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Effects of Superheavy Quarks and Leptons in Low-Energy Weak Processes $K_L \to \mu\bar{\mu}$, $K^+ \to \pi^+ \nu\bar{\nu}$ and $K^0 \leftrightarrow \bar{K}^0$

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We investigate potentially important effects due to the existence of superheavy quarks and leptons of the sequential type in higher-order weak processes at low energies. The second-order $\Delta S \neq 0$ neutral-current processes $K_L \to \mu\bar{\mu}$, $K^+ \to \pi^+ \nu\bar{\nu}$ and K_L-K_S mass difference are analysed allowing for fermions of masses comparable to or larger than the weak-boson mass in the Kobayashi-Maskawa scheme and in the general sequential scheme with an arbitrary number of generations. Possible connection between heavy-quark masses and light-heavy quark mixing are also examined. The requirement that the rare decay processes such as $K_L \to \mu\bar{\mu}$ and $K^+ \to \pi^+ \nu\bar{\nu}$ be absent up to order αG_F yields a rather stringent bound on the magnitude of light-heavy quark mixing: Such mixing has to be less than m_W/m_{Quark} times a factor much smaller than unity.

§ 1. Introduction

It is by now more or less established that there exist at least three generations of quarks and leptons. A striking feature of their spectra is that the fermion masses increase by large factors from one generation to the next. Therefore, it is by no means unrealistic to suspect that there exist in nature superheavy (as heavy as or heavier than the weak-bosons) quarks and leptons.[1),2)]

An interesting possibility is that such superheavy fermions may manifest themselves in low-energy weak processes through discernible effects as higher order corrections to the effective interactions involving only light fermions. This expectation will be fulfilled if superheavy fermions are not decoupled from low-energy processes, unlike in QED and QCD.[3)] Renormalization corrections to the weak-boson masses and some other problems have been studied from this viewpoint.[2),4)]

The purpose of this paper is to analyse the possible effects of superheavy quarks and leptons in the rare weak processes $K_L \to \mu\bar{\mu}$, $K^+ \to \pi^+ \nu\bar{\nu}$ and $K^0 \leftrightarrow \bar{K}^0$. These reactions occur due to effective $\Delta S \neq 0$ neutral-currents which are induced at the one-loop level of weak interactions. It will be shown that superheavy fermions are not decoupled from such low energy processes, giving potentially important effects. It should not be difficult to have an idea of how this non-decoupling of fermions of mass $m_F \gg m_W$ comes about. The Yukawa coupling of unphysical scalars (in R_ξ gauge) is proportional to m_F. Therefore, powers of scalar couplings could compensate inverse powers of m_F arising from superheavy fermion propagators. However, to know how important the resulting effects of superheavy fermions are, we have to make explicit calculations.

We take the $SU_L(2)\times U(1)$ model for unified weak and electromagnetic interactions[5] and consider a general scheme of an arbitrary number of generations N of quarks and leptons. Neutrinos are assumed to be massless. Only heavy quarks contribute to the processes $K_L\to\mu\bar{\mu}$ and $K^0\leftrightarrow\bar{K}^0$, while both heavy quarks and leptons come into play in the process $K^+\to\pi^+\nu\bar{\nu}$. For the former processes, the problem is to uncover the interplay between the masses of heavy quarks and the light-heavy quark mixing. For the latter, the problem is whether its decay rate can tell us something about the number and/or masses of charged heavy leptons which might exist in nature.

The effects of heavy quarks in the Weinberg-Salam model have been discussed in earlier works.[1),2),6)~10)] Heavy quarks much lighter than the weak-bosons were considered in Refs. 6)~8). There has so far been no attempt to analyse the effects of superheavy fermions in low-energy kaon processes taking account of all the weak interaction diagrams at the one-loop level (diagrams with unphysical scalar exchange, which were not considered in earlier works,[1),9)] will be shown to give dominant contributions for fermions much heavier than the weak-bosons).

The perturbation calculation of weak interactions would eventually break down in the limit of very heavy fermion mass m_F. This would occur for $m_F^2/m_W^2 \gtrsim \alpha/4\pi$, i.e., $m_F\gtrsim$ a few TeV. For $m_F\lesssim 1$ TeV, which are fermion masses we will consider in this paper, higher order weak corrections are still small and the perturbation calculation can be trusted. Small one-loop corrections could, however, have important effects on processes which are absent in the lowest order, such as $\Delta S\neq 0$ neutral-current processes.

