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Effects of expectation and noise on evolutionary games

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ABSTRACT

Considering the difference between the actual and expected payoffs, we bring a stochastic learning updating rule into an evolutionary Prisoners Dilemma game and the Snowdrift game on scale-free networks, and then investigate how the expectation level A and environmental noise κ influence cooperative behavior. Interestingly, numerical results show that the mechanism of promoting cooperation exhibits a resonance-like fashion including the coaction of A, κ and the payoff parameters. High cooperator frequency is induced by some optimal parameter regions. The variation of time series has also been investigated. This work could be of particular interest in the evolutionary game dynamics of biological and social systems.

1. Introduction

Ranging from biological systems to economic and social systems, cooperation can be found in almost all realistic systems. Thus, understanding the conditions for the emergence and persistence of cooperative behavior among selfish individuals becomes a central problem. Since the unselfish, altruistic actions apparently contradict Darwinian selection, many scientists from different communities resort to the game theory as a common framework to investigate this cooperative dilemma, especially the Prisoner's dilemma game (PDG) and the Snowdrift game (SDG) together with extensions involving evolutionary context [1–4].

In the original PDG, which has been considered as a general metaphor for studying cooperation among limited rational individuals, two players can simultaneously make two choices: to cooperate or to defect. For mutual cooperation both players receive the rewards R, but only the punishment P for mutual defection. A defector exploiting a cooperator gets an amount T (temptation to defect) and the exploited cooperator receives S (sucker's payoff). These elements satisfy the following two conditions: $T > R > P > S$ and $2R > T + S$. As a result, it is better to defect regardless of the opponents decision. This kind of game rules yields an unresolvable dilemma for limited rational players who just want to maximize their own income. The SDG differs from the PDG mainly in the order of P and S, as $T > R > S > P$. Thus, the best action depends now on the opponent: to defect if the other cooperates, but to cooperate if the other defects. This game often draws

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more biological interests. However, there still exists unstable cooperative behavior which is contrary to the observations in the real world. In the last few years, this disagreement has inspired numerous investigations of suitable extensions that enable cooperative behavior to emerge and persist [5]. For example, it is found that several rules for adopting strategies can enhance the cooperation, such as "Tit-for-tat" [6] and "win stay and lose shift" [7]. More interestingly, considering that aspiration or expectation is common in human behavior, the aspiration-based dynamical rules have been studied later [8–10]. An original work by Nowak and May showed that the evolutionary PDG on a simple spatial structure induces emergence and persistence of cooperation even with the co-existence of spatial chaos [11]. Since then, much attention has been given to the evolutionary games on different population structures, including on regular networks [12–20] and on complex networks [16,21–30].

It is well known that noisy and disordered processes can obtain surprising phenomena in the evolutionary games. Perc introduced the random disorder in the PDG matrix and found a resonant behavior that the cooperator frequency obtains maximal value at an intermediate disorder [17]. Ren and Wang investigated both the topological randomness and dynamical randomness and found that the mechanism of randomness promoting cooperation resembles a resonance-like fashion [27]. In a recent report, Chen and Wang adopted a new stochastic learning rule of appropriate payoff expectations in evolutionary PDG on Newman–Watts networks and they also found a resonance-like behavior [31]. However, they did not consider the coaction of expectation level and the environmental noise. Thus in this paper, we adopt their new strategy updating rule in both evolutionary PDG and SDG and then investigate the nontrivial dependance of cooperation level on the expectation level, the environmental noise and the payoff parameters. Here, the well known BA scale-free network is used to represent the population structure.

The paper is organized as follows. In the next section, we describe models of evolutionary games and the strategy updating rule used in this work. The simulation results and discussions are given in Section 3. And the paper is concluded by the last section.

