

# Effects of Quenching in $\Delta I = 1/2$ Kaon Decays\*

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We present the inconsistencies which arise in quenched QCD in  $\Delta I = 1/2$  non-leptonic kaon decays using chiral perturbation theory ( $\chi$ PT) to one loop. In particular we discuss how the lack of unitarity of the quenched theory invalidates the usual methods for the extraction of matrix elements from correlation functions.

## 1. INTRODUCTION

While different techniques have been developed in order to extract  $\Delta I = 3/2$   $K \rightarrow \pi\pi$  matrix elements from the lattice ([1,2]) and finite volume corrections can be controlled even in the quenched approximation (at least to one loop in  $\chi$ PT) ([2,3,4,5,6]), this is not true for the  $\Delta I = 1/2$  channel. In this case, in addition to the presence of large final state interactions (FSI), there are manifest inconsistencies due to quenching. The reason why the  $\Delta I = 3/2$  case is different will be explained in sec. 3, while the explicit calculation of the scalar form factor (which shares the same quenching artifacts as  $K \rightarrow \pi\pi$ ,  $\Delta I = 1/2$ ) is in ref. [7]. Some of these problems in the  $\chi$ PT framework were first discovered in refs. [5,8,9], our aim is to exhibit the full set of the quenching effects and to understand their origin. By computing matrix elements involving isoscalar  $\pi\pi$  scattering contributions (e.g.  $K \rightarrow \pi\pi$  or the scalar form factor) using quenched  $\chi$ PT ( $Q\chi PT$ ), in finite and infinite volume, with the insertion of different  $\Delta I = 1/2$  weak operators we observe that:

1) The strong phase shift is no longer universal since it depends on the choice of the weak opera-

tor.

2) The Lüscher quantization condition and the Lellouch-Lüscher ( $LL$ ) finite-volume correction factor for matrix elements are no longer valid.

3) The standard procedure for extracting matrix elements from time-independent ratios of finite-volume euclidean correlation functions (CF) fails, since the time dependence of the CF depends on the choice of the two-pion sink.

4) The  $\eta'$  double-pole produces new polynomial terms in  $t$  (time) and  $L$  (box size) which cause practical problems in the extraction of matrix elements.

## 2. FULL (UNQUENCHED) QCD

The unquenched  $K \rightarrow \pi\pi$  matrix element can be extracted by studying the time behaviour of a suitable four-point euclidean CF in finite volumes:

$$\begin{aligned} \sum_{\Omega_{\vec{q}}} \langle 0 | \pi_{\vec{q}}^-(t) \pi_{-\vec{q}}^+(t) H_W(0) K_0^\dagger(t_K) | 0 \rangle = \\ \sum_{\Omega_{\vec{q}}} \langle 0 | \pi_{\vec{q}}^-(0) \pi_{-\vec{q}}^+(0) | W \rangle \langle W | H_W(0) | K \rangle \times \\ \times \langle K | K_0^\dagger(0) | 0 \rangle e^{-m_K |t_K| - W|t|} + \dots, \end{aligned} \quad (1)$$

where  $\phi_{\vec{q}}(t) = \int d^3\vec{x} \phi(\vec{x}, t) e^{i\vec{q}\cdot\vec{x}}$ ,  $|W\rangle$  is a two-pion state with total energy  $W$ ,  $\Omega_{\vec{q}} = \{\vec{q} : |\vec{q}| \text{ fixed}\}$

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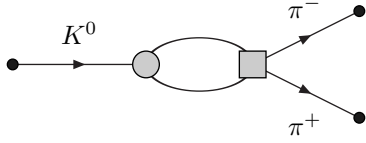


Figure 1. *The one-loop diagram which contains power corrections in the volume.*

and the large time limit has been taken while isolating the contribution with time behaviour  $e^{-W|t|}$ . In order to extract the matrix element  $\langle W|H_W(0)|K\rangle$  one has to divide by the factors corresponding to the kaon source and the two-pion sink extracted from the corresponding CF [2]. The validity of eq. (1) and Watson’s theorem rely on unitarity, and it is the absence of unitarity in the quenched theory which is responsible for most of the difficulties listed at the end of sec. 1. The calculation in infinite volume is straightforward and we will not discuss it further, except to point out that of the one-loop diagrams, it is only that in fig. (1) which can have physical intermediate states and hence an imaginary part (and  $1/L^n$  corrections in finite-volumes).

