EFFECTS OF SOLAR RADIATION PRESSURE AND AERODYNAMIC FORCES ON SATELLITE— ATTITUDE DYNAMICS AND THEIR UTILIZATION FOR CONTROL: A SURVEY

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ABSTRACT

The environment exerts an important influence on the performance of space systems. A brief review of most of the studies, presented over the past eighteen years, relating to the influence and the possible utilization of the solar radiation pressure and aerodynamic forces, with particular reference to attitude dynamics and control of satellites is presented here. The semi-passive stabilizers employing these forces show promise of long life, low power and economic systems, which though slower in response, compare well with the active controllers. It is felt that much more attention is necessary to the actual implementation of these ideas and devices: some of which are quite ingenious and unique.

Key words: Solar radiation pressure, rarefied atmosphere, earth satellites, attitude dynamics, semi-passive control, gravity-gradient stabilization, flexibility, orbital perturbations.

INTRODUCTION

The application of space technology for development is now well recognized. The success of the missions, however, depends, to a great extent, on the stability of orbits and satellite orientations. The environmental forces present a major source of errors, especially for passive controllers like gravity-gradient systems which can provide only small correcting torques but are gaining increasing interest because of their simplicity and economy. The effects have been a subject of considerable investigation over the past eighteen years with recent attempts at utilizing the perturbative forces to advantage. A brief account of some of the work relating to the attitude dynamics is included in the reviews by Roberson [1], Frye and Stearns [2], Ergin [3], Sabroff [4], and Shrivastava, Tschann and Modi [5]. A more complete, updated review of the investigations dealing with the environmental influence and their possible applications is presented here.

The orbital perturbation due to the environmental forces have been studied by a number of investigators [16–14 *et al.*]. Though very important this aspect falls beyond the scope of the present discussion. Here the attention is confined to attitude dynamics and control of satellites.

The relative magnitudes of torques due to various forces arising from gravity-gradient, solar radiations, earth and earth-reflected radiations atmospheric and magnetic forces, cosmic dust, etc., depend on the orbital elements and the satellite's shape, size, surface conditions, mass distribution and orientation. The actual systems are rather complex. For some simpler, shapes like plates, spheres and cylinders some of the torques have been mathematically modelled, through several assumptions, to a varying degree of accuracy by Roberson [15], Hall [16], Evans [17], Wiggins [18], Clancy and Mitchell [19], Hughes [20], Tidwell [21], Flanagan and Modi [22], and others. Figure 1, taken from Ref. [5], shows a comparison of maximum torques. at various altitudes, acting on a typical passive satellite (GEOS–A). It is apparent that for near-earth systems the atmospheric torques and for higher altitude those due to direct solar rodiation pressure become comparable to the control moment provided by the gravity-gradient. For larger area/mass ratios, typical of the next generation of satellites, both these influences would tend to be much more significant. As such any analysis and design must include these considerations.

Noting the importance, which is further substantiated by the flight experience of Echo [23], Vanguard [6], Explorer XII [24], Aloutte [25, 26], GEOS [27, 28], Proton–2 [29], SKYNET [30] and several other satellites, a large number of investigations have been undertaken. These can be broadly classified as under:

1. Attitude perturbations due to
   [*a*] Radiation pressure and [*b*] Aerodynamic forces.
2. Attitude stabilization and control using
   [*a*] Solar radiation pressure, [*b*] Aerodynamic forces.

The studies have essentially proceeded along two lines: either detailed analytical and numerical studies of simple models, or analysis of complex

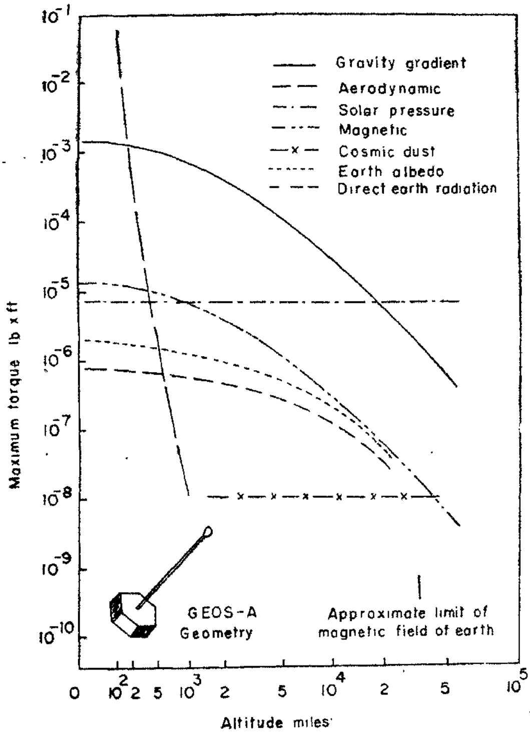

FIG. 1. Magnitude of environmental torques on a typical satellite[5].

systems using simplified equations. On the experimental and hardware development aspects the work is still in the preliminary stages though a number of controllers have been proposed and the feasibility is established by the flight of COSMOS-149 [31], which employed atmospheric forces for its pitch control, and Mariner-IV [32], which demonstrated the usefulness of the solar radiation pressure. In the following sections the important aspects of most of the papers published on the subject are briefly reviewed.

1.1. *Attitude Perturbations due to Radiation Pressure*

According to the quantum theory of electromagnetic radiation, the pressure exerted by the radiation flux incident on a surface is the rate of change of momentum of the photons. A totally absorbent surface placed normal to the incident flux experiences a pressure: $P = m_p c'$ where m_p is the mass of photons incident per unit area and c' is the velocity of light. From Einstein's energy relation, the pressure in terms of the solar constant S is

$$P = S/c'$$

The pressure is a function of surface reflectivity, absorptivity and transmissibility, as well as the position relative to the emitting body. These properties themselves are functions of the wavelength and angle of incidence. The variations, however, are usually low and may be ignored [22].