2. The model

A variety of recent researches have revealed that social networks are actually associated with small-world property and a scale-free, power-law degree distribution, $p(k) \sim k^{-\lambda}$ with $\lambda_{actor} = 2.3 \pm 0.1$ for the movie actor collaboration network [32], $\lambda_{science} = 2.1$ and 2.5 for the science collaboration network [33], etc. The standard Barabási–Albert (BA) scale-free network model [34], whose degree distribution is $p(k) \sim k^{-3}$, is generally considered suitable to represent the real population structure. In this model, starting from m_0 fully connected nodes and at every step one adds a new node with $m (m \leqslant m_0)$ edges that link to m different nodes already present in the system in such a way that the probability of being connected to the existing node i is proportional to its degree, i.e. $p_i = k_i / \sum_j k_j$, where j runs over all the nodes and k_i is the degree of node i.

For the original PDG, we can simplify the payoff matrix in accordance with common practice: let $T = b$, $R = 1$ and $P = S = 0$. b represents the advantage of defectors over cooperators. Generally,we can set $1 \leqslant b \leqslant 2$. For the SDG, we can simplify the model in the following way: let $R = 1$, $S = 1 - r$, $T = 1 + r$ and $P = 0$. $0 \leqslant r \leqslant 1$ indicates the rate of labor cost. Following Chen and Wang [31], we introduce the parameter A that indicates the expectation level of the players and each player calculates its own expectation payoff based on the parameter A. The expectation payoff of node i is $P_{Ai} = k_i * A$.

At each step of the evolution, all pairs of directly linked nodes engage in a single round of a given game and get relevant payoffs. The total payoff of player i is stored as P_i. When the node i is updated, it will compare P_i and P_{Ai} and reverse strategy with a probability based on the difference between real payoff and expectation payoff:

$$H_i = \frac{1}{1 + \exp[(P_i - P_{Ai})/\kappa]}. \tag{1}$$

Here $0 \leqslant \kappa < \infty$ characterizes the environmental noise, including bounded rationality, individual trials, errors in decision, etc. The expectation payoff P_{Ai} is used to evaluate whether a player is satisfied with its current strategy. This updating rule is indeed a stochastic alteration of the Win-Stay-Lose-Shift (WSLS) strategy. In this paper, we investigate the co-effect of noise κ and expectation level A on the stationary density of cooperator.

3. Simulation results and discussion

All the simulations below are carried out on BA scale-free networks with network size $N = 1000$ and $m = m_0 = 4$. Initially, strategies (C and D) are randomly distributed among the population. Equilibrium frequencies of cooperators are obtained by averaging over 3000 generations after a transient time of 10 000 generations. Each data is averaged by 30 runs on 30 different networks. A synchronous updating rule is adopted here.

Fig. 1 briefly shows the relationship of payoff parameters (b in PDG and r in SDG) and the cooperator frequency for different values of expectation level A and environmental noise κ. One can see that the cooperator frequency monotonically decreases with the increment of b (PDG) and r (SDG) no matter under what values of κ and A. Besides, κ and A also play notable roles: changing the value of A and κ can influence the cooperation level for fixed b and r. When $\kappa = 0.1$, the cooperator frequency has a non-monotonous dependance on A for both PDG and SDG and there exists appropriate payoff

