Geophys. J. R. astr. Soc. (1970) **19**, 367–393.

Effects of Major Seismic Events on the Rotation of the Earth

Ari Ben-Menahem and Moshe Israel

(Received 1969 October 9)

Summary

A spherical theory is advanced for perturbations of the Earth's rotation by major earthquakes and explosions. Explicit expressions are obtained for the dependence of the secular polar shift on the dimensions, depth, and location of the seismic event. Numerical results show that a single shallow earthquake of magnitude 8·5, occurring at a suitable latitude and with a favourable strike-azimuth, may suffice to maintain the Chandler wobble for about one year. Hence, it is deduced that earthquakes may at most account for 30 per cent of the observed secular polar shift.

1. Introduction

The equations of rigid gyroscopic motion were given by Euler in 1758. On the basis of this theory, he suggested in 1765 that the Earth might undergo a free precession with period of $A/(C-A)$ sideral days. Assuming this to be true, a spectator partaking in the Earth's motion should observe periodic changes in latitude with a period of about 10 months. However, no such period could be found. Instead, Chandler established in 1891 the existence of a 428-days period in the spectrum of the latitude variation. The lengthening of the period was explained by Newcomb, in 1892, to be the result of the Earth's elasticity. A theoretical verification was given by Love (1909), based on first-order theory of the figure of the Earth. This 14-month precessional motion of the instantaneous axis of rotation about the Earth's axis of figure is known today as the Chandler Wobble. (Fig. 1.)

Spectral analyses of latitude time series disclosed that the motion is damped with a 'Q' value between 30 and 40. The wobble must therefore be maintained by a certain source of energy. Munk & Macdonald (1960, p. 174) conclude that: 'The statistical properties of the latitude time series are those associated with a damped oscillator excited at random. . . . Irregular variations of the atmosphere are the most likely cause of the wobble.' Recently, however, observational evidence has been presented in support of the hypothesis that earthquakes may excite the wobble and produce the observed polar shift. To explain these arguments we shall recapitulate the theoretical background of the subject: The fundamental law of dynamics as experienced by earth-bound spectators is given by the Liouville equation (Munk & Macdonald 1960)

$$\mathbf{L} = \frac{d}{dt}[\boldsymbol{J}.\boldsymbol{\omega})+\mathbf{h}]+\boldsymbol{\omega}\times[\boldsymbol{J}.\boldsymbol{\omega})+\mathbf{h}], \tag{1.1}$$

where $\mathbf{L}$ is the applied torque, $\boldsymbol{J}$ stands for the inertia dyadic, $\boldsymbol{\omega}$ is the angular velocity of the Earth and $\mathbf{h}$ is the angular momentum imparted by a motion relative to the Earth.

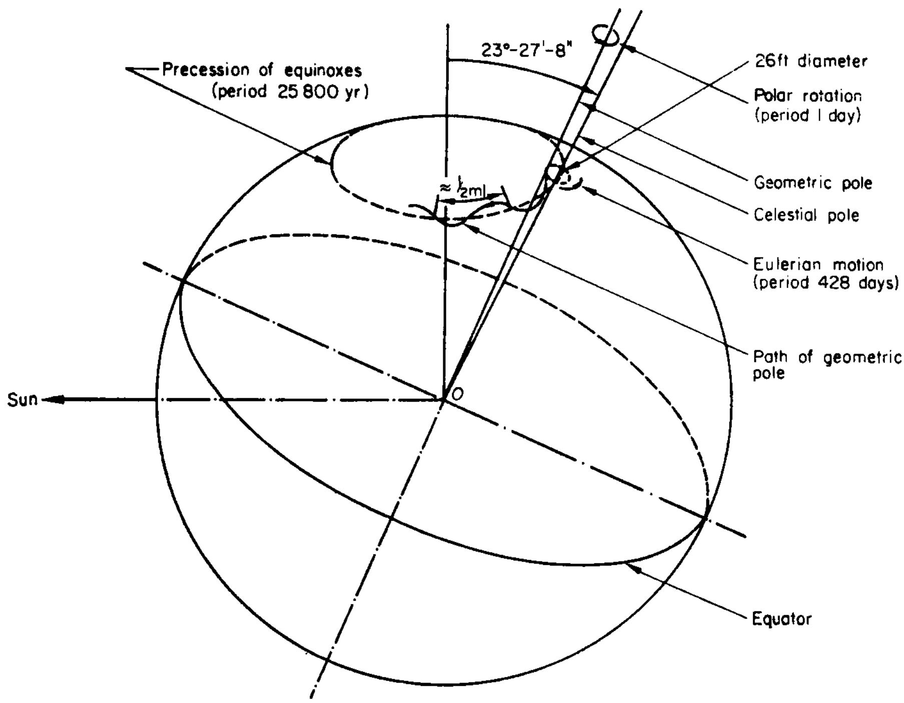

FIG. 1. Gyroscopic motion of the Earth (courtesy of Academic Press ' Gyrodynamics ' by R. N. Arnold and L. Maunder, 1961, p. 3, Fig. 1.2).

Considering the Earth as a symmetric top with moments of inertia $A = B \neq C$ referred to the principal axes, an approximate solution to equation (1.1) can be obtained. This solution is based on a perturbation scheme of Volterra (1895, 1898).

It is assumed that, due to some inner or outer perturbing agent, there occurs a change in the elements of the inertia dyadic

$$J_{ij} = (J_0)_{ij} + c_{ij}. \tag{1.2}$$

Let the corresponding change in the instantaneous axis of rotation of the Earth be

$$\boldsymbol{\omega} = \boldsymbol{\omega}_0 + \omega_0\,\mathbf{m}(t), \tag{1.3}$$

where $\boldsymbol{\omega}_0 = (0, 0, \omega_0)$ is the mean angular velocity vector of the Earth and $(m_1\, m_2\, m_3)$ are the direction cosines of the vectorial displacement of the rotation axis. A direct substitution of equations (1.2) and (1.3) into (1.1), followed by the neglection of terms of the order $(c_{ij}/C)^2$, $(m_i)^2$, $(h_i/\omega C)^2$, leads to a set of three linear equations of the first order in m_1 m_2 and m_3: One equation for m_3 and two coupled equations for m_1 and m_2. The component m_3 determines a variation in the length of the day and will be dealt with later. The equations for the components m_1 and m_2 can be conveniently arranged as a single equation in the complex scalar $m = m_1 + im_2$,

$$\frac{dm}{dt} - i\sigma_r m = -\frac{1}{A}\left[i\omega_0(c_{13}+ic_{23}) + \frac{d}{dt}(c_{13}+ic_{23})\right]. \tag{1.4}$$

where $\sigma_r = [(C-A)/A]\,\omega_0$ and $\mathbf{L} = \mathbf{h} = 0$ is assumed.

Assuming that the perturbation of the inertia–product function $c_{13}+ic_{23}$ is a step function in time† (Mansinha & Smylie 1967), a straightforward integration of equation (1.4) yields

$$m(t) = \frac{\omega_0}{A\sigma_0}\,[c_{13}+ic_{23}] - \frac{\omega_0+\sigma_0}{A\sigma_0}\,(c_{13}+ic_{23})\,e^{i\sigma_0 t}, \tag{1.5}$$

where $\sigma_r = K\sigma_0$, $K = 1{\cdot}43$ (Munk & MacDonald, p. 43, Table 6–2). Equation (1.4) is known as the wobble-equation. Its solution is composed of a time-independent part called the secular polar shift and a harmonic oscillation that constitutes the Chandler wobble. Since $\omega_0 \gg \sigma_0$, we may approximate equation (1.5) by

$$m(t) = K\,\frac{c_{13}+ic_{23}}{C-A}\,(1-e^{-i\sigma_0 t}). \tag{1.6}$$

This represents a circular motion of the vector $m_1\,\mathbf{e}_x + m_2\,\mathbf{e}_y$ with radius:

$$Ka\,|c_{13}+ic_{23}|/(C-A)$$

and period $2\pi/\sigma_0$. If no other perturbation appears, this motion will decay with time like $\exp\{-(\mathrm{Im}\,\sigma_0)\,t\}$ and the tip of the vector $\mathbf{m}(t)$ will describe an inward logarithmic spiral decreasing in amplitude by e^{-1} in about twenty years. However, as a new perturbation appears, the motion caused by it will superpose on the decaying old motion. Consider a case where the succession of the perturbing events is at a rate where their mean separation in time is small compared to the above-mentioned decay time. Let the wobble motion of the pole at a certain fiducial time $t = t_0$ be given by the rotating vector

$$m_0(t) = Z_0 + \gamma_0\,e^{-i\sigma_0(t-t_1)}$$

which is a circular motion with radius γ_0 about Z_0 as a centre. A new event occurring at a time $t = t_1 > t_0$ will, according to equation (1.6), add the motion

$$m_1(t) = K(c_{13}+ic_{23})(C-A)^{-1}[(1-e^{-i\sigma_0(t-t_1)}].$$

It is a simple matter to show that the sum of the two motions takes the form

$$m_0(t)+m_1(t) = \left\{Z_0 + K\,\frac{c_{13}+ic_{23}}{(C-A)}\right\} + Re^{-i\sigma_0(t-t^*)}, \tag{1.7}$$

where R and t^* depend on the constants of the two motions. Equation (1.7) says that the pole–path changes instantaneously from one circular arc to another. The radii of these arcs are γ_0 and R, respectively, and the distance between the radii is equal to the polar shift produced by the new event, namely $Ka\,|c_{13}+ic_{23}|(C-A)^{-1}$.

To be able to calculate the inertia products for a given earthquake one must evaluate first the deformation field induced in the Earth by a certain source model. Press (1965), made use of the elastic theory of dislocations to compute strains, tilts and displacements due to finite shear faults in a half-space earth model. He then showed that residual strains of the order of 10^{-8} caused by the Alaskan shock of 1964 March 28 could be explained by the elastic dislocation theory.‡ In the absence of a spherical theory Mansinha & Smylie (1967), used Press' displacement functions to calculate the elements of the inertia change-dyadic c_{ij} in a half-space earth model. Later Smylie & Mansinha (1968a), have applied this solution to interpret details

† This is not a rigorous statement. It is known, however, that ${}_0T_2$ and ${}_0S_2$ decay like $\exp[-(\pi t/QT)]$ where $Q \sim 300$. The corresponding amplitudes will decay by a factor of 10 in about 10 days, which is small compared to the period of Chandler's wobble. Hence, for the limited scope of the present paper, one may safety neglect the mass re-distribution time.

‡ Since then strain steps were recorded also from earthquakes with lower magnitudes. An example is shown in Fig. 2.

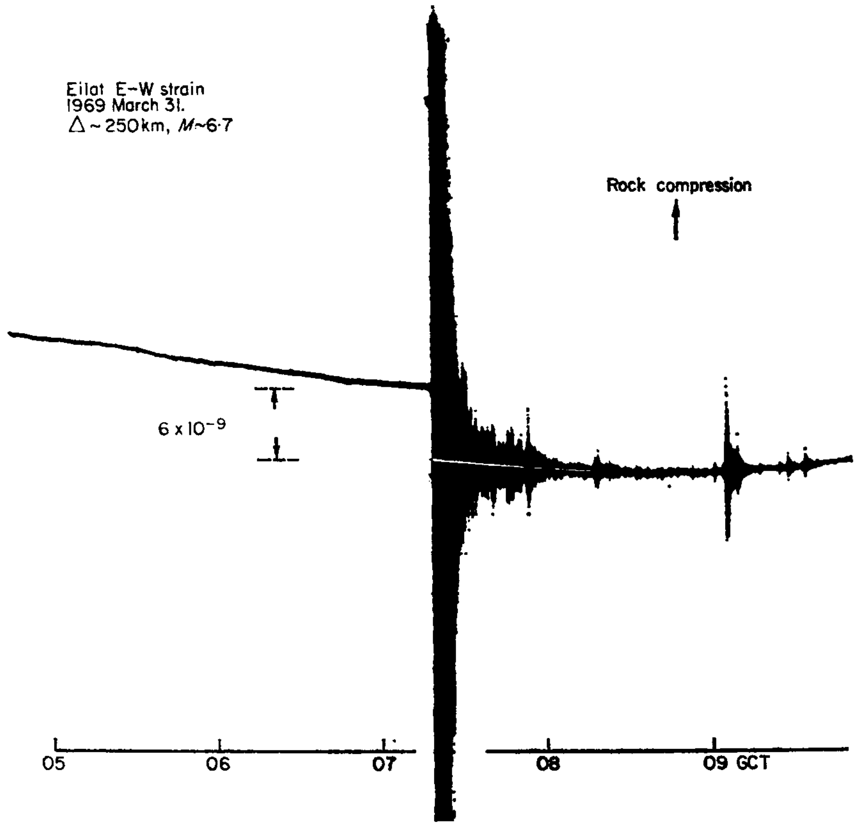

FIG. 2. Example of residual deformation in the Earth due to an earthquake in the intermediate magnitude range.

in the observed pole paths for the years 1957–1967. According to their conjecture there is a significant correlation between the occurrence of specific seismic events and pole–path breaks.

