Effects of Mechanical Rotation on Spin Currents

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We study the Pauli-Schrödinger equation in a uniformly rotating frame of reference to describe a coupling of spins and mechanical rotations. The explicit form of the spin-orbit interaction (SOI) with the inertial effects due to the mechanical rotation is presented. We derive equations of motion for a wave packet of electrons in two-dimensional planes subject to the SOI. The solution is a superposition of two cyclotron motions with different frequencies and a circular spin current is created by the mechanical rotation. The magnitude of the spin current is linearly proportional to the lower cyclotron frequency.

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Introduction.—Recently much attention has been paid to the control and generation of spin currents, i.e., the flow of electron spins in the field of spintronics [1]. Since the spin current is a nonconserved quantity, the utilization of spin currents is much more challenging than that of charge currents. A central concept of spintronics is the transfer of spin angular momentum based on angular momentum conservation. Experimental developments in the last decade have allowed us to exchange the angular momentum among conduction electron spin, local magnetization, and photon polarizations. These phenomena give birth to a variety of functions [2], and have accelerated the development of magnetic random access memory (MRAM) [3].

In this context, a remaining form of angular momentum carried by condensed matter systems is mechanical angular momentum due to the uniform rotation of a rigid body. Using this mechanical angular momentum in spintronics will permit the mechanical manipulation of spin currents. However, the effects of mechanical rotation on a spin current have not been demonstrated so far.

In this Letter, we derive the fundamental Hamiltonian with a coupling of spin currents and mechanical rotations from the generally covariant Dirac equation. The introduction of mechanical rotations involves extending our physical system from an inertial to noninertial frame. The dynamics of spin currents is closely related to the spin-orbit interaction (SOI), which results from taking the low energy limit of the Dirac equation.

Figure 1 illustrates the relation between mechanical rotation, magnetization, and spin current. The coupling of the magnetization and a spin current has been investigated extensively in terms of spin transfer torque [4,5], spin pumping [6], and spin motive force [7], i.e., the key technologies of spintronics. On the other hand, the coupling of a mechanical torque and the magnetization was studied long time ago. In the middle of the 1910s, the coupling of mechanical rotations and magnetization was investigated by Barnett [8], Einstein, and de Haas [9]. They measured the gyromagnetic ratio and the anomalous g factor of electrons before the establishment of modern quantum physics. Recently, several groups have detected the effects of mechanical rotations on nanostructured magnetic systems. Mechanical detection of ferromagnetic resonance spectroscopy has been recognized [10]; the Einstein–de Haas effect, rotation induced by magnetization, is observed in the submicron sized thin NiFe films deposited on a microcantilever [11], and the nanomechanical detection of a mechanical torque due to spin flips at the normal-ferromagnetic junction of a suspended nanowire has been reported [12]. There is theoretical work on the effects of a mechanical torque acting on a nanostructured magnetic system [13–16]. The Einstein–de Haas effect in Bose-Einstein condensates of atomic gases has been proposed [17].

Comparing to the well-established coupling of mechanical rotations and magnetization, and that of magnetization and spin currents, the direct coupling of mechanical rotations and spin currents has not been demonstrated. The main purpose of this Letter is to link the mechanical rotation with spin currents.

First, we introduce the Dirac equation in a uniformly rotating frame. In the low energy limit of this equation,

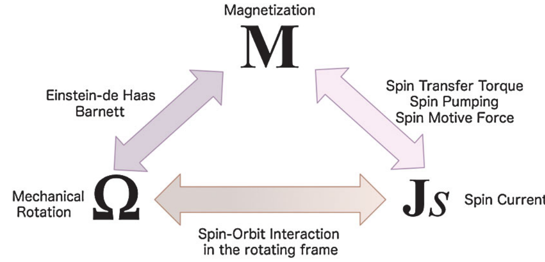

FIG. 1 (color). Angular momentum transfers between interacting systems.

we derive up to the order of $1/m^2$ with electron mass m the Pauli-Schrödinger Hamiltonian for a single electron including a SOI modified by a mechanical rotation. It is then straightforward to extend the derivation to condensed matter systems in a rotating frame by replacing the coupling parameter of the SOI in vacuum with that in materials [18,19]. We derive equations of motion for a wave packet subject to spin-dependent forces due to the SOI term and solve them in a particular case. The solution exhibits a circular pure spin current caused by mechanical rotation.