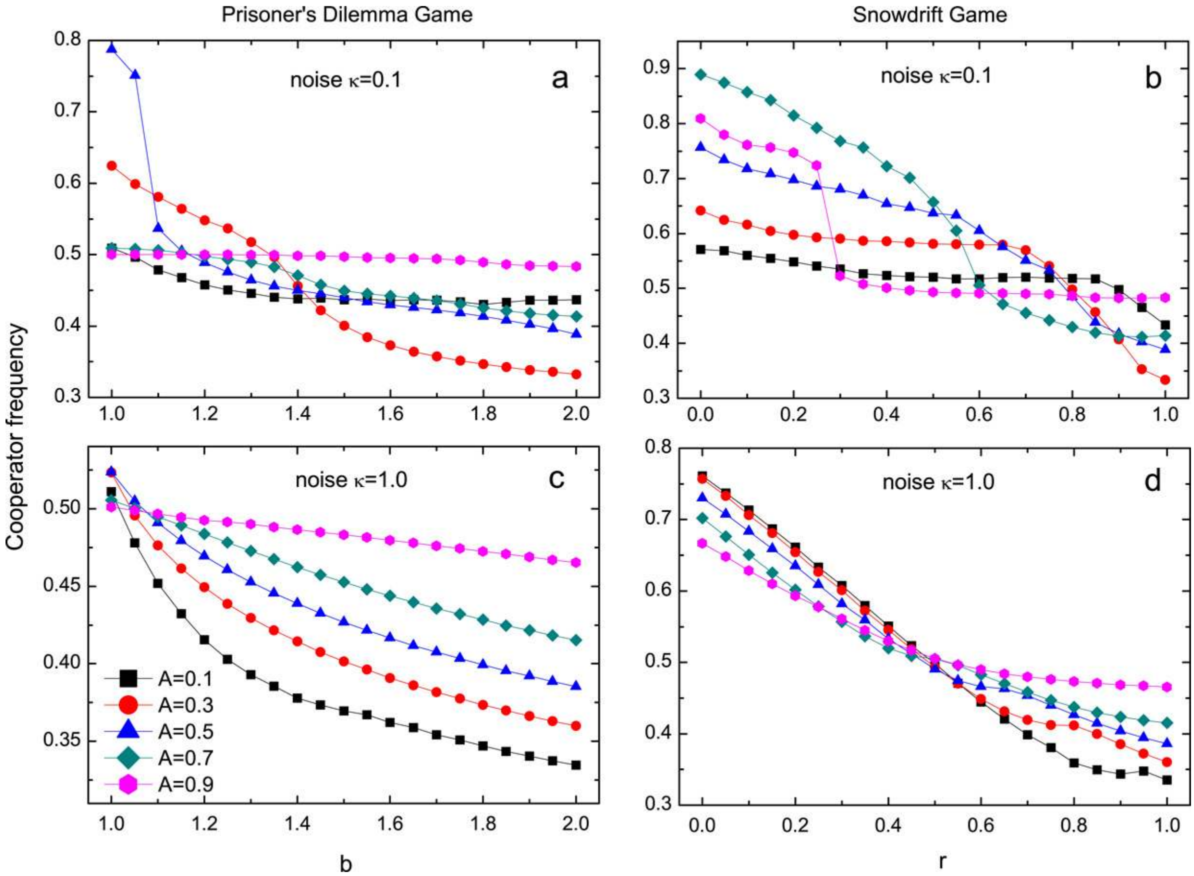


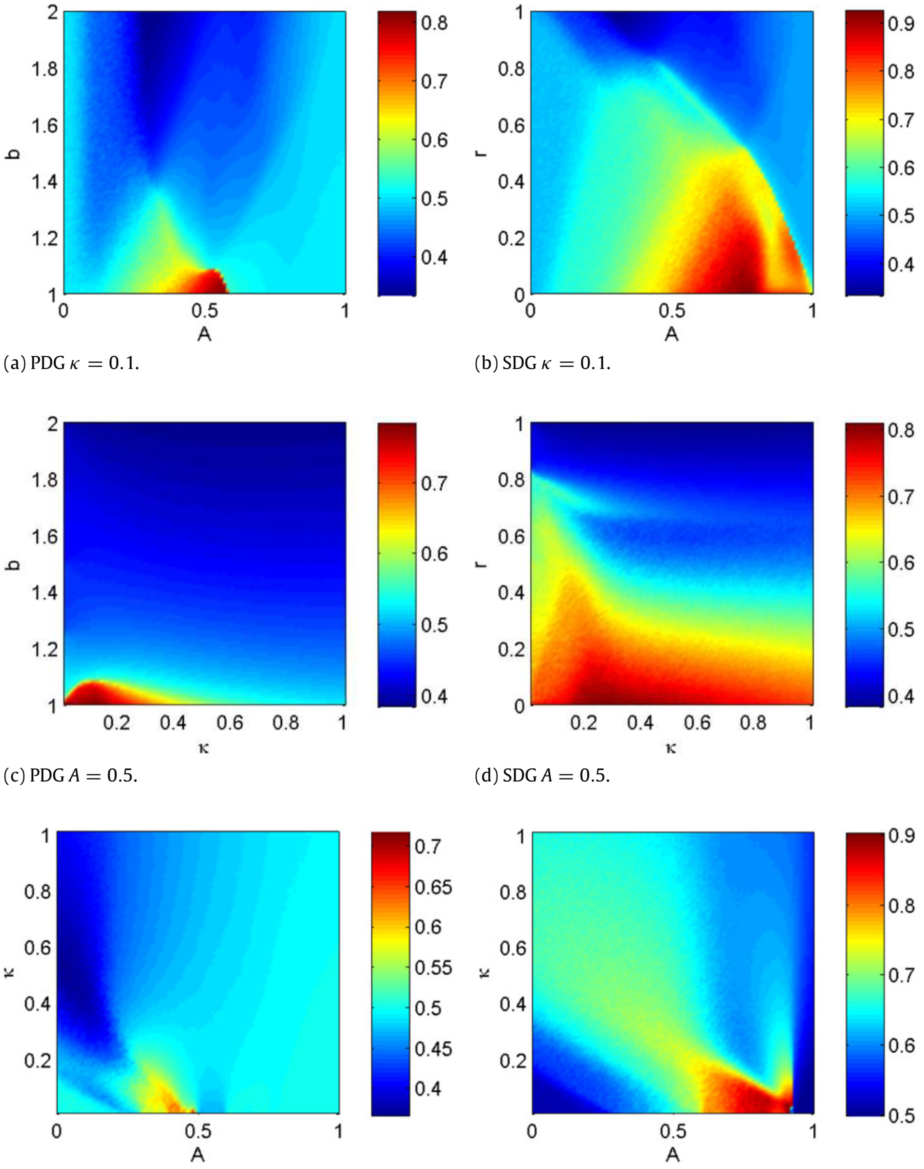
Fig. 1. (Color online) The cooperation frequency *VS* b (PDG) and r (SDG) for different values of A and κ.

expectations and environmental noises promoting cooperation. When $\kappa = 1.0$, for the PDG, the cooperator frequency monotonically decreases as a function of A; for the SDG, there exists a crossover point at $r = 0.5$: the cooperator frequency increases with an increment of A if $r > 0.5$ but decreases with an increment of A if $r < 0.5$. To investigate this phenomenon more precisely, we examine the effect in the $A \leftrightarrow b$, $A \leftrightarrow r$, $\kappa \leftrightarrow b$, $\kappa \leftrightarrow r$ and $A \leftrightarrow \kappa$ space respectively.

To quantify the effect of expectation level A to enhance the cooperation more precisely, we fixed $\kappa = 0.1$ to study the cooperator frequency as a function of A and b for PDG (A and r for SDG). As shown in Fig. 2(a), for the PDG, there exists an optimal resonant region that promotes cooperation at $0.4 < A < 0.6$ and $1.0 < b < 1.2$. This phenomenon reveals that the optimal cooperation level only occurs at certain intermediate expectation levels for some fixed b. For the SDG (Fig. 2(b)), the cooperation can be promoted by a larger range of A, especially when $0.5 < A < 1.0$. Interestingly, there exists a trigonal parameter space, which we call the "harmful region", at the upper right corner that can restrain cooperation. Whereafter, we will simply explain the non-trivial dependence of the cooperator frequency on A. For the small values of A, the total payoffs of nearly all players are larger than the expectation payoffs, so the players (including C players and D players) can hardly reverse their strategies. For the large values of A, the majority of players obtain total payoffs below the expectation payoff and players will change their strategies with a high probability. Hence the cooperator frequency keeps around 0.5. For the intermediate values of A, C players surrounded by C players can obtain higher payoffs and keep their strategies steadily, while the D players surrounded by D players can not be satisfied. Besides, although a D player surrounded by C players can receive a much higher payoff for both types of games, the C neighbors will probably reverse their strategies to D because their income can not meet the expectation level. Thus the pattern of C players surrounded by C players is more steady.