Ben-Menahem, Singh & Solomon (1969), calculated surface-deformation of a spherical earth model by finite dislocations. Their method was generalized in the present paper to evaluate the inertia changes due to earthquake and explosion sources in a spherical homogeneous earth model. Calculations were then made for certain specific events of outstanding magnitude. Polar shifts thus obtained show that a spherical model yields higher inertia changes than the corresponding half-space model.

2. Green's dyadic for a homogeneous sphere

Ben-Menahem & Singh (1968, referred to henceforth as Paper I), obtained the static Green's dyadic $\mathbf{G}_a(\mathbf{a}/\mathbf{r}_0)$ at the surface $r = a$ of a homogeneous elastic isotropic sphere with zero surface tractions. We wish to evaluate the more general dyadic $\mathbf{G}_a(\mathbf{r}/\mathbf{r}_0)$ valid everywhere within and on the sphere. To this end we start from the expression of $G_\infty(\mathbf{r}/\mathbf{r}_0)$ given in (Paper I, 3.14).

$$G_\infty(\mathbf{r}/\mathbf{r}_0) = \frac{1}{4\pi\mu} \sum_{l=1,1,0}^{\infty} \sum_{m=-l}^{l} \frac{(l-m)!}{(l+m)!} \left[\frac{\overset{*}{\mathbf{M}}{}^{+}_{m,l}(\mathbf{r}_0)\,\mathbf{M}^{-}_{m,l}(\mathbf{r})}{l(l+1)} + \frac{\overset{*}{\mathbf{N}}{}^{+}_{m,l-1}(\mathbf{r}_0)\,\mathbf{F}^{-}_{m,l-1}(\mathbf{r})}{l(2l-1)} + \frac{\overset{*}{\mathbf{F}}{}^{+}_{m,l+1}(\mathbf{r}_0)\,\mathbf{N}^{-}_{m,l+1}(\mathbf{r})}{(l+1)(2l+3)} \right], \quad (2.1)$$

where $r > r_0$. For $r < r_0$ one should interchange r and r_0. Here μ is the rigidity and **M**, **N** and **F** are the three independent vector solutions of the static Naviér equation (Paper I, 2.13–2.16). The summation over l for the first two dyadics of (2.1) starts from $l = 1$ while for the third dyadic it starts from $l = 0$.

Assume next, for $r > r_0$

$$G_a(\mathbf{r}/\mathbf{r}_0) = G_\infty(\mathbf{r}/\mathbf{r}_0) + \frac{1}{4\pi\mu} \sum_{l=1,1,0}^{\infty} \sum_{m=-l}^{l} \frac{(l-m)!}{(l+m)!} \left[\frac{\mathbf{X}_{m,l}(\mathbf{r}_0)\,\mathbf{M}^{+}_{m,l}(\mathbf{r})}{l(l+1)} + \frac{\mathbf{Y}_{m,l}(\mathbf{r}_0)\,\mathbf{F}_{m,l+1}(\mathbf{r})}{l(2l-1)} + \frac{\mathbf{Z}_{m,l}(r_0)\,\mathbf{N}^{+}_{m,l-1}(r)}{(l+1)(2l+3)} \right], \quad (2.2)$$

where **X**, **Y** and **Z** are three unknown source vectors, to be determined by the application of the boundary condition (Paper I, 4.2)

$$T(G_a).\mathbf{e}_r = 0 \text{ at } r = a. \quad (2.3)$$

Applying the operator T to the vectors **M**, **N** and **F**, we find that at $r = a$

$$\left.\begin{aligned}
\frac{1}{\mu} T(\mathbf{M}^+).\mathbf{e}_r &= \sqrt{[l(l+1)]}\,(l-1)\,a^{l-1}\,\mathbf{C}_{m,l},\\
\frac{1}{\mu} T(\mathbf{M}^-).\mathbf{e}_r &= -\sqrt{[l(l+1)]}\,(l+2)\,a^{-l-2}\,\mathbf{C}_{m,l},\\
\frac{1}{\mu} T(\mathbf{N}^+).\mathbf{e}_r &= 2(l-1)\,a^{2l-1}\sqrt{[l(l+1)]}\,\mathbf{B}_{m,l} + 2l(l-1)\,a^{l-2}\,\mathbf{P}_{m,l},\\
\frac{1}{\mu} T(\mathbf{N}^-).\mathbf{e}_r &= -2(l+2)\,a^{-l-3}\sqrt{[l(l+1)]}\,\mathbf{B}_{m,l} + 2(l+1)(l+2)\,a^{-l-3}\,\mathbf{P}_{m,l},\\
\frac{1}{\mu} T(\mathbf{F}^+).\mathbf{e}_r &= \frac{a^l}{\gamma}\,[2(l+1)^2-\gamma]\sqrt{[l(l+1)]}\,\mathbf{B}_{m,l} + \frac{a^l}{\gamma}\,[\gamma(l+1) + (2l^2-2l-8)(l+1)]\,\mathbf{P}_{m,l},\\
\frac{1}{\mu} T(\mathbf{F}^-).\mathbf{e}_r &= \frac{a^{-l-1}}{\gamma}\,[2l^2-\gamma]\sqrt{[l(l+1)]}\,\mathbf{B}_{m,l} - \frac{a^{-l-1}}{\gamma}\,[2l(l^2+3l-2)+\gamma]\,\mathbf{P}_{m,l}\\
\left\{\frac{\lambda}{\mu}\operatorname{div}\mathbf{F}^{\pm}\right\}\mathbf{e}_r &= \frac{\gamma-4}{\gamma}\left\{\begin{matrix}(l+1)(2l+3)\,r^l\\ l(2l-1)\,r^{-l-1}\end{matrix}\right\}\mathbf{P}_{m,l},
\end{aligned}\right\} \quad (2.4)$$

$\gamma = 4(1-\sigma)$,
σ = Poisson's ratio.

Substituting the results given in equation (2.4) into equation (2.2) and equating to zero the coefficients of the vectors $\mathbf{P}_{m,l}$, $\mathbf{B}_{m,l}$ and $\mathbf{C}_{m,l}$ [defined in Paper I (2.12)] we obtain after a number of steps,

$$\left.\begin{aligned}
\mathbf{X}_{m,l}(\mathbf{r}_0) &= \frac{(l+2)}{l(l+1)(l-1)}\, a^{-2l-1}\, \overset{*}{\mathbf{M}}{}^{+}_{m,l}(r_0),\\
\mathbf{Y}_{m,l}(\mathbf{r}_0) &= \frac{(l+2)}{2\Delta(l+1)(2l+3)\, a^{2l+3}}\, [\gamma(2l+1)\, \overset{*}{\mathbf{F}}{}^{+}_{m,l+1}(r_0)\\
&\qquad - a^2(l+1)(2l+3)\, \overset{*}{\mathbf{N}}{}^{+}_{m,l-1}(r_0)]\\
\mathbf{Z}_{m,l}(\mathbf{r}_0) &= \frac{1}{2\gamma\Delta l(l-1)(2l-1)\, a^{2l+1}}\, [a^2(2l+1)\{4(1-\sigma^2)\\
&\qquad + l(l-1)(l+2)(l+1)\}\, \overset{*}{\mathbf{N}}{}^{+}_{m,l-1}(r_0) - (l+2)\,\gamma l(l-1)\\
&\qquad (2l-1)\, \overset{*}{\mathbf{F}}{}^{+}_{m,l+1}(r_0)].
\end{aligned}\right\} \quad (2.5)$$

Hence, for $r > r_0$

$$\begin{aligned}
&G_a(\mathbf{r}/\mathbf{r}_0)\\
&= \frac{1}{4\pi\mu} \sum_{l=1,\,10,}^{\infty} \sum_{m=-l}^{l} \frac{(l-m)!}{(l+m)!} \left\{ \overset{*}{\mathbf{M}}{}^{+}_{m,l}(r_0)\, \frac{(l-1)\,\mathbf{M}^{-}_{m,l}(r) + (l+2)\, a^{-2l-1}\, \mathbf{M}^{+}_{m,l}(r)}{l(l+1)(l-1)} \right.\\
&\quad + \overset{*}{\mathbf{N}}{}^{+}_{m,l-1}(r_0)\, \frac{8\Delta(l-1)\,\mathbf{F}^{-}_{m,l-1}(r) + \Delta_1\, a^{-2l-1}\, \mathbf{F}^{+}_{m,l+1}(r) + (1/\gamma)\,\Delta_3\, a^{-2l+1}\, \mathbf{N}^{+}_{m,l-1}(r)}{8\Delta l(l-1)(2l-1)}\\
&\quad \left. + \overset{*}{\mathbf{F}}{}^{+}_{m,l+1}(r_0)\, \frac{8(l-1)\,\Delta\mathbf{N}^{-}_{m,l+1}(r) + \Delta_4\, a^{-2l-1}\, \mathbf{N}^{+}_{m,l-1}(r) + \gamma\Delta_2\, a^{-2l-3}\, \mathbf{F}^{+}_{m,l+1}(r)}{8\Delta(l-1)(l+1)(2l+3)} \right\},
\end{aligned} \quad (2.6)$$

where

$$\left.\begin{aligned}
\Delta_1 &= -4l(l-1)(2l-1)(l+2),\\
\Delta_2 &= 4(l-1)(l+2)(2l+1),\\
\Delta_3 &= l(l+1)\,\Delta_2 + 16(2l+1)(1-\sigma^2),\\
\Delta_4 &= -4(l+1)(l-1)(l+2)(2l+3),\\
\Delta &= l^2 + l(1+2\sigma) + (1+\sigma)
\end{aligned}\right\} \quad (2.7)$$

for $r < r_0$, one should interchange r and r_0 in equation (2.2).

3. The displacement field induced by a buried tangential dislocation

Consider a point-dislocation of area dS with a unit normal $\mathbf{n}_0$ on which a displacement vector $\mathbf{u}_0 = u_0\,\mathbf{e}_0$ is placed at the point $(r_0, 0, 0)$ inside a sphere of radius a. The ensuing dimensionless displacement field $\mathbf{U}(r) = \mathbf{u}_0(r)/u_0$ is given by the Volterra relation (Paper I, 5.1)

$$\mathbf{U}(r) = dS\{T_0(G_a) : \mathbf{e}_0\, \mathbf{n}_0\}, \quad (3.1)$$

where the operator T differentiates now in the source co-ordinate system, i.e., it operates only upon the source vectors in the expression (2.6) for $G_a(\mathbf{r}/\mathbf{r}_0)$. Since the details of this operation for $G_a(\mathbf{a}/\mathbf{r}_0)$ were described elsewhere (Paper I), we shall recapitulate here only the main points. Firstly we find it useful to introduce two angles δ and λ, which determine the orientation of the dislocation with respect to the source base $(\mathbf{e}_{r_0}, \mathbf{e}_{\phi_0}, \mathbf{e}_{\theta_0})$. (See Fig. 3). Thus, it can be shown that the dimensionless displacement due to an arbitrarily oriented tangential dislocation $[(\mathbf{e}_0 . \mathbf{n}_0) = 0]$ is given by a linear combination of three fundamental fields,

$$\mathbf{U}(r) = [\sin\delta \mathbf{U}_1 + \cos\delta \mathbf{U}_2^{**}] \cos\lambda + [\sin 2\delta \mathbf{U}_3 - \cos 2\delta \mathbf{U}_2] \sin\lambda, \qquad (3.2)$$

where $\mathbf{U}_1$ is the vertical strike-slip field ($\lambda = 0$, $\delta = 90°$), $\mathbf{U}_2$ is the vertical dip-slip field ($\lambda = 90°$, $\delta = 90°$), and $\mathbf{U}_3$ is the field for $\lambda = 90°$ and $\delta = 45°$. $\mathbf{U}_2^{**}$ is obtained from $\mathbf{U}_2$ by replacing ϕ by $[\phi - (\pi/2)]$ (counter-clockwise rotation of the $\mathbf{U}_2$ field by 90° about the z-axis).

