Effects of Small Defects and Nonmetallic Inclusions on the Fatigue Strength of Metals*

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The fatigue behaviors of metals containing small defects or inclusions are so complicated that the prediction of the influence of defects or inclusions is very difficult. However, recent advances in fracture mechanics have led to quantitative treatment of this problem. In this paper, first, the characteristics of the effects of small defects and inclusions are discussed by reviewing the existing literature, which mostly reports the defect and inclusion problems qualitatively. Next, it is revealed that the clue to solving the problems quantitatively is the concept that defects and inclusions are virtually equivalent to cracks from the viewpoint of fatigue strength. A method of evaluating the fatigue limits of metals containing small defects based on this concept is introduced. Finally, it is shown that the method is useful in predicting the lower limit of fatigue strength of high-strength steels containing nonmetallic inclusions which cause a distinct decrease in fatigue strength and a large scatter of fatigue strength.

Key Words : Fatigue, Defects, Nonmetallic Inclusions, Small Cracks, High-Strength Steels, Projected Area of Defects, Vickers Hardness, Fisheyes, Statistics of Extreme Values, Threshold Stress Intensity Factor Range, Fatigue Limit

1. Introduction

The importance of the influence of small defects and nonmetallic inclusions on fatigue of metals has been recognized for a long time. So many investigations have been carried out in this field that it is rather difficult to make an exact and impartial survey. The complication of the affecting mechanisms and inclusion configurations has prevented the establishment of a reliable quantitative method of evaluating the effects of small defects and nonmetallic inclusions. However, the problems of small cracks have recently attracted attention in the field of the fracture mechanics approach to fatigue, and the correlation between a small crack and a small defect has been studied(1)~(3). Recent studies in this direction seem to suggest that we search for a clue to the solution of inclusion problems. Although problems of small cracks may be new in a sense, those of small defects and nonmetallic inclusions are not necessarily new from a historical viewpoint. Some engineers who are well versed in practical phenomena may have detailed (but qualitative) knowledges of the influence of small defects and inclusions. However, it is extremely important to note that the effects of small defects and inclusions on fatigue strength are essentially the problem of small cracks, and that therefore, the effects can be evaluated quantitatively and also can be unified only by regarding small defects and inclusions as small cracks(1)~(6).

This paper reviews early and recent studies on the effects of inclusions and small defects on the fatigue strength of metals and presents a new concept for treating the effects uniformly. In particular, a successful application of the new concept will be shown in the prediction of fatigue strength of high-strength steels.

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2. Previous Studies on the Effects of Defects and Inclusions

There have already been many reviews on this subject, reflecting the importance of these problems(6)~(13), and some of them are rather old. Nevertheless, as the essential points in the study of the effects of defects and inclusions have not changed in the last decades, we believed it would be helpful to make a thorough review of the related literature.

Some results of previous studies contradict each other, though others present similar conclusions. The comparison and review of these results from the viewpoint of fracture mechanics may provide us with a clue to the solution of the problems. Although a defect can be considered to be equivalent to an inclusion with no rigidity, the effect of defects and that of inclusions will not be distiguished in this section.

In many early reports, the correlation between the cleanliness of steels and fatigue strength was investigated, but the results were not satisfactory(14)~(18). On the other hand, Atkinson(16) introduced the Fairey inclusion counts, which take into account the number, size and the stress concentration factor of nonmetallic inclusions, and successfully demonstrated that the Fairey inclusion counts had a very good correlation with the plane and rotating bending fatigue strength of En 24 steel, the equivalent of SAE 4340 steel. This method is based on the idea that all inclusions contribute to some extent to reduce the fatigue strength and that the extent is counted by the proposed rule. However, it must be noted that the experiments show that fatigue strength is mostly determined by the effect of one crucial inclusion.

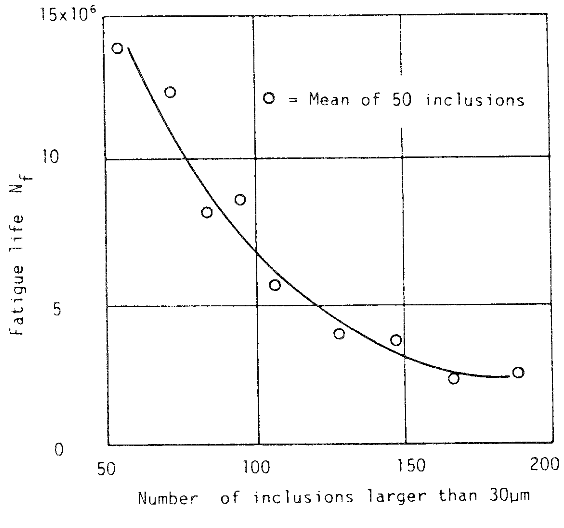


Fig. 1 Correlation between the numbers of oxide inclusions larger than 30 μm and the fatigue life of bearings(17)

Figure 1 shows the results of the fatigue tests for ball bearings conducted by Uhrus(17). This figure indicates that only the oxide inclusions that were more than 30 μm in diameter should be counted to evaluate the fatigue life of ball bearings.

As shown in Fig. 2, Duckworth and Ineson(19) reported that the effect of the same size inclusions could be different depending on where they were situated in the cross section of the specimen (especially on the surface or in the interior), and also that inclusions less than a certain size did not affect the fatigue strength of the metals. Kawada and Kodama(11), Tanaka and Funahashi(20) and de Kazinczy(18) reported similar results. There were some investigations which could not find any influence of inclusions on the fatigue strength of high-strength steels(14),(15),(21)~(25). Ineson et al.(14) showed that the ratio of the fatigue strength and the ultimate tensile strength could be decreased from 0.5 to 0.3 by increasing the ultimate tensile strength from 85 tons/in^2 (1 172 MPa) to 125 tons/in^2 (1 724 MPa) by heat treatments with the same material containing inclusions. These results suggest that the ultimate tensile strength must be higher than a critical value for the effect of inclusions to be exhibited, and in fact there are some papers that report the critical tensile strengths(10),(19),(26),(27). These critical values should have been affected by the size of inclusions, but no discussion on this point can be found in the literature.