Dirac Equation in a uniformly rotating frame.—According to Einstein's principle of equivalence, gravitation cannot be distinguished from noninertiality. In the general relativity, both gravitational and inertial effects are expressed by a metric and a connection in a curved space-time. Dynamics of a spin-1/2 particle in a curved space-time is described by the generally covariant form of the Dirac equation [20]:

$$[\gamma^\mu(p_\mu - qA_\mu - \Gamma_\mu) + mc/\hbar]\Psi = 0, \tag{1}$$

where c and $\hbar$ are the velocity of light and the Planck constant, m and $q = -e$ are the mass and charge of an electron, $A_\mu = (A_0, \mathbf{A})$ is the gauge potential, and Γ_μ the spinor connection (see [20], for example). The coordinate-dependent Clifford algebra in the curved space-time $\gamma^\mu = \gamma^\mu(x)$ is satisfying $\{\gamma^\mu(x), \gamma^\nu(x)\} = 2g^{\mu\nu}(x)$ with the metric $g^{\mu\nu}(x)$ $(\mu, \nu = 0, 1, 2, 3)$. In a uniformly rotating frame of reference, of which the angular velocity with respect to an inertial frame is $\mathbf{\Omega}(t)$, the coordinate transformation from the rotating frame to the inertial frame is $d\mathbf{r}' = d\mathbf{r} + (\mathbf{\Omega} \times \mathbf{r})dt$. The space-time line element is given by $ds^2 = [-c^2 + (\mathbf{\Omega} \times \mathbf{r})^2]dt^2 + 2(\mathbf{\Omega} \times \mathbf{r})dtd\mathbf{r} + d\mathbf{r}^2$. The metric in a uniformly rotating frame becomes $g_{00} = -1 + (\mathbf{\Omega} \times \mathbf{r}/c)^2$, $g_{0i} = g_{i0} = (\mathbf{\Omega} \times \mathbf{r}/c)_i$, $g_{ij} = \delta_{ij}$ $(i, j = 1, 2, 3)$. From this metric, we obtain the Clifford algebra and the spinor connection in the rotating frame as $\gamma^0(x) = i\beta$, $\gamma^i(x) = i\beta\alpha_i - (\mathbf{\Omega} \times \mathbf{r}/c)_i$, $\Gamma_0 = \mathbf{\Omega} \cdot \mathbf{\Sigma}/2c$, $\Gamma_i = 0$, where $\beta = \begin{pmatrix} I & O \\ O & -I \end{pmatrix}$ and $\boldsymbol{\alpha} = \begin{pmatrix} O & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & O \end{pmatrix}$ are the Dirac matrices and $\mathbf{\Sigma}$ is the spin operator for 4-spinor defined by $\mathbf{\Sigma} = \frac{\hbar}{4i}\boldsymbol{\alpha} \times \boldsymbol{\alpha}$ with the Pauli matrix $\boldsymbol{\sigma}$. Thus, Eq. (1) can be rewritten as

$$\begin{aligned} i\hbar\frac{\partial\Psi}{\partial t} &= H\Psi, \\ H &= \beta mc^2 + c\boldsymbol{\alpha} \cdot \boldsymbol{\pi} + qA_0 - \mathbf{\Omega} \cdot (\mathbf{r} \times \boldsymbol{\pi} + \mathbf{\Sigma}), \end{aligned} \tag{2}$$

where $\boldsymbol{\pi} = \mathbf{p} - q\mathbf{A}$ is the mechanical momentum and $\mathbf{r}$ is position vector from the rotation axis. It is well known that, in classical mechanics, the Hamiltonian in the rotating frame has the additional term $\mathbf{\Omega} \cdot (\mathbf{r} \times \boldsymbol{\pi})$ reproducing the inertial effects: Coriolis, centrifugal, and Euler forces [21]. The term $\mathbf{\Omega} \cdot \mathbf{\Sigma}$ is the so-called spin-rotation coupling found in Ref. [22] and also discussed in the context of neutron interferometry in a stationary laboratory on Earth [23]. The last term of Eq. (2), $\mathbf{\Omega} \cdot (\mathbf{r} \times \boldsymbol{\pi} + \mathbf{\Sigma})$, can be regarded as a quantum mechanical generalization of the inertial effects [23] obtained by replacing the mechanical angular momentum $\mathbf{r} \times \boldsymbol{\pi}$ with total angular momentum $\mathbf{r} \times \boldsymbol{\pi} + \mathbf{\Sigma}$.