The existence of the force has been known for a long time. Its magnitude being small [10^{-5} Newtons/m^2, for a perfectly reflecting surface] the designers of early satellites did not pay much attention. The attitude and orbital perturbations of Echo and Vanguard and spin-up of Explorer XII, which were later attributed to solar radiation pressure by Bryant [23], Evans [17], and Fedor [24], exhibited the necessity of a careful consideration of this effect in design and operation of even small satellites, especially those having only small control torques. Hall [16] derived an empirical force expression for direct solar radiation of the form $F = PAC_f$, where A is the surface area and C_f is the force coefficient which is found to be less than two for several convex shapes. McElvain [33] obtained an analytical expression for direct solar radiation torque and determined, for two geometries, the change in the satellite's angular momentum necessary to maintain a specified orientation. In a later study [34] ways to minimize the solar torque on a gravity-stabilized satellite, ATS, were suggested. The method involves optimal balance of surfaces and their characteristics. Karymov [35] derived equations of librational motion of a sun-orbiting satellite and analysed the stability of equilibrium along the local vertical. The derivations, however, are needlessly complicated. Clancy and Mitchell [19] undertook a more complete analysis accounting for three major sources of radiations, namely, the sun, earth and reflections from the earth and its atmosphere. The dynamical behaviour of the system is found as the motion of and about the angular momentum vector. In addition to the inherent limitations of the analysis, the resulting force expressions, given in an integral form, have to be evaluated numerically. The computation times involved appear to be very large and may render a comprehensive study of attitude dynamics virtually impossible [36].

A precise evaluation of these forces was undertaken by Flanagan and Modi [22] with particular reference to a flat plate model of a satellite, executing planar librations in an ecliptic orbit. The closed form character of the resulting expressions for the forces makes them ideally suited for satellite attitude dynamics studies. The system response under only direct solar radiations was analysed in detail using W.K.B.J. approach [37]. Later in conjunction with the concept of integral manifold in state-space, they were able to establish the altitude bounds where various environmental forces become significant [36, 38–40]. Figure 2, one of their results, shows the rapid reduction in the stability bounds and maximum allowable eccentricity for a relatively more stable gravity-gradient satellite. Modi and Kumar [41–43] concentrated upon direct solar radiation pressure, acting on a more realistic model—a cylindrical satellite. For the planar librations the system was analysed using digital as well as analog simulations [41]. The cross-plane motions in circular orbits were also included in a later study [42]. Here again the radiation reduces the stability region significantly. It may be mentioned here that a possibility of excitation of resonant motion across the orbital plane, due to radiations, was suggested for GEOS–II [27]. The observations corfirmed it [28]. The importance of the influence on librational dynamics of a slowly spinning system is also established recently by Modi and Pande [46]. All these analyses, through extensive computations. clearly point out that the radiation pressure can play as important a role as the system inertia, orbital eccentricity, and spin, if any.

The designers, who often want a simple working model of a complex system, may find the study by Hodge [45], who, using the model given by Tidwell [21] for gravity, magnetic, aerodynamic, eddy-current and solar pressure torques on a rolling hexagonal satellite, pointed out the adequacy of a linearised analysis for short-term [1/3 orbit] predictions. This is mainly because the effects are small.

All the studies mentioned above assume the satellite to be rigid. The actual systems, especially those being designed now are far from this idealization. Their elasticity coupled with heating and presence of environmental forces can significantly affect the performance and success of the missions. Such a possibility, even for a small satellite containing elastic parts [*e.g.*, Aloutte and Explorer XX] was established by Etkins and Hughes [25]. A large number of studies that followed emphasized the behaviour further and resulted in many improvements in the components like booms and panels which may undergo very large deflections [46]. The combined effect of solar radiation pressure and differential heating influence the system response and stability rather adversely [47–48 *et, al*].

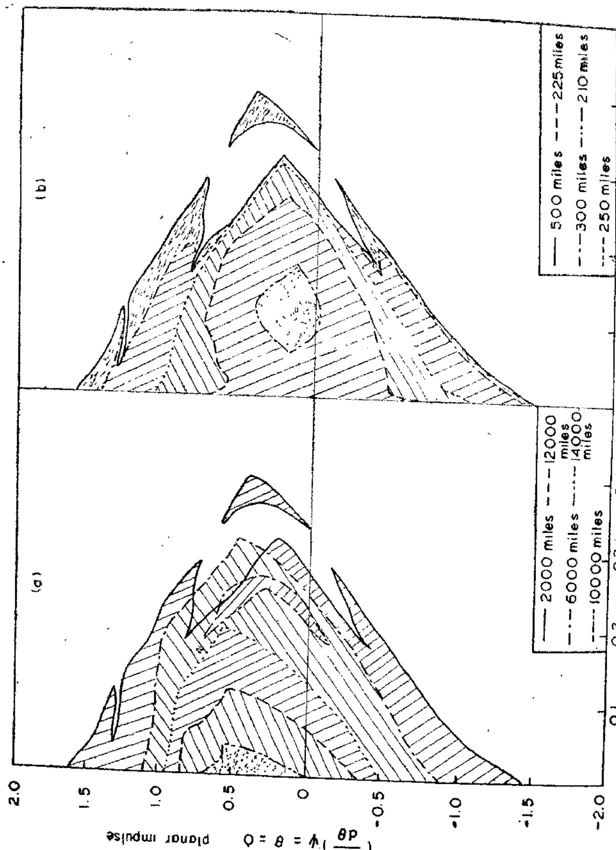

FIG. 2. Stability charts for planar librations of GEOS-A showing (*a*) Direct solar radiation at higher altitudes, (*b*) Aerodynamic effect at lower altitude.

The vast amount of literature, dealing with flexibility-control-interaction problems, thermoelastic behaviour of space structures, and analyses of flexible satellites subjected to environmental forces are well reviewed in a number of reports, *i.e.*, NASA special publication [49], which is briefly summarized by Noll *et al.* [50], and papers by Ashley [51], Likins and Bouver [52], Hughes [53] and most recently by Modi [54]. These should prove valuable to the designers.

