, Seyed-Yagoobi et al. 1989a, Seyed-Yagoobi
15 et al. 1989b), it has recently emerged as a potential driving mechanism for micropumps
16 due to its miniaturization potential (Choi and Kim 1995, Fuhr et al. 1992). Further, EHD
17 pumps have the ability to pump a variety of liquids for applications in various research
18 fields (Crowley et al. 1990). A particular advantage of these pumps is that for traveling-
19 wave induction EHD pumps, no modifications are necessary to enable pumping of
20 different types of liquids other than a change in frequency of excitation (Fuhr et al. 1994).

21 Melcher and Firebaugh (Melcher and Firebaugh 1967), in an early EHD model, used
22 the electric shear approach to calculate the flow profile due to induction EHD. This
23 analytical solution neglected the time and space variation in induced charge density and
24 assumed quasi-static flow. Later models have considered a similar approach using the
25 electric shear stress but have considered developing flow as well (Seyed-Yagoobi et al.
26 1989a, Seyed-Yagoobi et al. 1989b). Combining two promising scalable pumping
27 mechanisms – EHD with nozzle-diffuser elements and vibrating diaphragm actuation –
28 Singhal and Garimella devised a micropump with the potential for direct integration into
29 an active chip for heat removal (Singhal and Garimella 2005a, Singhal and Garimella
30 2005b, Singhal and Garimella 2007). A force density approach, which can predict the

1 induced charge density as a function of both space and time, was used in developing a
2 numerical model of the pumping scheme such that the transient nature of traveling-wave
3 induction EHD could be simulated (*i.e.*, charge induction, relaxation, and flow initiation).
4 Temperature field-controlled induction EHD, in which the temperature gradient is
5 controlled using Peltier elements, has also been discussed (Felten et al. 2006). Flow
6 velocities were shown to increase by increasing the temperature-dependent conductivity
7 gradient for relatively low applied voltages. Numerical schemes have been presented for
8 traveling wave-induced electrothermal fluid flows in which material selection and device
9 geometry play an important role in driving forces (Perch-Nielsen et al. 2004). Similar
10 traveling wave devices, used for dielectrophoresis with conductive fluids, can result in
11 Joule heating and temperature gradients where electrohydrodynamic forces become
12 significant when driven at frequencies equal to the charge relaxation frequency.
13 Illumination methods for experimental characterization of electrothermal flows can also
14 contribute to temperature gradients and affect results (Green et al. 2001).

15 Early analytical models have represented the sinusoidally varying voltage at the
16 boundary as a perfect sinusoid (Melcher and Firebaugh 1967). Subsequent numerical
17 models have typically been validated by comparison against these early analytical models.
18 However, practical implementations of this pumping scheme (as well as numerical model
19 domains) necessitate a discretization of the voltage boundary. In the present work, we
20 consider the impact of the use of such discretized electrode arrays on flow rate in
21 traveling-wave induction EHD pumps. The performance achievable with these discrete
22 electrodes is compared to that obtained with the ideal case of a truly sinusoidal voltage
23 boundary condition. Optimization of the respective lengths of the electrode and the inter-
24 electrode spacing as well as the number of phases used for actuation is explored to
25 increase the flow rate for a given wavelength and dielectric material. We also discuss
26 fluid selection considerations and provide flow rates that can be achieved using a range of
27 fluid types.

28 **2. EHD PUMPING CAPABILITY**

29 EHD pumping has progressively found favor in applications where integration is
30 paramount. Past induction EHD studies typically include dielectric fluids (Bohinsky and

1 Seyed-Yagoobi 1990) along with selected organic fluids (Crowley et al. 1990). Here, we
2 provide a comparison of dielectric fluids (common to the electronics industry) and some
3 common electrolyte solutions (common to the biological community) using the analytical
4 treatment developed by Melcher and Firebaugh (Melcher and Firebaugh 1967). An
5 approach of this type could easily be adapted to evaluate fluids specific to other
6 applications.

7 **2.1 Analytical treatment**

8 Induction EHD pumping is generated through the interaction of an electric field with
9 charges induced in the fluid medium. Induction of charge in the bulk of a liquid occurs in
10 the presence of a gradient in the temperature (and corresponding electrical conductivity)
11 of the liquid through anisotropic heating. These charges can then be attracted or repelled
12 by a traveling-wave potential established on the boundary of the channel so that the fluid
13 moves with the charges due to viscous drag (see Figure 1). Melcher and Firebaugh
14 presented a 2D analytical solution of the velocity profile for this pumping mechanism
15 (Melcher and Firebaugh 1967). Some important limiting assumptions of their model are
16 as follows: effects of charge transport by the fluid are small, the gradient of the electrical
17 conductivity is uniform (linear), and the traveling wave period is small so that thermal
18 relaxation can be neglected (resulting in a constant temperature profile) and a time-
19 averaged electric stress can be used. Under these assumptions, the velocity profile is
20 described by equation 18 in their paper. If we further assume that the viscosity is uniform
21 throughout the channel, the viscosity can be removed from the integration terms and the
22 velocity profile easily evaluated numerically.

23 **2.2 Working fluid considerations**

24 Using the 2D velocity profile of Melcher and Firebaugh, we compare the fluid flows
25 generated for several electrolytic fluids. Each set of conditions is numerically integrated
26 using equation 18 from their paper with the additional assumption of constant viscosity
27 (Melcher and Firebaugh 1967). Table I lists the dimensions and parameters common to
28 the cases that will be considered in the comparisons. A wavelength of 54 μm was also
29 used for all fluid comparison calculations of this section. We select a 10°C temperature

1 difference across the channel depth for all cases; alternatively, this comparison could be
 2 conducted for a given imposed heat flux. For the 10°C temperature difference assumed
 3 here, the corresponding heat flux through a water layer of thickness 50 μm ($k_{f,w} = 0.6$
 4 $\text{W/m}\cdot\text{K}$) would be approximately 12 W/cm^2 , neglecting convective effects.

