Effects of Random Inhomogeneity on Radar Measurements and Rain Rate Estimation

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***Abstract*—We study the sampling effect on radar measurements of inhomogeneous media and the resultant rain estimation. A two-level drop size distribution (DSD) model is proposed, in which DSD parameters are assumed to be variable for representing the sampling effects. The dependence of statistical moments on the variation of DSD parameters are calculated and applied to radar-based rain estimation.**

***Index Terms*—Author, please supply your own keywords or send a blank e-mail to keywords@ieee.org to receive a list of suggested keywords..**

I. Introduction

It is known that most natural media (rain and cloud) are random and inhomogeneous. Randomness and inhomogeneity are generally related to wavelength and sample volume respectively in radar meteorology. Meteorological radars measure electromagnetic wave scattered by a target region containing many small scatterers (particles or spatial inhomogeneities such as turbulence). The resultant backscattered signal has random characteristics. Statistical moments of the random signal are obtained by averaging over a train of pulses. If the random change of particle position during sample time is larger or comparable to wavelength and causes a random phase of the scattered wave, the medium is called a random medium. Otherwise, it is a partially random or deterministic medium. The condition of randomness has been studied in [1]. In the case of the inhomogeneity of random media, radar can only measure the spatial variation with a scale larger than the size of a sample volume (resolution). While the variation inside the sample volume can not be measured, small scale (local) inhomogeneity does contribute to the radar measurements and introduces bias when compared with that of a homogeneous medium. Currently, data fusion has been widely used in remote sensing, which combines measurements from different sensors which might have different sample volumes. So it is important to understand the sampling effects when the data sets with differnet sample volume are merged.

For random media with small scale inhomogeneity, measured wave statistics such as radar reflectivity (Z) not only depend on sample time but also on sample volume. Sample volume affects data interpretation and parameter retrieval of target medium from radar measurements. Nonuniformity of a reflectivity field has been recognized as a factor affecting the precision of rain estimation because of the nonlinear relation between reflectivity and rain rate (R) [2]. Recently, effects of nonuniform beam filling and reflectivity gradient have attracted attention in the field of radar meteorology. The bias of radar estimated rain rate is studied by comparing radar and gauge measurements (G). Previous studies show that the ratios of gauge and radar rainfalls (G/R) decrease with range in the mean [3]–[7]. Assuming that time-averaged gauge measurements are accurate, the decrease in G/R translates to overestimation of the radar-based estimate. The increase in the radar-based rain estimate might be due to the increase in radar sample volume with the range. Preliminary analysis of the radar data collected in Florida by NCAR S-band polarization radar (S-Pol) shows that the radar estimated rain rate depends on sample volume. The estimated rain with spatially averaged radar reflectivity can be two times as that without spatial averaging in regions with reflectivity gradients. We will study sampling effects on radar measurements and rain estimation based on a two-level DSD model.

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II. Model and Formulation

Meteorological radar measures statistical moments of scattered wave field from a target region filled with random scatterers. For a discrete random medium such as rain, the wave moments depend on moments of raindrop size distribution. For example, radar reflectivity (Z) is defined as the sixth moment ($Z = \langle D^6 \rangle$) and attenuation is proportional to the third moment ($A \propto \langle D^3 \rangle$) for scatterers that satisfy Rayleigh scattering approximation. For random media with small scale inhomogeneity, statistical moments depend on the DSD which is related to the size of sample volume, while radar sample volume increases with range. Thus, the effect of small scale inhomogeneity on radar measurements can be investigated by answering the question: How does one model a variable DSD and calculate the statistical moments while accounting for the sampling effect?

A. General Two-Level DSD Model

Rain and cloud DSDs change as a function of time and space as well as size of sample volume. If we assume that local DSDs can be specified, the statistical moments of the DSD change as the sample volume increases; and as a result, the DSD for the entire volume is different from a local DSD. To represent these changes in DSD, we use a two-level DSD model, as follows.