Proceeding as in Paper I, we express the fields $\mathbf{U}_i$ ($i = 1, 2, 3$) in the form:

Case I, strike-slip fault ($\lambda = 0, \delta = 90°$)

$$\left.\begin{aligned}
U_r^{(1)} &= \Omega \sum_{l=2}^{\infty} \beta_l^{(1)}(x)\, P_l^2(\cos\theta) \sin 2\phi, \\
U_\theta^{(1)} &= \Omega \sum_{l=2}^{\infty} \left[2 f_l^{(1)}(x) \frac{P_l^2(\cos\theta)}{\sin\theta} + g_l^{(1)}(x) \frac{\partial P_l^2(\cos\theta)}{\partial\theta}\right] \sin 2\phi, \\
U_\phi^{(1)} &= \Omega \sum_{l=2}^{\infty} \left[f_l^{(1)}(x) \frac{\partial P_l^2(\cos\theta)}{\partial\theta} + 2 g_l^{(1)}(x) \frac{P_l^2(\cos\theta)}{\sin\theta}\right] \cos 2\phi.
\end{aligned}\right\} \qquad (3.3)$$

Case II, dip-slip fault ($\lambda = 90°, \delta = 90°$)

$$\left.\begin{aligned}
U_r^{(2)} &= \Omega \sum_{l=1}^{\infty} \beta_l^{(2)}(x)\, P_l^1(\cos\theta) \sin\phi, \\
U_\theta^{(2)} &= \Omega \sum_{l=1}^{\infty} \left[f_l^{(2)}(x) \frac{P_l^1(\cos\theta)}{\sin\theta} + g_l^{(2)}(x) \frac{\partial P_l^1(\cos\theta)}{\partial\theta}\right] \sin\phi, \\
U_\phi^{(2)} &= \Omega \sum_{l=1}^{\infty} \left[f_l^{(2)}(x) \frac{\partial P_l^1(\cos\theta)}{\partial\theta} + g_l^{(2)}(x) \frac{P_l^1(\cos\theta)}{\sin\theta}\right] \cos\phi
\end{aligned}\right\} \qquad (3.4)$$

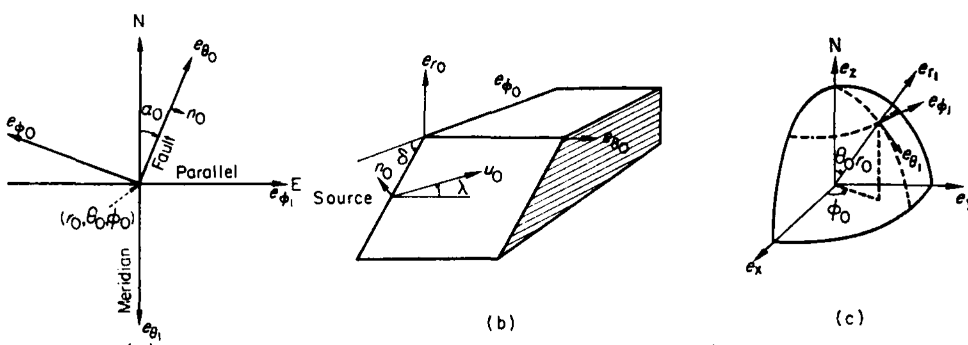

FIG. 3. Geometry of a displacement dislocation-source in a spherical earth: (*a*) Spherical co-ordinate-system at the source $(\mathbf{e}_{\theta_1}, \mathbf{e}_{\phi_1}, \mathbf{e}_{r_1})$ and the 'epicentral' co-ordinate-system $(\mathbf{e}_{\theta_0}, \mathbf{e}_{\phi_0}, \mathbf{e}_{r_0})$. (*b*) 'Epicentral' co-ordinate-system and source-parameters (λ, δ, u_0). (*c*) 'Geographic' co-ordinate-system $(\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z)$ and spherical co-ordinate-system at the source $(\mathbf{e}_{\theta_1}, \mathbf{e}_{\phi_1}, \mathbf{e}_{r_1})$.

Case III, dip-slip on a 45° plane ($\lambda = 90°$, $\delta = 45°$)

$$\left.\begin{aligned} U_r^{(3)} &= \Omega \sum_{l=1}^{\infty} \beta_l^{(0)}(x)\, P_l(\cos\theta) + \Omega \sum_{l=2}^{\infty} \beta_l^{(3)}(x)\, P_l^2(\cos\theta) \cos 2\phi, \\ U_\theta^{(3)} &= \Omega \sum_{l=1}^{\infty} g_l^{(0)}(x) \frac{\partial P_l(\cos\theta)}{\partial\theta} + \Omega \sum_{l=2}^{\infty} \left[2 f_l^{(3)}(x) \frac{P_l^2(\cos\theta)}{\sin\theta} \right. \\ &\qquad \left. + g_l^{(3)}(x) \frac{\partial P_l^2(\cos\theta)}{\partial\theta} \right] \cos 2\phi, \\ U_\phi^{(3)} &= -\Omega \sum_{l=2}^{\infty} \left[f_l^{(3)}(x) \frac{\partial P_l(\cos\theta)}{\partial\theta} + 2 g_l^{(3)}(x) \frac{P_l^2(\cos\theta)}{\sin\theta} \right] \sin 2\phi, \end{aligned}\right\} \quad (3.5)$$

with

$$\left.\begin{aligned} \Omega &= \frac{dS}{4\pi a^2}, \qquad \mathbf{U} = \frac{\mathbf{u}}{u_0}, \\ t &= \frac{r_0}{a}, \qquad x = \frac{r}{a}, \qquad 0 \leqslant x \leqslant 1. \end{aligned}\right\} \quad (3.6)$$

In these expressions, θ is the usual colatitude angle and ϕ is the azimuth angle taken counter-clockwise from the fault's-strike. The radial functions $\beta_l^{(i)}(x)$, $f_l^{(i)}(x)$ and $g_l^{(i)}(x)$ have the following meaning. For $x > t$, $l > 1$:

Case I

$$\left.\begin{aligned} \beta_l^{(1)} &= -\frac{t^{l-2}}{\gamma\Delta_R} \left[\frac{E_1(x)}{l(2l-1)} + \frac{(l+1+\gamma)\, E_2(x)}{(l+1)(2l+3)} t^2 \right], \\ g_l^{(1)} &= -\frac{t^{l-2}}{\gamma\Delta_R} \left[\frac{E_3(x)}{l(2l-1)} + \frac{(l+1+\gamma)\, E_4(x)}{(l+1)(2l+3)} t^2 \right], \\ f_l^{(1)} &= -\frac{t^{l-1}}{l(l+1)(l-1)} [(l-1)\, x^{-l-1} + (l+2)\, x^l]. \end{aligned}\right\} \quad (3.7)$$

Case II

$$\left.\begin{aligned} \beta_l^{(2)} &= -\frac{t^{l-2}}{\gamma\Delta_R} \left[\frac{2(l-1)\, E_1(x)}{l(2l-1)} + \frac{\{2(l+1)^2-\gamma\}\, E_2(x)}{(l+1)(2l+3)} t^2 \right], \\ g_l^{(2)} &= -\frac{t^{l-2}}{\gamma\Delta_R} \left[\frac{2(l-1)\, E_3(x)}{l(2l-1)} + \frac{\{2(l+1)^2-\gamma\}\, E_4(x)}{(l+1)(2l+3)} t^2 \right], \\ f_l^{(2)} &= \frac{-t^{l-1}}{l(l+1)} [(l-1)\, x^{-l-1} + (l+2)\, x^l]. \end{aligned}\right\} \quad (3.8)$$

Case III

$$\left.\begin{aligned} &\beta_l^{(3)} = -\tfrac{1}{2}\beta_l^{(1)}, \qquad g_l^{(3)} = -\tfrac{1}{2} g_l^{(1)}, \qquad f_l^{(3)} = -\tfrac{1}{2} f_l^{(1)}, \\ &\beta_l^{(0)} = \frac{t^{l-2}}{2\gamma\Delta_R} \left[\frac{3(l-1)\, E_1(x)}{(2l-1)} + \frac{l(3l+5-\gamma)\, E_2(x)}{(2l+3)} t^2 \right], \\ &g_l^{(0)} = \frac{t^{l-2}}{2\gamma\Delta_R} \left[\frac{3(l-1)\, E_3(x)}{(2l-1)} + \frac{l(3l+5-\gamma)\, E_4(x)}{(2l+3)} t^2 \right], \end{aligned}\right\} \quad (3.9)$$

where

$$\left.\begin{aligned} E_1(x) &= l(l+\gamma-1)\,\Delta_R\,x^{-l}+(l+2-\gamma)(l+1)\,\Delta_1\,x^{l+1}+l\Delta_3\,x^{l-1},\\ E_2(x) &= -(l+1)\,\Delta_R\,x^{-l-2}+(l+1)(l+2-\gamma)\,\Delta_2\,x^{l+1}+l\Delta_4\,x^{l-1},\\ E_3(x) &= -(l-\gamma)\,\Delta_R\,x^{-l}+(l+1+\gamma)\,\Delta_1\,x^{l+1}+\Delta_3\,x^{l-1},\\ E_4(x) &= \Delta_R\,x^{-l-2}+(l+1+\gamma)\,\Delta_2\,x^{l+1}+\Delta_4\,x^{l-1},\\ \Delta_R &= 8(l-1)\,\Delta. \end{aligned}\right\} \quad (3.10)$$

For $x < t,\ l > 1$:

Case I

$$\left.\begin{aligned} \beta_l^{(1)} &= -\frac{x^{l-1}}{\gamma\Delta_R}\left[\frac{D_3(t)}{2l-1}+\frac{(l+2-\gamma)\,D_4(t)}{2l+3}\,x^2\right],\\ g_l^{(1)} &= -\frac{x^{l-1}}{\gamma\Delta_R}\left[\frac{D_3(t)}{l(2l-1)}+\frac{(l+1+\gamma)\,D_4(t)}{(l+1)(2l+3)}\,x^2\right],\\ f_l^{(1)} &= -\frac{x^l}{l(l+1)(l-1)}\,[(l-1)\,t^{-l-2}+(l+2)\,t^{l-1}]. \end{aligned}\right\} \quad (3.11)$$

Case II

$$\left.\begin{aligned} \beta_l^{(2)} &= -\frac{x^{l-1}}{\gamma\Delta_R}\left[\frac{D_1(t)}{2l-1}+\frac{(l+2-\gamma)\,D_2(t)}{2l+3}\,x^2\right],\\ g_l^{(2)} &= -\frac{x^{l-1}}{\gamma\Delta_R}\left[\frac{D_1(t)}{l(2l-1)}+\frac{(l+1+\gamma)\,D_2(t)}{(l+1)(2l+3)}\,x^2\right],\\ f_l^{(2)} &= -\frac{x^l}{l(l+1)}\,[t^{l-1}-t^{-l-2}](l+2). \end{aligned}\right\} \quad (3.12)$$

Case III

$$\left.\begin{aligned} &\beta_l^{(3)} = -\tfrac{1}{2}\beta_l^{(1)}, \qquad g_l^{(3)} = -\tfrac{1}{2}g_l^{(1)}, \qquad f_l^{(3)} = -\tfrac{1}{2}f_l^{(1)},\\ &\beta_l^{(0)} = \frac{x^{l-1}}{2\gamma\Delta_R}\left[\frac{lF_1(t)}{2l-1}+\frac{(l+2-\gamma)\,F_2(t)}{2l+3}\,x^2\right],\\ &g_l^{(0)} = \frac{x^{l-1}}{2\gamma\Delta_R}\left[\frac{F_1(t)}{2l-1}+\frac{(l+1+\gamma)\,F_2(t)}{(l+1)(2l+3)}\,x^2\right], \end{aligned}\right\} \quad (3.13)$$

where

$$\left.\begin{aligned} D_1(t) &= (2l^2-\gamma)\,\Delta_R\,t^{-l-1}+[2(l+1)^2-\gamma]\,\Delta_1\,t^l+2(l-1)\,\Delta_3\,t^{l-2},\\ D_2(t) &= -2(l+2)\,\Delta_R\,t^{-l-3}+[2(l+1)^2-\gamma]\,\Delta_2\,t^l+2(l-1)\,\Delta_4\,t^{l-2},\\ D_3(t) &= \frac{1}{t}\,E_3(t); \quad D_4(t) = \frac{1}{t}\,E_4(t),\\ F_1(t) &= -(l+1)(3l-2+\gamma)\,\Delta_R\,t^{-l-1}+(l+1)(3l+5-\gamma)\,\Delta_1\,t^l\\ &\qquad +3(l-1)\,\Delta_3\,t^{l-2},\\ F_2(t) &= 3(l+1)(l+2)\,\Delta_R\,t^{-l-3}+3l(l-1)\,\Delta_4\,t^{l-2}\\ &\qquad +l(l+1)(3l+5-\gamma)\,\Delta_2\,t^l. \end{aligned}\right\} \quad (3.14)$$

Next, for $x > t,\ l = 1$:

Case II

$$\left.\begin{aligned}\beta_1^{(2)} &= \frac{t}{5\gamma}[(8-\gamma)x^{-3}+5(2-\gamma)-6(3-\gamma)x^2],\\ g_1^{(2)} &= -\frac{t}{10\gamma}[(8-\gamma)x^{-3}-10(2-\gamma)+6(2+\gamma)x^2],\\ f_1^{(2)} &= -\tfrac{3}{2}xt^2.\end{aligned}\right\} \quad (3.15)$$