Furthermore, it was reported that the same inclusion could cause different effects on the fatigue strength depending on the loading directions(8),(12),(26),(28)~(31). These results indicate that the shape and the size of inclusions are the important factors. Summarizing many reviews on this subject,we can say that the factors that should be accounted for are as follows:

(1) The shape of inclusions,
(2) The adhesion of inclusions to the matrix,

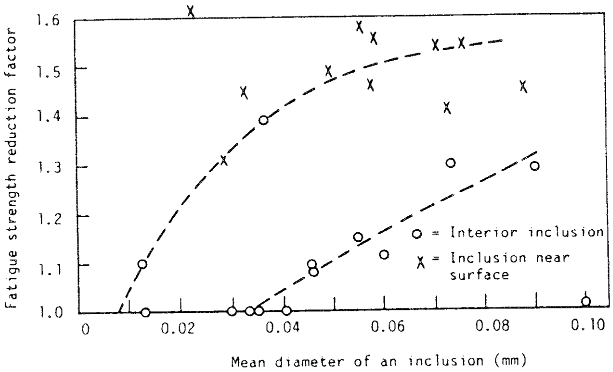


Fig. 2 Relationship between the mean diameter of an inclusion at fracture origin and the fatigue strength reduction factor(19)

(3) The elastic constants of the inclusions and matrix.

All these factors are related to the stress concentration factors and the stress distribution around the inclusions. Many efforts have been made to quantitatively evaluate the stress concentration factors of inclusions by assuming that their shapes were sphere or ellipsoid, but these assumptions are just rough estimations, because a slight deviation from the geometry or small protrusions could greatly affect the stress concentration factors. Using the stress concentration factors for the estimation of the fatigue strength of steels is not only unreasonable but also impractical since the inclusions found at the center of fisheyes in high-strength steels have various shapes and some are far from spherical or ellipsoid(19),(26),(32)~(45). Another misunderstanding is to assume the stress concentration factor to be less than unity when an inclusion and matrix have perfect adhesion, though this is correct at the end of the principal axis of an inclusion which is perpendicular to the loading direction. At the pole in the loading direction, however, the stress concentration factor is more than unity(46)~(48), and a fatigue crack would initiate here(44)~(56). Adhesion of inclusions to the matrix is usually not perfect and often there are some gaps(26),(52) between them : i. e. there are intrinsic cracks in the material, and the stress concentration factors are useless in this case. The detailed observations of Lankford and Kusenberger(49), Morris et al.(50), Morris(51) Kung and Fine(52), Lankford(53),(54), and Eid and Thomason(55) are useful for understanding the crack initiation mechanisms at inclusions. Thus, in high-strength steels, voids are easily produced by the separation of the interface between an inclusion and a matrix. These steels may be considered an equivalent material containing many voids or cracks with the size of inclusions. Therefore, it is extremely important to investigate the effect of a very small hole with the same size as inclusions for the determination of the critical size affecting fatigue strength and for understanding the mechanism of fatigue strength reduction.

In order to study the effects of small surface defects, Murakami and his colleagues(1)~(3),(58)~(65) have conducted systematic experiments using specimens containing artificial small holes with diameters ranging from 50 μm to 200 μm. Figure 3 shows some results of their experiments which indicate the increase in fatigue limit with decreasing size of artificial surface holes and the existence of a critical size of a hole exerting no influence on fatigue strength, though the critical size is dependent on materials. Since the sizes of the artificial holes on the surfaces of these specimens are very small in comparison with specimen size, and the hole shapes are approximately geometrically similar, their stress concentration factors are considered almost equal. Nevertheless, Fig. 3 shows that the fatigue strength varies distinctly depending on hole size. This clearly demonstrates the unreasonableness of fatigue strength evaluation based on the stress concentration factor alone.

Concerning the existence of the critical defect size and the mechanism of its nondamaging effect, Murakami and Endo(58),(59) and Murakami et al.(60) explained the correlation with the maximum size l_0 of nonpropagating cracks which were observed on the surface of unnotched specimens at fatigue limit. Yamada et al.(66) reported a similar result. It may be regarded as established, by recent investigations, that the fatigue limit of a carbon steel is not the critical stress under which no cracks appear, but the threshold stress where the fatigue crack developed under the stress level stops propagating. In other words, a steel specimen at the fatigue limit can include the defect of a fatigue cracks, and the fatigue limit may be considered to be determined by the condition of the propagation of the newly developed defect which is induced by repeated stress in an ordinarily defect-free specimen(58). As a natural consequence, it is expected that the fatigue limit of steel specimens having original cracks or defects in a matrix, the size of which is less than the critical size of the cracks found at the fatigue limit of unnotched specimens, is approximately of the same order of magnitude as the fatigue limit of the materials in a defect-free state. In other words, a material that is called defective from the standpoint of fatigue strength is defective only when the material has a flaw from which a separation originates or crack propaga-

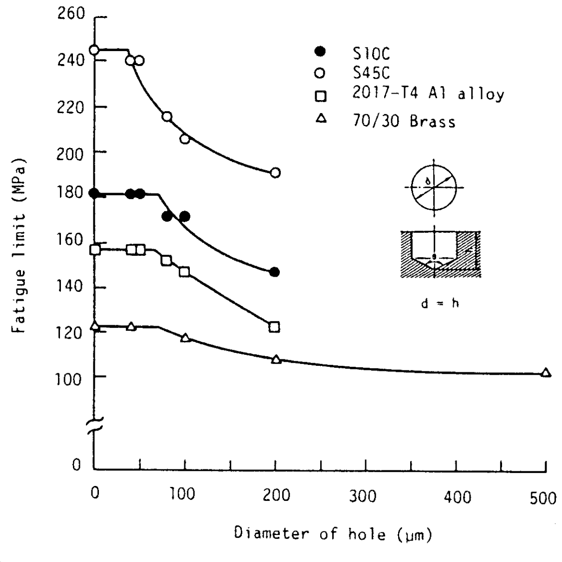


Fig. 3 Effect of a small drilled hole on fatigue strength(60),(62)

tion continues. From the above discussions, the maximum size l_0 of nonpropagating cracks which are observed at the fatigue limit of unnotched specimens should be considered an extremely important quantity when we investigate the fatigue behavior of materials containing defects or inclusions.