5 Fluid properties play an important role in determining EHD pump operating
 6 conditions. The induced charge relaxation time τ is characterized by the permittivity ε
 7 and electrical conductivity σ of the fluid as $\tau = \varepsilon/\sigma$. Ideally, the traveling wave frequency
 8 should correspond to the charge relaxation time ($f_{ideal} = 1/2\pi\tau$) such that the charges are
 9 able to fully transit from one electrode to the next, without long residence times before
 10 the voltages assigned to the electrode array change. If the frequency is much lower,
 11 induced charges reach their equilibrium position opposite the traveling wave quickly and
 12 the tangential shear exerted on the fluid is small. At high frequencies, the relaxation
 13 process inhibits charge from accumulating at the surface, again reducing the shear stress
 14 (Crowley 1980).

15 Table II lists important properties for several electrolytic fluids. Electrolytes are
 16 commonly used in EHD studies with properties tailored by the level of doping in the
 17 aqueous solution. A range of doping molarities are listed for both KCl and NaCl with
 18 accompanying electrical properties. As a comparison, water is also included in Table II.
 19 Unless otherwise noted, electrical conductivity values were obtained using a temperature
 20 coefficient (α) and the following relation based on a known conductivity (σ) at a
 21 reference temperature, T_0 (Fuhr et al. 1992).

$$22 \quad \sigma(T) = \sigma(T_0) [1 + \alpha(T - T_0)] \quad (1)$$

23 For the electrolyte solutions, temperature coefficients depend on the doping
 24 concentration, with higher coefficients for lower doping. However, this change is
 25 relatively small. Temperature coefficients can play a significant role in changing the
 26 conductivity gradient and corresponding velocities. Further, as doping increases, the
 27 electrical conductivity increases and the dielectric constant decreases. Thus for higher
 28 concentration solutions, the charge relaxation time significantly decreases and optimum
 29 traveling frequencies extend well into the MHz range. Using the average velocity at the
 30 ideal frequency as the basis for comparison (Table II), we see that with increasing doping

1 concentration of KCl and NaCl, the average velocity decreases slightly. While the
2 decrease in velocity does not appear to be significant, the ideal frequency of the traveling
3 wave does increase considerably to accommodate the increase in electrical conductivity.

4 One possible unit of fluid comparison that emerges from Melcher and Firebaugh's
5 analysis is the term $\varepsilon\Delta\sigma/\mu\sigma_{avg}$, where μ is the fluid viscosity. For electrolytes, this value
6 does not vary by orders of magnitude, thereby resulting in comparable pumping velocities
7 (despite changes in frequency). However, when the temperature coefficient varies
8 significantly from fluid to fluid it can result in significantly different velocities at a given
9 fluid's ideal frequency.

10 High concentration electrolyte solutions have the potential of causing significant
11 Joule heating and have been excluded from the calculations. However, in some instances
12 Joule heating may be used as the primary mechanism for establishing the temperature
13 gradient for induction electrohydrodynamics when heat removal is not necessarily the
14 target application. For use in biological fluid systems, the standard 0.01 M phosphate
15 buffered saline (PBS) commonly used as a biological buffer (0.137 M NaCl, 0.0027 M
16 KCl) has the potential of being driven without an external heating mechanism since Joule
17 heating can be considerable at large voltages.

18 The increase of salt concentration in aqueous solution can cause a corresponding
19 increase in the electrolysis of water. Combining Faraday's law of electrolysis (Serway et
20 al. 2005) with the ideal gas law, it is possible to estimate the volume of gas generated
21 from electrolysis. For the ~0.01 M aqueous solutions in Table II, electrolysis can become
22 significant as the volume of gas generated begins to fill the pumping volume when left
23 stagnant. However, as the liquid is pumped, the gas generated due to electrolysis would
24 travel with the transported liquid such that buildup of gas in the pump volume could be
25 mitigated.

26 **3. NUMERICAL MODEL OF EHD PUMPING**

27 To address some of the limitations of the analytical approach to modeling EHD, a
28 transient numerical model was developed using the force-density approach that can
29 predict the induced charge density as a function of both space and time. The geometry
30 and boundary conditions associated with discrete electrodes are also accommodated.

1 A quasi-static analysis is used for modeling the induced fluid motion due to EHD, in
 2 which an electrostatic field is applied to a fluid containing electrically charged particles.
 3 The continuity equation takes the standard incompressible form with velocity \vec{v} ,

$$4 \quad \vec{\nabla} \cdot \vec{v} = 0. \quad (2)$$

5 The incompressible Navier-Stokes equation for conservation of momentum is modified to
 6 account for Coulomb forces acting on the charged particles in the fluid

$$7 \quad \rho \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \vec{\nabla} \vec{v} \right) = -\vec{\nabla} p + \left(\vec{\nabla} \cdot \vec{\tau}_{ij} \right) + \vec{F}_b + q\vec{E} \quad (3)$$

8 in which ρ is density, p is pressure, τ is shear stress and F_b is body force. The
 9 electrostatic field \vec{E} is given by

$$10 \quad \vec{E} = -\vec{\nabla} \phi. \quad (4)$$

11 Gauss's law may be used to relate the electric potential ϕ to the charge density q ,

$$12 \quad q = \vec{\nabla} \cdot (\varepsilon \vec{E}) \quad (5)$$

13 where ε is the fluid permittivity. Thus the Coulomb force is represented as:

$$14 \quad q\vec{E} = (-\varepsilon \vec{\nabla}^2 \phi) \cdot (-\vec{\nabla} \phi) = \varepsilon \vec{\nabla}^2 \phi \cdot \vec{\nabla} \phi. \quad (6)$$

15 Assuming no species reactions, conservation of charge takes the form,

$$16 \quad \frac{\partial q}{\partial t} + \vec{\nabla} \cdot \vec{J} = 0 \quad (7)$$

17 where the current density \vec{J} is comprised of conduction, convection, and diffusion of
 18 charge (diffusivity D), as

$$19 \quad \vec{J} = \sigma \vec{E} + q\vec{v} - D\vec{\nabla} q. \quad (8)$$

20 The resulting fluid transport equations are given below for an incompressible fluid after
 21 incorporating the Coulomb forces and neglecting body forces.

