

## CHAPTER 151

### Effects of Short-crested Waves on the Scouring Around the Breakwater

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The aim of this research is to investigate qualitatively the influence of short-crested waves on the scouring around the breakwater through mainly some laboratory studies. At first, we succeed to observe clearly in laboratory some sedimentary bed forms such as troughs, holes, triangle bars and longitudinal bars under short-crested wave actions. Then we elucidate the association of their formation mechanism with theory of short-crested waves, also indicate its effect on scouring at the toe of breakwaters. In addition, it is shown that the breaking wave height of short-crested waves is certainly higher than that of two-dimensional standing waves. Finally we demonstrate some facts of failure of breakwater caused by short-crested wave breaking basing on some field results.

#### 1. Introduction

As the breakwater is built in the sand bed region the incident waves and their reflected waves are combined into Short-crested waves system. The vortex motion and macro-turbulence structure under the wave system can scour deep holes and result in subsidence of the breakwater. This phenomenon is well known to the practicing engineers.

A theoretical solution presented by Fuchs (1952), based upon linear theory, had been applied to short-crested wave system. Later, the extensive work by Chappellear (1961) had improved the accuracy of Fuch's approximation, but with the difficulty of immediate application due to very length formulation. A more sophisticated second order theory had been proposed by Hamada (1965) by means of the secondary interactions. A third order approximation to the short-crested waves was derived by Hsu (1978). In addition, that breaking wave height of short-crested waves is certainly higher than that of two dimensional progressive waves was shown by Halliwell and Machen (1981). The breaking wave height obtained from the third approximation of Hsu & Siloverster (1982) matches well with that result by Halliwall (1981). The aim of this paper is to investigate qualitatively influence of short-crested waves on the scouring around the breakwater through mainly the movable model tests studies.

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2. Summary of short-crested wave theory

2.1 Basic properties of short-crested waves

When waves approach and are reflected from vertical breakwater, they create a surface undulation of a diamond pattern, known as short-crested waves, as shown in Fig. 1. With a coordinate system of x being in the direction of advancing the combined crest, Y transversally to the celerity vector of the combined wave and Z positive upward from the still water surface, Fuchs (1952) derived the first order approximation as:

$$\phi = \frac{ag \cos h r}{m C_s \cos h(rd)} \cos m (X - C_s t) \cos nZ \dots \dots \dots (2-1)$$

$$m = \frac{2\pi}{L} \quad n = \frac{2\pi}{L'} \quad r^2 = m^2 + n^2$$

$$Y_s = - \left[ \frac{1}{g} \frac{\partial \phi}{\partial t} \right]_{y=0} = a \sin m (X - C_s t) \cos nZ \dots \dots \dots (2-2)$$

$$C_s^2 = \frac{gr}{m^2} \tanh h (r \cdot d)$$

$$= \frac{gL}{2\pi} \sqrt{1 + \left(\frac{L}{L'}\right)^2} \tanh \left[ \frac{2\pi d}{L} \sqrt{1 + \left(\frac{L}{L'}\right)^2} \right] \dots \dots \dots (2-3)$$

where  $\phi$ ,  $Y_s$  and  $C_s$  are velocity potential, surface profile and wave velocity respectively.

$$C_s = \frac{gI}{2\pi} \sqrt{1 + \left(\frac{L}{L'}\right)^2} \tanh \left[ \left(\frac{2\pi d}{L}\right) \sqrt{1 + \left(\frac{L}{L'}\right)^2} \right] \dots \dots \dots (2-4)$$

because  $\sqrt{1 + \left(\frac{L}{L'}\right)^2} \geq 1$  then

$$C_s \geq \left(\frac{gI}{2}\right) \tanh \frac{2\pi d}{L} \dots \dots \dots (2-5)$$

Eq.(2-5) Show that the combined celerity of short-crested wave is larger than wave celerity of incident waves.