The values of l_0 differ depending on matrices. Table 1 shows the values of l_0 for various materials(62). A general trend in Table 1 is a decrease in the value of l_0 with increasing static strength or hardness. This trend implies that the fatigue strength of high-strength steels could be influenced by very small defects or inclusions and that the origins of fatigue failure in such materials are mostly defects or inclusions rather than fatigue cracks nucleated at slip bands or grain boundaries. These characteristics of high-strength steels are the main cause of the relatively low fatigue strength for their high hardness. It is known that there is a good correlation between the rotating bending or uniaxial fatigue strength σ_{w0} of unnotched specimens and the ultimate tensile strength σ_B or Vickers hardness Hv for low- or medium-strength steels. The uniaxial fatigue strength can be formulated as follows based on the experimental results(67)~(69).

$$\sigma_{w0}=1.6\mathrm{Hv}\pm 0.1\,\mathrm{Hv} \qquad (\mathrm{Hv}<400)\ , \qquad (1)$$

where σ_{w0} is in MPa and Hv is in kgf/mm^2.

However in the higher range of Vickers hardness numbers or ultimate tensile strength, this linear correlation no longer holds and there is more scatter in the fatigue strength. Figure 4 shows a typical example of these tendencies, which were first indicated by Garwood et al.(69).

In order to investigate fatigue behaviors of steels with high hardness, Murakami and Endo(59) conducted rotating bending fatigue tests on quenched (Hv=650) and quenched-and-tempered (Hv=520) 0.46% C steel using specimens which contained artificial small holes with diameters ranging from 40 μm to 200 μm. From the experimental results, they reached the following conclusions :

Table 1 The length l_0 of the maximum nonpropagating crack observed at the fatigue limit of unnotched specimen

Materials	ℓ_0 (μm)	σ_{w0} (D=6)	$\frac{\sigma_{w0}}{\sigma_B}$	$\frac{\sigma_{w0}}{\sigma_s}$
S10C: Annealed	∿100	18.5	0.51	0.88
S45C: Annealed	∿ 50	25.0	0.45	0.86
WT80C	∿ 60	44.0	0.54	0.59
S45C: Q. & Temp.	∿ 20	80	—	—
S45C: Quenched	∿ 20	90	—	—

D : Diameter of specimen

σ_{w0} : Fatigue limit of unnotched specimen

σ_B : Ultimate tensile strength

σ_s : Lower yield stress

(1) Even a very small hole 40 μm in diameter can become the cause of a distinct decrease in the fatigue limit of steels having Hv>500.

(2) When slip bands in microstructure become the origin of fatigue fracture, the empirical formula (Eq.(1)) holds in the case of hard steels. In this case, the fatigue limit reaches the upper limit, which is referred to as the ideal fatigue limit by Yamamoto et al.(70), Maikuma et al.(71) and Su et al.(72) These conclusions are consistent with the aforementioned discussion and the experimental results in Table 1. Recently, Yamamoto et al.(70), Maikuma et al(71) and Su et al.(72) conducted similar fatigue tests and verified more clearly the previous conclusions on the basis of the detailed microscopical observations. Therefore, for the achievement of high fatigue strength, the steel-making process must be improved so that the inclusion size will become a little smaller than the value of l_0 in Table 1. In fact, as shown in Fig. 5, Saito and Ito(73) indicated a distinct improvement of fatigue strength of ultraclean spring steels with Hv>500. Although Saito and Ito did not control the inclusion size in their steel-making process, improvement of the cleanliness of the materials resulted in a decrease in the inclusion size, as shown in Figs. 6(b) and (c).

3. A Representative Geometrical Parameter for Defects and Inclusions, and a Representative Material Parameter for Matrices

The conclusions arrived at in the previous chapter can be summarized as follows :

(1) Although high-strength steels or high-hard-

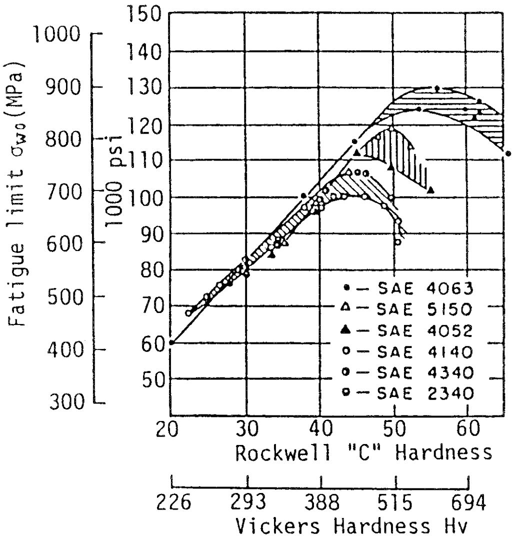


Fig. 4 Correlation between hardness and fatigue limit(69)

ness steels are very sensitive to inclusions and small defects, the fatigue strength increases with increasing cleanliness and decreasing size of inclusions and defects.

(2) When we regard high-strength steels as materials containing many small cavities of nonmetallic inclusions, the characteristics of the fatigue behaviors can be clearly explained.