22 Conservation of mass:

$$23 \quad \vec{\nabla} \cdot \vec{v} = 0 \quad (9)$$

24 Conservation of momentum:

$$25 \quad \rho \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \vec{\nabla} \vec{v} \right) = -\vec{\nabla} p + \left(\vec{\nabla} \cdot \vec{\tau}_{ij} \right) + \varepsilon \vec{\nabla}^2 \phi \cdot \vec{\nabla} \phi \quad (10)$$

26 Conservation of charge:

$$\frac{\partial q}{\partial t} + \vec{\nabla} \cdot (q\vec{v}) = \vec{\nabla} \cdot (\sigma\vec{\nabla}\phi + D\vec{\nabla}q) \quad (11)$$

1 The finite element software package FIDAP (Fluent 1998) was used to numerically
 2 model these transport equations. At each time step the voltage assigned to discrete
 3 electrodes along the boundary (used to provide a spatially varying, sinusoidal voltage
 4 along the wall) is updated using user-defined functions. Discretization of the boundary is
 5 discussed in more detail in sections that follow. The magnitude of the assigned voltage
 6 varies sinusoidally with time and with neighboring electrodes offset in time. The
 7 neighboring electrode phase shift depends on the number of electrodes used to represent
 8 the spatially varying, sinusoidal voltage. Thus, the voltage at the neighboring electrodes
 9 varies continuously in time providing a traveling wave with the neighboring, phase-
 10 shifted electrodes. Both voltage and charge distributions are tracked throughout the
 11 calculation in response to the changing voltage boundary condition. Figure 2 illustrates
 12 these boundary conditions and computational domain for a 3-phase, repulsion-type,
 13 induction EHD device.

14 A zero pressure gradient condition is imposed at the inlet and outlet and a subroutine
 15 was used to define and apply the traveling-wave voltage boundary condition. The voltage
 16 and velocity distributions in the fluid are initially set to zero for transient startup and
 17 application of the boundary conditions. An imposed temperature difference is applied at
 18 the upper and lower channel walls. With the top and bottom walls at a uniform
 19 temperature, the temperature field must be fully developed, in which case the energy
 20 equation reduces to a linear temperature profile. Viscous dissipation can be neglected
 21 since $Pr \cdot Ec \ll 1$. The temperature-dependent conductivity varies through the channel
 22 depth in a linear fashion (see also equation 1). While permittivity also varies with
 23 temperature, we have neglected permittivity variation in the aqueous solution used here
 24 since $\frac{1}{\varepsilon} \left(\frac{\partial \varepsilon}{\partial T} \right)$ is roughly 0.4% K^{-1} and $\frac{1}{\sigma} \left(\frac{\partial \sigma}{\partial T} \right)$ is roughly 2.2% K^{-1} (Green et al. 2001,
 25 Lide 2001, Perch-Nielsen et al. 2004). Further discussion of the validation of this model
 26 is provided in (Singhal and Garimella 2005b).

1 **4. TREATMENT OF DISCRETE ELECTRODES**

2 In electrohydrodynamic devices that incorporate a traveling-wave voltage boundary
3 condition, the manner in which the voltage is treated in the model has a significant impact
4 on the behavior of the fluid flow. A true sinusoidal voltage condition cannot be achieved
5 at the boundary in practice. In order to fabricate a boundary used to provide a
6 sinusoidally varying voltage, an ideal sinusoid must be discretized into a number of
7 neighboring electrodes with separately assigned voltages; in addition, to avoid shorting,
8 these neighboring electrodes must be isolated from one another. The resulting boundary
9 condition therefore has important differences compared to an ideal sinusoidal voltage
10 condition.

11 The basic design considered here is that of a wide, flat channel with electrodes
12 placed on the underside of the top wall of the channel. Further, the electrodes cover the
13 entire width and length of the channel. While the presence of electrodes on only one side
14 of the channel renders the domain three-dimensional, 3D models are computationally
15 prohibitive since very small node spacings are required to resolve the short wavelengths
16 present in microscale devices. In the following computations, we assume a 2D
17 representation of a 50 μm deep, 0.8 cm wide, and 1.0008 cm long channel, in view of the
18 large channel width-to-depth ratio. A linear temperature profile is assumed through the
19 channel depth. For all subsequent numerical calculations, the first electrolytic solution
20 listed in Table II (2.2×10^{-5} M KCl (1)) is used at a driving voltage of 200 V. This fluid
21 is characterized by an average conductivity of 5.43×10^{-4} S/m and dielectric constant of
22 80 resulting in a charge relaxation time and frequency of 1.30 μs and 122 kHz,
23 respectively. For a driving voltage of 200 V, the temperature rise due to Joule heating
24 can be shown to be minimal using the scaling approach outlined by Ramos et al., but
25 without neglecting convection effects (Ramos et al. 1998). Electrolysis can also be
26 shown to be minimal using the approach suggested in section 2.2.