The water partial velocities in X,Y and Z direction under the short-crested waves are as follows:

$$u = \frac{\partial \phi}{\partial X} = \frac{ga}{C_s} \frac{\cosh [r(Y+d)]}{\cosh (rd)} \sin m (X - C_s t) \cos nZ$$

$$v = \frac{\partial \phi}{\partial Y} = - \frac{gar}{mC_s} \frac{\sinh [r(Y+d)]}{\cosh (rd)} \cos m (X - C_s t) \cos nZ$$

$$w = \frac{\partial \phi}{\partial Z} = \frac{gan}{mC_s} \frac{\cosh [r(Y+d)]}{\cosh (rd)} \cos m (X - C_s t) \sin nZ \dots \dots \dots (2-6)$$

The orbital of water partial is :

$$X = \left(\frac{H}{2}\right) \frac{\cos \left(\frac{2\pi}{L}\right)(Y+d) \sqrt{1 + \left(\frac{L}{L'}\right)^2}}{\sqrt{1 + \left(\frac{L}{L'}\right)^2} \sinh \left(\frac{2\pi d}{L}\right) \sqrt{1 + \left(\frac{L}{L'}\right)^2}} \left[ \cos \left(\frac{2\pi Z}{L'}\right) \sin \left(\frac{2\pi X}{L}\right) \right]$$

$$\begin{aligned}
 Y &= \left(\frac{H}{2}\right) \frac{\sinh\left(\frac{2\pi}{L}\right)(Y+d) \sqrt{1+\left(\frac{L}{L_r}\right)^2}}{\sinh\left(\frac{2\pi}{L}\right) \sqrt{1+\left(\frac{L}{L_r}\right)^2}} \left[\cos\left(\frac{2\pi Z}{L_r}\right)\cos\left(\frac{2\pi X}{L}\right)\right] \\
 Z &= \left(\frac{H}{2}\right)\left(\frac{L}{L_r}\right) \frac{\cosh\left(\frac{2\pi}{L}\right)(Y+d) \sqrt{1+\left(\frac{L}{L_r}\right)^2}}{\sqrt{1+\left(\frac{L}{L_r}\right)^2} \sinh\left(\frac{2\pi d}{L}\right) \sqrt{1+\left(\frac{L}{L_r}\right)^2}} \left[\sin\left(\frac{2\pi Z}{L_r}\right)\cos\left(\frac{2\pi X}{L}\right)\right] \dots(2-7)
 \end{aligned}$$

The water particles follow elliptical paths which are angled to the horizontal.

$$\tan \beta = \frac{n \tan(nZ)}{r \tanh r(Y+d)} \dots\dots\dots (2-8)$$

Furthermore, also derived the free surface profile to the second-order approximation as follow:

$$\begin{aligned}
 \phi &= A \cosh r(d+Y) \cos m(X - C_S t) \cos n Z + B \cosh 2r(d+Y) \cdot \\
 &\quad \sin 2m(X - C_S t) \cos 2nZ + D \cosh 2m(d+Y) \sin 2m(X - C_S t)
 \end{aligned}$$

$$A = \frac{am}{r} \frac{\cosh r(d+Y)}{\sinh r d} \cos n Z$$

$$B = -\frac{3}{16} \frac{A^2 r^2}{C_S m \sin^2 h^2 r d}$$

$$D = \frac{A}{8 m C_S} \frac{4 n^2 \cosh^2 r d - 3 r^2}{2 \cosh 2 m d - (m \sinh 2 m d / r \tanh r d)} \dots\dots\dots (2-9)$$

$$\begin{aligned}
 Y_S &= a \sin m(X - C_S t) \cdot \cos nZ + \frac{a^2 r^2}{4 \sinh 2 r d} (2 \sinh^2 r d - 1 \\
 &\quad + \frac{3 \cosh 2 r d}{\sinh^2 r d}) \cdot \cos 2 m(X - C_S t) \cdot \cos 2 n Z \\
 &\quad + \frac{a^2(m^2 - n^2 \cosh 2 r d)}{4 r \sinh 2 r d} \cdot \cos 2 nZ + \frac{a^2}{4 r \sinh 2 r d} \\
 &\quad \left[ \frac{(3r^2 - 4 n^2 \cosh^2 r d) \cosh 2 m d}{\cosh 2 m d - (m \sinh 2 m d / 2 r \tanh r d)} - m^2 \cosh^2 r d \right. \\
 &\quad \left. + 3 r^2 \sinh^2 r d + n^2 \cosh^2 r d \right] \cdot \cos 2m(X - C_S t) \dots\dots\dots (2-10)
 \end{aligned}$$