Conclusion (2) suggests the possiblity of finding a representative geometrical parameter to evaluate the effects of defects and inclusions on fatigue strength. Such a parameter will enable one not only to analyze quantitatively the influences of defects and inclusions from the standpoint of fatigue strength, but also to give quantitative guidelines for the control of inclusions in metallurgical processes. Recent studies by Murakami and Usuki(5) Natsume et al.(74) and Murakami et al.(4),(75) in this direction will be introduced in this chapter.

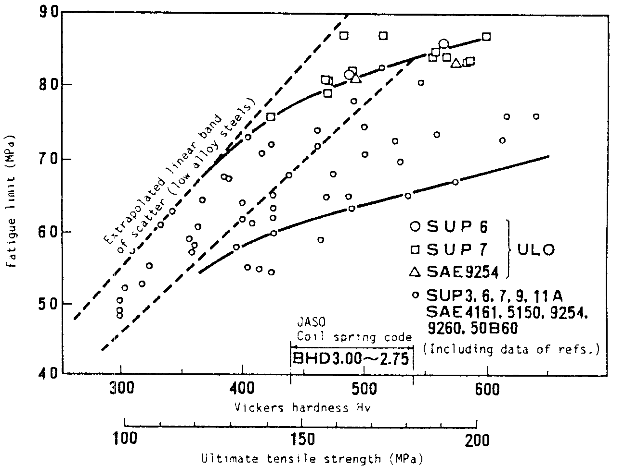
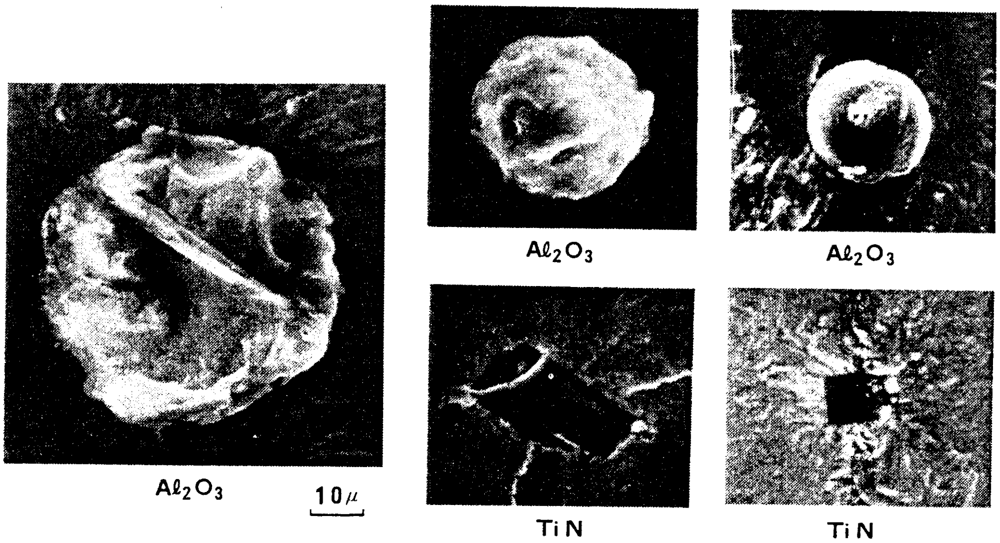


Fig. 5 High fatigue strength achieved in ULO automobile suspension steels(73)

3.1 A representative geometrical parameter for defects, inclusions and small cracks

The crucial cause for the absence of a unifying method of the evaluation of defects, inclusions and small cracks was an incorrect understanding of the fatigue threshold condition of specimens or structures. Murakami and Endo(1),(2) and Murakami, Kodama and Konuma(4) showed how to overcome this difficulty by considering the fatigue limit not as the critical condition for crack initiation but as the condition for the nonpropagation of a crack emanating from defects, cracks, and inclusions. For example, the fatigue limit of a structural component containing a small defect must not be treated as a notch problem in which the critical condition of crack initiation is questioned, but should be understood as a problem of a crack which emanates from the defect and stops propagating. Only the interpretation of problems in this manner leads one to find the geometrical parameter for defects, cracks, inclusions and even sharp notches.

It is reasonable to seek the geometrical parameter from the standpoint that the effects of shapes and sizes of cracks on fatigue strength may be correlated with stress intensity factors, especially with the maximum stress intensity factor along the three-dimen-



(a) lower
Hv=534

(b) middle
Hv=614

(c) upper
Hv=588

Fig. 6 Nonmetallic inclusions observed at fatigue fracture origin(73). Pictures show typical inclusions contained in each group of specimens and the fatigue limit of each group is at (a) lower point of scatter band, (b) middle point of scatter band and (c) upper point of scatter band.

sional crack front. Previous studies(2),(4),(5),(76)~(78) concerning this problem can be summarized as the following approximate equations :

Internal crack : $K_{I\max}=0.5\sigma_0\sqrt{\pi\sqrt{\text{area}_i}}$ (2)

Surface crack : $K_{I\max}=0.65\sigma_0\sqrt{\pi\sqrt{\text{area}_s}}$, (3)

where σ_0 : the maximum principal stress, area_i : the area of an internal crack, and area_s : the area of a surface crack.

The error in Eqs. (2) and (3) may be estimated to be less than 10 per cent. Equations (2) and (3) imply that the square root of the crack area projected onto the plane perpendicular to the maximum tensile stress should be adopted as the most relevant geometrical parameter for three-dimensional cracks.