1.2. *Attitude Perturbations due to Aerodynamic Forces*:

A number of studies indicate that the earth's atmosphere, though rarefied, plays a significant role in the satellite dynamics even at the altitudes of about 600–800 km. Due to large speeds of vechicles the drag becomes quite significant. The physics of the interaction with the free molecular flow is rather complex. The force depends on a number of surface and environmental parameters, including temperatures, reflectivity, Knudsen number, angle of incidence, etc. [55]. A simple model, extending relations in the continuum flow, can be made with the drag coefficient determined experimentally. In most cases the coefficient turns out to be about 2. With this approximation many studies relating to the equilibrium, response and stability of satellites of various shapes have been carried out.

Debra [56] discussed the variation of equilibrium for two gravity-stabilized satellites. Beletskii [57] treated the forcee as a small perturbative source acting on a rapidly spinning system. Through a linearized analysis Schrello [58] pointed out that the aerodynamic torques may exceed gravity gradient moment even at altitudes nearing 500 km. Evans [17] presented the disturbances in the fundamental form of pressure and shear stresses. The influence of a constant moment acting on a gravity stabilized system was determined through an infinitesimal analysis by Garber [60]. More directly Sarychev [60, 61] with a particular reference to the Russian Satellites, derived equations of motion and determined the necessary and adequate conditions for asymptotic stability of the eigen oscillations which are also caused by the rotating atmosphere. Meirovitch and Wallace [62] established the regions of guaranteed stability for a slowly spinning system with a constant aerodynamic force. For two configurations, the stability of equilibrium positions was established using Liapunov's direct method. Morozov [63, 64] included the magnetic forces and found conditions for stability of the steady state behaviour of a gyrostat.

Most of the studies discussed above consider a simpler case of axsymmetric satellites moving in circular orbits. A majority of the actual systems, however, do possess inertial as well as geometric non-symmetries. In such cases, the interaction with the atmosphere may result in instabilities, as shown by Nurre [65]. The center of mass was still assumed to be in the plane of geometric symmetry. Frik [66], extending it to an arbitrarily shaped body, found that if the aerodynamic forces were conservative at least one stable equilibrium would exist. For non-conservative forces no stable position is possible.

The planar librations of gravity-stabilized satellite modelled like a plate moving in an elliptic orbit and subjected to various radiations and aerodynamic forces were investigated by Flanagan and Modi [40]. A more complete analysis, accounting for the transverse motion as well, for a cylindrical satellite moving in an arbitrary orbit and subjected to gravity gradient and aerodynamic torques is presented by Shrivastava and Modi [67–71]. After establishing the stability of equilibrium through infinitesimal as well as Liapunov's approach, detailed response and stability studies were carried out using the digital [67] and analog simulations [68]. The design plots of the allowable disturbances for non-tumbling motion indicate a rapid reduction in the stability region. The concept of integral manifold is successfully extended to the axisymmetric satellites in circular orbits. Three distinct types of motions corresponding to the fundamental and other periodic solutions are noted. The critical Hamiltonians representing the strength of the maximum permissible disturbance for many systems are established through Floquet's theory [70]. An approximate solution in terms of elliptic functions is also found using the constant first integral of the system [71]. Under the combined effect of eccentricity and atmosphere the equilibrium changes continuously and the response gets much more compilcated. The stability bounds also shrink rapidly [72] [Fig. 3]. This puts a severe limitation on the usefulness of gravity gradient system for near-earth systems. Even for relatively stable dual-spin spacecraft of the type of SAS–A the effects of environment could not be ignored [73, 74]. Small attitude perturbations, mainly due to the atmosphere, were noted for this satellite which was put in a 500 km circular orbit.

The studies mentioned above are mostly deterministic in their approach and assume a rather simple model for a complex and uncertain phenomenon. Better results may be possible if one uses statistical methods. The study by Sheporaitis [76], who finds a parametric stability region using stochastic Liapunov function and gives probability estimates of the convergence of uilieqbrium position, should be of interest to the investigators.

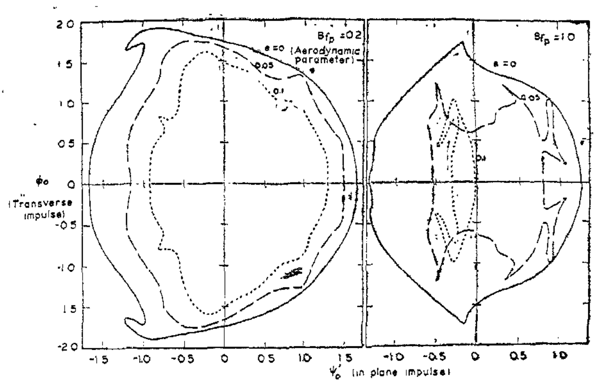

FIG. 3. Effect of aerodynamic torque and orbit eccentricity on the allowable impulsive distrubance for stable coupled motion of slender satellite[78].

All the results, particularly the latter few, indicate the significant role the forces due to the rarefied atmosphere can play, and emphasize their careful consideration in design.

2.1. *Attitude Stabilization and Control Using Solar Radiation Pressure*:

The solar radiation pressure is found to affect the performance of the spacecrafts adversely. A judicious design of the system can change its role-making it useful for stabilization and control. Such a possibility was first suggested by Garwin [76] who proposed " Solar-Sailing " for interplanetary missions. Though requiring a huge surface, it was shown to take less time than the chemical rockets for the distant planets. It was also thought to be simpler in implementation and operation than its competitor: ion propulsion. Sohn [77] suggested the use of a weathervane type solar attitude stabilizer. Frye and Stearns [2] thought of a trailing cone for the purpose. These were felt to be rather combursome appendages and Newton [78] preferred a satellite to be a big sphere having two types of coatings —one portion reflecting and the other absorbing. The local heating in such a system, however, could create problems. A possibility of increasing the effectiveness of the available force by focussing it through a set of reflecting and collecting mirrors was suggested by Hibbard [79]. Ule [80] designed and built an array of wind-mill type mirrors with corner reflectors which could maintain the axis of rotation sun-oriented and could regulate the spin-rate. In absence of damping the effectiveness of all these systems would be limited. Accord and Nikalas [81] evolved a unique passive stabilizer, technically feasible, which could also provide damping, through a thermo-mechanical phase-lag component. No velocity sensor is necessary. The system, now patented [82], is said to be better than the conventional ones. Donlin and Randall ' s model [83] is essentially similar. Mar and Vigneron [84] thought of using the shape deformations of a balloon satellite of the type of Echo-II