Let us use a conditional probability density function $n(D|\mathbf{P})$ to represent a local DSD, where D is the equivolume diameter of a particle, $\mathbf{P}$ is the collection of parameters used to define the DSD, the vertical bar denotes conditioning. The DSD parameters can be random variables of space and time. Then the DSD for the entire volume and period can be written as

$$n(D) = \frac{\int\int n[D|\mathbf{P}(\vec{r},t)] W(\vec{r},t) d\vec{r} dt}{\int\int W(\vec{r},t) d\vec{r} dt} \tag{1}$$

where the weighting function is $W(\vec{r},t)$.

In general, it is difficult to characterize the random function $\mathbf{P}(\vec{r},t)$. To simplify the formulation, we assume that the stationary random field and the small-scale fluctuation (the correlation length of DSD parameters is much smaller that the size of sampling volume). Using ergodicity of stationary random field, the space-time averaging in (1) can be replaced by probability averaging, as given by

$$n(D) = \int p(\mathbf{P}) n(D|\mathbf{P}) d\mathbf{P} \tag{2}$$

where $\mathbf{P}$ is the probability density function (pdf) of the DSD parameters within the sample volume.

Statistical moments of diameter D are of interest because they directly related to radar measurements. For the two-level DSD model, the mth moment is

$$E(D^m) = E_{\mathbf{P}}\left[E(D^m|\mathbf{P})\right] \tag{3}$$

i.e.,

$$\langle D^m \rangle = \int p(\mathbf{P}) \left[\int D^m n(D|\mathbf{P}) dD \right] d\mathbf{P}. \tag{4}$$

The statistical moment of D is simply an average of all local moments corresponding to all conditional parameters $\mathbf{P}$ if $E(D^m|\mathbf{P})$ is considered as a local moment.

B. Two-Level Gamma DSD

In characterizing rain DSD, early studies commonly used exponential distributions with one or two parameters was commonly used because of its simplicity. This includes the Marshall-Palmer spectrum and the Laws-Parsons spectrum. Some observations, however, indicate that natural rain DSD contains fewer of both very large and very small drops than exponential distribution [9]. Ulbrich (1983) suggested the use of Gamma distribution for representing rain drop spectra. For a two-level model, it becomes

$$n(D|N_0, \mu, \Lambda) = N_0 D^{\mu} \exp(-\Lambda D). \tag{5}$$

The Gamma DSD with three parameters (offset parameter: N_0, shape parameter: μ, and slope parameter: Λ) is capable of describing a broader variation in rain drop size distribution than an exponential distribution, which is a special case of the Gamma distribution with $\mu = 0$.

In previous studies, the three parameters of the Gamma distribution are assumed to be constant within a sample volume and are obtained by fitting the DSD measurements using curve-fitting or moment method. It is noticed that the DSD changes during storm evolution. Recent study also indicates that the DSD evolves as a function of sample time and volume [6]. Our approach is to assume the DSD parameters are random functions of time and space. It is expected that their variances increase as sample time and volume become large. For a large sample volume, the rain DSD is the result of averaging over all possible parameters as given by

$$n(D) = \int\int\int p(N_0, \mu, \Lambda) N_0 D^{\mu} \exp(-\Lambda D) dN_0 d\mu d\Lambda. \tag{6}$$

To simplify the integral in (6), we write the three conditional parameters as sums of their respective means and fluctuations as

$$\begin{aligned} N_0 &= \bar{N}_0 + N_1 \\ \mu &= \bar{\mu} + \mu_1 \\ \Lambda &= \bar{\Lambda} + \Lambda_1 \end{aligned} \tag{7}$$

and assume fluctuations of the three parameters are independent of each other. Equation (6) becomes

$$\begin{aligned} n(D) = \bar{N}_0 D^{\bar{\mu}} \exp\left(-\bar{\Lambda} D\right) &\int p(\mu_1) D^{\mu_1} d\mu_1 \\ &\cdot \int p(\Lambda_1) \exp(-\Lambda_1 D) d\Lambda_1. \end{aligned} \tag{8}$$