When a specimen has a three-dimensional defect other than a planar crack, the fatigue limit is determined by the threshold condition of the crack emanating from the defect(1),(2),(58). In this case, the initial three-dimensional shape of the defect is not directly correlated with the stress intensity factor. Rather, the planar domain (area) which is occupied by projecting the defect onto the plane perpendicular to the maximum principal stress should be regarded as the equivalent crack, and the stress intensity factor should be evaluated from the equivalent crack (see Fig. 7(b)). Here, it should be noted that the area of a crack emanating from a three-dimensional defect occupies only a small portion of the total projected area(1),(2), (see Fig. 7(a)), and accordingly, the area of the equivalent crack which should be used for Eqs. (2) and (3) may be estimated from the projected area of the initial defect. With regard to an estimation of $\sqrt{\text{area}}$ for irregularly shaped cracks, very slender cracks and very deep cracks, some simple corrections in the estimation of $\sqrt{\text{area}}$ are necessary(2) for the application of Eqs. (2) and (3).

On the basis of this hypothesis, previous data on rotating bending fatigue were analyzed using the parameter $\sqrt{\text{area}}$. The artificial defects investigated in the study were very small artificial holes(1)~(3),(58)~(65),(79)~(82) with diameters ranging from 40 μm to 500 μm and depths greater than 40 μm, as well as very small and very shallow notches(80),(81),(83)~(93) with depths ranging from 5 μm to 300 μm, very shallow circumferential cracks(93) with depths greater than 30 μm, and finally a Vickers hardness indentation (80) 72 μm in surface length. The shapes of defects and cracks considered are shown in Fig. 8. The effects of work hardening and residual stress resulting from the introduction of drilled holes were examined and found to be small(1). In these tests, almost all notched specimens were electropolished after introduction of the notches(80),(81),(83)~(93) , and the cracked specimens were annealed after introduction of the fatigue cracks(93). Accordingly the effect of work hardening can be expected to be negligible.

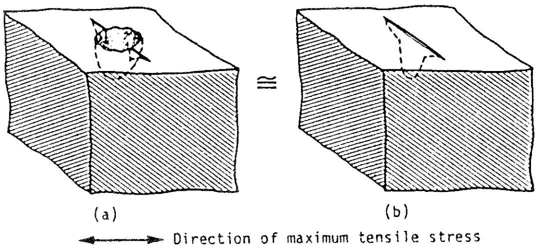


Fig. 7 Defect with cracks and its equivalent crack(2)

The relationship between the threshold stress intensity factor range ΔK_{th} and $\sqrt{\text{area}_s}$ is illustrated in Fig. 9. The data in the figure are adopted from the aforementioned references. The adoption of the new parameter $\sqrt{\text{area}}$ characterizes the threshold behavior very well, particularly for the data on very small cracks.

For the region $\sqrt{\text{area}_s}\leqq 1\,000$ μm , the relationship between ΔK_{th} and $\sqrt{\text{area}}$ is approximately linear and the following equation holds regardless of the nature of the material :

$$\Delta K_{\text{th}}\propto(\sqrt{\text{area}_s})^{1/3}. \qquad (4)$$

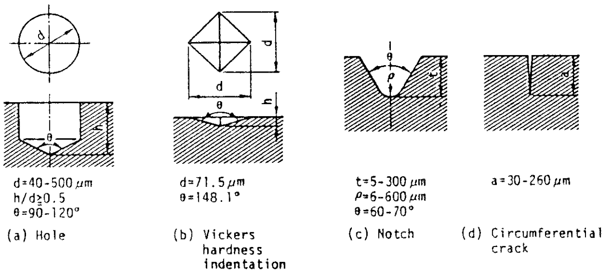


Fig. 8 The shapes of defects and cracks investigated in ref. 2

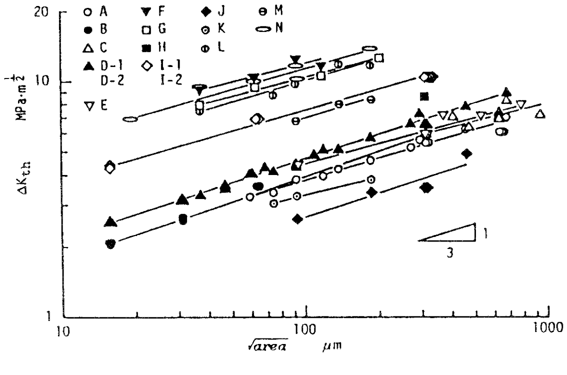


Fig. 9 Relationship between ΔK_{th} and $\sqrt{\text{area}}$ for various defects and cracks. Letters correspond to the materials given in ref. 2

Equation[4] indicates that the dependence of fatigue limit σ_w on defect size is expressed by the following equation:

$$\sigma_w \propto 1/(\sqrt{\text{area}_s})^{1/6} \qquad (5)$$

If we simply set $\sqrt{\text{area}_s} \rightarrow 0$, we have $\sigma_w \rightarrow \infty$. But in reality this never happens, because the lower limit of $\sqrt{\text{area}_s}$ over which Eq. (5) is valid is related to the maximum size of nonpropagating cracks l_0 observed in unnotched specimens. Therefore, it may be concluded that $\sqrt{\text{area}}$ is promising as the representative geometrical parameter.

3.2 Vickers hardness as a representative material parameter

Although the question of what material parameter best represents fatigue characteristics is substantially different from the one of what material parameter is most convenient for engineers, a material parameter which can be easily measured should be chosen for the analysis of the data. Fortunately, the good correlation between Vickers hardness and the fatigue limit of low- and medium-strength steels is empirically well established[67],[68],[94],[95]. Therefore, choosing Hv as the material parameter and investigating the data in Fig. 9 in detail, we can confirm the following formula to hold approximately:

$$\Delta K_{th} \propto (\text{Hv}+120). \qquad (6)$$

The reason why ΔK_{th} is not simply proportional to Hv is presumably beacuse the occurrence of nonpropagating cracks may have a different dependency. In other words, a crack is likely to show nonpropagating behavior in soft materials while, on the contrary, it is difficult to find nonpropagating cracks at the fatigue limit of hard steels (Only short nonpropagating cracks can be observed within a narrow range of stress amplitude)[59],[80].