Case III

$$\beta_1^{(0)} = -\beta_1^{(2)}; \qquad g_1^{(0)} = -g_1^{(2)}. \quad (3.16)$$

Finally, for $x < t$, $l = 1$:

Case II

$$\left.\begin{aligned}\beta_1^{(2)} &= \frac{1}{5\gamma}t^{-2}[5(\gamma-2)(1-t^3)+6(3-\gamma)t^{-2}(1-t^5)x^2],\\ g_1^{(2)} &= \frac{1}{5\gamma}t^{-2}[5(\gamma-2)(1-t^3)+3(2+\gamma)t^{-2}(1-t^5)x^2],\\ f_1^{(2)} &= \tfrac{3}{2}t^{-2}[t^{-1}-t^4]x.\end{aligned}\right\} \quad (3.17)$$

Case III

$$\left.\begin{aligned}\beta_1^{(0)} &= -\frac{1}{\gamma}[(1+\gamma)t^{-2}+(2-\gamma)t+\tfrac{6}{5}(\gamma-3)(\tfrac{3}{2}t^{-4}+t)x^2],\\ g_1^{(0)} &= -\frac{1}{\gamma}[(1+\gamma)t^{-2}+(2-\gamma)t-\tfrac{3}{5}(\gamma+2)(\tfrac{3}{2}t^{-4}+t)x^2].\end{aligned}\right\} \quad (3.18)$$

4. The inertia integrals in the ' epicentral ' co-ordinate system

Our previous results give the deformation of a spherical earth model due to a dislocation at $(r_0, 0, 0)$. Let us now introduce an axis of rotation that goes through the centre of mass and the dislocation source. The inertia dyadic $\boldsymbol{J}$ about this axis for the undeformed sphere is given by the volume integral over all mass elements dm in the relevant volume V,

$$\boldsymbol{J} = \int_V \{I(\mathbf{r}.\mathbf{r})-\mathbf{rr}\}\,dm, \quad (4.1)$$

where I is the unit dyadic and $\mathbf{r}$ is the position vector at any point, before the deformation. After the deformation this point moves to the location $\mathbf{r}+\mathbf{u}$ where $\mathbf{u}(r, \theta, \phi)$ is the displacement vector. Neglecting second order terms in the quantity $|\mathbf{u}|/|\mathbf{r}|$, it follows immediately that the corresponding change in the inertia dyadic amount to

$$\boldsymbol{\Theta} = \int_V \{2(\mathbf{r}.\mathbf{u})I-(\mathbf{ru}+\mathbf{ur})\}\,dm. \quad (4.2)$$

Next, we set up a cartesian co-ordinate system with unit vectors $(\mathbf{e}_{\theta_0}, \mathbf{e}_{\phi_0}, \mathbf{e}_{r_0})$ in such a way that the $\mathbf{e}_{r_0}$ axis coincides with the rotation axis, the $\mathbf{e}_{\theta_0}$ axis points in the direction of the fault's azimuth ($\phi = 0$) and the $\mathbf{e}_{\phi_0}$ axis in a direction opposite to the normal $\mathbf{n}_0$ of the source element dS, on a vertical fault. (Fig. 3). Denoting by $\Theta_{ij} = \Theta_{ji}$ the cartesian elements of the 'inertia-change' dyadic with respect to this co-ordinate system, we have

$$\Theta_{ij} = \mathbf{\Theta} : \mathbf{e}_i \mathbf{e}_j, \tag{4.3}$$

where the indices 1, 2, 3 correspond to the axes θ_0, ϕ_0 and r_0 respectively and the symbol (:) signifies the double dot product.

Inserting therefore the expressions

$$\left.\begin{aligned} \mathbf{e}_{\theta_0} &= \mathbf{e}_r \sin\theta\cos\phi + \mathbf{e}_0 \cos\theta\cos\phi - \mathbf{e}_\phi \sin\phi, \\ \mathbf{e}_{\phi_0} &= \mathbf{e}_r \sin\theta\sin\phi + \mathbf{e}_\theta \cos\theta\sin\phi + \mathbf{e}_\phi \cos\phi, \\ \mathbf{e}_{r_0} &= \mathbf{e}_r \cos\theta - \mathbf{e}_\theta \sin\theta, \\ dm &= \rho_0 r^2 \sin\theta\, dr\, d\theta\, d\phi, \quad \mathbf{u} = u_r \mathbf{e}_r + u_\theta \mathbf{e}_\theta + u_\phi \mathbf{e}_\phi, \end{aligned}\right\} \tag{4.4}$$

into equation (4.2), we obtain with the aid of (4.4) the following dimensionless integrals for the elements of the 'inertia-change' dyadic,

$$\begin{aligned} \frac{1}{A_0}\Theta_{11} = {} & \frac{15}{4\pi}\left[\frac{u_0}{a}\right]\int_{x_0}^{1} x^3\,dx \int_0^{\pi} \sin\theta\,d\theta \int_0^{2\pi} U_r(x,\theta,\phi)\,d\phi \\ & - \frac{15}{4\pi}\left[\frac{u_0}{a}\right]\int_{x_0}^{1} x^3\,dx \int_0^{\pi} \sin^3\theta\,d\theta \int_0^{2\pi} U_r(x,\theta,\phi)\cos^2\phi\,d\phi \\ & - \frac{15}{8\pi}\left[\frac{u_0}{a}\right]\int_{x_0}^{1} x^3\,dx \int_0^{\pi} \sin 2\theta\sin\theta\,d\theta \int_0^{2\pi} U_\theta(x,\theta,\phi)\cos^2\phi\,d\phi \\ & + \frac{15}{8\pi}\left[\frac{u_0}{a}\right]\int_{x_0}^{1} x^3\,dx \int_0^{\pi} \sin^2\theta\,d\theta \int_0^{2\pi} U_\phi(x,\theta,\phi)\sin 2\phi\,d\phi, \end{aligned} \tag{4.5}$$

$$\begin{aligned} \frac{1}{A_0}\Theta_{22} = {} & \frac{15}{4\pi}\left[\frac{u_0}{a}\right]\int_{x_0}^{1} x^3\,dx \int_0^{\pi} \sin\theta\,d\theta \int_0^{2\pi} U_r(x,\theta,\phi)\,d\phi \\ & - \frac{15}{4\pi}\left[\frac{u_0}{a}\right]\int_{x_0}^{1} x^3\,dx \int_0^{\pi} \sin^3\theta\,d\theta \int_0^{2\pi} U_r(x,\theta,\phi)\sin^2\phi\,d\phi \\ & - \frac{15}{8\pi}\left[\frac{u_0}{a}\right]\int_{x_0}^{1} x^3\,dx \int_0^{\pi} \sin 2\theta\sin\theta\,d\theta \int_0^{2\pi} U_\theta(x,\theta,\phi)\sin^2\phi\,d\phi \\ & - \frac{15}{8\pi}\left[\frac{u_0}{a}\right]\int_{x_0}^{1} x^3\,dx \int_0^{\pi} \sin^2\theta\,d\theta \int_0^{2\pi} u_\phi(x,\theta,\phi)\sin 2\phi\,d\phi, \end{aligned} \tag{4.6}$$

$$\begin{aligned} \frac{1}{A_0}\Theta_{33} = {} & \frac{15}{4\pi}\left[\frac{u_0}{a}\right]\int_{x_0}^{1} x^3\,dx \int_0^{\pi} \sin^3\theta\,d\theta \int_0^{2\pi} U_r(x,\theta,\phi)\,d\phi \\ & + \frac{15}{8\pi}\left[\frac{u_0}{a}\right]\int_{x_0}^{1} x^3\,dx \int_0^{\pi} \sin 2\theta\sin\theta\,d\theta \int_0^{2\pi} U_\theta(x,\theta,\phi)\,d\phi, \end{aligned} \tag{4.7}$$

$$\frac{1}{A_0}\Theta_{12} = -\frac{15}{8\pi}\left[\frac{u_0}{a}\right]\int_{x_0}^{1} x^3\,dx\int_0^{\pi}\sin^3\theta\,d\theta\int_0^{2\pi}U_r(x,\theta,\phi)\sin 2\phi\,d\phi$$

$$-\frac{15}{16\pi}\left[\frac{u_0}{a}\right]\int_{x_0}^{1} x^3\,dx\int_0^{\pi}\sin 2\theta\sin\theta\,d\theta\int_0^{2\pi}U_\theta(x,\theta,\phi)\sin 2\phi\,d\phi$$

$$-\frac{15}{8\pi}\left[\frac{u_0}{a}\right]\int_{x_0}^{1} x^3\,dx\int_0^{\pi}\sin^2\theta\,d\theta\int_0^{2\pi}U_\phi(x,\theta,\phi)\cos 2\phi\,d\phi, \quad (4.8)$$

$$\frac{1}{A_0}\Theta_{13} = -\frac{15}{8\pi}\left[\frac{u_0}{a}\right]\int_{x_0}^{1} x^3\,dx\int_0^{\pi}\sin 2\theta\sin\theta\,d\theta\int_0^{2\pi}U_r(x,\theta,\phi)\cos\phi\,d\phi$$

$$-\frac{15}{8\pi}\left[\frac{u_0}{a}\right]\int_{x_0}^{1} x^3\,dx\int_0^{\pi}\cos 2\theta\sin\theta\,d\theta\int_0^{2\pi}U_\theta(x,\theta,\phi)\cos\phi\,d\phi$$

$$+\frac{15}{16\pi}\left[\frac{u_0}{a}\right]\int_{x_0}^{1} x^3\,dx\int_0^{\pi}\sin 2\theta\,d\theta\int_0^{2\pi}U_\phi(x,\theta,\phi)\sin\phi\,d\phi, \quad (4.9)$$

$$\frac{1}{A_0}\Theta_{23} = -\frac{15}{8\pi}\left[\frac{u_0}{a}\right]\int_{x_0}^{1} x^3\,dx\int_0^{\pi}\sin 2\theta\sin\theta\,d\theta\int_0^{2\pi}U_r(x,\theta,\phi)\sin\phi\,d\phi$$

$$-\frac{15}{8\pi}\left[\frac{u_0}{a}\right]\int_{x_0}^{1} x^3\,dx\int_0^{\pi}\cos 2\theta\sin\theta\,d\theta\int_0^{2\pi}U_\theta(x,\theta,\phi)\sin\phi\,d\phi$$

$$-\frac{15}{16\pi}\left[\frac{u_0}{a}\right]\int_{x_0}^{1} x^3\,dx\int_0^{\pi}\sin 2\theta\,d\theta\int_0^{2\pi}U_\phi(x,\theta,\phi)\cos\phi\,d\phi, \quad (4.10)$$

$$A_0 = \frac{8\pi}{15}\rho_0 a^5, \quad (4.11)$$

where a is the radius of a homogeneous sphere of density ρ_0 and x_0 is a certain reference level to be chosen later.

The evaluation of the integrals (4.5)–(4.10) is straightforward; substituting the expressions for the displacements as given in equations (3.3)–(3.5) and making use of the orthogonality relations

$$\left.\begin{aligned}&\int_0^{2\pi}\cos m\phi\sin n\phi\,d\phi = 0,\\ &\int_0^{2\pi}\sin m\phi\sin n\phi\,d\phi = \int_0^{2\pi}\cos m\phi\cos n\phi\,d\phi = \pi\delta_{mn},\end{aligned}\right\} \quad (4.12)$$

for all integral values of m and n, we find that the only contributions to the Cases I and II arise from the elements Θ_{12}, Θ_{23}, respectively, whereas contributions to Case III arise only from the elements Θ_{11}, Θ_{22} and Θ_{33}.