27 In attraction-type induction EHD, the fluid at the lowest temperature which has the
28 lowest electrical conductivity is closer to the electrodes where the most intense electric
29 field is present in the microchannel (see Figure 1a). The fluid follows the traveling
30 potential wave in the same direction as the induced charges are “attracted” towards the

1 traveling wave. In the case of repulsion-type induction EHD, the region of high electrical
2 conductivity also has a high electric field resulting in charges being repelled in a direction
3 opposite to that of the traveling wave (see Figure 1b). In the computations here, we
4 consider repulsion-type EHD; the computed velocities would be essentially the same for
5 attraction-type EHD as well. Increasing the node count from 70,749 to 129,129 resulted
6 in a difference of less than 5% in the velocity, providing satisfactory mesh independence.
7 Therefore, the coarser mesh was used in the following computations.

8 Figure 3 illustrates three possible ways in which the sinusoidal boundary condition
9 may be approximated. First, an idealized sinusoid may be used in which each node along
10 the boundary is assigned a voltage corresponding to $V(x) = \sin(2\pi x/\lambda)$. In this
11 configuration, neighboring electrodes are assumed to operate without any electrical
12 interference between neighbors. Such a condition is not achievable in practice since there
13 would need to be some isolation between neighboring electrodes in a real device. The
14 remaining two curves represented in Figure 3 illustrate isolation of neighboring electrodes
15 with a dielectric for 3-phase representations of a sine curve using a zero voltage and zero
16 voltage flux (normal to the wall) boundary condition.

17 **4.1 Idealized-sinusoid boundary**

18 Consider first a microscale repulsion EHD case where the voltage boundary condition is
19 modeled using a discretized sine wave with no spacing between electrodes. As the
20 number of nodes increases, this boundary condition approaches the perfect sinusoid
21 condition commonly used in analytical models. The idealized-sinusoid boundary
22 condition is produced using 36 electrodes of 1.5 microns width for a total wavelength of
23 54 μm (case 1 in Table III). The entire potential wave is divided into 36 phases with a
24 different voltage applied to each phase/electrode. The voltage is then updated at each
25 time step to achieve a traveling potential wave.

26 The difference in the velocity profiles resulting from the perfect sinusoid boundary
27 condition from Melcher and Firebaugh's model and the idealized-sinusoid boundary
28 condition using the current model (case 1, Table III) is shown in Figure 4. Integrating
29 these profiles and dividing by the depth we obtain an average velocity of 1.79 m/s for the
30 analytical result and 1.66 m/s for numerical case 1, for a difference of 7%.

1 Melcher and Firebaugh's model is designed for re-entrant channel geometries and
 2 limited to steady solutions by using a time-averaged, electric shear stress. Using the
 3 force-density approach, the current numerical model is capable of re-entrant geometries
 4 or non-zero pressure gradient scenarios. The model can predict the induced charge
 5 density as a function of both space and time, thus capturing flow transients due to
 6 induction EHD. Moreover, the effect of different voltage profiles, as well as of the
 7 presence of separate electrodes, can be considered in the present model. Unlike Melcher
 8 and Firebaugh, convection of charge is also taken into account. The finely discretized
 9 traveling wave used in our numerical results, although approaching a perfect sinusoidal
 10 condition, is also expected to have some deviation from the analytical solution with a
 11 sinusoidal boundary condition.

12 The simplified version of the analytical solution presented by Melcher and Firebaugh
 13 further assumes that the wavelength is much larger than the channel depth. This length-
 14 scale limitation in the analytical model and postulation about the variation of the potential
 15 and charge field profiles within the fluid make the current numerical model (using a
 16 force-density approach) applicable to a wider range of device modeling.

17 However, neither a perfect sinusoid nor the idealized sinusoid of case 1 can be
 18 experimentally achieved. Fabrication of such devices requires electrical isolation
 19 between neighboring electrodes at different potentials. Therefore, in practice, further
 20 relaxation of the sinusoidal boundary condition assumption is necessary.

21 **4.2 Discretized-sinusoid boundary with isolation**

22 In a real traveling-wave device, neighboring electrodes are electrically isolated from each
 23 other using dielectric films and/or substrates. For the same wavelength, inclusion of
 24 isolation regions in the boundary effectively reduces the electrode coverage area and
 25 density as compared to the idealized sinusoid of case 1. Two possible choices for the
 26 boundary condition in the spacing region between electrodes can be considered. Either
 27 the voltage can be set to zero, or the flux of voltage normal to the boundary
 28 ($dV/dy|_{wall} = -\vec{E}_y = 0$) can be set to zero. The former condition implies that there is an
 29 immediate transition to ground state beyond the electrode region, while the latter
 30 condition allows a gradual transition in voltage from one electrode to the next in the x

1 direction but restricts any voltage gradient in the y direction. The zero voltage flux
2 condition results in a boundary condition that is more like an ideal sinusoid wave than the
3 zero voltage isolation condition, and also more closely resembles a practical realization of
4 the boundary comprised of a dielectric material. The only possible means of achieving
5 the alternate zero voltage spacing condition would be to insert ground electrodes between
6 the sinusoidally varying driving electrodes. However this is impractical and would also
7 increase the total wavelength since the driving and ground electrodes would still need
8 isolation spacing between them.