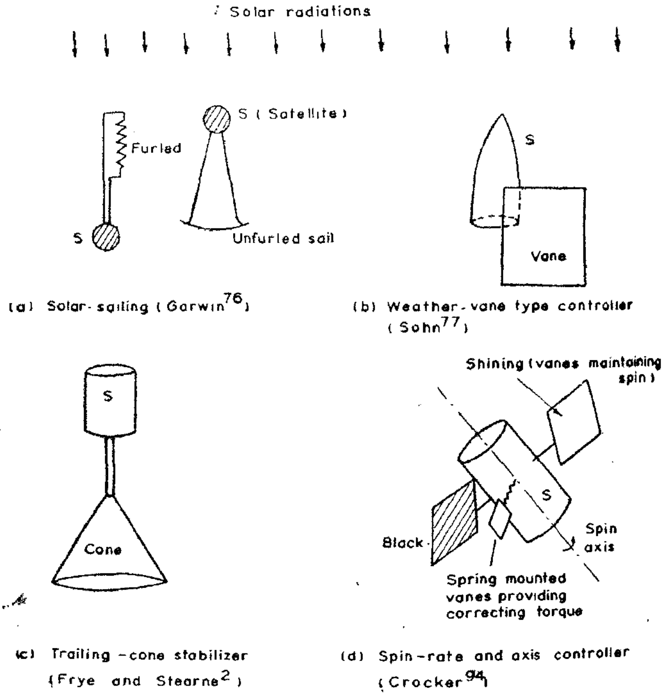

FIG. 4 Some concepts of solar controllers.

for spin-up. Galitskaya and Kisler [85] analysed a set of panels for three-axes stabilization and qualitatively established their optimum inclinations for the maximum utilization of the solar pressure. A number of theoretical studies [86–91] carried out at M.I.T. consider the dynamics, generally through linearization, of both spinning and non-spinning sun-orbiting spacecrafts stabilized by the solar radiation pressure. Aside from the conventional dampers a possibility of using the lag in thermal reradiation from the spacecraft's surface to produce non-conservative torques is also shown. The last of the above studies evolves a workable design of a three-axes controller for a small probe. Dzhumanoliev [92] analysed the small oscillations of a sun-pointing spacecraft with two coatings which result in a net control moment. A system of connected bodies to impart damping to a sun-pointing solar stabilizer is suggested by Merrick *et al.* [93]. To maintain spin and axis-orientation Crocker [94] proposed two sets of shining and black paddles. The spring-mounted set provides the desired change automatically. The preliminary analysis ignoring the gravity effects and assuming existence of a damper, shows good response. Figure 4 presents a few of the concepts developed above.

It is recognized that the gravity-oriented systems need damping to be effective. As discussed earlier they are also susceptible to even small disturbances. Mallach [95] using a phase-plane analysis and average torques considered the use of a solar pressure damper, Recently, Modi *et al.*, who had established the detrimental influence of radiations through the detailed nonlinear response and stability analyses, looked into the possibility of its utilization. Several semipassive systems involving a controlled change of area, moment arm or angle of incidence have been evolved [Fig. 5]. The first one was a simple velocity sensitive pitch damper. Its success both in circular and elliptic orbits under large external disturbances [96] lead to the development of a velocity- and position-sensitive controller which could stabilize the satellite about any desired in-plane orientation [97, 98]. The difficulties in making area changes through unfurlable material, etc., in space, were overcome by changing the moment arm [42]. This idea was later extended to coupled motion in circular orbit both in the ecliptic as well as other planes [99]. The response of several representative cases is analysed.

For dual-spin stabilized spacecrafts perhaps the only study on these lines is by Modi and Pande [100]. Nutation damping and attitude control is shown to be possible through a set of rotatable panels. The generalized anlysis for vehicles in any orbit, with a reference to INTELSAT-IV and Anik–1 shows the transient time to be only 1/8 orbit with the steady-state errors

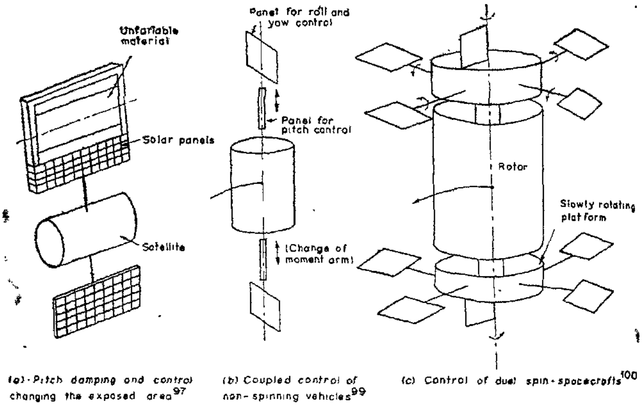

FIG. 5. Typical configuration of semi-passive solar controllers.

completely removed through a modified control function. The performance of this and other systems is further improved by numerical optimization [97, 101]. Though quite effective and promising these systems need attitude and angular-rate sensors and on-board computation. This may be difficult for smaller satellites.

It may be mentioned that a solar pressure controller has been successfully tested, despite some initial difficulties, on Mariner IV spacecraft [32, 81, 102]. This success may pave the way for implementation of the numerous ideas presented above.

2.2. *Aerodynamic Controllers*:

The aerodynamic forces have been shown to have a substantial influence on the attitude motion of the near-earth systems. Unlike solar radiation pressure, the literature on the advantageous applications of these forces is rather small. This may be due to the complex nature of the aerodynamic forces, rotation of atmosphere and strong dependence of density on height, season, sun-position and local changes [14]. The lack of a complete understanding of the atmospheric model also adds to the limited effort in their utilization. Yet, the few studies that have been made do show a feasibility of evolving a good aerodynamic attitude controller, which might also add to the life of the near-earth system. Such possibilities were indicated qualitatively in the early analyses by Debra and Stearns [103], Wall [104] and Schiello [105]. The success of aerodynamic pitch control, with the roll and yaw stabilized by gyros, was exhibited by COSMOS–149 [31]. In a model suggested by Hoffer [106] the gyros are replaced by a set of moving masses. Here the planar librations are reduced by a pair of drag flaps operating on an on-off control. This system may be difficult to implement because of the large changes in inertia. Ravindran and Hughes [107, 108] optimized, through linearization, a set of aerodynamic controller paddles for a satellite in a circular orbit. The system provides stabilization along the local horizontal. Using Liapunov's criteria the stability of such a controller is studied by Flanagan and Rangarajan [109].