Pauli-Schrödinger equation in a rotating frame.—In the low energy limit, the Dirac equation in a flat space-time reduces to the Pauli-Schrödinger equation by the Foldy–Wouthuysen–Tani transformation [24,25], which block diagonalizes the Hamiltonian and is the systematic expansion yielding relativistic corrections in any order of the inverse mass, $O(1/m^n)$ $(n = 1, 2, \cdots)$. We divide the Hamiltonian (2) into even and odd parts denoted by $\mathcal{E}$ and $\mathcal{O}$, respectively; $H = \beta mc^2 + \mathcal{E} + \mathcal{O}$, $\mathcal{E} = qA_0 - \mathbf{\Omega} \cdot (\mathbf{r} \times \boldsymbol{\pi} + \mathbf{\Sigma})$, $\mathcal{O} = c\boldsymbol{\alpha} \cdot (\mathbf{p} - q\mathbf{A})$. By successive transformations, the Hamiltonian up to the order of $1/m^2$ becomes

$$\begin{aligned} H = \beta\left[mc^2 + \frac{\mathcal{O}^2}{2mc^2} - \frac{\mathcal{O}^4}{8m^3c^6}\right] + \mathcal{E} \\ - \frac{1}{8m^2c^4}[\mathcal{O}, [\mathcal{O}, \mathcal{E}] + i\hbar\dot{\mathcal{O}}]. \end{aligned} \tag{3}$$

Neglecting the rest energy in Eq. (3), the Pauli-Schrödinger equation for the upper component of Dirac spinors in the rotating frame is obtained by

$$i\hbar\frac{\partial\psi}{\partial t} = H_{\mathrm{PR}}\psi, \tag{4}$$

$$H_{PR} = H_K + H_Z + H_I + H_S + H_D, \tag{5}$$

$$H_K = \frac{1}{2m}\boldsymbol{\pi}^2 + qA_0, \tag{6}$$

$$H_Z = \mu_B\boldsymbol{\sigma} \cdot \mathbf{B}, \tag{7}$$

$$H_I = -\mathbf{\Omega} \cdot (\mathbf{r} \times \boldsymbol{\pi} + \mathbf{S}), \tag{8}$$

$$H_S = \frac{\lambda}{2\hbar}\boldsymbol{\sigma} \cdot [\boldsymbol{\pi} \times q\mathbf{E}' - q\mathbf{E}' \times \boldsymbol{\pi}], \tag{9}$$

$$H_D = -\frac{\lambda}{2}\mathrm{div}[q\mathbf{E}'], \tag{10}$$

with $\mu_B = q\hbar/2m$, $\lambda = \hbar^2/4m^2c^2$, $\mathbf{S} = (\hbar/2)\boldsymbol{\sigma}$, and

$$\mathbf{E}' = \mathbf{E} + (\mathbf{\Omega} \times \mathbf{r}) \times \mathbf{B}. \tag{11}$$

Equation (4) is the 2-spinor equation for a single electron in the rotating frame and the Hamiltonian is a 2×2 matrix operator. In this expansion, the Hamiltonian to the order of $1/m$ is given by $H_K + H_Z + H_I$. The spin-independent H_K contains the kinetic energy and the potential energy. The Zeeman energy H_Z contains the g factor of the electron equal to 2, which, combined with H_K, yields the coupling with magnetic fields, $(q/2m)(\mathbf{r}\times\boldsymbol{\pi}+2\mathbf{S})\cdot\mathbf{B}$. This contrasts with Eq. (8): the mechanical rotation couples to the total angular momentum of the electron $\mathbf{r}\times\boldsymbol{\pi}+\mathbf{S}$. The inertial effects, i.e., Coriolis, centrifugal, and Euler forces, are reproduced by H_I and $\boldsymbol{\Omega}\cdot\mathbf{S}$ is the spin-rotation coupling term. The expansion of the order of $1/m^2$ yields H_S and H_D, which are the SOI and Darwin terms with the mechanical rotation, respectively. In the absence of the mechanical rotation, $\boldsymbol{\Omega}=0$, these terms reduce to the usual SOI and Darwin terms in a flat space-time. In the case of $\boldsymbol{\Omega}\neq 0$, we find that "the electric force" $q\mathbf{E}$ is modified by an additional term $(\boldsymbol{\Omega}\times\mathbf{r})\times\mathbf{B}$. This can be interpreted as Lorentz boost with the rotating velocity $\boldsymbol{\Omega}\times\mathbf{r}$. The modified SOI term H_S is responsible for the mechanical manipulation of the spin current as shown below.