9 Figure 5 compares the idealized-sinusoid boundary condition (case 1) with the 3-
10 phase (cases 2-6) and 5-phase (case 7) traveling-wave conditions that include isolation
11 regions between electrodes and represents the transient startup response after pump
12 activation through quasi-steady state conditions. First, consider the comparison of a 3-
13 phase sinusoid in which $6\ \mu\text{m}$ electrodes are separated by $12\ \mu\text{m}$ isolation spacings (cases
14 2-3) to the idealized-sinusoid boundary condition (case 1). With the voltage between
15 electrodes held to zero for the zero-voltage isolation (case 2), the magnitude of the
16 average potential (voltage per unit area) at the top wall of the channel is significantly
17 reduced, resulting in much smaller flow rates. For the zero voltage flux (case 3), the
18 transition in voltage from one electrode to the next yields a larger average potential at the
19 channel top wall, thus increasing charge induction and fluid transport. This is illustrated
20 by the comparison provided in the voltage contour plots of Figure 6a and b for the 36-
21 phase boundary condition (case 1) and the 3-phase, zero flux inter-electrode spacing
22 boundary condition (case 3). At a given location in the depth of the channel, the
23 magnitude of the voltage and corresponding electric field are clearly lower for the 3-phase
24 cases.

25 **4.3 Electrode coverage and density**

26 To modify the electric field so that high volume flow rates can be achieved for the same
27 total area available for EHD induction, either the voltage can be increased or the electrode
28 spacing decreased; both are different approaches for increasing the electric field. In
29 practice, there is a limit to the reduction in spacing at a given operating voltage that can
30 be achieved between neighboring electrodes while still isolating them using a dielectric

1 material. Clearly, this limit is dependent on the breakdown electric field of the dielectric
2 material used for isolation and for the fluid. Consider cases 3-5 in Table III and Figure 5.
3 Here the wavelength is held constant but the fraction of the surface area that is covered
4 with electrodes (electrode coverage area) is varied for the same operating conditions. The
5 result is that for wider electrodes in the same three-phase system, the flow rate is
6 effectively increased. The total electrode coverage area is increased by decreasing the
7 total spacing area, resulting in an increased magnitude of the voltage experienced through
8 the depth of the channel. Specifically, by widening the electrodes from 3 to 6 μm while
9 keeping the pitch constant at 18 μm , the electrode coverage area increases by 16.7% but
10 results in a 41.8% increase in flow rate. The increase appears to be roughly linear with
11 coverage area for a total increase of 78.2% when the electrode width is increased from 3
12 to 9 μm . Of course, breakdown of the dielectric or fluid due to the presence of very high
13 electric fields would limit the extent to which the electrode width can be increased at a
14 fixed wavelength. Similarly, Joule heating or electrolysis (for aqueous solutions) must
15 also be considered with an increase in electric field strength.

16 Another means for increasing the electric field is simply by reducing the wavelength
17 of the traveling wave while keeping the assigned electrode voltage magnitudes constant.
18 Case 6 is characterized by an electrode width of 6 μm and spacing of 9 μm for a
19 wavelength of 45 μm . Compared to case 3, which has a longer wavelength but the same
20 electrode width, an increase of 17.6% in net flow rate is observed in Figure 5. The
21 increase can be attributed to the denser packing of electrodes yielding greater total
22 electrode coverage area. The case 6 flow rate with a 40% coverage area clearly falls
23 between the results for the 33% coverage in case 3 and the 50% coverage in case 5.

24 **4.4 Phase representation**

25 As seen in Figure 5, the idealized-sinusoidal voltage boundary condition at the wall
26 produces a much higher flow rate than do the three-phase cases 2-6. One difference from
27 case 1 is that cases 2-6 have a decreased potential in the inter-electrode spacing region.
28 Another difference is the phase representation of the 36-phase design of case 1. While
29 designs that exclude spacing between electrodes (as in the idealized-sinusoid case 1) are
30 not experimentally achievable since electrical isolation between electrodes is required, the

1 electrode design can be modified to bring about a more symmetric and sinusoidal
2 potential at the boundary.

3 Numerical case 7 (Table III) has a 5-phase electrode configuration with 6 μm
4 electrodes and 6 μm spacing. The 60 μm wavelength in this case is slightly larger than
5 the 54 μm for cases 1-5; however, the electrode coverage area is the same as that for case
6 5 (50%). Despite the coverage area being identical to that in case 5, the distribution of
7 the assigned potential is such that the boundary more closely approximates a sinusoidal
8 condition in case 7 and the flow rate is increased. This is evident in the comparison of
9 the voltage contour plot for case 7 (Figure 6c) with that for the 36-phase, idealized-
10 sinusoid (Figure 6a). Further, comparison with the 3-phase case (Figure 6b) reveals that
11 the additional two phases significantly alter the magnitude and distribution of the voltage
12 field. Of particular note is the fact that even though case 7 has a slightly larger
13 wavelength (which would decrease the flow rate for the same frequency), the more
14 sinusoidal-like boundary condition has a more profound effect on the velocity than that
15 observed by simply changing the coverage area. For the same coverage area, the flow
16 rate increases 73.4% from case 5 to case 7 by redistributing the electrode coverage area to
17 approach a more symmetric boundary condition by employing a higher phase
18 representation at the boundary.

19 **5. CONCLUSION**

20 Electrohydrodynamic pumping scales favorably as device size is reduced and may
21 provide a viable option for fluid delivery mechanisms in electronics cooling or
22 biomedical devices. The achievable flow rate can be increased through working fluid
23 selection, device scaling, and increased electrode density, as well as by assigning
24 electrode potentials in such a way as to approach a symmetric, sinusoidal boundary
25 condition. Flow rate appears to increase roughly linearly with an increase in electrode
26 coverage area. Further, by using five phases instead of three to represent the sinusoidal
27 traveling wave (for the same coverage area), an increase in flow rate of approximately
28 73.4% is observed. For applications in which the anisotropic heating is a part of the
29 device environment (as in the case of electronics cooling), the energy required for
30 establishing the required temperature gradient across the liquid layer would not add to the

1 power required for device operation.

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5 I/UCRC, and the Indiana 21st Century Research and Technology Fund.