2.2 Water partical motions under the short-crested waves system.  
 Basing on the Fuths theory, Silvester (1974) sketched the schematic diagram of water particle path under short-crested wave action, as Fig. 2. From this figure, the water particle motions can be described as

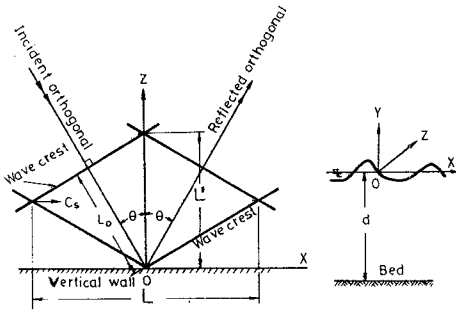


Fig. 1 Short-crested wave formed in front of a vertical wall.

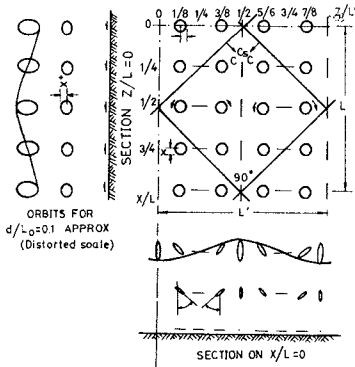


Fig. 2 Water particle orbits and mass transport under the short-crested wave. (Silverster, 1974)

follows.

- (1) The wave profile along the centreline of the crests, at  $Z/L' = 0, 1/2, 1$ , etc., is more likely to be that of a solitary wave.
- (2) Half-way between these crest alignment, at  $Z/L'=1/4, 3/4$ , etc., the orbital motion are rectilinear and are transverse to wave propagation.
- (3) Between these again, at  $Z/L' = 1/8, 3/8, 5/8$ , etc., the water particles follow elliptical paths which are angled to the horizontal.

The influence of these complex water motions on a sedimentary bed have been examined by. Tanalca and ozasa (1972), and Silvester (1974).

2.3 The breaking wave height wave height of short-crested wave. There are two different theorem on the breaking wave height of short-crested wave so far. One is solitary wave theorem, another is progressive wave theorem.

(A) Solitary wave theorem:

Halliwell and Machen (1981), basing upon work of Milies (1977) , derived the breaking wave height of short-crested as follows: when

$$C_s/C_1 \div \text{Sec } \theta_1$$

$$C = \sqrt{g(n+d)} \dots\dots\dots(2-11)$$

$$\frac{d + n_3}{d + n_1} = \text{Sec } \theta_1 \dots\dots\dots(2-12)$$

$$\frac{n_3}{n_1} = \left( \frac{d}{n_1} + 1 \right) \sec^2 \theta_1 - \frac{d}{n_1} \dots\dots\dots (2-13)$$

$$\therefore n_3 \geq n_1 \dots\dots\dots (2-14)$$

Let the breaking wave condition be  $U_{max}/C_s = 1$ , since  $C_s > C$ , there are greater  $U_{max}$  than that of incident progressive wave with celectiy  $C$ . That is why the breaking wave height of short-crested wave is higher than that of progressive wave.

(B) The interaction of progressiv wave theorem.

Hsu & Silverster (1982), basing the third order approximation and taking the  $U_{max}/C_s=1$  as a condition of breaking wave, got the breaking wave height which is larger than that of progressive wave and depend on the angle of interaction of two progressive wave.

### 3. Laboratory experiments

In order to investegate the effects of short-crested wave on the scouring around the breakwater, the movable bed model expeniments were performed in a wave basin with 60 m length, 43 m width and 1 m depth. A vertical breakwater was built in wave basin for producing a reflected wave train . The scouring at the toe of the breakwater has been since before one of the serious problems confronting the Taichung engineers. The laboratory studies are therefore based on the model test of Taichung Harbour located in Mid-western Taiwan. Fig. 3 depictes a schematic sketch of the experimental hydraulic model employed.