### 2.1. Finite volume

Evaluating eq. (1) to one loop in  $\chi$ PT on a finite volume (FV) one obtains, up to exponentially small terms in the volume,

$$\frac{e^{-m_K|t\kappa|}}{2m_K} \frac{e^{-2E|t|}}{(2E)^2} A_\infty(E)[1+T], \quad (2)$$

$$T = -\Delta W t + \frac{\partial A_\infty}{\partial W} \Delta W + \frac{b_s}{L^3} + \frac{c_0 z(0)}{(EL)^3} + \frac{c_1 z(1)}{EL},$$

where  $A_\infty$  is the corresponding infinite volume result,  $T$  parameterizes FV corrections,  $\Delta W$  is the energy shift ( $\propto 1/L^3$ ),  $E^2 = \bar{q}^2 + m_\pi^2$  and  $z(i)$  are defined for example in [2,3]. The first two terms in  $T$  shift the two-pion energy in the time exponential and in the argument of  $A_\infty$  respectively, the third term depends on the sink used to annihilate the two pions (it is cancelled when

the matrix element is extracted [2]) and finally the last two terms represent the chiral expansion of the  $LL$  finite volume factor to this order. The validity of eq. (2) relies on the unitarity of the theory.

## 3. QUENCHED QCD

We now consider the determination of the matrix elements in quenched QCD, restricting our attention to the diagram in fig. (1), which is the one leading to anomalous effects. In particular it receives two unphysical contributions: (i) ghosts which cancel internal quark loops, (ii)  $\eta'$  with double-pole insertions<sup>(3)</sup>. They lead to a number of inconsistencies:

(i) The ghosts are a manifestation of the lack of unitarity. Phase shifts, energy shifts and finite volume corrections are no longer universal, the factor corresponding to the sink cannot be removed and the extraction procedure in eq. (1) cannot be applied.

(ii)  $\eta'$  double-poles lead to a more severe and unphysical infrared structure. In infinite volume they appear as divergences in the chiral limit (the so-called “quenched chiral logs”, see for example [5]) and in the singular limit at threshold [7,10]. In finite volumes they produce terms with an anomalous behaviour in time and volume (see below and [7,8,9]).

### 3.1. The scalar form factor

To study these effects consider the time-like scalar form factor of a scalar and isoscalar operator  $S$ , which shows all the effects of  $K \rightarrow \pi\pi$ ,  $\Delta I = 1/2$  but has a simpler chiral structure. The corresponding CF at one loop in  $Q\chi$ PT can be written as follows:

$$\langle 0|\pi_{\bar{q}}^+(t)\pi_{-q}^-(t)S(0)|0\rangle = \frac{e^{-2E|t|}}{(2E)^2} A_\infty[1+T], \quad (3)$$

$$T = \frac{a_1 t + a_2 t^2 + a_3 t^3}{(EL)^3} + \frac{b_0 + b_1 t + b_2 t^2}{(EL)^3} + \frac{c_0 z(0)}{(EL)^3} + \frac{c_1 z(1)}{EL} + c_2 z(2)(EL) + c_3 z(3)(EL)^3.$$

<sup>3</sup> Both these contributions do not appear to one loop in the  $\Delta I = 3/2$  case because of total isospin conservation.