Turning next to the integrals over the colatitude angle θ, we use the orthogonality properties of the associated Legendré polynomials to show that:

$$\left.\begin{aligned}
&\int_0^\pi \sin^2\theta\cos\theta P_l^1(\cos\theta)\,d\theta = \tfrac{4}{5}\delta_{2l},\\
&\int_0^\pi \left[\cos 2\theta\, P_l^1(\cos\theta)+\sin\theta\cos\theta\,\frac{\partial P_l^1(\cos\theta)}{\partial\theta}\right]d\theta = 0,\\
&\int_0^\pi \left[\cos\theta\, P_l^1(\cos\theta)+\cos 2\theta\sin\theta\,\frac{\partial P_l^1(\cos\theta)}{\partial\theta}\right]d\theta = \tfrac{24}{5}\delta_{2l},\\
&\int_0^\pi \sin^3\theta\, P_l^2(\cos\theta)\,d\theta = \tfrac{16}{5}\delta_{2l},\\
&\int_0^\pi \left[\sin 2\theta\, P_l^2(\cos\theta)+\sin^2\theta\,\frac{\partial P_l^2(\cos\theta)}{\partial\theta}\,d\theta\right] = 0,\\
&\int_0^\pi \left[\sin\theta\cos\theta\,\frac{\partial P_l^2(\cos\theta)}{\partial\theta}+2P_l^2(\cos\theta)\right]\sin\theta\,d\theta = \tfrac{48}{5}\delta_{2l},\\
&\int_0^\pi \sin^3\theta\, P_l(\cos\theta)\,d\theta = \tfrac{4}{3}\delta_{l0}-\tfrac{4}{15}\delta_{2l},\\
&\int_0^\pi P_l(\cos\theta)\sin\theta\,d\theta = 2\delta_{0l}.
\end{aligned}\right\}\quad(4.13)$$

Performing the integrations over the angles θ and ϕ we arrive at the conclusion that all contributions from Cases I, II, II** and III arise from the spherical harmonic $l = 2$ only.† The sole non-zero contributions to the 'inertia-change' dyadic are therefore,

For Case I

$$\Theta_{12} = -6k_0\int_{x_0}^{1} x^3[\beta_2^{(1)}(x)+3g_2^{(1)}(x)]\,dx.$$

For Case II

$$\Theta_{23} = -3k_0\int_{x_0}^{1} x^3[\beta_2^{(2)}(x)+3g_2^{(2)}(x)]\,dx.$$

For Case II**

$$\Theta_{13} = -\Theta_{23}.$$

† The absence of higher even orders may seem peculiar. This cut-off however, is determined by the order of the source. Thus, a source of the j-th order will excite all even harmonics up to $l = j$.

For Case III

$$\left.\begin{aligned}\Theta_{11} &= k_0 \int_{x_0}^{1} x^3[\beta_2^{(0)}(x)+3g_2^{(0)}(x)+3\beta_2^{(1)}(x)+9g_2^{(1)}(x)]\,dx\\ &= -\tfrac{1}{2}[\Theta_{33}+\Theta_{12}],\\ \Theta_{22} &= k_0 \int_{x_0}^{1} x^3[\beta_2^{(0)}(x)+3g_2^{(0)}(x)-3\beta_2^{(1)}(x)-9g_2^{(1)}(x)]\,dx\\ &= -\tfrac{1}{2}[\Theta_{33}-\Theta_{12}],\\ \Theta_{33} &= -2k_0 \int_{x_0}^{1} x^3[\beta_2^{(0)}(x)+3g_2^{(0)}(x)]\,dx,\end{aligned}\right\} \tag{4.14}$$

where

$$k_0 = A_0\left[\frac{dS}{4\pi a^2}\right]\left[\frac{u_0}{a}\right], \quad \Theta_{ij} = \Theta_{ji}. \tag{4.15}$$

Note that the radial functions $f_l(x)$, that arise from the toroidal motion, were eliminated in the integration process. Hence, it becomes clear that a change in the inertia moment of the Earth due to a tangential dislocation is caused only by the zero frequency limit of the spheroidal mode of the second degree.

Note also that the inertia-change dyadic $\boldsymbol{\Theta}$ is represented by the symmetric matrix

$$\begin{bmatrix} -\tfrac{1}{2}(\Theta_{12}+\Theta_{33}) & \Theta_{12} & -\Theta_{23}\\ \Theta_{12} & \tfrac{1}{2}(\Theta_{12}-\Theta_{33}) & \Theta_{23}\\ -\Theta_{23} & \Theta_{23} & \Theta_{33}\end{bmatrix}. \tag{4.16}$$

5. Evaluation of the inertia integrals in the ' Geographic ' co-ordinate system

Following Munk & Macdonald (1960), we set a 'Geographic' frame with its origin at the Earth's mass-centre. The z-axis is aligned with the Earth's axis of rotation, the x-axis is drawn through the Greenwich meridian and the y-axis points at 90° east of Greenwich.

The results of the previous section are valid only for the special case in which the epicentre lies on the Earth's rotation axis. In order to solve the problem for a source that is situated arbitrarily with respect to the axis we must apply an orthogonal transformation which brings the 'epicentral' axes into coincidence with the 'geographical' axes.

To enact this transformation we consider a source at a point (r_0, θ_0, ϕ_0) where θ_0 is the colatitude and ϕ_0 is its longitude drawn eastward from Greenwich. Let α_0 be the fault's azimuth, taken clockwise from north toward the fault's strike. (Figs 3 and 4).

If we denote by $(\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z)$ the unit vectors of the 'geographic' system, by $(\mathbf{e}_{\theta_1}, \mathbf{e}_{\phi_1}, \mathbf{e}_{r_1})$ the unit vectors of the spherical system at the source and by $(\mathbf{e}_{\theta_0}, \mathbf{e}_{\phi_0}, \mathbf{e}_{r_0})$ the unit vectors of the 'epicentral' system at the source, we then find that the following relations hold

$$\begin{bmatrix}\mathbf{e}_x\\ \mathbf{e}_y\\ \mathbf{e}_z\end{bmatrix} = \begin{bmatrix}\cos\phi_0\cos\theta_0 & -\sin\phi_0 & \cos\phi_0\sin\theta_0\\ \sin\phi_0\cos\theta_0 & \cos\phi_0 & \sin\phi_0\sin\theta_0\\ -\sin\theta_0 & 0 & \cos\theta_0\end{bmatrix}\begin{bmatrix}\mathbf{e}_{\theta_1}\\ \mathbf{e}_{\phi_1}\\ \mathbf{e}_{r_1}\end{bmatrix}. \tag{5.1}$$

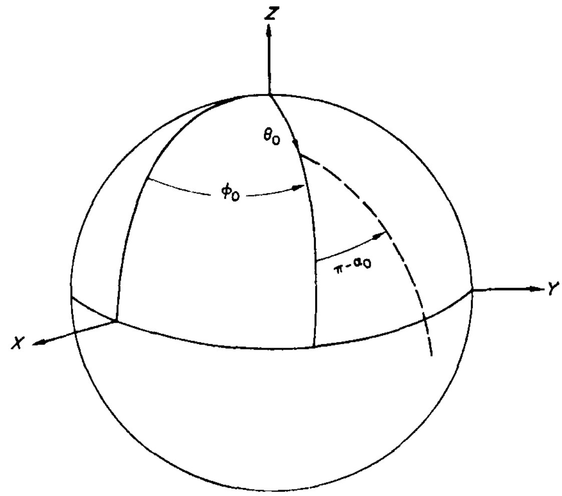

FIG. 4. Source's Azimuth (ϕ_0), colatitude (θ_0) and fault's azimuth (α_0) as the Euler angles needed to coincide the ' epicentral ' system with the ' geographic ' system.

$$\begin{bmatrix} \mathbf{e}_{\theta_1} \\ \mathbf{e}_{\phi_1} \\ \mathbf{e}_{r_1} \end{bmatrix} = \begin{bmatrix} -\cos\alpha_0 & -\sin\alpha_0 & 0 \\ \sin\alpha_0 & -\cos\alpha_0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{e}_{\theta_0} \\ \mathbf{e}_{\phi_0} \\ \mathbf{e}_{r_0} \end{bmatrix} \tag{5.2}$$

Thus, the inertia dyadic C in the geographic system is related to the inertia dyadic $\boldsymbol{\Theta}$ in the epicentre system through the relation

$$C = T\Theta T^{-1} = T\Theta \tilde{T}, \tag{5.3}$$

or explicitly

$$c_{13} = (t_{11}\,t_{31})\,\Theta_{11} + (t_{12}\,t_{32})\,\Theta_{22} + (t_{13}\,t_{33})\,\Theta_{33} + (t_{12}\,t_{31} + t_{11}\,t_{32})\,\Theta_{12} + (t_{13}\,t_{31} + t_{11}\,t_{33})\,\Theta_{13} + (t_{13}\,t_{32} + t_{12}\,t_{33})\,\Theta_{23}, \tag{5.4}$$

$$c_{23} = (t_{21}\,t_{31})\,\Theta_{11} + (t_{22}\,t_{32})\,\Theta_{22} + (t_{23}\,t_{33})\,\Theta_{33} + (t_{22}\,t_{31} + t_{21}\,t_{32})\,\Theta_{12} + (t_{23}\,t_{31} + t_{21}\,t_{33})\,\Theta_{23} + (t_{22}\,t_{33} + t_{23}\,t_{32})\,\Theta_{23}, \tag{5.5}$$

and a similar expression for c_{33}.

The elements t_{ij} of T are

$$\left.\begin{aligned} t_{11} &= -\sin\phi_0 \sin\alpha_0 - \cos\phi_0 \cos\theta_0 \cos\alpha_0, \\ t_{12} &= +\sin\phi_0 \cos\alpha_0 - \cos\phi_0 \cos\theta_0 \sin\alpha_0, \\ t_{13} &= \cos\phi_0 \sin\theta_0, \\ t_{21} &= +\cos\phi_0 \sin\alpha_0 - \sin\phi_0 \cos\theta_0 \cos\alpha_0, \\ t_{22} &= -\cos\phi_0 \cos\alpha_0 - \sin\phi_0 \cos\theta_0 \sin\alpha_0, \\ t_{23} &= \sin\phi_0 \sin\theta_0, \\ t_{31} &= +\sin\theta_0 \cos\alpha_0, \\ t_{32} &= +\sin\theta_0 \sin\alpha_0, \\ t_{33} &= \cos\theta_0, \\ 0 \leqslant \theta_0 &\leqslant \pi, \quad 0 \leqslant \phi_0 \leqslant 2\pi, \quad 0 \leqslant \alpha_0 \leqslant 2\pi. \end{aligned}\right\} \tag{5.6}$$

Here the indices 1, 2, 3 correspond to the geographic axes x, y, z respectively.

Substituting the proper expressions from (5.6) into (5.4) and (5.5) we have

Case I ($\lambda = 0°$, $\delta = 90°$)

$$\left.\begin{aligned} c_{13} &= [\sin\phi_0 \sin\theta_0 \cos 2\alpha_0 - \tfrac{1}{2}\cos\phi_0 \sin 2\theta_0 \sin 2\alpha_0]\,\Theta_{12},\\ c_{23} &= -[\cos\phi_0 \sin\theta_0 \cos 2\alpha_0 + \tfrac{1}{2}\sin\phi_0 \sin 2\theta_0 \sin 2\alpha_0]\,\Theta_{12},\\ c_{33} &= [\sin^2\theta_0 \sin 2\alpha_0]\,\Theta_{12}. \end{aligned}\right\} \quad (5.7)$$

Case II ($\lambda = 90°$, $\delta = 90°$)

$$\left.\begin{aligned} c_{13} &= [\sin\phi_0 \cos\theta_0 \cos\alpha_0 - \cos\phi_0 \cos 2\theta_0 \sin\alpha_0]\,\Theta_{23},\\ c_{23} &= -[\cos\phi_0 \cos\theta_0 \cos\alpha_0 + \sin\phi_0 \cos 2\theta_0 \sin\alpha_0]\,\Theta_{23},\\ c_{33} &= [\sin 2\theta_0 \sin\alpha_0]\,\Theta_{23}. \end{aligned}\right\} \quad (5.8)$$

*Case II*** ($\lambda = 0°$, $\delta = 0°$)

$$\left.\begin{aligned} c_{13} &= [\sin\phi_0 \cos\theta_0 \sin\alpha_0 + \cos\phi_0 \cos 2\theta_0 \cos\alpha_0]\,\Theta_{23},\\ c_{23} &= [-\cos\phi_0 \cos\theta_0 \sin\alpha_0 + \sin\phi_0 \cos 2\theta_0 \cos\alpha_0]\,\Theta_{23},\\ c_{33} &= -(\sin 2\theta_0 \cos\alpha_0)\,\Theta_{23}. \end{aligned}\right\} \quad (5.9)$$

Case III ($\lambda = 90°$, $\delta = 45°$)

$$\left.\begin{aligned} c_{13} &= \tfrac{1}{2}[\sin\phi_0 \sin\theta_0 \sin 2\alpha_0 + \tfrac{1}{2}\cos\phi_0 \sin 2\theta_0 \cos 2\alpha_0]\,\Theta_{12}\\ &\qquad + \tfrac{3}{4}\sin 2\theta_0 \cos\phi_0\,\Theta_{33},\\ c_{23} &= -\tfrac{1}{2}[\cos\phi_0 \sin\theta_0 \sin 2\alpha_0 - \tfrac{1}{2}\sin\phi_0 \sin 2\theta_0 \cos 2\alpha_0]\,\Theta_{12}\\ &\qquad + \tfrac{3}{4}\sin 2\theta_0 \sin\phi_0\,\Theta_{33}\\ c_{33} &= P_2(\cos\theta_0)\,\Theta_{33} - \tfrac{1}{2}\sin^2\theta_0 \cos 2\alpha_0\,\Theta_{12}. \end{aligned}\right\} \quad (5.10)$$