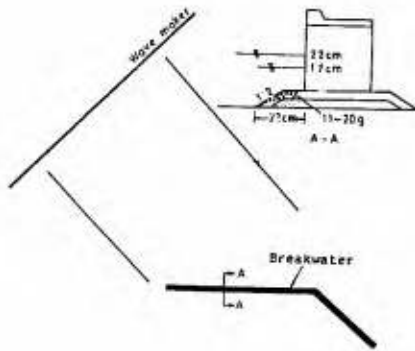


Fig. 3 Schematic diagram of the experimental model.



Fig. 4 Short-crested wave formed in front of the breakwater.

The ratios of model employed were that the breakwater, wave period, wave velocity, and wave pressure were  $1/64$  and the rock weight was  $1/64^3$ . The sand used for movable bed was directly taken from the natural beach in the vicinity of Taichung Harbour and its mean median diameter  $D_{50}$  was 0.22 mm.

It takes about one hour of time to complete a run of experiments. Measurements performed were mainly included wave height distribution, breaking wave height and overall bed form. The bed forms were simultaneously obtained by photographing and level measuring. In order to investigate the scouring, when performing the experiments, we paid particularly attention to the situation as the short-crested waves are breaking.

#### 4. Results and discussions

The runs of model test, the results of laboratory studies and the field data survey are shown in Table 1, 2 and 3. The short-crested wave formed in front of the breakwater was shown in Fig. 4 and Fig. 5. A typical result of bed forms under the action of short-crested waves are depicted in Fig 6 and 7. It can be clearly seen from this figure that the sedimentary bed is scoured by the breaking short-crested waves. As shown in Fig. 8 and Fig. 9, concerning the bed form in detail near the breakwater, deep holes and triangle bar are generally formed at the place of wave breaking. In Fig. 10 the sand ripples appear in the front of the breakwater and in Fig 11, the longitudinal bar and trough appear around the corner of the breakwater. If the breaking short-crested wave height is even greater, the impinging wave breaking will produce a large scouring hole just at the toe of the breakwater, as shown in Fig. 12. (1) The breaking wave height resulting from the short-crested wave is greater than that of incident wave and this result matches that of Hsu & Silvester. (1982), as shown in Fig. 13 and 14.

(2) The scouring in front of breakwater under the short-crested wave is strong and serious. The characteristics of the scouring as follows.

- (a) at  $Z/L'=0, 1/2, 1, \dots$ , the sand bed were scoured and the sediments were transported in the X direction.
- (b) at  $Z/L'=1/4, 3/4, \dots$  there are the back and forth motion of water particles along the Z direction. There are no sediment transported in the X direction
- (c) at  $Z/L'=1/8, 3/8, 5/8, 7/8, \dots$  etc., the swirling motion of water particles have much contribution in expediting the sediment movement. The moving sediments are transported into the adjoining regions.

(3) The mechanism of various sedimentary bed forms may be explained as follows:

- (a) at  $Z/L'=0, 1/2, 1, \dots$  etc, the sediments are transported along the short-crested wave progressing. The greater the wave height is, the more the transportation is. Then, in this condition, the trough is formed along these lines ( $Z/L'=0, 1/2, 1$  etc), as shown in fig. 1.
- (b) at  $Z/L'=0, 1/4, 1, \dots$  only a few sediments are transported along transverse and longitudinal direction. In this characteristic, the longitudinal bar was formed.
- (c) When the breaking waves take place, the sand bed is formed

Table 1: The wave conditions of model test

Run	Wave period (sec)	Incident angle ( $\theta_i$ )	Water depth (m)	Wave height (m)
1	1.25	35°	0.22	0.098
2	1.25	45°	0.17	0.098
3	1.25	45°	0.22	0.098
4	1.25	45°	0.17	0.124
5	1.25	30°	0.165	0.111
6	1.25	60°	0.21	0.123
7	1.25	45°	0.21	0.075
8	1.00	45°	0.205	0.110

Table 2: The breaking wave heights obtained from the results of experimental tests