For the general case of non-linear, coupled motion in an arbitrary orbit, Modi and Shrivastava [72] proposed several schemes of rotatable flaps which use both lift and drag. The velocity-sensitive controller results in an effective damping and stabilization of a gravity-gradient system. Later, an aerodynamic controller which stabilizes the system about any arbitrary spatial orientation is evolved using a velocity- and position-sensitive system. Through parametric optimization [110] the transient time is reduced to less than 1/3 orbit and the steady-state amplitude [in elliptic orbits] to smaller than 1°.

Noting the success of solar controllers and realizing the limitations of aerodynamic systems in elliptic orbits where the usable force may be significant only near the perigee, a simple hybrid control which employs atmospheric forces at lower altitudes and solar pressure at upper levels is developed [111]. The analysis is general enough to be applicable for aerodynamic, solar or a hybrid system. A modified control relation removes the steady-state errors even for large eccentricities. A similar system for a spinning satellite is also analysed [112, 113].

A schematic presentation of the above controllers is shown in Fig. 6.

The studies clearly indicate a need for further investigation of the possibilities of use of these semi-passive controllers because of their promise as light weight, low power, long life systems.

Concluding Remarks

The review of the literature suggests a considerable interest both in the influence of the environmental forces and their advantageous utilization for attitude control of satellites. A majority of the studies are, however

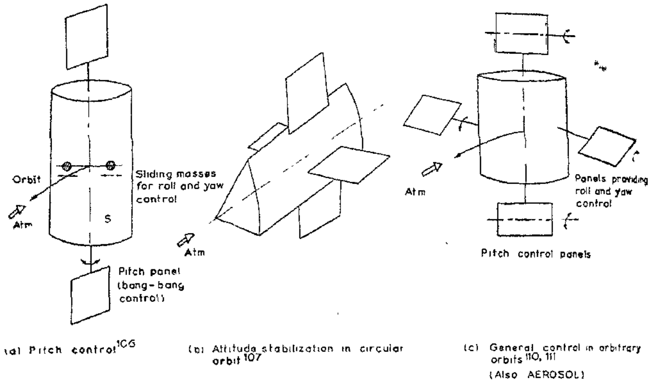

FIG. 6. Use of atmosphere for attitude stabilization and control.

theoretical or qualitative in nature. For the successful implementation of the promising ideas considerable work on hardware development and testing needs to be done.

It may be mentioned that this report does not cover the related important areas of orbital perturbations and corrections, radiation heating, and flexibility. In designing a system these too must be carefully considered.

An attempt has been made here to highlight the most important aspects of the studies available in the recent open literature. It is recognized that many important reports, particularly those in languages other than English may have been left out. It is felt, however, that this report may draw the attention of designers to the importance of the effects and may also stimulate thinking about some of the new concepts.

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REFERENCES

[1] Roberson, R. E. .. A review of the current status of satellite attitude control. *Journal of Astronautical Sciences*, 1959, **7**, 25.

[2] Frye, W. E. and Stearns, E. V. B. Stabilization and attitude control of satellite vehicles. *ARS Journal*, 1959, **29**, 927–931.

[3] Ergin, E. I. .. Current status of progress in attitude control. *Guidance and Control-II, Prog. in Astronautics and Aeronautics*, Academic Press, 1964, **13**, 7–36.

[4] Sabroff, A. E. .. Advanced spacecraft stabilization and control techniques. *Journal of Spacecraft and Rockets*, 1968, **5**, 1377–1393.

[5] Shrivastava, S. K., Tschann, C. and Modi, V. J. Librational dynamics of earth orbiting satellites A brief review. *Proc. XIV Cong. of Theo. and App. Mech. Kurukshetra, India*, 1969, pp. 284–306.

[6] Musen, P. .. The influence of solar radiation pressure on the motion of an artificial satellite. *Journal of Geophysical Research*, 1960, **65**, 1391.

[7] Parkinson, R. W., Jones, H. M. and Shapiro, I. I. Effects of solar radiation pressure on earth satellite orbits. *Science*, 1960, **131**, 920.

[8] Bryant, R. W. .. The effect of solar radiation pressure on the motion of an artificial satellite. *Astronautical Journal*, 1961, **66**, 430.

[9] Kozai, Y. .. Effects of solar radiation pressure on the motion of an artificial satellite. *Smithsonian Contribution to Astrophysics*, 1963, **6**, 109.

[10] Poliakhova, E. N. .. Solar radiation pressure and the motion of an earth satellite. *AIAA Journal*, 1963, **1**, 2893–2909.

[11] Adams, W. M. Jr. and Hodge, W. F. Influence of solar radiation pressure on orbital eccentricities of a gravity-gradient-oriented lenticular satellite. *NASA TND*-2715, 1965.

[12] Baker, R. M. L. Jr. .. Radiation on a satellite in the presence of partly diffuse and partly specular reflecting body. *COSPAR–IAU–IUTAM Symposium on Trajetories of Artificial Celestial Bodies*, Ed. Kovalevsky, Springer–Verlag, Paris, 1965.

[13] King–Hele, D. G. and Walker, J. G. Contraction of orbits due to drag in a spherically symmetric rotating atmosphere. *Space Research*, North Holland Publishers, **2**, 918.

[14] Jensen, J., Townsend, G., Kork, J. and Kaft, D. *Design Guide to Orbital Flight*. McGraw–Hill, New York, 1962.

[15] Roberson, R. E. .. Attitude control of a satellite vehicle A an outline of the problem. *Proc. VIII Int. Astro. Cong.*, Wein-Springer-Verlag, Berlin, 1958, pp. 317–319.