The integrations in (8) can be performed for specific pdf of μ_1 and that of Λ_1. For Gaussian distributions of μ_1 and Λ_1, we have

$$n(D) = \bar{N}_0 D^{\bar{\mu}} \exp\left(-\bar{\Lambda} D\right) \exp\left(\frac{(\ln D)^2 \sigma_{\mu}^2}{2}\right) \exp\left(\frac{D^2 \sigma_{\Lambda}^2}{2}\right) \tag{9}$$

and for uniform distributions of μ_1 and Λ_1, we get

$$\begin{aligned} n(D) = \bar{N}_0 D^{\bar{\mu}} \exp\left(-\bar{\Lambda} D\right) &\frac{D^{\sqrt{3}\sigma_{\mu}} - D^{-\sqrt{3}\sigma_{\mu}}}{2\sqrt{3}\sigma_{\mu} \ln D} \\ &\cdot \frac{\exp\left(\sqrt{3}\sigma_{\Lambda} D\right) - \exp\left(-\sqrt{3}\sigma_{\Lambda} D\right)}{2\sqrt{3}\sigma_{\Lambda} D}. \end{aligned} \tag{10}$$

We notice that neither (9) nor (10) are Gamma distributions anymore. That is because the fluctuation of DSD changes the overall drop spectrum. To further study sampling effects, we calculate the statistical moments as described in the next section.

C. Statistical Moments

We first look at the conditional mth moment for a set of specified parameters as

$$\begin{aligned} \overline{D^m} &= \int D^m n(D|N_0, \mu, \Lambda) dD \\ &= N_0 \Lambda^{-(\mu+m+1)} \Gamma(\mu + m + 1) \end{aligned} \tag{11}$$

where $\overline{\cdots}$ represents conditional averaging over particle size.

Radar measurements, however, are collective effects from all possible conditional DSDs in the sample volume. The mth statistical moment is the result of the conditional mth moment averaged over the conditional parameters (N_0, μ and Λ). Using (7) and the assumption of independent random variables, we have

$$\begin{aligned} \langle D^m \rangle &= \left\langle \overline{D^m} \right\rangle_{N_0, \mu, \Lambda} \\ &= \int\int\int p(N_0, \mu, \Lambda) \overline{D^m} \\ &\qquad \cdot (N_0, \mu, \Lambda) dN_0 d\mu d\Lambda \\ &= \int\int\int p(N_1) p(\mu_1) p(\Lambda_1) \left(\bar{N}_0 + N_1\right) \\ &\qquad \cdot \left(\bar{\Lambda} + \Lambda_1\right)^{-(\bar{\mu}+\mu_1+m+1)} \\ &\qquad \cdot \Gamma(\bar{\mu} + \mu_1 + m + 1) dN_1 d\mu_1 d\Lambda_1 \\ &= \bar{N}_0 \int\int p(\mu_1) p(\Lambda_1) \left(\bar{\Lambda} + \Lambda_1\right)^{-(\bar{\mu}+\mu_1+m+1)} \\ &\qquad \cdot \Gamma(\bar{\mu} + \mu_1 + m + 1) d\mu_1 d\Lambda_1. \end{aligned} \tag{12}$$

The fluctuation of N_o has no effect on the statistical moments because of its linear relation with the moments. Equation (12) is calculated both numerically and analytically. To analyze the effects of DSD variation on the statistical moment, we compare them without any variation which is the conditional moment at the mean value of the parameters, as given by

$$\langle D^m \rangle_u = \bar{N}_0 \bar{\Lambda}^{-(\bar{\mu}+m+1)} \Gamma(\bar{\mu} + m + 1) \tag{13}$$

where the subscript u denotes uniform, i.e., absence of variation.