Combining Eqs. (4) and (6) and applying the least squares method to the data in Fig. 9, we have

$$\Delta K_{th} = 3.3 \times 10^{-3}(\text{Hv}+120)(\sqrt{\text{area}_s})^{1/3} \qquad (7)$$

$$\sigma_w = 1.43(\text{Hv}+120)/(\sqrt{\text{area}_s})^{1/6}, \qquad (8)$$

where the units of the quantities are ΔK_{th} : MPa·m$^{1/2}$, $\sqrt{\text{area}_s}$: μm, Hv :kgf/mm^2 and σ_w :MPa.

Figure 10 shows the comparison of the experimental data in Fig. 9 and the correlation by Eq. (7). It is encouraging to note that the various data for Hv ranging from 70 to 720 are well represented by Eq. (7) (The details of the data are tabulated in ref. 2). Thus, we can use Hv as the representative material parameter.

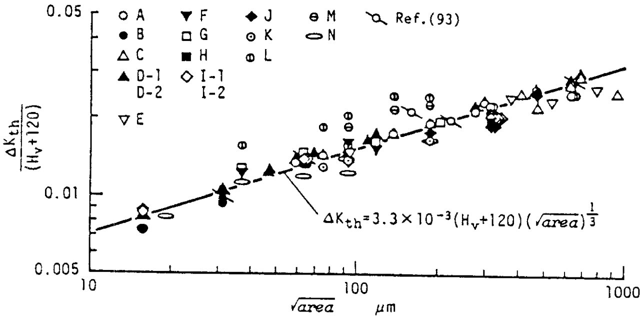
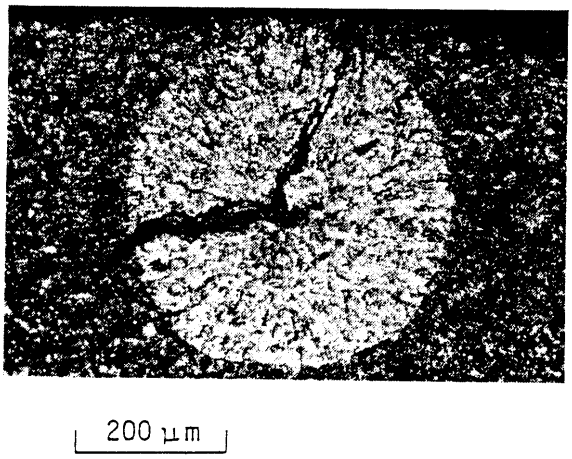


Fig. 10 Relationship between $\Delta K_{th}/(\text{Hv}+120)$ and $\sqrt{\text{area}}$[2]. Letters correspond to the materials given in ref. 2

4. Application of a Fatigue Strength Prediction Equation for Defects to Fatigue Fracture from Nonmetallic Inclusions

4.1 Fatigue fracture from nonmetallic inclusions and fatigue strength at inclusions

It is quite common for so-called fisheye marks (see Fig. 11) to be observed at the fatigue fracture origin of high-strength steels[4]~[6],[26],[36]~[45],[73]~[75]. This implies that a fatigue crack is initially nucleated at the interface between an inclusion and a matrix[54] or that inclusions are first cracked[55],[96], and that the fatigue crack thereafter propagates into the matrix. As already described, the fatigue limit of unnotched specimens of low- and medium-strength steels is not the critical stress for crack initiation but the threshold stress for non- propagation of a crack which is nucleated usually at slip bands or grain boundaries (see references of ref. 58). When cracks or sharp notches exist in specimens, nonpropagating cracks are always observed at the tip of cracks or sharp notches at the fatigue limit of most materials. These characteristics of fatigue limit must be taken into consideration for



Hv=352, σ=490MPa, N_f=5.39x10^6, Distance from surface h=280 μm.

Fig. 11 A typical example of fisheye[5]
The material is SAE10L45

evaluation of the fatigue strength of high-strength steels.

In other words, inclusions must be regarded as virtually equivalent to defects or voids, because the stresses in nonmetallic inclusions are relieved by the existence of cracks (free surface) at the interface between the inclusion and the matrix or cracks interior of inclusions. Considering these characteristics of inclusions. Murakami et al.[(4)] and Murakami and Usuki[(5)] emphasized the importance of the concept that inclusions should be regarded as initial defects or cracks in order to solve the fatigue strength problems of high-strength steels. In this concept, as in the case of defects, the geometrical effect of inclusions on fatigue strength is evaluated by the area of inclusions projected onto a plane perpendicular to the maximum tensile stress. The application of Murakami and Endo's method for cracks and defects[(2),(65)] (see Fig. 10, Eqs. (7) and (8)) to inclusions is expected to be promising.

The fatal inclusion, i. e. the inclusion at the fatigue fracture origin, is not necessarily the biggest one, because the magnitude of stress at inclusions, i. e. the location of inclusion in a specimen, is also an important factor to be considered.

When the fatal inclusion exists at the surface of a specimen, ΔK_{th} and σ_w can be predicted by Eqs. (7) and (8). In this evaluation, area$_s$ is the area of the inclusion projected onto the plane perpendicular to the maximum tensile stress. However, if the fatal inclusion exists in a subsurface layer and is almost in touch with the free surface, as shown in Fig. 12, area$_s$ should be estimated with the dotted convex contour line rather than the original contour line of the inclusion.

For interior inclusions, the stress intensity factors equation (Eq. (3)) for interior cracks of arbitrary shape is used to derive the fatigue strength prediction equations by modifying Eqs. (7) and (8).