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Table I. Parameters used in analytical and numerical EHD calculations.

Symbol	Property	Value
h	Channel height	50 [μm]
L	Channel length	1.0008 [cm]
ΔT	Wall temperature difference	10 [$^{\circ}\text{C}$]
ρ	Density	987 [kg/m^3]
μ	Viscosity	5.28×10^{-4} [$\text{N}\cdot\text{s}/\text{m}^2$]

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Table II. Fluid properties and corresponding average velocities generated with representative liquids at a driving voltage of 50 V. Calculation for additional fluids can be similarly carried out following equation 18 from (Melcher and Firebaugh 1967) assuming constant viscosity.

Fluid	Temperature coefficient	Average conductivity [S/m]	Dielectric constant	Average velocity (at f_{ideal}) [cm/s]	Reference
Water	2.6 - 3.0%	1.89×10^{-5}	80	51.6	(Janssen and Warmoeskerken 1987)
2.2×10^{-5} M KCl (1)	0.82% ^a	5.43×10^{-4}	80	11.2	(Fuhr et al. 1992)
2.2×10^{-5} M KCl (2)	2.11% ^b	5.44×10^{-4}	80	24.0	(Horiba 2008)
0.013 M KCl	2.11%	0.2632	77.9	23.4	(Barthel et al. 1995, Horiba 2008, Lide 2001)
0.001 M NaCl	2.19%	0.0206	78.1	24.0	(Barthel et al. 1995, Horiba 2008, Lide 2001)
0.011 M NaCl	2.19%	0.1995	77.7	23.9	(Barthel et al. 1995, Horiba 2008, Lide 2001)

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^a Based on selected conductivity values in the range provided by (Fuhr et al. 1992)

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^b Based on extrapolated data from (Horiba 2008)

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Table III. Various cases of sinusoidal voltage boundary condition and geometry for repulsion-type EHD considered in the computations at a driving voltage of 200 V

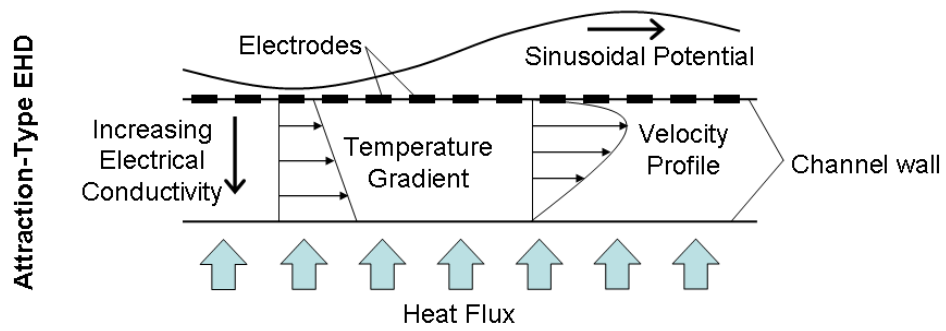
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Case	Boundary condition in inter-electrode space	Number of phases	Electrode width / spacing [μm]	Wavelength [μm]
1	Idealized-sinusoid	36	1.5 / 0	54
2	Zero voltage	3	6 / 12	54
3	Zero voltage flux	3	6 / 12	54
4	Zero voltage flux	3	3 / 15	54
5	Zero voltage flux	3	9 / 9	54
6	Zero voltage flux	3	6 / 9	45
7	Zero voltage flux	5	6 / 6	60

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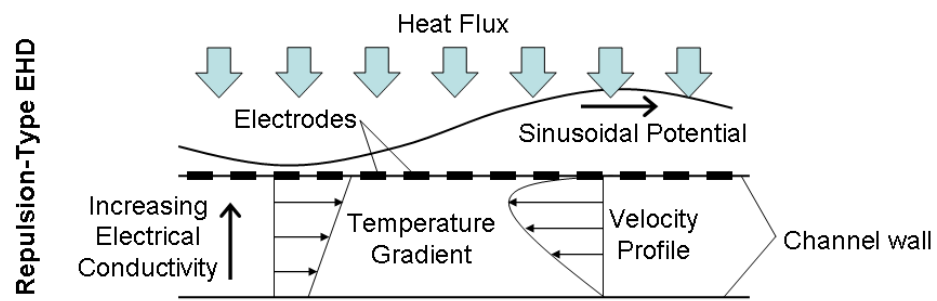
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(a)



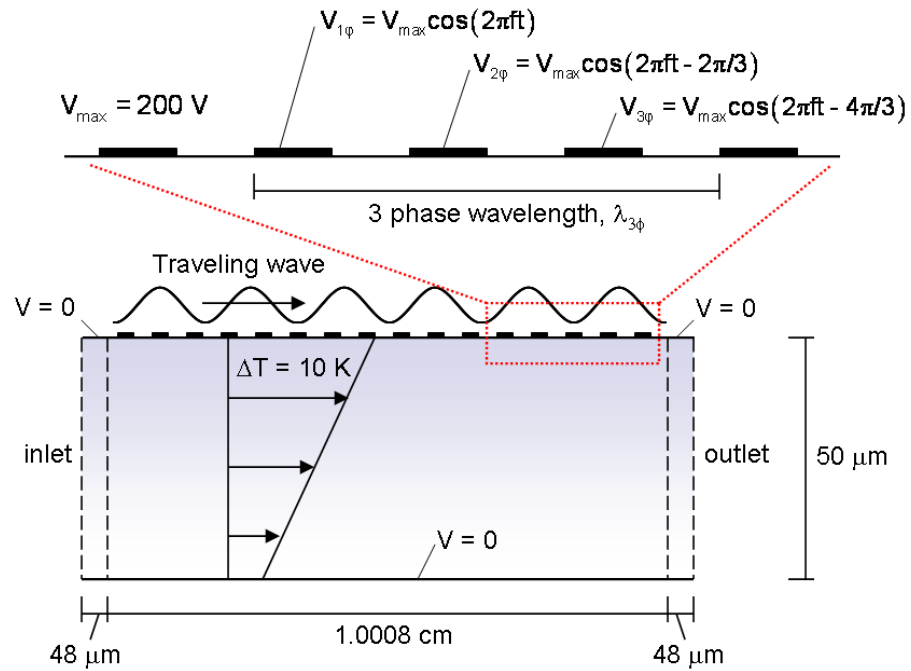
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(b)