The ratio between (12) and (13) for radar reflectivity (Z $= \langle D^6 \rangle$) and rain rate can be calculated. Rain rate is measured as accumulated rain water per unit time in $\mathrm{mm\,hr^{-1}}$. It is calculated as $7.125 \times 10^{-3} \langle D^{3.67} \rangle$, and units of D and $n(D)$ are in mm and $\mathrm{m^{-3} mm^{-1}}$ [8]. The statistical distribution of DSD parameters μ and Λ is assumed to be uniform in the numerical calculations. The parameters used for the calculation are $\bar{\mu} = 2$ and $\bar{\Lambda} = 3\ \mathrm{mm^{-1}}$. The results of reflectivity ratio (Z/Z_u) and rain rate ratio (R/R_u) are shown in Fig. 1(a) and (b), respectively. We see that the ratios are more sensitive to σ_{Λ} than σ_{μ}. That is because μ is a shape parameter of the

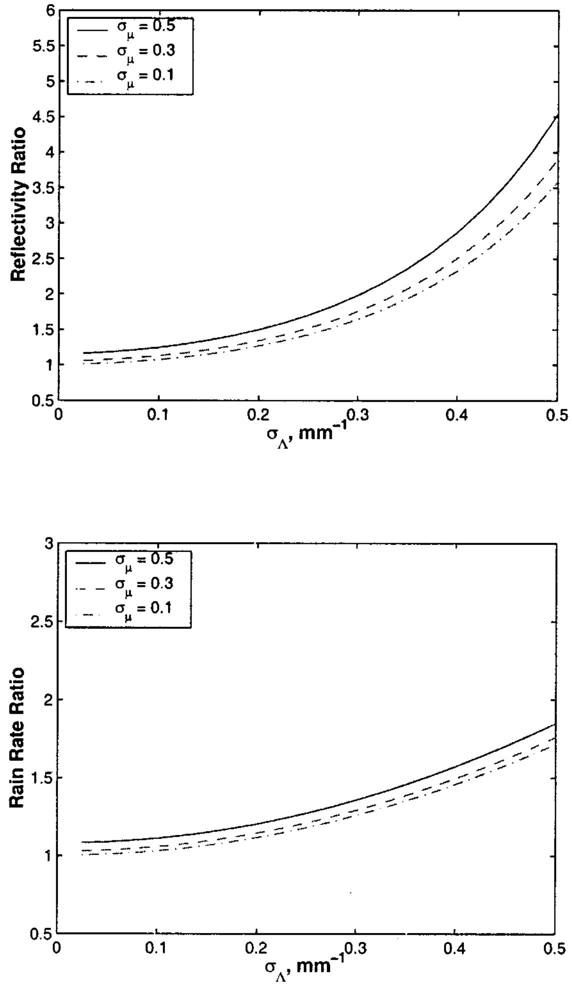

Fig. 1. Dependence of moment ratio on the standard deviation of DSD parameters. (a) Reflectivity and (b) rain rate. Parameters used for the calculations are $\bar{\mu} = 2$ and $\bar{\Lambda} = 3$ mm^{-1}.