Renormalization of SOI.—In vacuum, the contribution of H_S to H_I is negligible, provided that the dimensionless spin-orbit coupling parameter $\eta_{SO}=\lambda(mv)^2/\hbar^2=(v/2c)^2\ll 1$. However, the effect from the SOI can be enhanced in condensed matter systems yielding renormalization of the coupling λ with that of materials. Using Fermi momentum $\hbar k_F$ as mv, $\eta_{\rm SO}$ equals to $\tilde{\lambda}k_F^2$ where $\tilde{\lambda}$ is a enchanced spin-orbit coupling parameter. The renormalization depends on detailed electronic structures and electron correlations [26,27]. In the case of Pt, the dimensionless coupling $\eta_{\rm SO}$ is estimated as 0.59 using the nonlocal measurement of the spin Hall effect [28,29]. Electrons in a noninertial frame cannot distinguish rotational effects $(\boldsymbol{\Omega}\times\mathbf{r})\times\mathbf{B}$ in Eq. (9) from electric fields $\mathbf{E}$. Therefore, the coupling constant of $(\boldsymbol{\Omega}\times\mathbf{r})\times\mathbf{B}$ is renormalized in the same manner as that of $\mathbf{E}$. Consequently, the effect due to the SOI in a rotating frame can be sizable effects as shown below in the large SOI systems [26,27,30,31].

Circular spin current because of mechanical rotation.—To clarify physical meanings of the Pauli-Schrödinger equation in a uniformly rotating frame, we investigate the equations of motion for operator $\mathbf{r}$, $m\ddot{\mathbf{r}}=\mathcal{F}$, where a quantum mechanical "force", $\mathcal{F}=[m\mathbf{v},H_{\rm PR}]/i\hbar+m\partial\mathbf{v}/\partial t$, with $\mathbf{v}=[\mathbf{r},H_{\rm PR}]/i\hbar$. Ehrenfest's theorem leads to equations of motion for an electron wave packet by taking the expectation values with a certain Heisenberg state $|\psi\rangle$ [19]. Since the full expression for $\mathcal{F}$ is lengthy, in this Letter we show a particular case: $\mathbf{E}=\mathbf{0}$, $\mathbf{B}=(0,0,B)$, $\boldsymbol{\Omega}=(0,0,\Omega)$, and B a constant. In this case, the in-plane forces $\langle\mathcal{F}\rangle_\perp=(\langle\mathcal{F}_x\rangle,\langle\mathcal{F}_x\rangle,0)$ are spin diagonal and we focus on the electron motion in the xy plane. Up to the order of Ω/ω_c with $\omega_c=qB/m$, the equations of motion for the center of mass of the electron wave packet is

$$\ddot{\mathbf{R}}_\pm+a_\pm\tau_y\dot{\mathbf{R}}_\pm-b_\pm\mathbf{R}_\pm=0,\qquad(12)$$

where $\mathbf{R}_+$, $\mathbf{R}_-$ are the wave packet's position vector of up- and down-spin electron, the 90° rotation operator in the xy plane $\tau_y=\begin{pmatrix}0&1\\-1&0\end{pmatrix}$ with $a_\pm=(1\pm\kappa)\omega_c$, $b_\pm=\pm\kappa\omega_c^2$, $\kappa=\eta_{\rm SO}(\hbar\Omega/2\epsilon_F)$. κ is the dimensionless parameter which separates the electron motion into fast and slow modes. This parameter consists of the dimensionless SOI coupling $\eta_{\rm SO}$ and the ratio of spin-rotation coupling energy $\hbar\Omega$ to the Fermi energy $\epsilon_F=\hbar^2k_F^2/2m$. Because of $|\kappa|\ll 1$ and $a_\pm\approx\omega_c$, we obtain the solution of Eq. (12) as

$$\mathbf{R}_\pm(t)=e^{\omega_c t\tau_y}\tau_y\mathbf{R}_\pm^{(1)}+e^{\pm\kappa\omega_c t\tau_y}\mathbf{R}_\pm^{(2)},\qquad(13)$$