The transitions $K_L\to\mu\bar{\mu}$, $K^+\to\pi^+\nu\bar{\nu}$ and $K^0\leftrightarrow\bar{K}^0$ are described by effective $\Delta S\neq 0$ neutral-current interactions of light quarks and leptons. The effective four-Fermion Lagrangian can be obtained from the sum of Z-exchange and box-diagram amplitudes. We will take the free-quark model to compute these second-order (in weak interactions) diagrams. Strong interaction corrections may be taken into account in the standard manner in QCD, and they are known not to be significantly large.[11)]

The effective Lagrangian for light quarks and leptons is obtained in § 2. The Z-exchange and box-diagram amplitudes are computed in §§ 2.1 and 2.2. The result is given in the arbitrary R_ξ gauge.[13)] Only the final expressions are given in the text, while more details are given in Appendices A, B and C. Section 3 contains an analysis of the effects of heavy quarks and leptons in the decay rates of $K_L\to\mu\bar{\mu}$ and $K^+\to\pi^+\nu\bar{\nu}$ and in the K_L-K_S mass difference. Here, a quantitative analysis is made in the Kobayashi-Maskawa model.[14)] In § 4 we speculate on possible connection between heavy quark masses and light-heavy quark mixing in the limit of large quark masses.

§ 2. Effective Lagrangian for $d\bar{s}\to\mu\bar{\mu}$, $d\bar{s}\to\nu\bar{\nu}$ and $d\bar{s}\to s\bar{d}$

Consider the sequential scheme with an arbitrary number of generations N, and denote the j-th quark doublet of charge $2/3$ and $-1/3$ and the j-th lepton doublet by $\begin{pmatrix} u_j \\ d_j \end{pmatrix}$ and $\begin{pmatrix} \nu_j \\ L_j \end{pmatrix}$ respectively. In this notation, $u=u_1$, $d=d_1$ and $\nu_e=\nu_1$, $e=L_1$. We will assume that all neutrinos are massless, so that there is no mixing among leptons. The hadronic charged current takes the form

$$J_\mu=\sum_{j,k}\bar{u}_j\gamma_\mu(1-\gamma_5)U_{jk}d_k\,, \tag{2·1}$$

where U_{jk} is an $N\times N$ unitary matrix.

The computation of the amplitudes for $K_L\to\mu\bar{\mu}$, $K^+\to\pi^+\nu\bar{\nu}$ and $K^0\leftrightarrow\bar{K}^0$ amounts to finding the effective Lagrangian for the elementary processes $d\bar{s}\to\mu\bar{\mu}$, $d\bar{s}\to\nu\bar{\nu}$ and $d\bar{s}\to s\bar{d}$[15)~18)] and the evaluation of the matrix elements thereof. There are two classes of diagrams that contribute to these elementary processes: One consists of Z exchange diagrams generated by the induced $d\bar{s}Z$ coupling (Fig. 1), the other of the box-diagram in which weak-bosons $W^\pm$ and unphysical scalar $\phi^\pm$ are exchanged (box-diagrams for $d\bar{s}\to\nu\bar{\nu}$ are shown in Fig. 2).

Throughout the paper, the external fermion masses and momenta will be set to zero compared with the internal fermion masses. The one-loop amplitudes for the three elementary processes are then described concisely in terms of the effective four-Fermion interaction, which takes the form

$$\begin{aligned}\mathcal{L}_{\text{eff}}=\frac{G_F}{\sqrt{2}}\{&x[4\,\bar{s}_L\gamma_\mu d_L(\tilde{C}\bar{\mu}_L\gamma^\mu\mu_L-\sum_{i=1}^{N}\tilde{D}_i\bar{\nu}_{Li}\gamma^\mu\nu_{Li})+\tilde{E}(\bar{s}_L\gamma_\mu d_L)^2]\\&+(\alpha/4\pi)[\tilde{H}_1\,\bar{s}_L\gamma_\mu d_L+\tilde{H}_2\Box^{-1}\partial^\nu(m_s\,\bar{s}_L\sigma_{\mu\nu}d_L\\&\qquad+m_d\,\bar{s}_R\sigma_{\mu\nu}d_R)]\bar{\mu}\gamma^\mu\mu\}+\text{h.c.}\end{aligned} \tag{2·2}$$

with

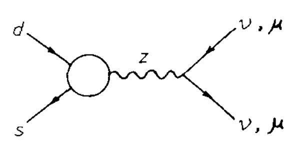


Fig. 1. The Z exchange diagram contributing to $d\bar{s}\to\nu\bar{\nu}$ and $d\bar{s}\to\mu\bar{\mu}$.