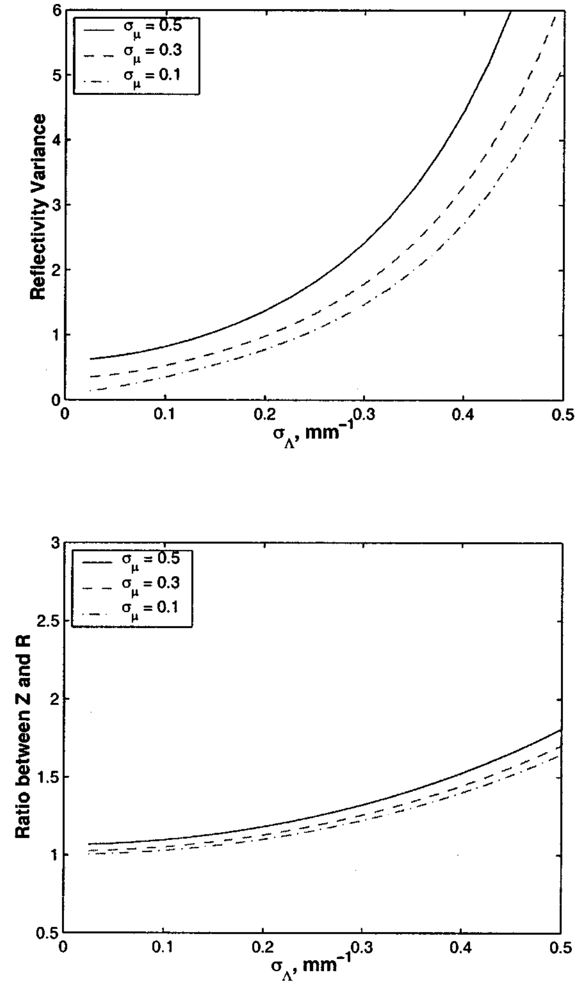

Fig. 2. Dependence of variance on the standard deviation of DSD parameters. (a) Reflectivity and (b) rain rate. Parameters used for the calculations are $\bar{\mu} = 2$ and $\bar{\Lambda} = 3$ mm^{-1}.

DSD, while Λ is directly related to particle size. The ratios increase as the standard deviation of DSD parameters (σ_μ and σ_Λ) increase, but the reflectivity ratio increases faster than the rain rate ratio because the DSD moment of reflectivity is higher than that of the rain rate of the 3.67th moment. This leads to overestimation of rain rate using radar reflectivity. Application of these results for rain rate estimation is discussed in the next section.

The fluctuation of the moments is also studied by calculating their variances. The second moment of the mth conditional moment is calculated as follows:

$$\left\langle \overline{D^m}^2 \right\rangle = \int\int\int p(N_1)p(\mu_1)p(\Lambda_1)\left(\bar{N}_0 + N_1\right)^2 \cdot \left(\bar{\Lambda} + \Lambda_1\right)^{-2(\bar{\mu}+\mu_1+m+1)} \cdot \Gamma^2(\bar{\mu} + \mu_1 + m + 1) dN_1 d\mu_1 d\Lambda_1. \quad (14)$$

Then the variance of the moments due to DSD variation is calculated by

$$\mathrm{var}\left(D^m\right) = \left\langle \overline{D^m}^2 \right\rangle - \left\langle \overline{D^m} \right\rangle^2. \quad (15)$$

The numerical result of the relative variance for radar reflectivity and rain rate is shown in Fig. 2(a) and (b), respectively. Again, the variances increase as the variation of DSD parameters (σ_μ and σ_Λ) increase. This effect has been observed in the results of Anagnostou *et al.* [4]. The experimental results were plotted as a variance of logarithm of G/R ratio as a function of the distance from radar shown in Fig. 4 of the paper by Anagnostou *et al.* [4].

III. Application to Radar Rain Estimation

The two-level DSD model is applied to rain rate estimation from radar measurement. Data used in this study was collected in east-central Florida during the summer of 1998, when NCARs S-Pol radar was deployed in a special experiment (PRECIP98) to evaluate the potential of polarimetric radar for estimating rain in a tropical environment. The experiment was conducted in conjunction with the National Aeronautics and Space Administration (NASA) Tropical Rainfall Measuring Mission (TRMM). Radar samples were made at intervals of 20 s to 2 min and had a radial resolution of 0.15 km and 1° beam. The region is covered by a number of gauges at various distances from the radar. Radar rain estimation based on reflectivity and differential reflectivity (Z_{DR}) has been studied. The $R(Z)$ and $R(Z, Z_{DR})$ relations used for the rain estimation were obtained by numerical simulation using equilibrium raindrop shape and obtained DSDs [6]

$$R(Z) = 0.014 Z^{0.7645} \quad \text{mm hr}^{-1}. \quad (16)$$

$$R(Z, Z_{DR}) = 0.007\,08 Z^{0.9535} Z_{DR}^{-4.9542} \quad \text{mm hr}^{-1}. \quad (17)$$