Run	Wave period (sec)	Incident angle $\theta_i$	Water depth $d_{sb}$	Breaking wave height $H_{sb}$	Wave length $L_1$	$\frac{d_{sb}}{gT^2}$	$\frac{H_{sb}}{ds}$	$\frac{H_{bs}}{L_1}$	$\frac{d_{sb}}{L_1}$
1	1.25	35°	0.22	0.3093	1.663	0.0144	1.41	0.186	0.132
2	1.25	45°	0.17	0.258	1.495	0.011	1.518	0.173	0.114
3	1.25	45°	0.22	0.3232	1.663	0.0144	1.47	0.194	0.132
4	1.25	45°	0.17	0.297	1.495	0.011	1.74	0.199	0.114
5	1.25	30°	0.165	0.2317	1.476	0.0107	1.404	0.157	0.112
6	1.25	60°	0.21	0.2575	1.632	0.0137	1.226	0.157	0.129
7	1.25	45°	0.21	0.3003	1.632	0.0137	1.43	0.184	0.129
8	1.00	45°	0.205	0.2582	1.222	0.021	1.26	0.211	0.168

Table 3: The breaking wave heights of short-crested wave obtained from the field data survey, Taichung Harbor Bureau, 1973.

Wave period (sec) T	Incident angle $\theta_i$	Water depth $d_{sb}$	Breaking wave height $H_{sb}$	Wave length $L_1$	$\frac{d_{sb}}{gT^2}$	$\frac{H_{sb}}{ds}$	$\frac{H_{bs}}{L_1}$	$\frac{d_{sb}}{L_1}$
10		13	18	156	0.01	1.38	0.115	0.083





Fig. 5 Short-crested wave breaking in front of the breakwater.

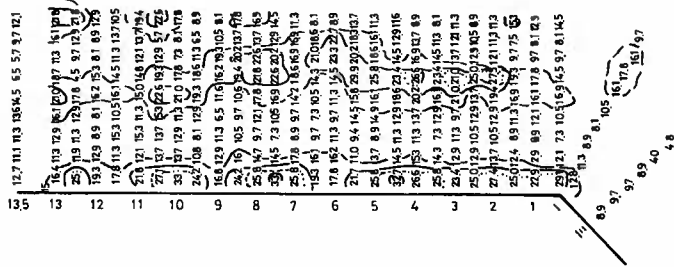


Fig. 6 Short-crested wave height distribution in front of breakwater. Incident wave period = 1.25 sec, incident angle = 45°, incident wave height = 0.098m, water depth in front of breakwater = 22 cm.

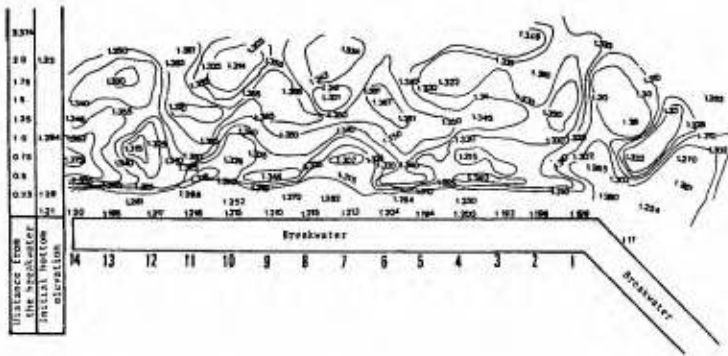


Fig. 7 Contours of bottom topography under short-crested wave actions.  
 incident wave period=1.25sec., incident angle=45°.  
 incident wave height=0.098m, water depth at the breakwater=22cm.

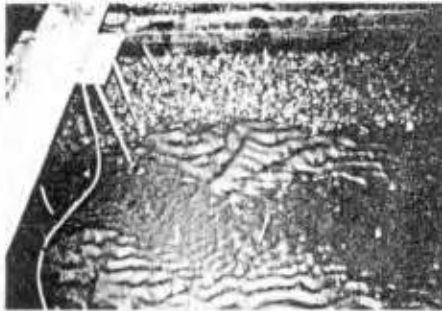


Fig. 8 Triangle sand bar and deep hole in the front of the breakwater.



Fig. 9 Triangle sand bar under the short-crested wave breaking.



Fig. 10 The sand ripples under the short-crested wave which did not break.