Hence a high cooperation level emerges. From what has been discussed above, it is not difficult to draw the conclusion that there exists an appropriate intermediate level A which can induce maximum cooperator frequency.

Since Fig. 1 shows that the environmental noise κ plays an important role on the cooperative behavior during the evolution, to further demonstrate the effect of κ, next we will investigate the relationship of κ and the cooperator frequency under fixed A. Fig. 2(c) depicts the co-action of k and b for PDG, and Fig. 2(d) depicts coaction of k and r for SDG. One can see that at the region of $0 < \kappa < 0.4$ and $1.0 < b < 1.1$, the cooperative behavior is highly promoted. For the SDG, the parameter region promoting cooperation is much larger and the highest cooperator frequency is obtained under $0.2 < \kappa < 0.6$ and $0 < r < 0.2$. $\kappa = 0$ denotes the completely deterministic learning and $\kappa = +\infty$ denotes the completely random strategy learning and thus ignores the neighbor information. The finite positive values of κ incorporate



(a) PDG $\kappa = 0.1$.

(b) SDG $\kappa = 0.1$.

(c) PDG $A = 0.5$.

(d) SDG $A = 0.5$.

(e) PDG $b = 1.2$.

(f) PDG $r = 0.2$.

Fig. 2. (Color online) The cooperation frequency as *VS* A, κ, b (PDG) and r (SDG).

the uncertainties in the strategy adoption, i.e., the better player's strategy is readily adopted, but there is also a small probability to adopt the worse strategy, which is more realistic. In the present work, the cooperation level has a nonlinear dependence on κ, which is in accordance with previous researches [35].

In Fig. 2(e) and (f), to investigate the combined effect of both A and κ, we fix $b = 1.2$ and $r = 0.2$ and focus on the cooperator frequency depending on A and κ together. For the PDG, the cooperation is slightly promoted in a small parameter space of $0.3 < A < 0.5$ and $0 < \kappa < 0.2$. But for the SDG, the cooperation is promoted in the whole parameter space except for a trigonal region at the lower left corner, especially when $A > 0.5$ and $0 < \kappa < 0.2$.

Moreover, it is found that besides the promotion of cooperation, the stochastic updating rule can also induce surprising evolution time series. In the following, we will examine the time series of PDG and SDG in detail.

Fig. 3 shows the time series of PDG and SDG under different values of A and κ. Fig. 3(a) shows the first 400 steps of $A = [0.01, 0.5, 1.5]$ under $\kappa = 1.0$ and $b = 2.0$. One can see that the time series of $A = 1.5$ is very interesting: the "ping-pong effect" is observed since $T = 350$ and the fluctuation amplitude enlarges from [0.48, 0.52] to [0.03, 0.91] with the evolution process. Fig. 3(c) represents the last 50 steps of the evolution under $A = 1.5$ and $\kappa = 1.0$. We can easily find that the "ping-pong effect" emerges for all values of b and the amplitude increases with the increment of b. The difference between the maximum and minimum values of the cooperator density is 0.04, 0.2 and 0.91 for $b = 1.0$, 1.5 and