where $\mathbf{R}_\pm^{(1)}=\dot{\mathbf{R}}_\pm(0)/\omega_c$ and $\mathbf{R}_\pm^{(2)}=\mathbf{R}_\pm(0)-\dot{\mathbf{R}}_\pm(0)/\omega_c$. The first term corresponds to the rapid cyclotron motion due to the Lorentz force, $q\mathbf{v}\times\mathbf{B}$, with frequency ω_c and radius $|\mathbf{R}_\pm^{(1)}|$. The second term describes the slow circular motion with the velocity $\mathbf{v}_d^\pm=\pm R\kappa\omega_c\hat{\phi}$ where $\hat{\phi}$ is the azimuthal unit vector and radius $R=|\mathbf{R}_\pm^{(2)}|$, which is caused by spin-dependent central forces due to the SOI and the mechanical rotation. Let us consider an initial condition in which $|\mathbf{R}_+(0)|=|\mathbf{R}_-(0)|$ and $|\dot{\mathbf{R}}_+(0)|=|\dot{\mathbf{R}}_-(0)|$. Though both up- and down-spin electrons move on a circle around the z axis with radius R, each propagates in the opposite direction due to spin dependence of the second term of Eq. (13) which originates from the SOI with a mechanical rotation. This solution shows that the mechanical rotation causes a circular (pure) spin currents in a rotating frame (Fig. 2). The spin current is obtained as $\mathbf{J}_s=\sum_{\sigma=\pm}\sigma en_\sigma\mathbf{v}_d^\sigma=2enR\kappa\omega_c\hat{\phi}$ with the electron density $n_\sigma=n$. In the case of $B\approx 1$ T, $\Omega\approx 1$ kHz, $\eta_{\rm SO}\approx 0.59$, $k_F\approx 10^{10}$ m^{-1}, and $R\approx 0.1$ m, the spin current $|J_s|$ becomes about 10^8 A/m^2. This can be investigated using spin detection methods such as nonlocal spin valves [32], the inverse spin Hall effect [33] and the real-time imaging method [34].

The generation of the circular spin currents can be interpreted as an analogy of the drift of charged particles in electromagnetic fields. The average velocity of motion of a charge in crossed a magnetic field $\mathbf{B}$ and an external force $\mathbf{F}$ is given by the drift velocity $\mathbf{v}_d=\mathbf{F}\times\mathbf{B}/qB^2$

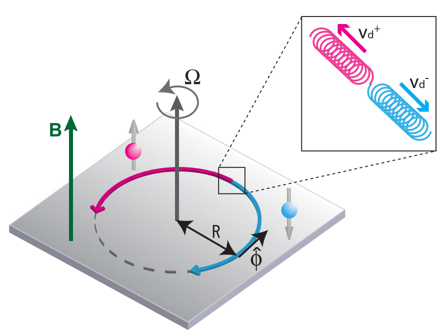

FIG. 2 (color). Schematic illustration of electrons' trajectories under mechanical rotation $\boldsymbol{\Omega}$ and a magnetic field $\mathbf{B}$. Solution of equations of motion for wave packet is a superposition of two cyclotron motions with different frequencies. The drift velocity of the up-(down-) electron is $\mathbf{v}_d^+(\mathbf{v}_d^-)$ parallel to the azimuthal direction denoted by $\hat{\phi}$.

[35]. In our case, $\mathbf{F}$ corresponds to the spin-dependent force $mb_{\pm}\mathbf{R}_{\pm} = \pm m\kappa\omega_c^2\mathbf{R}_{\pm}$. Thus, the spin-dependent drift velocity is $\mathbf{v}_d^{\pm} = \pm m\kappa\omega_c^2\mathbf{R}_{\pm} \times \mathbf{B}/qB^2$, reproducing the previous result obtained from Eq. (13).

Conclusion.—We have derived the Pauli-Schrödinger equation in a uniformly rotating frame of reference thereby describing the coupling of spin to mechanical rotations. This equation involves the spin-orbit interaction augmented by a mechanical rotation, which reveals a mechanism for the quantum mechanical transfer of angular momentum between a rigid rotation and a spin current. Using the semiclassical equations of motion for electrons with spin-dependent forces, a circular spin current is predicted. The magnitude of the spin current is linearly proportional to the angular velocity of the mechanical rotaiton, a magnetic field, and the spin-orbit coupling strength. It should be noted that starting from the generally covariant Dirac equation is essential when treating spintronics in accelerated frames. The present formalism offers a route to ''spin mechatronics'', viz., a strong coupling of mechanical motion with spin and charge transport in nanostructures.

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