6 **Figure 1.** Schematic illustration of (a) attraction- and (b) repulsion-type induction EHD configurations for
 7 fluids in which the electrical conductivity increases with temperature.

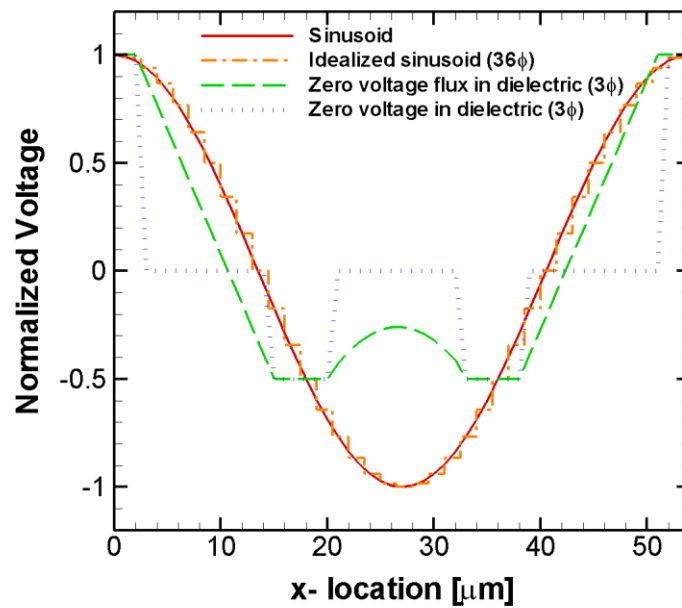
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Figure 2. Computational domain for a representative 3-phase, repulsion-type, induction EHD device.

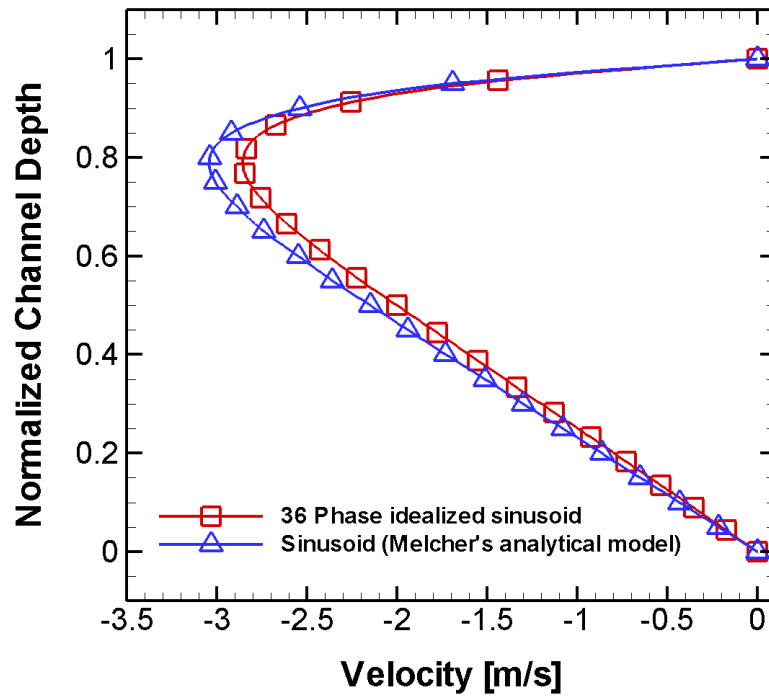
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3 **Figure 3.** Approximate methods of representing an ideal, sinusoidal traveling wave (wavelength $54 \mu\text{m}$) at
 4 an arbitrary time instant: a discretized 36-phase wave, a 3-phase wave with zero voltage boundary condition
 5 in the inter-electrode dielectric space, and a 3-phase wave with zero voltage flux boundary condition in the
 6 inter-electrode dielectric space. The curves representing 3-phase waves have $6 \mu\text{m}$ wide electrodes and 12
 7 μm wide inter-electrode dielectric spaces. For the zero flux boundary condition, the transitions in voltage
 8 shown in the dielectric layers are purely schematic.