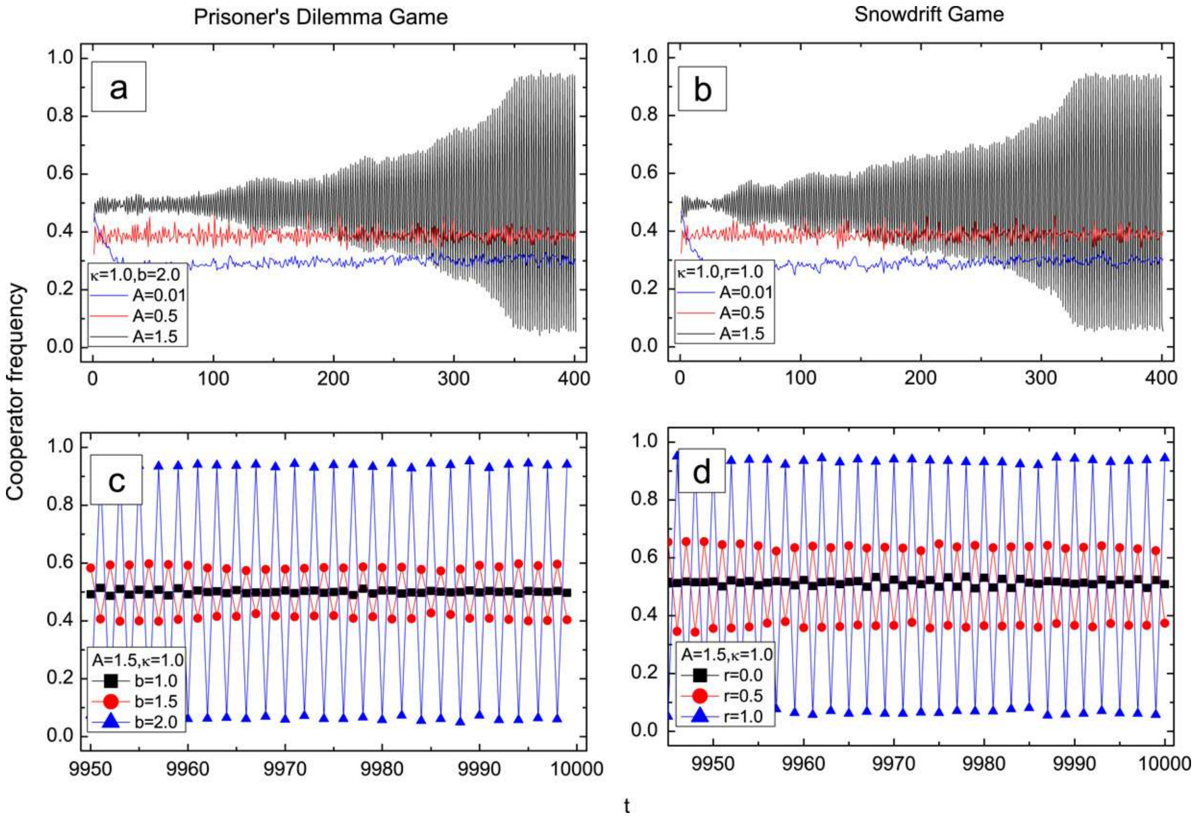


Fig. 3. (Color online) The long-range time series of PDG under different values of A and κ.

2.0, respectively. For the SDG, as shown in Fig. 3(b) and (d), the situation is nearly the same as the PDG, such as the "ping-pong effect" and the variety of fluctuation amplitude. We have also examined the time series under the random sequential updating mechanism and similar oscillation is observed. In the present rule of evolution, players don't learn from neighbors. Thus the synchronous and random updating mechanisms should produce the same result.

4. Conclusion

In summary, we have investigated how the co-action of expectation level A and environmental noise κ influences the cooperation behavior by introducing a stochastic Win-Stay-Lose-Shift strategy updating rule with appropriate payoff expectations into the evolutionary Prisoner's Dilemma game and the Snowdrift game on BA scale-free networks. Numerical results demonstrate that there exist some optimal areas of randomness, resulting in high cooperator frequency. Moreover, the evolution processes also have a strong dependence on the updating rule. When A is large, the time series resembles the "ping-ping effect" for both the PDG and the SDG, and the fluctuation amplitude is determined by b and r (b for the PDG, r for the SDG). This work may be helpful to understand the cooperative behavior in biological and social systems.

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