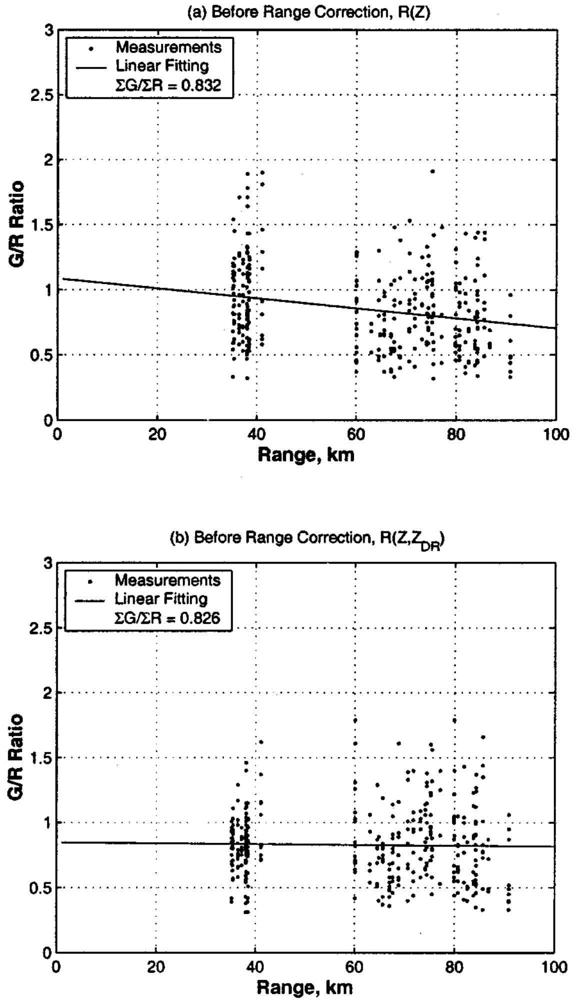

Fig. 3. Gauge-radar rain rate ratio without range correction (a) for $R(Z)$ estimation and (b) for $R(Z, Z_{DR})$ estimation.