It remains to integrate over the radial functions from the level x_0 to the surface according to equations (4.14) and (4.15). Using the expressions for $\beta_2^{(i)}(x)$ and $g_2^{(i)}(x)$ from equations (3.7)–(3.18) we get

$$\begin{aligned} F_1(t, x_0, \sigma) &= \int_{x_0}^{1} x^3\,dx[\beta_2^{(1)}(x) + 3g_2^{(1)}(x)] = \frac{15+10\sigma}{28+20\sigma}\left[\frac{\sigma+2}{3+2\sigma}t^2 - 1\right]\\ &\quad + x_0^5\,\frac{(7+5\sigma)(1-2\sigma)\,t^{-3} + 5(7-\sigma^2) - 6(7-4\sigma)\,t^2}{12(1-\sigma)(7+5\sigma)}\\ &\quad + \tfrac{3}{7}x_0^7\,\frac{(7+5\sigma)\,t^{-5} - 42 + 10(7-4\sigma)\,t^2}{12(1-\sigma)(7+5\sigma)}. \end{aligned} \quad (5.11)$$

$$\begin{aligned} F_2(t, x_0, \sigma) &= \int_{x_0}^{1} x^3\,dx[\beta_2^{(2)}(x) + 3g_2^{(2)}(x)] = \frac{15+10\sigma}{14+10\sigma}(t^2 - 1)\\ &\quad + x_0^5(1-t)\,\frac{6(7+2\sigma)(1+t) + (1+\sigma)(7+5\sigma)(1+t+t^2)\,t^{-3}}{6(1-\sigma)(7+5\sigma)}\\ &\quad - \tfrac{6}{7}x_0^7(1-t)\,\frac{5(7+2\sigma)(1+t) + 2(7+5\sigma)(1+t+t^2+t^3+t^4)\,t^{-5}}{6(1-\sigma)(7+5\sigma)}, \end{aligned}$$

(5.12)

$$F_3(t, x_0, \sigma) = \int_{x_0}^{1} x^3\,dx[\beta_2^{(0)}(x) + 3g_2^{(0)}(x)] = \frac{45+30\sigma}{28+20\sigma}\left[1 - \frac{10+7\sigma}{9+6\sigma}t^2\right]$$

$$+x_0^5\,\frac{(5\sigma^2-3\sigma)+6(7+4\sigma)\,t^2+2(2-\sigma)\,t^{-3}(7+5\sigma)}{4(1-\sigma)(7+5\sigma)}$$

$$+x_0^7\,\frac{63-15(7+4\sigma)\,t^2-9(7+5\sigma)\,t^{-5}}{14(1-\sigma)(7+5\sigma)}. \qquad (5.13)$$

6. Inertia products for finite sources

Evaluation of the inertia products for faults of finite extent require two additional integrations. Restricting our analysis to vertical faults only, we shall obtain the proper results by the following integration scheme: Consider a fault with vertical extent $1 \geqslant t \geqslant \tau$ striking $N\alpha_0 E$ along a great circle path with horizontal extent $\varepsilon_0 \geqslant \varepsilon \geqslant 0$ from $P(\theta_0, 0)$ to $Q(\theta_1, \phi_1)$. We substitute for the area-element dS in equations (5.7) and (5.8) the operator

$$a^2 \int_{\tau}^{1} t\,dt \int_{0}^{\varepsilon_0} d\varepsilon, \qquad (6.1)$$

and start with the radial integeration over t. This soon yields

$$\int_{\tau}^{1} F_1(t)\,t\,dt = \left\|\frac{15+10\sigma}{14+10\sigma}\,\frac{t^2}{4}\left[\frac{\sigma+2}{3+2\sigma}\,\frac{t^2}{2} - 1\right]\right\|_{\tau}^{1}$$

$$+x_0^5\left\|\frac{(6\sigma-\frac{21}{2})\,t^4+\frac{1}{2}(35-5\sigma^2)\,t^2-(7-9\sigma-10\sigma^2)\,t^{-1}}{12(1-\sigma)(7+5\sigma)}\right\|_{\tau}^{1}$$

$$+\tfrac{3}{7}x_0^7\left\|\frac{\frac{1}{4}(70-40\sigma)\,t^4-21t^2-\frac{1}{3}(7+5\sigma)\,t^{-3}}{12(1-\sigma)(7+5\sigma)}\right\|_{\tau}^{1}, \qquad (6.2)$$

$$\int_{\tau}^{1} F_2(t)\,t\,dt = \left\|\frac{15+10\sigma}{28+20\sigma}\,(\tfrac{1}{2}t^2-1)\,t^2\right\|_{\tau}^{1}$$

$$+x_0^5\left\|\frac{3(7+2\sigma)\,t^2(1-\frac{1}{2}t^2)-(1+\sigma)(7+5\sigma)(\frac{1}{2}t^2+t^{-1})}{6(1-\sigma)(7+5\sigma)}\right\|_{\tau}^{1}$$

$$-\tfrac{6}{7}x_0^7\left\|\frac{\frac{5}{2}(7+2\sigma)\,t^2(1-\frac{1}{2}t^2)-(7+5\sigma)(t^2+\frac{2}{3}t^{-3})}{6(1-\sigma)(7+5\sigma)}\right\|_{\tau}^{1}. \qquad (6.3)$$

To enact the angular integration we apply the basic formulae of spherical trigonometry to the triangle OPQ (O being the north pole) in order to express the trigonometric functions of the angles θ, α and $\phi-\phi_0$ in terms of the functions of the angle ε.

Eventually one finds for

Case I

$$\int_0^{\varepsilon_0} c_{13}\, d\varepsilon = \Theta_{12} \int_0^{\varepsilon_0} d\varepsilon \left[\sin\alpha_0 \sin\varepsilon \frac{m^2-1}{m^2+1} + \frac{2m}{m^2+1} \right.$$

$$\left. \times (\cos\theta_0 \cos\alpha_0 \sin\varepsilon - \cos\varepsilon \sin\theta_0) \times (\cos\theta_0 \cos\varepsilon + \sin\theta_0 \cos\alpha_0 \sin\varepsilon) \right], \quad (6.4)$$

$$m = (-\sin\varepsilon \cot\theta_0 + \cos\alpha_0 \cos\varepsilon)/\sin\alpha_0, \quad (6.5)$$

$$\int_0^{\varepsilon_0} c_{23}\, d\varepsilon = -\Theta_{12} \int_0^{\varepsilon_0} d\varepsilon \left[(\cos\varepsilon \sin\theta_0 - \cos\theta_0 \cos\alpha_0 \sin\varepsilon) \frac{m^2-1}{m^2+1} \right.$$

$$\left. + \frac{2m}{1+m^2} \sin\alpha_0 \sin\varepsilon (\cos\theta_0 \cos\varepsilon + \sin\theta_0 \cos\alpha_0 \sin\varepsilon \right]. \quad (6.6)$$

Case II

$$\int_0^{\varepsilon_0} c_{13}\, d_\varepsilon = \Theta_{23} \int_0^{\varepsilon} \left[\frac{m(\cos\theta_0 \cos\varepsilon + \sin\theta_0 \sin\varepsilon \cos\alpha_0) - m_1}{\sqrt{(1+m^2)}.\sqrt{(1+m_1^2)}} \right.$$

$$\left. + 2 \sin\theta_0 \sin\alpha_0 (\cos\varepsilon \sin\theta_0 - \cos\theta_0 \cos\alpha_0 \sin\varepsilon) \right] d\varepsilon,$$

$$m_1 = (\sin\theta_0 \cot\varepsilon - \cos\theta_0 \cos\alpha_0)/\sin\alpha_0, \quad (6.7)$$

$$\int_0^{\varepsilon_0} c_{23} = d_\varepsilon - \Theta_{23} \int_0^{\varepsilon} \left[\frac{m_1 m_2 (\cos\theta_0 \cos\varepsilon + \sin\theta_0 \cos\alpha_0 \sin\varepsilon) + 1}{\sqrt{(1+m^2)}.\sqrt{(1+m_1^2)}} \right.$$

$$\left. - 2 \sin^2\alpha_0 \sin\varepsilon \sin\theta_0 \right] d\varepsilon.$$

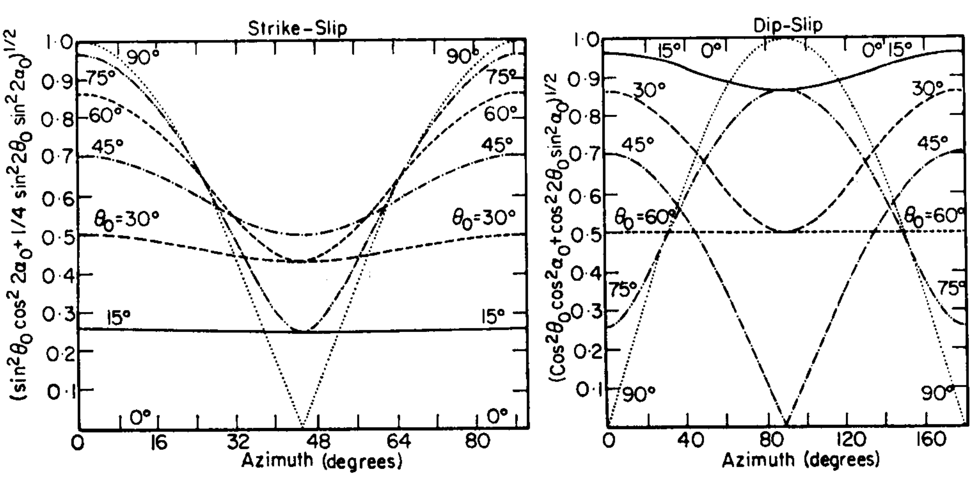

FIG. 5. Dependence of angular factor of the inertia products on fault's azimuth (α_0) and colatitude (θ_0) for vertical dip-slip and strike-slip point-dislocations in a spherical earth model.

7. The inertia integrals for an explosion source

Using the theory of Ben-Menahem & Singh (1968, pp. 446–447), we obtain the following expressions for the field induced in a homogeneous sphere by a buried explosion:

for $x > t$:

$$u_r = -\frac{E_0}{a^2}\left[x^{-2}+2x\frac{1-2\sigma}{1+\sigma}\right] - \frac{E_0}{a^2(1+\alpha)} t\cos\theta[2(1+\sigma)x^{-3}$$

$$+(4+\sigma)+3(1-4\sigma)x^2] + \frac{E_0}{a^2}\sum_{l=2}^{\infty}(l+1)P_l(\cos\theta)\frac{t^l}{D}\{2(l+2)$$

$$\times(2l+1)(l+2-\gamma)x^{l+1}-2l(l+2)(2l+3)x^{l-1}-Dx^{-l-2}\}, \qquad (7.1)$$

$$u_\theta = -\frac{E_0}{a^2}\frac{t\sin\theta}{1+\sigma}[(1+\sigma)x^{-3}-(4+\sigma)+3x^2(3-2\sigma)]$$

$$+\frac{E_0}{a^2}\sum_{l=2}^{\infty}\frac{\partial P_l(\cos\theta)}{\partial\theta}\frac{t^l}{D}\{2(l+2)(2l+1)(l+1+\gamma)x^{l+1}$$

$$-2(l+1)(l+2)(2l+3)x^{l-1}+Dx^{-l-2}\}, \qquad (7.2)$$

$$u_\phi = 0,$$

and for $x < t$:

$$u_r = -\frac{2E_0}{a^2}\frac{1-2\sigma}{1+\sigma}x - \frac{E_0}{a^2(1+\sigma)}\cos\theta\left[t\{(4+\sigma)+3x^2(1-4\sigma)\}-\frac{1+\sigma}{t^2}\right]$$

$$+\frac{E_0}{a^2}\sum_{l=2}^{\infty}\frac{P_l(\cos\theta)}{D}[Dlx^{l-1}t^{-l-1}+2(l+1)(l+2)t^l\{(2l+1)$$

$$\times(l+2-\gamma)x^{l+1}-2l(2l+3)x^{l-1}\}], \qquad (7.3)$$

$$u_\theta = -\frac{E_0}{a^2(1+\sigma)}\sin\theta\left[t\{3x^2(3-2\sigma)-(4+\sigma)\}+\frac{1+\sigma}{t^2}\right]$$

$$+\frac{E_0}{a^2}\sum_{l=2}^{\infty}\frac{1}{D}\frac{\partial P_l(\cos\theta)}{\partial\theta}[Dx^{l-1}t^{-l-1}+2(l+2)t^l\{(2l+1)$$

$$\times(l+1+\gamma)x^{l+1}-(l+1)(2l+3)x^{l-1}\}], \qquad (7.4)$$

$$u_\phi = 0.$$

Here $D = 4\Delta$ and E_0 is a source parameter of dimensions (length)^3. It has the physical significance of yield-energy per unit pressure in the exploding cavity. (Latter *et al.* 1961).