Consequently, the fatigue strength prediction equations for each case are classified as follows:

[Surface inclusions]

$$\sigma_w = 1.43(\mathrm{Hv}+120)/(\sqrt{\mathrm{area}_s})^{1/6}, \tag{9}$$

[Subsurface inclusions]

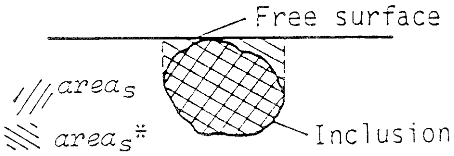


Fig. 12 An inclusion just below the free surface
area$_s^*$: projected area of an inclusion
area$_s^*$: virtual area

$$\sigma_w = 1.40(\mathrm{Hv}+120)/(\sqrt{\mathrm{area}_s})^{1/6}, \tag{10}$$

[Interior inclusions]

$$\sigma_w = 1.56(\mathrm{Hv}+120)/(\sqrt{\mathrm{area}_i})^{1/6}. \tag{11}$$

The applicability of these equations can be examined by comparing the stresses at fisheyes of fractured specimens with the predicted fatigue strength at the locations of the fatal inclusions.

Table 2 shows the experimental data[(4)] of a rotating bending fatigue test on three kinds of bearing steels and the predicted fatigue limit σ_w' at the location of nonmetallic inclusions. When the stress amplitude σ' at the location of an inclusion is greater than the predicted fatigue limit σ_w', the inclusion is expected to become the fracture origin. The values of σ'/σ_w in Table 2 are all greater than 1.0, and the fatigue failure of specimens coincides with the predictions.

Table 3 shows similar results[(4)] on heat-treated 0.34% C and 0.55% C steels. In these data, the values of σ'/σ_w' are greater than 1.0 except for in two cases, in which they are very close to 1.0, i. e., 0.974 and 0.995. Therefore, it may be concluded that the predictions are quite accurate.

4.2 Prediction of the upper and the lower limit of fatigue strength of high-strength steels containing nonmetallic inclusions

When nonmetallic inclusions affect the fatigue strength, a specimen has its specific fatigue strength because of the existence of inclusions. This characteristic, particularly in high-strength steels, makes quantitative prediction of the fatigue limit difficult. One method of solving this problem is to inspect all inclusions contained in specimens or structures nondestructively and to predict the fatigue limit of an individual specimen or structure. However, this method is not only unrealistic but also will not work because it is extremely difficult to measure the shape, size and location of nonmetallic inclusions accurately, even with the most advanced method of nondestructive inspection.

Nordberg[(97)] proposed a method of predicting the distribution of fatigue strength using the statistical analysis of inclusion distribution together with the notch effect theory in which inclusions are regarded as elliptical notches. This method predicts the probability of fatigue fracture, because the fatigue strength of an individual specimen is indeterminate. However, the mathematical treatment of this method is very complicated and it may be inconvenient for practical use. Various mathematical treatments of inclusion distribution are discussed by Dehoff and Rhines[(98)].

Murakami et al.[(4)] and Murakami and Usuki[(5)] proposed a method of predicting the upper and the lower limit of fatigue strength. The upper limit of fatigue strength is obtained when fatigue fracture is not affected by defects or inclusions, and according to previous experimental results(67),(68), the value can be estimated by

$$\sigma_{wu} \cong 1.6\ \mathrm{Hv}. \qquad (12)$$

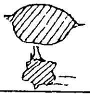
The lower limit σ_{wl} of fatigue strength can be obtained when a large inclusion is located in contact with the surface of a specimen(5),(74). In order to estimate the expected maximum size of inclusions contained in a definite number of specimens, Murakami and colleagues(4),(5),(74),(75) applied the statistics of

Table 3 Size and location of inclusions and fatigue limit predicted by Eq.(11)

Materials	Hv	σ	N_f	$\sqrt{area}$	h	σ'	σ_w'	σ'/σ_w'
S35C	570	724	$4.02^{\times 10^6}$	1134	42	716	599	1.20
	610	713	4.40	2204	200	681	600	1.14
	672	717	3.23	641	66	706	721	0.974
	655	735	2.19	1023	50	727	679	1.07
	638	724	1.08	1960	70	712	629	1.13
	657	686	1.48	1254	30	681	669	1.02
S55C	782	887	9.35	473	250	838	842	0.995
	775	918	2.37	769	290	858	802	1.07
	797	897	2.35	750	143	868	824	1.05
	801	896	4.40	491	110	874	857	1.02
	803	892	7.87	1257	375	817	803	1.02
	831	910	4.01	1257	175	874	819	1.05

The units are the same as the quantities in Table 2.

Table 2 Size and location of inclusions and fatigue limit predicted by Eq.(11)

Materials & Hv	Nominal stress at surface σ(MPa)	Cycles to failure N_f	Inclusion size $\sqrt{area}$(μm^2)	Distance from surface h(μm)	Shape of inclusions	Nominal stress at inclusion σ'(MPa)	Fatigue limit predicted by Eq.(11) σ_w'(MPa)	σ'/σ_w'
Steel N Hv $\simeq$ 734	981	$254.36^{\times 10^4}$	962	316		907	752	1.21
	981	120.05	1343	370		895	731	1.22
	932	429.54	1154	390		846	740	1.14
	883	1280.50	962	120		858	752	1.14
	981	192.51	1343	38		971	731	1.33
	932	296.64	1501	420		839	724	1.16
	912	134.21	808	63		898	763	1.18
	883	277.34	416	14		879	806	1.09
	883	729.50	857	295		821	759	1.08
Steel S Hv $\simeq$ 758	1030	125.25	577	310		954	806	1.18
	1030	556.77	254	140		995	863	1.15
	981	422.95	231	28		973	870	1.12
	981	898.01	99	74		963	934	1.03
	981	175.51	156	10		978	899	1.09
	1030	224.64	346	74		1011	841	1.20
	1030	30.52	491	24		1023	817	1.25
	1030	26.48	804	110		1003	784	1.28
	971	735.45	836	350		960	782	1.14
Steel V Hv $\simeq$ 685	981	50.11	1409	80		961	686	1.40
	981	39.21	858	170		941	715	1.32
	932	683.38	1056	570		807	703	1.15
	883	138.24	2859	200		841	647	1.30
	981	75.06	962	350		900	708	1.27
	932	160.35	654	240		878	732	1.20
	932	11.12	2206	100		909	661	1.38
	932	23.40	10147	1030		706	582	1.21
	834	420.00	4882	600		715	619	1.16