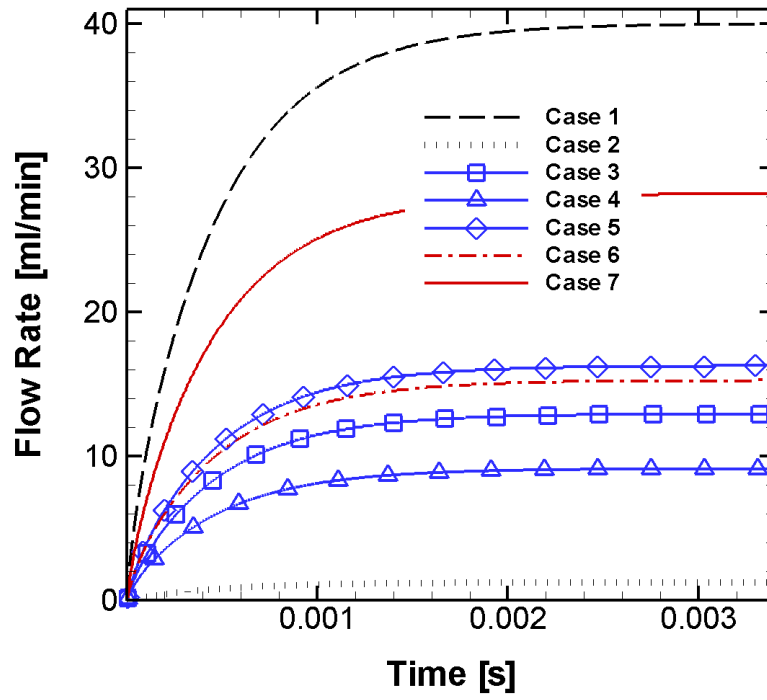
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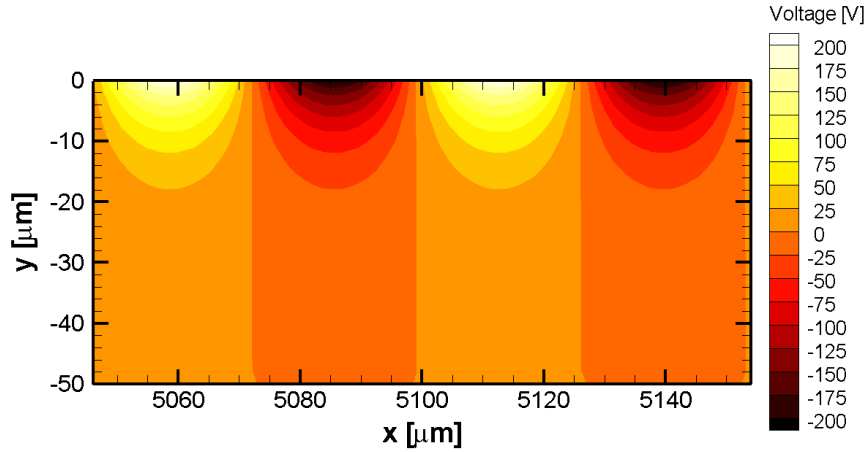
3 **Figure 4.** Comparison of Melcher's analytical result with numerical modeling results of a discretized
4 sinusoidal potential boundary condition with 36 phases (case 1, Table III).

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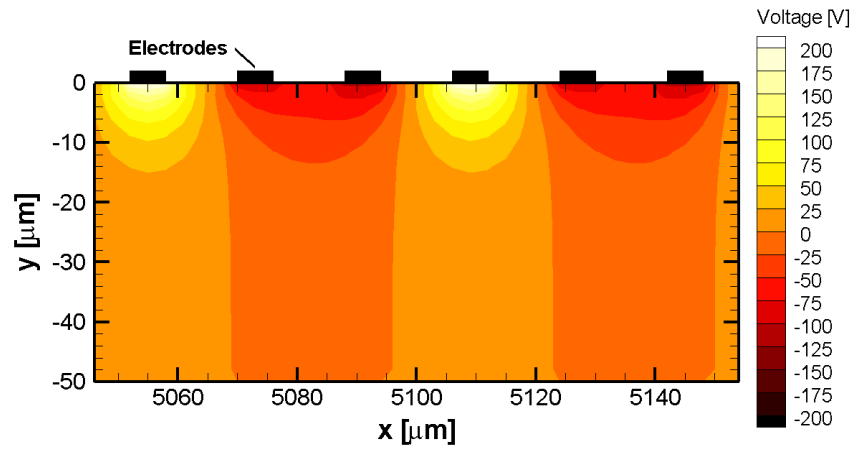
3 **Figure 5.** Calculated net flow rates for cases 1-7, Table III. Case 1 is an idealized, 36-phase sinusoid;
 4 cases 2-6 are 3-phase sinusoids; case 7 is a 5-phase sinusoid boundary condition.



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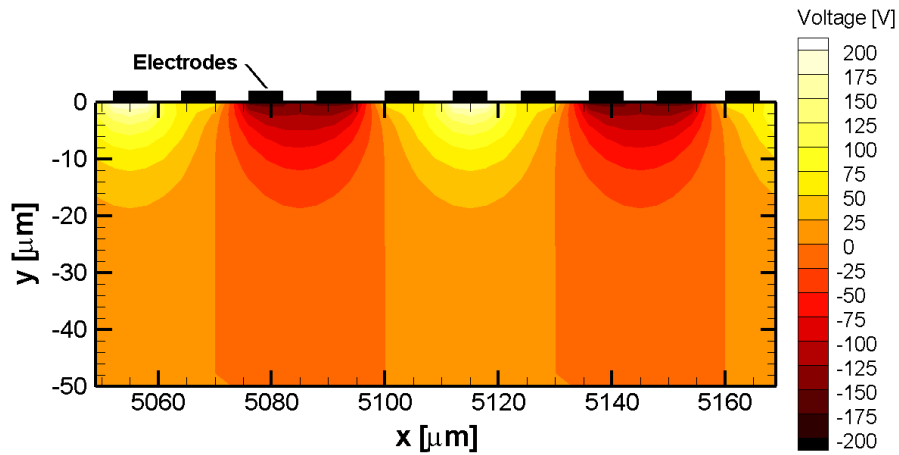
(a)



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(b)



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(c)

7 **Figure 6.** Voltage contour plots over a width of two wavelengths at a time corresponding to a maximum
 8 voltage of 200 V for (a) case 1 with a 36-phase, discretized boundary, (b) case 3 with a 3-phase, zero flux
 9 inter-electrode spacing, and (c) case 7 with a 5-phase, zero flux inter-electrode spacing (see also Table III).