Substituting the displacement functions $u_r(x, t, \theta)$ and $u_\theta(x, t, \theta)$ into the integrands of equations (4.5)–(4.10) we obtain non-zero contributions for $l = 0$ and $l = 2$ only. The contribution due to $l = 0$ is found to be

$$\Theta_{11}^{(0)} = \Theta_{22}^{(0)} = \Theta_{33}^{(0)} = -A_0\frac{E_0}{a^3}\left[5(1-t^2)+4\frac{1-2\sigma}{1+\sigma}(1+x_0^5)\right]$$

$$\Theta_{12}^{(0)} = \Theta_{13}^{(0)} = \Theta_{23}^{(0)} = 0. \qquad (7.5)$$

Likewise, the contribution from $l = 2$ amounts to

$$\Theta^{(2)}_{11} = \Theta^{(2)}_{22} = -\tfrac{1}{2}\Theta^{(2)}_{33} = A_0 \frac{E_0}{a^3}\left[t^{-3}(t^5 - x_0{}^5) + \frac{6t^2}{7+5\sigma}\{5(1-x_0{}^7) - 7(1-x_0{}^5)\}\right]$$

$$\Theta^{(2)}_{12} = \Theta^{(2)}_{13} = \Theta^{(2)}_{23} = 0. \qquad (7.6)$$

To find the elements of the transformed inertia dyadic c_{ij}, we make use of equations (5.1) and (5.2). Taking $\alpha_0 = \pi$ on account of the symmetry of the explosion source, we find,

$$\left.\begin{aligned} c_{13} &= [\tfrac{3}{4}\cos\phi_0 \sin 2\theta_0]\,\Theta^{(2)}_{33} \\ c_{23} &= [\tfrac{3}{4}\sin\phi_0 \sin 2\theta_0]\,\Theta^{(2)}_{33} \\ c_{33} &= [P_2(\cos\theta_0)]\,\Theta^{(2)}_{33} + \Theta^{(0)}_{33}. \end{aligned}\right\} \qquad (7.7)$$

In particular, for $t = 1$ (surface source), $x_0 = \frac{3000}{6371}$ and $\sigma = 0{\cdot}275$

$$\left.\begin{aligned} |c_{13} + ic_{23}| &= A_0 \frac{2E_0}{3a^3}|\sin 2\theta_0| \\ c_{33} &= -\frac{A_0 E_0}{a^3}[1{\cdot}41 - 0{\cdot}86\, P_2(\cos\theta_0)]. \end{aligned}\right\} \qquad (7.8)$$

Note that c_{33} is negative for all values of θ_0.

8. Applications to various seismic events

Equations (5.7)–(5.10) and (7.8) state the dependence of the inertia changes on the position and the intrinsic parameters of the source. To be able to apply these formulae to specific events we have calculated the basic functions that determine the radial and angular dependence of the inertia integrals.

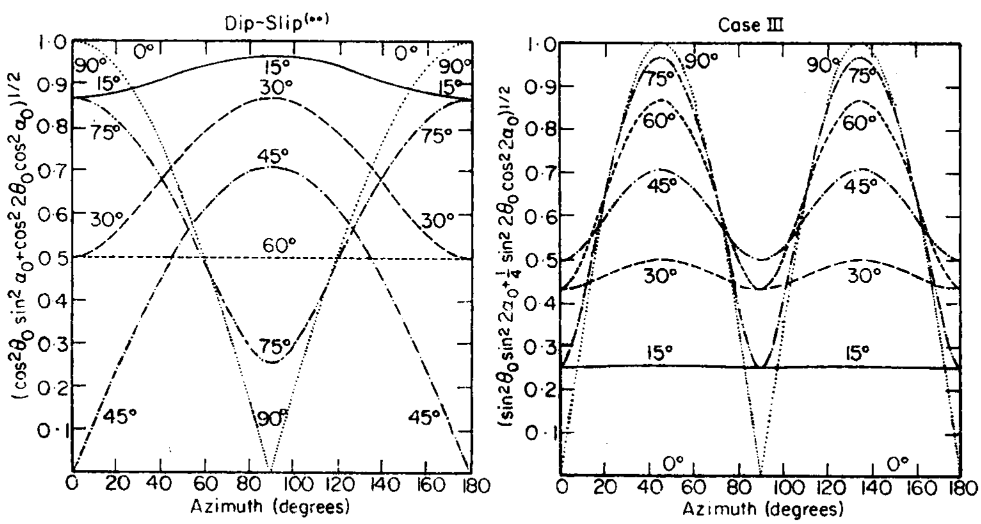

Fig. 6. Dependence of angular factor of the inertia products on fault's azimuth (α_0) and colatitude (θ_0) for horizontal dip-slip and 45° dip-slip point-dislocations in a spherical earth model.

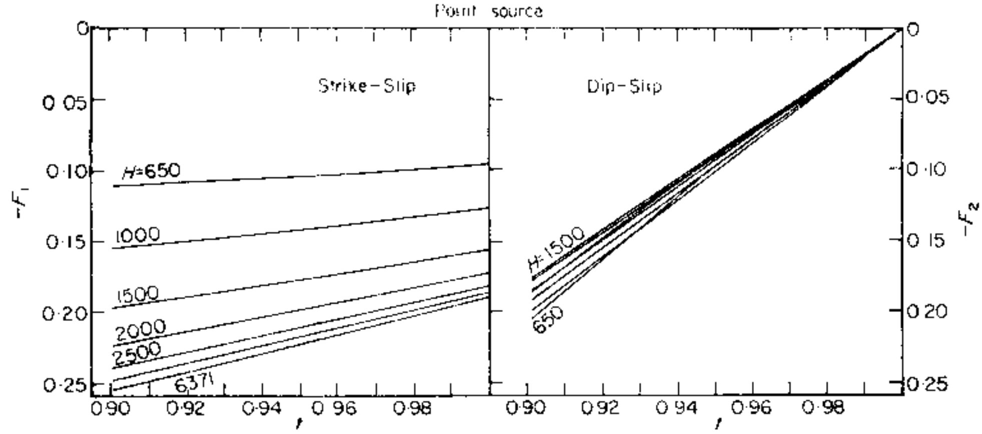

FIG. 7. Dependence of radial factor of the inertia products on source's depth (t) for various values of the lower integration limit (H, in km).

Results for a point dislocation-source are shown in Figs 5–7. Suppose that we wish to calculate the inertia changes for the Mongolian earthquake of 1957 December 4 (Ben-Menahem & Toksöz 1962; Florensov & Solonenko 1965). Its parameters were $\theta_0 = 45{\cdot}3°$, $\phi_0 = 99{\cdot}4°$, $\alpha_0 \cong 100°$, Dip $\cong 50°$, Slip $= 30°$, $u_0^{(\mathrm{I})} = 5\,\mathrm{m}$, $u_0^{(\mathrm{II})} = 3\,\mathrm{m}$, $t \cong 0{\cdot}992$, $dS = (300 \times 50)\,\mathrm{km}^2$. Treating it first as a point source centred on an area dS, we notice from Fig. 5 that the dip-slip component does not contribute to the wobble. For the strike-slip component, the angular function (Fig. 5) has the value 0·7 and the radial function (Fig. 7) for $H = 3000$ km is $F_1 = -0{\cdot}2$. Equations (5.7–10) then yield $|c_{13}+ic_{23}| \sim 3{\cdot}6 \times 10^{26}\,\mathrm{kg\text{-}m}^2$.

More accurate results are obtained by using the finite source model (Figs 8 and 9). However, for a fault of the above dimensions and depth the difference is not significant.

Other examples are given in Tables 1, 2 and 3. Table 1 shows that the spherical theory yields results that are higher by a factor of 3, then the corresponding 'half-space approximation' results of Mansinha & Smylie (1967). Source-mechanism of the Alaskan earthquake has recently been derived by Ben-Menahem & Rosenman

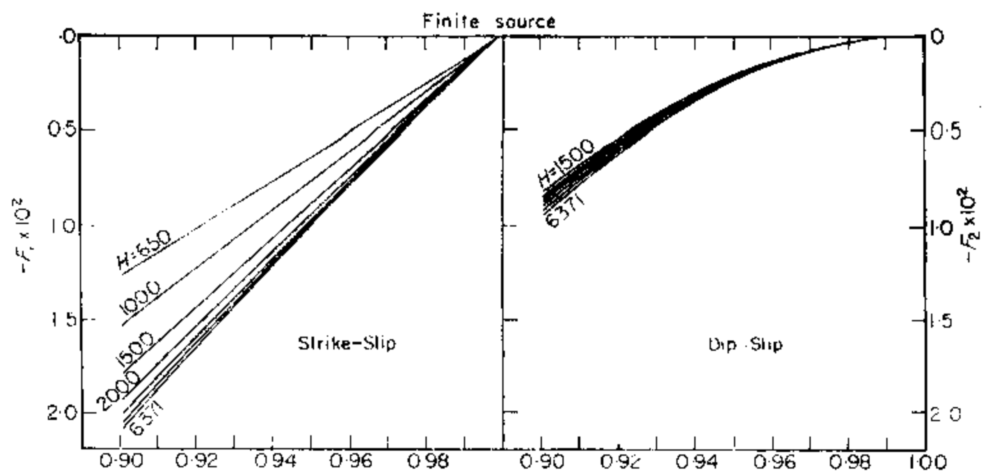

FIG. 8. Dependence of radial factor on the inertia products on source's vertical extension (τ) for various values of the lower integration limit (H, in km).

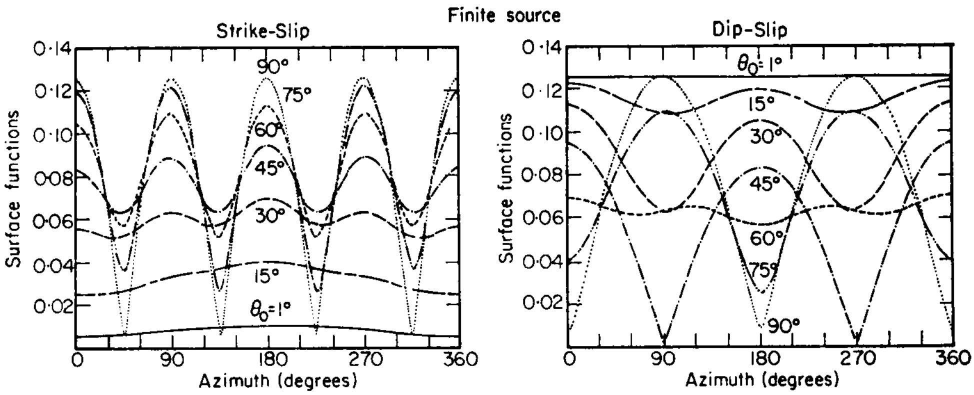

FIG. 9. Dependence of angular factor of the inertia products on fault's azimuth (α_0) and colatitude of fault's tip (θ_0) for a particular fault-length. (ε_0 = 800 km).

(1970, in preparation), from the radiation patterns of Long-Period Rayleigh and Love waves. Analysis of this data disclosed that the source responsible for the dynamic surface-wave field was a vertical dip-slip fault, 700 km in length.

A close inspection of equations (5.7)–(5.10) and (7.8) shows that the strength of the inertia changes depend on the location of the epicentre with respect to the pole of rotation. Amongst shallow events, the dip-slip type fault is a poor perturbator of the Earth's rotation. Explosions and strike-slip faults, on the other hand, cause extremal perturbations if they occur at the 'right places' on the globe. To exhibit this possibility we have chosen two major seismic events. The Colombia–Ecuador earthquake is considered by Gutenberg (1956) and Richter (1958) as the greatest shock since 1896. Adopting Gutenberg's value for its energy (2×10^{25} erg), and the magnitude–fault length–displacement relation (Iida 1965; King & Knopoff 1968), we find a fault length of 700 km together with a maximum offset of 15 m. The results of the calculations are shown in Table 2. Here we notice a difference between the point-source approximation and the finite source model for a deep dip-slip fault.

The other major seismic event is the Krakatoa volcanic explosion (Sunda Strait, 6·10° S, 105·42° E). Pekeris (1939), estimated the energy of the generated air waves to be of the order of $W = 10^{24}$ erg. Harkrider & Press (1967), concluded that 'an explosion of ($W =$) 100–150 megatons is required to produce the equivalent of the Krakatoa pressure disturbance'. Translating their result to ergs we come up with 4×10^{24} erg which is similar to Pekeris' result.