Fig. 11 The longitudinal bar and trough around the corner of the breakwater.

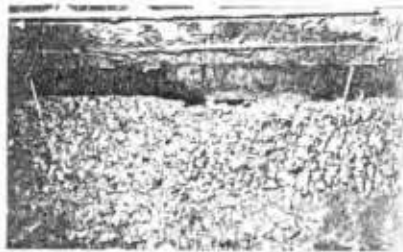


Fig. 12 Scouring hole at the toe of the breakwater.

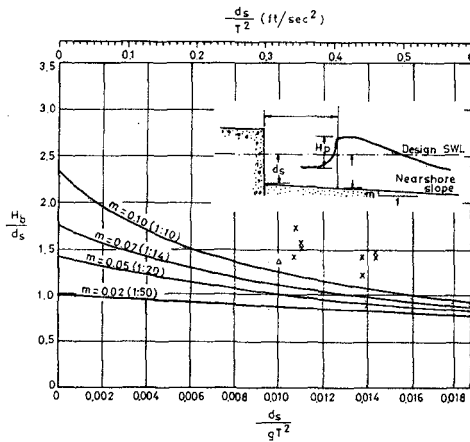


Fig. 13 Compare the breaking wave height of short-crested wave with that from Shore Protection Manual (1977).  
 - : the results by Weggel  
 X : the lab results by authors  
 Δ : the field data

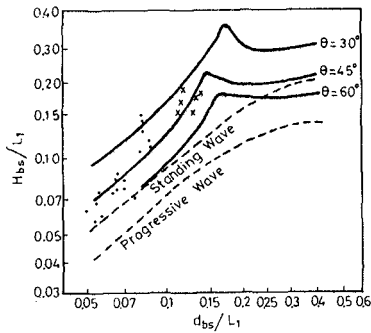


Fig. 14 Compare the breaking wave height of short-crested wave with that of standing wave, progressive wave, the numerical solution by Hsu & Silvester and the lab results by Halliwell.  
 • : the lab results by Halliwell  
 - : the numerical solution by Hsu & Silvester  
 X : the lab results by authors

- into deep hole under the breaking waves, and the sand bed, by the deep hole, was formed triangle bar.
- (d) Furthermore, for comparison, the sea bottom contours around the corner of the breakwater obtained from the field data survey is depicted in Fig. 15, revealing the same tendency as that of laboratory.

### 5. Conclusions and Remarks

From the laboratory experiments and field observation, the scouring effect, resulted from the short-crested wave system, on the foundation of breakwater can be drawn:

- (1) Oblique incident waves and their reflected waves are formed into short-crested waves system in front of the breakwater. This short-crested waves progress along the breakwater and gradually swell up. Swelling up along the breakwater is greater than that of breaking wave height limit, the short-crested wave are broken. The breaking short-crested wave height is larger than that of progressive wave, and belong to plunging breaking wave. This breaking waves result in serious scouring around the breakwater.
- (2) The characteristics of water particle motions under the short-crested wave system enforced the sediment transport and form into special bed forms such as troughs, holes, triangle bars and longitudinal bars.
- (3) While the breaking waves happen, the vortex motions penetrate to the bed in the vicinity of the breakwater scour the foundation of the breakwater, and result in subsidence of the breakwater.
- (4) It's suggested that when the breakwater is designed or built, the influence of short-crested wave should be considered.
- (5) However, in this paper, only the qualitative results were reported.

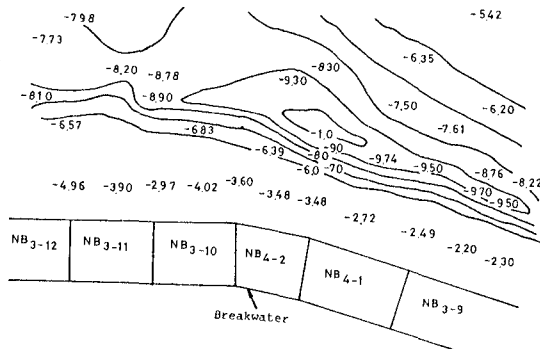


Fig. 15 Bottom contours around the corner of the breakwater for field case.

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