[16] Hall, H. B. .. The effect of radiation force on satellites of convex shape. *NASA TND*-604, 1961.

[17] Evans, W. J. .. Aerodynamic and radiation disturbance torques on satellites having complex geometry. *Journal of Astronautical Sciences*, 1962, **9**, 93–99.

[18] Wiggins, L. E. .. Relative magnitudes of space-environment torques on a satellite. *AIAA Journal*, 1964, **2**, 770–771.

[19] Clancy, T. F. and Mitchell, T. P. Effects of radiation forces on the attitude of an artificial earth satellite. *AIAA Journal*, 1964, **2**, 517–524.

[20] Hughes, W. G. .. Attitude control: environmental torques on a satellite. *ESRO Space Technology*, 1966, Vol. **11**.

[21] Tidwell, N. W. .. Modelling of environmental torques of a spin stabilized spacecraft in a near-earth orbit. *Journal of Spacecraft and Rockets*, 1970, **7** 1425–1433.

[22] Flanagan, R. C. and Modi, V. J. Radiation forces on a flat plate in ecliptic near-earth orbits. *CASI Transactions*, 1970, **3**, 147–158

[23] Bryant, R. W. .. A comparison of theory and observation of the Echo I satellite. *NASA TND*-1124, 1962.

[24] Fedor, F. W. .. The effect of solar radiation pressure on the spin of explorer XII. *NASA TND*-1855, 1963.

[25] Etkin, B. and Hughes, P. C. Explanation of the anomalous spin behaviour of satellite with long flexible antennae. *Journal of Spacecraft and Rockets*, 1967, **4**, 1139–1145.

[26] Hughes, P. C. and Cherchas, D. B. Influence of solar radiation on the spin behaviour of satellites with long flexible antennae. *CASI Transactionn*, 1969, **2**, 53–57.

[27] Whisnant, J. M. and Anand, D. K. Roll resonance for a gravity gradient satellite. *Journal of Spacecraft and Rockets*, 1968, **5**, 743–744.

[28] Whisnant, J. M., Waszkiewicz, P. R. and Piscane, V. L. Attitude performance of the GEOS–II gravity-gradient spacecraft. *Journal of Spacecraft and Rockets*, 1969, **6**, 1379–1384.

[29] Beletskii, V. V. .. Interaction of the aerodynamic stream with a satellite according to an analysis of the motion of 'Proton–2' about its center of mass. *Cosmic Research*, 1970, **8**, 475–483.

[30] Davison, G. J. and Mersan, R. H. The effect of direct solar radiation on the attitude of the SKYNET spacecraft *JBIS*, 1973, **26**, 228–291.

[31] Sarychev, V. A. .. Aerodynamic stabilization of satellites. *Proc. Int. Colloquim on Attitude Changes and Stabilization of Satellites, Paris*, 1968, pp. 177–179.

[32] Scull, J. R. .. Mariner IV revisited—or the tale of the ancient Mariner. *XX Int. Astronautical Cong. Argentina*, 1969.

[33] McElvain, R. J. .. Effects of solar radiation pressure upon satellite attitude control. *ARS Preprint No.* 1918–61, 1961.

[34] McElvain, R. J. and Schavartz, L. Minimization of solar radiation pressure effects for gravity-gradient stabilized satellites. *Journal of Basic Engg. Trans. ASME*, *D*88, 1966, **2**, 444–450.

[35] Karymov, A. A. .. Stability of rotational motion of a geometrically symmetrical artificial satellite of the sun in the field of light pressure forces. *Journal of Appl. Maths. and Mechanics (P.M.I.M.)*, 1964, **28**, 923–930.

[36] Flanagan, R. C. .. Effect of environmental forces on the attitude dynamics of gravity oriented satellites. *Ph.D. Thesis*, Univ. of British Columbia, 1969.

[37] Flanagan, R. C. and Modi, V. J. Attitude dynamics of a gravity oriented satellite under the influence of solar radiation pressure. *Aeronautical Journal, R.A.S.*, 1970, **74**, 835–841.

[38] Modi, V. J. and Flanagan, R. C. Effect of environmental forces on the attitude of gravity oriented satellites: Part I: High altitude orbits. *Aeronautical Journal, R.A.S.*, 1971, **75**, 783–793.

[39] Modi, V. J. and Flanagan, R. C. Effect of environmental forces on the attitude of gravity oriented satellites: Part II: Intermediate orbit accounting for earth radiation. *Aeronautical Journal, R.A.S.*, 1971, **75**, 846–849.

[40] Flanagan, R. C. and Modi, V. J. Effect of environmental forces on the attitude dynamics of gravity oriented satellites: Part III: close-earth orbits accouning for aerodynamic forces. *Aeronautical Journal R.A.S.*, 1972, **76**, 34–40.

[41] Modi, V. J. and Kumar, K. Librational dynamics of gravity oriented satellite under the influence of solar radiation pressure. *Proc. V Int. Symp. on Computer Aided Engg., Waterloo*, 1971, p. 359.

[42] Modi, V. J. and Kumar, K. Coupled librational dynamics and attitude control of satellites in presence of solar radiation pressure *Astronautical Research, Ed.* Napolitano *et al.*, D. Reidel Pub. Co., 1971, pp. 37–52.

[43] Kumar, K. .. Effect of solar radiations on the attitude dynamics of gravity-oriented satellites. *Ph.D. Thesis*, Univ. of British Columbia, 1972.

[44] Modi, V. J. and Pande, K. C. Solar pressure induced librations of spinning axi symmetric satellites. *Journal of Spacecraft and Rockets*, 1973, **10**, 615.

[45] Hodge, W. F. .. Effect of environmental torques, on the short term attitude prediction for a rolling wheel spacecraft in a sun-synchronous orbit. *NASA TND*–6583, 1972.

[46] Fixler, S. Z. .. Effect of solar environment and aerodynamic drag on structural booms in space. *Journal of Spacecraft and Rockets*, 1967, **4**, 315–321.

[47] Hughes, P. C. .. Attitude dynamics of a three axis stabilized satellite with a large solar array. *The Journal of Astronautical Sciences*, 1972, **20**, 166–189.