We shall assume that the seismic yield of the Krakatoa event was of the same magnitude as its acoustic yield. Bullard (1962), quotes a calculation of Verbeck in which it is stated that about five cubic miles of material was blown out of Krakatoa during the eruption. This amounts to a volume of $\frac{1}{3}(4\pi)\,b^3 = E_0 = 1{\cdot}7\times10^{16}\,\text{cm}^3$ [using, as a check, the relation $P = (\gamma-1)\,W/\frac{1}{3}(4\pi)\,b^3$ (Latter *et al.* 1961), we find a pressure of $P = 100$ Atm just prior to the explosion]. Thus, having estimated E_0, we evaluated c_{33} as shown in Table 3. The corresponding ΔT was derived from $\Delta T = (c_{33}/C)\times T_0$ where $T_0 = 86\,400$ sec and $C \approx 8\times10^{37}\,\text{kg m}^2$.

From equations (7.8) we may deduce the effect of a hypothetical underground nuclear explosion with a yield of 100 MT at $\theta_0 = 40°$. Taking again $E_0 = 1{\cdot}7\times10^{16}\,\text{cm}^3$ we find that $|c_{13}+ic_{23}| = 3\times10^{27}$ kg-m^2.

It is clear from the numerical results of Tables 1–3 that both the polar shift and the change in the length of day increase with the depth of the source. For a strike-slip fault the increase is linear with depth while for a dip-slip fault the increase is proportioned to the square of the depth.

Table 1

Calculated Wobble Parameters for the Alaskan Earthquake of March 28, 1964, 03 36 13 UT

Model*		Point-Source approximation in spherical Earth†		Finite-Source in spherical Earth†		Finite-Source in flat Earth‡	
Case	D km	$\lvert c_{13}+ic_{23}\rvert$ kg m^2	Polar shift cm	$\lvert c_{13}+ic_{23}\rvert$ kg m^2	Polar shift cm	$\lvert c_{13}+ic_{23}\rvert$ kg m^2	Polar shift cm
Strike Slip	120	$6{\cdot}22\times10^{27}$	22·2	$6{\cdot}60\times10^{27}$	23·6		
	200	$9{\cdot}90\times10^{27}$	35·3	$1{\cdot}04\times10^{28}$	37·2		
	400	$2{\cdot}08\times10^{28}$	74·4	$2{\cdot}15\times10^{28}$	77·2		
	600	$3{\cdot}39\times10^{28}$	121·1	$3{\cdot}43\times10^{28}$	122·6		
Dip Slip	120	$5{\cdot}58\times10^{26}$	2·0	$5{\cdot}39\times10^{26}$	1·9		
	200	$1{\cdot}36\times10^{27}$	4·9	$1{\cdot}30\times10^{27}$	4·6		
	400	$5{\cdot}39\times10^{27}$	19·2	$5{\cdot}03\times10^{27}$	17·9	$1{\cdot}92\times10^{27}$	4·8
	600	$1{\cdot}17\times10^{28}$	41·5	$1{\cdot}17\times10^{28}$	41·5		
III	120	$3{\cdot}90\times10^{27}$	13·9				
	200	$6{\cdot}01\times10^{27}$	21·5				
	400	$1{\cdot}19\times10^{28}$	42·5				
	600	$1{\cdot}95\times10^{28}$	69·6				

* $H = 3000$ km, $u_0 = 5$ metres, $D = 2h$, $\theta_0 = 28{\cdot}9°$, $\alpha_0 = 225°$, $\phi_0 = 212{\cdot}4°$

† $dS = (700\times D)$ km^2

‡ As calculated by Mansinha & Smylie 1967.

Table 2

Calculated Wobble Parameters for the Colombia–Equador Earthquake of* 1906, January 31, 12 36·0 GCT (M = 8·9)

Model†		Point-Source approximation in spherical Earth‡		Finite source in spherical Earth‡	
Case	D km	$\lvert c_{13}+ic_{23}\rvert$ kg m^2	Polar shift cm	$\lvert c_{13}+ic_{23}\rvert$ kg m^2	Polar shift cm
Strike Slip	120	$1{\cdot}58\times10^{28}$	56·5	$1{\cdot}76\times10^{28}$	62·9
	200	$7{\cdot}02\times10^{28}$	250·3	$6{\cdot}97\times10^{28}$	248·8
	400	$1{\cdot}47\times10^{29}$	524·8	$1{\cdot}44\times10^{29}$	514·8
	600	$2{\cdot}40\times10^{29}$	858·0	$2{\cdot}29\times10^{29}$	818·0
Dip Slip	120	$7{\cdot}29\times10^{24}$	—	$2{\cdot}94\times10^{25}$	—
	200	$9{\cdot}81\times10^{25}$	—	$3{\cdot}58\times10^{26}$	—
	400	$3{\cdot}89\times10^{26}$	1·4	$1{\cdot}39\times10^{27}$	
	600	$9{\cdot}27\times10^{26}$	3·3	$3{\cdot}21\times10^{27}$	
III	120	$2{\cdot}75\times10^{26}$	1·0		
	200	$1{\cdot}22\times10^{27}$	4·3		
	400	$2{\cdot}57\times10^{27}$	9·2		
	600	$4{\cdot}19\times10^{27}$	14·9		

* Rudolph & Szirtes 1911: Gutenberg 1956: Richter 1958
† $H = 3000$, $U_0 = 15$ metres, $D = 2h$, $\theta_0 = 89°$, $\alpha_0 = 1°$
‡ $dS = (700\times D)\,\mathrm{km}^2$

9. Discussion

The perturbing effects of single major seismic events on the Earth's rotation have been formulated analytically and then calculated for a few typical cases. In light of these results it is natural to investigate the extent to which earthquakes are able to maintain the Chandler wobble. The answer depends on the dissipation rate of the wobble as well as on the rate of occurrence of the appropriate seismic events. We shall adopt a value of 15 cm/year for the most probable secular polar shift (Mansinha & Smylie 1967). From Tables 1 and 2 we deduce that the highest value for the polar shift attainable from a shallow earthquake having the parameters:

$$M_1 = 8\tfrac{1}{2} \quad 2h \sim 80\,\mathrm{km},\ b \sim 650\,\mathrm{km},\ u_0 \sim 5\,\mathrm{m},\ E \sim 5\times10^{24}\,\mathrm{erg}$$

amount to $S_1 = 15$ cm/year. From a list of earthquakes with $M_1 \geqslant 8{\cdot}5$ (Richter 1958), we find that earthquakes of this magnitude occur on the average once in four years. Furthermore, Gutenberg (1956), calculated an annual average of 6×10^{24} erg. Assume a hypothetical situation in which all the annual seismic energy is released through a single shallow strike-slip rupture on a meridional fault located at the equator. This shock, then, will suffice to drive the wobble forever. Clearly, the geographical distribution of earthquake epicentres and the dependence of the wobble magnitude on the strike and source parameters will result in a cumulative wobble which is smaller than the threshold of 15 cm/year.

Mansinha & Smylie (1967) tried to estimate the total effect of all earthquakes on the wobble by applying a two-dimensional random walk model to account for the frequency–magnitude dependence of earthquakes. Using their result with $S_1 = 15$ cm/year, we obtain an excitation which is 20 per cent of the observed. From our Figs 5 and 6 we find that the variability of the angles α_0 and θ_0 introduce a factor of uncertainty which lies between 1 and 0·5. Let us assume that all shocks are either of the dip-slip type or the strike-slip type, both mechanisms occurring with equal

Table 3

Calculated changes in the axial moment of inertia and the corresponding changes in length of day

Model (H = 3000 km) Case	D km	Alaska earthquake 1964 March 28 C_{33} kg m^2	ΔT μ s	Colombia–Equador earthquake 1906 January 31 C_{33} kg m^2	ΔT μ s	Krakatoa eruption 1883 August 27* C_{33} kg m^2	ΔT μ s
Strike Slip	50	$1·57\times10^{27}$	1·70	$7·09\times10^{26}$			
	120	$3·43\times10^{27}$	3·69	$1·54\times10^{27}$	1·65		
	200	$5·47\times10^{27}$	5·87	$2·45\times10^{27}$	2·63		
	400	$1·15\times10^{28}$	12·30	$5·14\times10^{27}$	5·52		
	600	$1·87\times10^{28}$	20·10	$8·38\times10^{27}$	8·99		
Dip Slip	50	$-1·01\times10^{26}$					
	120	$-4·60\times10^{26}$					
	200	$-1·12\times10^{27}$	− 1·20	Negligible			
	400	$-4·45\times10^{27}$	− 4·77				
	600	$-1·06\times10^{28}$	−11·40				
Explosion source	$D = 0$					$-1·6\times10^{28}$	−17·0

* or underground nuclear explosion of 100 MT at the same latitude

probability. We then find that the total estimated wobble is about 10 per cent of the observed. However, it is rather likely that a theoretical model incorporating a liquid core and gravity will boost the calculated wobble by a factor of 2–3. Altogether the expected wobble will be 20–30 per cent of the observed.

Calculated changes in length of day that resulted from three events of magnitude $M \geqslant 8\frac{1}{2}$ are shown in Table 3. Markowitz & Guinot (1968) state that 'sudden changes in acceleration occurred about September 1957 and January 1962. No correlation with the motion of the mean pole is evident. However, another 20 years of additional observation are needed for more details'. It means that events such as the Chilean earthquake of May 1960 and the Alaskan earthquake of March 1964 were not sufficient to excite measurable changes in the length of day. It may also imply that the mass readjustment time is not small relative to the length of day.

Acknowledgments

This Research has been sponsored by the Air Force Cambridge Research Laboratories under contract AF61(052)–954, through the European Office of Aerospace Research, OAR, U.S.A.F., as part of the Advanced Research Projects Agency's Project VELA-UNIFORM.

Department of Applied Mathematics,
The Weizmann Institute of Science,
Rehovot, Israel.

References

Ben-Menahem, A. & Singh, S. J., 1968. Eigenvector expansions of Green's dyads with applications to geophysical theory, *Geophys. J. R. astr. Soc.*, **16,** 417–452.

Ben-Menahem, A., Singh, S. J. & Solomon, F., 1969. Static deformation of a spherical earth model by internal dislocations, *Bull. seism. Soc. Am.*, **59,** 813–853.

Ben-Menahem, A. & Toksöz, M. N., 1962. Source-mechanism from spectra of long-period seismic surface-waves, Part I, *J. geophys. Res.*, **67,** 1943–1955.

Bullard, M. F., 1962. *Volcanoes*, University of Texas Press, Austin.

Florensov, N. A. & Solonenko, V. P., (editors) 1965. *The Gobi–Altai Earthquake.* Translated from Russian, Israel Program for Scientific Translations, Jerusalem.

Gutenberg, B., 1956. Great earthquakes, 1896–1903. *Trans. Am. Geophys. Un.*, **37,** 608–614.

Harkrider, D. & Press, F., 1967. The Krakatoa air–sea waves: an example of pulse propagation in coupled systems, *Geophys. J. R. astr. Soc.*, **13,** 149–159.

Iida, K., 1965. Earthquake magnitude, earthquake fault and source dimensions, *J. earth Sci., Nagoya Univ.*, **13,** 115–132.

King, C. & Knopoff, L., 1968. Stress drop in earthquakes, *Bull. seism. Soc. Am.*, **58,** 249–257.

Latter, A. L., LeLevier, R. E., Martinelli, E. A. & McMillan, W. G., 1961. A method of concealing underground nuclear explosions, *J. geophys. Res.*, **66,** 943–946.

Love, A. E. H., 1909. The yielding of the Earth to disturbing forces, *Proc. R. Soc. Lond., A,* **82,** 73–88.

Mansinha, L. & Smylie, D. E., 1967. Effect of Earthquakes on the Chandler wobble and the secular polar shift, *J. geophys. Res.*, **72,** 4731–4743.

Markowitz, W. & Guinot, B., (editors) 1968. *Continental Drift, Secular Motion of the Pole, and Rotation of the Earth*, Reidel Publishing Company, Holland.

Munk, W. H. & Macdonald, G. F. J., 1960. *The Rotation of the Earth*, Cambridge University Press, London.

Pekeris, C. L., 1939. The propagation of a pulse in the atmosphere, *Proc. R. Soc., Lond. Ser. A.*, **171,** 434–449.

Press, F., 1965. Displacements, strains and tilts at teleseismic distances, *J. geophys. Res.*, **70**, 2395–2412.

Richter, C. F., 1958. *Elementary Seismology*, Freeman and Co., San-Francisco.

Rudolph, E. & Szirtes, S., 1911. Das Kolumbianische Erdbeben am 31, Januar, 1906. *Beitr. Geophys.*, **11**, 132–199, 207–275.

Smylie, D. E. & Mansinha, L., 1968a. Earthquakes and the observed motion of the rotation pole, *J. geophys. Res.*, **73**, 7661–7673.

Smylie, D. E. & Mansinha, L., 1968b. *J. geophys. Res.*, **73**, 4738.

Volterra, Vito, 1895. Sui moti periodici de polo terrestre, *Atti Accad. Torino*, **30**, 547–561.

Volterra, Vito, 1898. Sur la théorie des variation des latitude, *Acta Math.*, **22**, 201–357.