Notice that the term  $\partial A_\infty/\partial W$  is not present because in this case  $A_\infty$  is a constant. The  $a_1$  term would be the quenched energy shift, but it now depends on the choice of operator  $S$  and receives contributions from  $\eta'$  double-poles. The remaining  $a_i$ 's come from the  $\eta'$  double-pole insertions giving an anomalous behaviour in time. The terms proportional to the  $b_i$ 's depend on the sink and are different from the corresponding ones in full QCD. A consequence of the lack of unitarity is that they cannot be eliminated using the procedure described in sec. 2. The terms proportional to the  $c_i$ 's correspond to the finite-volume corrections. They are no longer universal and the last two-terms, which come from the  $\eta'$  double pole, appear prima-facie to diverge in the large-volume limit. We now discuss this point further.

#### 4. VOLUME DEPENDENCE

The apparent volume dependence in eq. (3) is rather strange. By evaluating the matrix element in infinite volume and euclidean metric no divergences appear away from the chiral limit, and one would hence expect that the large volume limit of the finite volume matrix element should exist. In spite of appearances this could be the case, since at fixed physical momenta  $\vec{q}$  (so that the excitation level  $n$  grows as the volume is increased), the  $z(i)$ 's scale with the volume, for instance  $z(0) = -\nu \propto L^2$  <sup>(4)</sup>. By dimensional analysis and numerical evaluation, it is possible to verify that the  $z(i)$ 's decrease with  $L$  fast enough to compensate the powers of  $L$  appearing in eq. (3). However we do not yet have a proof of this since it is not possible to perform the large volume limit at fixed physical energy without the quantization condition.

#### 5. CONCLUSIONS

We have shown that quenching artifacts in  $K \rightarrow \pi\pi$ ,  $\Delta I = 1/2$  matrix elements are very important and might invalidate the usual procedure for extracting physical matrix elements from lattice CF. Problems related to the non-unitarity of the quenched theory also affect  $K \rightarrow \pi\pi$  ma-

trix elements with  $\Delta I = 3/2$ , but they do not appear at one loop in  $\chi PT$  and are therefore less severe.

The problems we have raised do not necessarily have a solution within the quenched approximation, which, it must be remembered, is an intrinsically unphysical theory. However, it is worth exploring whether there is some practical way forward, which is not obviously inconsistent.

Quenched QCD is perturbatively reproduced in the  $\chi PT$  framework by enlarging the set of particle states to contain new unphysical mesons. This extension can be realized in two different, but perturbatively equivalent, ways either with the supersymmetric enlargement of the chiral group [5] or with the replica method [11]. We are currently investigating properties of the enlarged set of states and the behaviour of weak matrix elements between those states, including the unphysical ones [7]. In particular, we are exploring whether it might be possible that some of those universal relations of full QCD mentioned in sec. 1 can be generalized to the quenched approximation, once the extended set of external states is considered.

#### REFERENCES

1. SPQCDR Collaboration, Ph.Boucaud et al., *Nucl. Phys. Proc. Suppl.* **106** (2002), 323
2. C.-J.D. Lin *et al.*, [hep-lat/0208007](#)
3. L. Lellouch and M. Lüscher, *Commun. Math. Phys.* **219** (2001), 31
4. C.-J.D. Lin, G. Martinelli, C.T. Sachrajda, M. Testa, *Nucl. Phys.* **B619** (2001), 467 and *Nucl. Phys. Proc.Suppl.* **109** (2002), 218
5. C. Bernard and M. Golterman, *Phys. Rev.* **D46** (1992), 853
6. M.F.L. Golterman and K.-C. Leung, *Phys. Rev.* **D56** (1997), 2950
7. C.-J.D. Lin *et al.*, in preparation
8. C. Bernard and M. Golterman, *Phys. Rev.* **D53** (1996), 476
9. M.F.L. Golterman and E. Pallante, *Nucl. Phys. Proc. Suppl.* **83** (2000), 250
10. G. Colangelo and E. Pallante, *Nucl. Phys.* **B520** (1998), 433
11. P.H. Damgaard and K. Splittorff, *Phys. Rev.*

<sup>4</sup>where  $\nu = \sum_{\Omega_{\vec{q}}} \text{with } \Omega_{\vec{q}} = \{\vec{n} \in \mathbb{Z}^3 : |2\pi\vec{n}/L| = |\vec{q}|\}$

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