[48] Modi, V. J. and Kumar, K. Librational dynamics of a satellite with thermally flexed appendages. *AAS/AIAA Astrodynamics Specialists Conf. Vail, Colorado*, 1973.

[49] Modi, V. J. and Kumar, K. .. Effect of structural flexibility on spacecraft control systems *NASA SP*–8016, 1969.

[50] Noll, R. B., Deyst, J. J. and Spenny, C. H. .. A sturvey of structural flexibility effects on spacecraft control system. *AIAA 7th Aerospace Science Meeting*, New York, Paper No. 69–116, 1969.

[51] Ashley, H. .. Observations on the dynamic behaviour of large flexible bodies in orbit. *AIAA Journal*, 1967, **5**, 460–469.

[52] Likins, P. W., and Bouvier, H. K. .. Attitude control of non-rigid spacecraft. *Astronautics and Aeronautics*, 1971, **9**, 64–71.

[53] Hughes, P. C. .. Recent advances in the attitude dynamics of spacecraft with flexible solar arrays. *CAS Journal*, 1973, **19**, 165–171.

[54] Modi, V. J. .. Attitude dynamics of satellites with flexible appendages—brief review. *Journal of Spacecraft and Rockets*, 1974, **11**, 743–751.

[55] Schaaf, S. A., and Chambré, P. L., .. *Flow of Rarefied Gases*, Princeton Univ. Press, 1961, pp. 17-24.

[56] Debra, D. B. .. The effect of aerodynamic forces on satellite attitude. *The Journal of Astronautical Sciences*, 1959, **6**, 40-45.

[57] Beletskii, V. V. .. Motion of an artificial earth satellite about the center of mass. *Artificial Earth Satellite*, Kurnosova, Plenum Press, New York, 1960. **1**, 30–59.

[58] Schrello, D. M. .. Aerodynamic influence on satellite librations. *ARS Journal* 1961, **31**, 442–444.

[59] Garber, T. B. .. Influence of constant disturbance torques on the motion of gravity-gradient stabilized satellites. *AIAA Journal*, 1963, **1**, 968–969.

[60] Sarychev, V. A. .. The influence of the atmosphere drag on gravity attitude stabilization of artificial earth-satellites (In Russian) *Kosmicheskie Issledovaniia*, 1964, **2**, 667–678.

[61] Sarychev, V. A. .. Dynamics of a satellite gravitational stabilization system with consideration of atmospheric resistance. *Proc. XI Int. Cong. Appl. Mech.*, Springer-Verlag, Berlin, 1966.

[62] Meirovitch, L. and Wallace, F. B., Jr. .. On the effect of aerodynamic and gravitational torques on the attitude stability of satellites. *AIAA Journal*, 1966, **4**, 2196–2202.

[63] Morozov, V. M. .. Stability of the motion of a gyrostat under the action of gravitational, magnetic and aerodynamic forces. *Cosmic Research*, 1967, **5**, 727–732.

[64] Morozov, V. M. .. Stability of the relative equilibrium of satellite acted upon by gravitational, magnetic and aerodynamic moments. *Cosmic Research*, 1969, **7**, 356–361.

[65] Nurre, G. S. .. Effect of aerodynamic torque on an asymmetric gravity stabilized satellite. *Journal of Spacecraft and Rockets*, 1968, **6**, 1046–1050.

[66] Frik, M. A. .. Attitude stability of satellites subjected to gravity-gradient and aerodynamic torques. *AIAA Journal*, 1970, **8**, 1780.

[67] Shrivastava, S. K. and Modi, V. J. Effect of atmosphere on attitude dynamics of axisymmetric satellites. *Proc. XX Int. Astro. Cong.*, 1972, pp. 535–562, Pergamon Press/PWN–Polish Scientific Publishers.

[68] Shrivastava, S. K., Modi, V. J. and Soudack, A. Analog study of atmospheric effect on satellite librations. *Proc. XV Cong. of Theoretical and Appl. Mech*, Ranchi, India, 1970, pp. 353–368.

[69] Shrivastava, S. K. .. On coupled librational dynamics of gravity-oriented axisymmetric satellites. *Ph.D. Thesis*, Univ. of British Columbia, 1970.

[70] Modi, V. J. and Shrivastava, S. K. On the limiting regular stability and periodic solutions of a gravity oriented system in the presence of atmosphere. *CASI Transactions*, 1972, **5**, 5–10.

[71] Modi, V. J. and Shrivastava, S. K. Approximate solution for coupled librations of an axisymmetric satellite in a circular orbit. *AIAA Journal*, 1971, **9**, 1212-1214.

[72] Modi, V. J. and Shrivastava, S. K. Librations of gravity-oriented satellites in elliptic orbits through atmosphere. *AIAA Journal*, 1971, **9**, 2208–2215.

[73] Anand, D. K., Whishanant, J. M. and Sturmanis, M. Attitude perturbations of a slowly spinning multi-body satellite. *Journal of Spacecraft and Rockets*, 1969, **6**, 324.

[74] Fueschel, P. G., Bainum, P. M. and Grunberger, P. J. Attitude motion of a nutatoinally damped dual-spin spacecraft in the presence of near-earth environment. *Journal of Spacecraft and Rockets*, 1971, **8**, 913.

[75] Sheporaitis, L. P. .. Stochastic stability of a satellite influenced by aerodynamic and gravity-gradient torques. *AIAA Journal*, 1971, **9**, 218–222.

[76] Garwin, R. L. .. Solar Sailing—A practical method of propulsion within the solar system. *Jet Propulsion*, 1958, **28**, 188–190.

[77] Sohn, R. L. .. Attitude stabilization by means of solar radiation pressure. *ARS Journal*, 1959, **29**, 371–373.

[78] Newton, R. R. .. Stabilizing a spherical satellite by radiation pressure. *ARS Journal*, 1960, 30, 1173.

[79] Hibbard, R. R. .. Attitude stabilization using focussed radiation pressure. *ARS Journal*, 1961, **31**, 844–845.