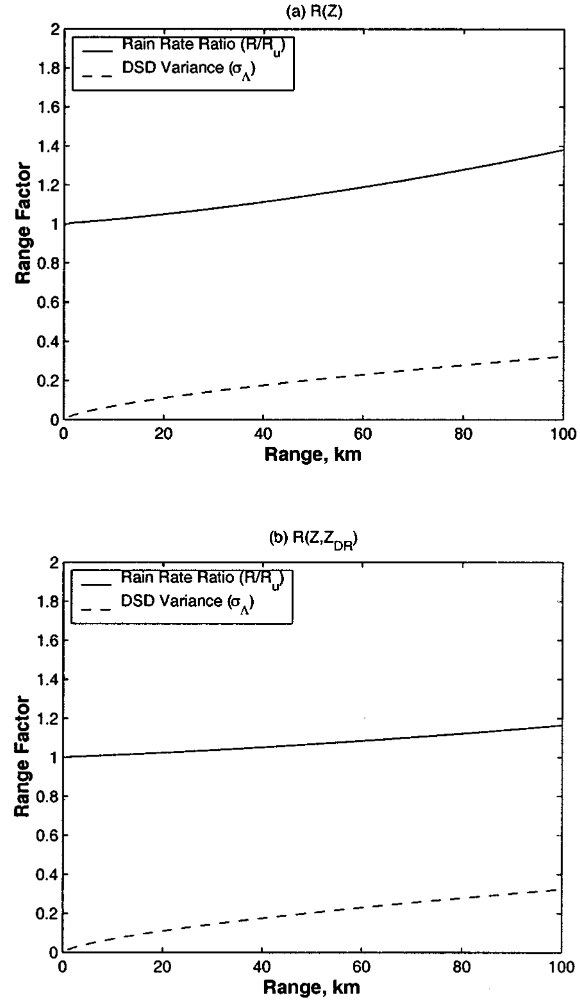

Fig. 4. DSD variation model and calculated range factor (a) for $R(Z)$ estimation and (b) for $R(Z, Z_{DR})$ estimation.

Radar/Gauge comparison is made by taking the ratio of mean rain accumulation for each storm at each gauge location and shown in Fig. 3 as a function of range. Fig. 3(a) is the result obtained using the $R(Z)$ relation. The scatter plot shows a decreasing trend as the range increases with a slope of -0.003 km^{-1}, indicating that the radar tends to overestimate rain at farther ranges. This could be due to the large radar sample volume at farther range. Fig. 3(b) is the result obtained using the $R(Z, Z_{DR})$ relation. The fitted line is close to horizontal and it shows less range dependence. This is because both Z and Z_{DR} increase as sampling volume, i.e., range, and hence, the ratio $Z^{0.9535}/Z_{DR}^{-4.9542}$ is immune to the range effect. The range dependence of Z and Z_{DR} tend to cancel each other.

To apply the two-level DSD model, we assume the variance of Λ as

$$\sigma_\Lambda = c\sqrt[3]{V} = Cr^{2/3} \quad \text{mm}^{-1} \tag{18}$$

where

- V sample volume;
- r range from radar location in km;
- c and C proportionality constants.

For the data shown in Fig. 3, we estimated a value of 0.015 mm^{-1}km$^{-2/3}$ for C would eliminate the range-dependent bias. A general methodology for estimating the value of σ_Λ for a specified radar sampling will be investigated in future. σ_μ is assumed to be zero because the rain ratio is more sensitive to σ_Λ than σ_μ, as shown in Fig. 1. The means of the DSD parameters are obtained from the video-disdrometer measurements as $\bar{\mu} = 2.2$ and $\bar{\Lambda} = 3.6$ mm^{-1}. The reflectivity (Z) and differential reflectivity (Z_{DR}) can be calculated based on the previous work [8]. The calculated Z and Z_{DR} are used in (16) and (17) for estimating rainrate with and without DSD variation. The range factors are defined as the ratio of rain rate with DSD variation and that without DSD variation. These factors are calculated and shown in Fig. 4. The function of σ_Λ is also shown in the same figure. Fig. 4(a) is for $R(Z)$ estimated rain rate and Fig. 4(b) is for $R(Z, Z_{DR})$ estimation. The range factor for $R(Z, Z_{DR})$ estimation increases with range slower than that for $R(Z)$ estimation, which is consistent with the observations shown in Fig. 3. This is because the range dependences of Z and Z_{DR} tend to cancel each other.

Fig. 5 shows the revised G/R ratios, correcting the radar estimated rain rate by normalizing the range factor discussed earlier. The corrected G/R ratio for $R(Z)$ estimation exhibits almost no range dependence, as shown in Fig. 5(a), and the accumulated rain ratio between gauge and radar measurements (ΣG$/\Sigma$R ratio) is 0.991, which has almost no bias compared with the earlier value of 0.832 in the absence of range correction. The ΣG$/\Sigma$R for $R(Z, Z_{DR})$ estimation is not improved, as shown in Fig. 5(b). This might be because the negative power in the $R(Z, Z_{DR})$ relation (17) over accounts the Z_{DR} contribution. A more accurate retrieval algorithm may improve the result.