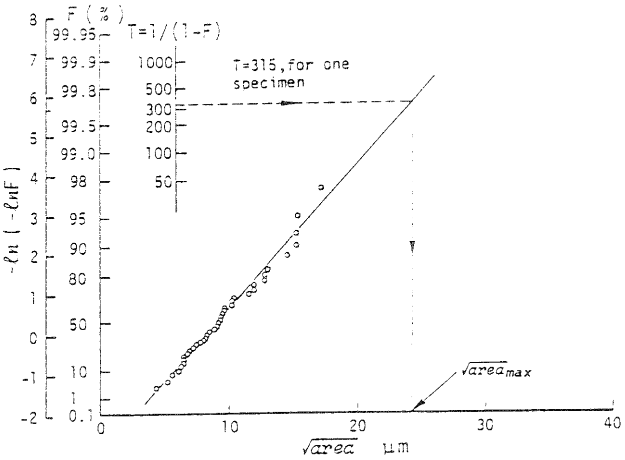


Fig. 13 Cumulative frequency of $\sqrt{\text{area}}_{\text{max},j}$
The material is SAE10L45.
The unit test area S_0 is 0.482 mm^2.

extreme values[(99)] to the distribution of inclusions.

Figure 13 shows the distribution of the maximum size $\sqrt{\text{area}}_{\text{max}}$ of inclusions observed in the unit area $S_0(=0.482\,\text{mm}^2)$ of SAE10L45. Taking the return period T for N specimens as $T=NS/S_0$, the expected maximum size of inclusion $\sqrt{\text{area}}_{\text{max}}$ can be determined by the procedure indicated in the figure.

Introducing Hv and $\sqrt{\text{area}}_{\text{max}}$ to Eq. (10), the lower limit σ_{wl} of fatigue strength for N specimens is predicted as in Fig. 14. The lower limit of experimental data is well predicted by the equation. Murakami and colleagues[(4),(5),(74),(75)] have applied this method to various materials and have obtained good results. The accurate prediction of the lower limit of fatigue strength will be very useful not only for the design of machine elements or structure but also for the improvement of the grade and quality of materials. The line of the lower limit of fatigue strength in Fig. 14 is more helpful for the determination of reliable allowable stress than taking an ambiguous safety factor. Figure 13 provides a guide for the control of nonmetallic inclusions in the steel-making process.

5. Conclusions

Although the problems of defects and nonmetallic inclusions in metal fatigue are very complicated, it is particularly important to view these problems from the perspective that defects and inclusions are virtually equivalent to small cracks. This concept will help one to understand various fatigue phenomena caused by defects and inclusions. Although the problems of residual stress and strain, and the interaction effect of many inclusions are not discussed in this paper, the current conclusions may be summarized as follows.

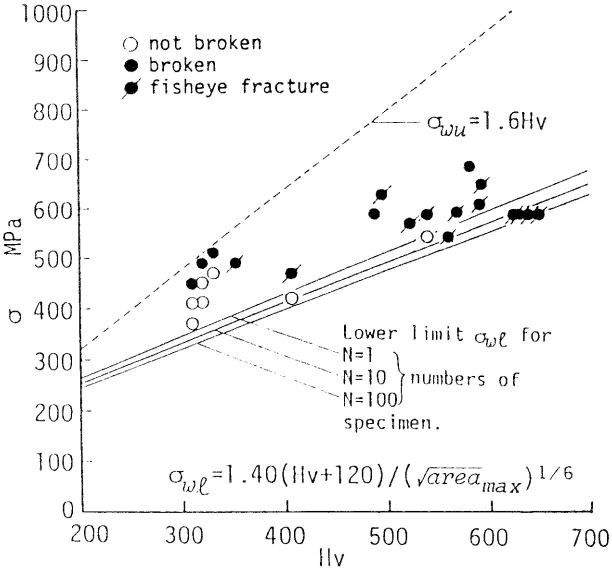


Fig. 14 Comparison of the predicted lower limit of fatigue strength
σ_{wl} with experimental data[(5)]
The material is SAE10L45.

(1) Only defects and inclusions larger than a critical size have harmful effects on the fatigue strength of metals. The critical size is concerned with the length l_0 of small nonpropagating cracks which are observed at the fatigue limit of unnotched specimens. The values of l_0 are dependent on the hardness of the matrix, and it is a general trend that hard metals have smaller values of l_0.

(2) The fatigue limit of unnotched specimens is approxiamtely proportional to Hv for Hv$\leqq$400. When the value of Hv increases beyond 400, the fatigue limit shows a distinct drop from the fatigue strength expected from the empirical formula. This is because nonmetallic inclusions in hard materials become the origin of fatigue fracture.

(3) Surface nonmetallic inclusions have a more harmful effect than interior inclusions. Different effects of inclusions with various shapes and sizes at various locations cause a large scatter of fatigue in the high hardness region. Therefore, it is almost impossible to predict the fatigue limit of individual specimens and machine components.

(4) The lower limit σ_{wl} of scatters of fatigue strength can be predicted by Murakami and colleagues' method, in which Hv and the square root of the projected area ($\sqrt{\text{area}}$) of inclusions are the crucial influential parameters. The value of σ_{wl} depends on the maximum inclusion ($\sqrt{\text{area}}_{\text{max}}$) which is expected to be contained in specimens or machine components.

(5) Statistics of extreme values can be used to predict $\sqrt{area}_{max}$ of inclusions. The statistical distribution of extreme values of $\sqrt{area}$ may be used as a guideline for the control of inclusion size in the steel-making processes.

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