[80] Ule, L. A. .. Orientation of spinning satellites by radiation pressure. *AIAA Journal*, 1963, **1**, 1575–1578.

[81] Accord, J. D. and Nikalas, J. C. Theoretical and practical aspects of solar pressure attitude control for interplanetary spacecraft. *Guidance and Control II, Advances in Astronautical Sciences*, Ed. Langford and Mundo, Academic Press. 1964, 73-101,

[82] Nikalas, J. C. *et al.* .. Attitude control for spacecrafts. *Patent*, *JPL*, California Institute of Technology.

[83] Donlin, T. J. and Randall, J. C. A solar-vane actuation system for spacecraft attitude control. *ASME Mech. Conf. Lafayette*, 1964, A65-10602.

[84] Mar J. and Vigneron, F. R. Passive spin propulsion of larger flexible spherically shaped satellites by the solar radiation field. *IUTAM/COSPAR/IAV Joint Symp. on Trajectories of Artificial Bodies*, 1965, pp. 151–163, Spring–Verlag.

[85] Galitskaya, E. B. and Kiseler, M. I. Radiation control of the orientation of space probes. *Cosmic Research*, 1965, **3**, 298–301.

[86] Carrel, J. R. and Limburg, R. C. Dynamics of a solar pressure stabilised satellite. *M.S. Thesis, MIT*, 1965, N65-32703.

[87] Peterson, C. A. .. Use of thermal re-radiation effects in spacecraft attitude control. *CSR-TR-66-3, MIT Center for Space Research*, 1966, N66-27738.

[88] Colombo, G. .. Passive stabilization of a sunblazer probe by means of radiation pressure torque. *CSR-CR-66-5, MIT Center for Space Research*, 1966.

[89] Harrington, J. C. .. The dynamics of a spinning solar pressure stabilised satellite with precession damping. *CSR-TR-66-6, MIT Center for Space Research*, N67-12064, 1966.

[90] Falcovitz, J. .. Attitude control of a spinning sun-orbiting spacecraft by mean of a grated solar sail. *CSR-TR-66-17, MIT Center for Space Research*, 1966.

[91] Naguyen, L. T. .. Three axis orientation and stabilisation of a probe using solar pressure. *M.S. Thesis, MIT*, N70-40999, 1970.

[92] Dzhumanoliev, N. D. *et al.* Small oscillations of black and white bodies stabilised by radiation pressure. *Cosmic Research*, 1967, **5**.

[93] Merrick, V. K. Moran, F. J. and Tinling, B. E. A technique for passive attitude control of solar oriented interplanetary spacecraft. 1968, *NASA TND*-4641.

[94] Crocker, M. C. .. Attitude control of a sun-pointing spinning spacecraft by means of solar radiation pressure. *Journal of Spacecraft and Rockets*, 1970, **7**, 357.

[95] Mallach, E. G. .. Solar pressure damping of the librations of a gravity-gradient oriented satellite. *AIAA Student Journal*, 1966, p. 4.

[96] Modi, V. J. and Flanagan, R. C. Librational damping of a gravity-oriented system by using solar radiation pressure. *Aeronautical Journal, R.A.S.*, 1971, **75**, 560–564.

[97] Modi, V. J. and Tschann, C. On the attitude and librational control of a satellite using solar radiation pressure. *Proc. XXI Int. Astro. Cong.*, 1971, pp. 84–100, North Holland Pub. Co.

[98] Modi, V. J. and Tschann, C. The solar radiation damping of a gravity-oriented satellite using W.K.B. Method. *Israel Journal of Tech*, 1973, **11**, 53–61.

[99] Modi, V. J. and Kumar, K. Attitude control of satellites using the solar radiation pressure. *Journal of Spacecraft and Rockets*, 1972, **9**, 711–713.

[100] Modi, V. J. Pande, K. C. and Kumar, K. On the optimized attitude contiol of space vehicles using solar radiation pressure. *Proc. VI Symp. of IFAC, U.S.S.R.*, 1973.

[101] Modi, V. J. and Pande, K. C. Solar pressure control of a dual spin satellite. *Journal of Spacecraft and Rockets*, 1973, **10**, 355–361.

[102] Kopf, E. H. Jr. .. Solar pressure effect on spacecraft attitude after gas system depletion. *JPL, Calif. Inst.* of *Technology*, 1966, N67–15707.

[103] Debia, D. and Stearns, E. V. *Attitude Control, Elec. Engg.*, 1958, **77**, 1088.

[104] Wall, J. K. .. The feasibility of aerodynamic attitude stabilisation of a satellite vehicle. *ARS Controllable Satellite Conf., MIT, Cambridge, Mass.*, 1959.

[105] Schrello, D. M. .. Dynamic stability of aerodynamically responsive satellites, *J. Aerospace Sciences*, 1962, **29**, 442–444.

[106] Hoffer, E. P. .. Time optimal semiactive attitude control for the pitch motion of a satellite. *XXII Int. Astro. Cong., Brussels*, Belgium, 1971.

[107] Ravindran, R. .. Optimal aerodynamic attitude stabilization of near earth satellites. *Ph.D. Thesis*, Univ. of Toronto, 1971.

[108] Ravindran, R. and Hughes, P. C. Optimal aerodynamic attitude stabilisation of near earth satellites. *Journal of Spacecraft and Rockets*, 1972, **9**, 499.

[109] Flanagan, R. C. and Rangarajan, R. Liapunov stability analysis and attitude response of a passively stabilized space system. *Astronautica Acta*, 1973, **18**, 21.

[110] Modi, V. J. and Shrivastava, S. K. On the optimised performance of a semipassive aerodynamic controller. *AIAA Journal*, 1973, **11**, 1080–1085.

[111] Modi, V. J. and Shrivastava, S. K. AEROSOL: A semipassive hybrid control system. *Israel Journal of Technology*, 1975, **13**, 63–72.

[112] Modi, V. J. and Pande, K. C. Aerodynamic solar hybrid attitude control of near-earth satellites. To be published.

[113] Pande, K. C. .. Attitude control of spinning satellites using environmetal forces. *Ph.D. Thesis*, Univ. of British Columbia, 1973.