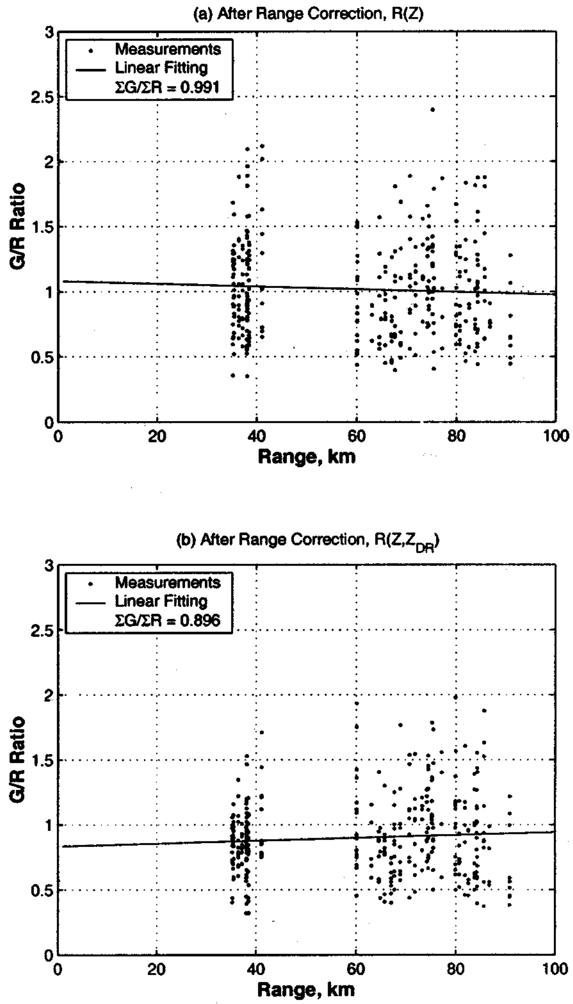

Fig. 5. Gauge-radar rain rate ratio with range correction (a) for $R(Z)$ estimation and (b) for $R(Z, Z_{DR})$ estimation.

IV. Summary and Discussions

In this paper, we have studied sampling effects on radar measurements and on parameter retrievals. A two-level DSD model is proposed and illustrated for calculating statistical moments. The numerical results show that the variation of DSDs contribute to the overestimation of statistical moments and biases the parameter retrieval. A range correction has been applied to radar estimated rain rate for accurate rain rate estimation based on the two-level Gamma DSD model. Although the two-level DSD model is proposed to include the DSD variation, the effects due to incomplete beam filling can also be compensated, since incomplete beamfilling will alter the DSD and hence the parameters of the DSD.

In this study, we randomize DSD parameters for including sampling effects, while their fluctuations are assumed to be independent random variables. Some of the observations show that DSD parameters may be mutually dependent [9]. But we have not seen high correlation among the fluctuations of DSD parameters in literature. The $N_0 - \mu$ relation shown in Ulbrich's paper varies an order of magnitude for a specific μ. The fluctuations in DSD parameters are even less correlated. The independence among the fluctuations of DSD parameters can be a valid assumption for some cases. Even if the correlation among the fluctuations of DSD parameters are found, the two-level model does not lose its generality. In that case, the correlation among them can be included in the joint probability density function of the fluctuation of the DSD parameters and (6) and (12) are still valid. Another approach to deal with the parameters with correlation is to use the transformed parameters which are less correlated instead of Gamma parameters. Normalized Gamma distribution will be helpful for further study.

Range correction for rain estimation is obtained by assuming that the DSD variance depends on sample volume. Further studies are needed to evaluate and improve the approach. The spatial correlation of DSD parameters was neglected and the spatial averaging was replaced by probability averaging in this paper. This can be included in future studies. Fixed relations were used for rain estimation and accounting DSD variation, while rain DSD changes from storm to storm, causing large scatter in G/R ratio and overestimation of rain based on the $R(Z, Z_{DR})$ relation. These variations may be addressed by using different DSD parameters and their variances as a function of rain storm and using a more accurate retrieval algorithm.

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