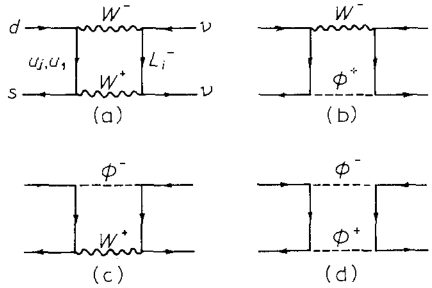


Fig. 2. The box-diagrams for $d\bar{s}\to\nu\bar{\nu}$.

$$\varkappa \equiv (\alpha/4\pi)/\sin^2\theta_W, \tag{2·3}$$

where θ_W is the Weinberg angle.

On the second line of the above equation, the first term gets contributions from both Z exchange and photon exchange, while the last two arise from photon exchange. These terms do not contribute to the $K_L \to \mu\bar{\mu}$ decay. They are included in the effective Lagrangian for $d\bar{s} \to \mu\mu$ for completeness and will be discussed only very briefly.

The coefficients $\tilde{C}$ and $\tilde{D}_i$ in $\mathcal{L}_{\rm eff}$ are given, with the aid of the unitarity relation $\sum_{j=1}^{N} U_{js}^* U_{jd}=0$, by the sum of $N-1$ heavy quark $(u_2, u_3, \cdots, u_N)$ contributions,

$$\tilde{C}(\{x_j\}) = \sum_{j=2}^{N} U_{js}^* U_{jd} \bar{C}(x_j, x_1=0),$$
$$\tilde{D}(\{x_j\}, y_i) = \sum_{j=2}^{N} U_{js}^* U_{jd} \tilde{D}(x_j, x_1=0; y_i), \tag{2·4}$$

where $x_j \equiv m_{uj}^2/m_W^2$ and $y_i \equiv m_{Li}^2/m_W^2$. As for the term for $d\bar{s} \to s\bar{d}$, $\tilde{E}$ is given by the sum of $(N-1)^2$ terms

$$\tilde{E}(\{x_j\}) = \sum_{j,k=2}^{N} U_{js}^* U_{jd} U_{ks}^* U_{kd} \bar{E}(x_j, x_k). \tag{2·5}$$

In Eqs. (2·4) and (2·5), the light quark (u_1) contribution is rearranged into other $N-1$ heavy quark contributions so that ultraviolet divergences cancel out by the GIM mechanism.[12] We will give below the result of our calculation of the coefficients, $\bar{C}$, $\bar{D}$ and $\bar{E}$ for two classes of diagrams separately.

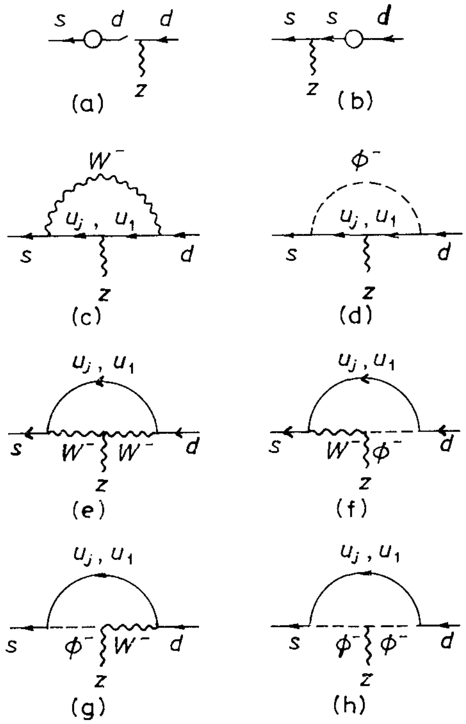
2.1. *Z exchange diagrams*

The one-loop diagrams for the induced $d\bar{s}Z$ coupling are shown in Fig. 3. The blob in the diagrams 3(a) and 3(b) represents the self-energy part of the $d \leftrightarrow s$ transition. The one-loop diagrams for this transition are shown in Fig. 4. The induced $d\bar{s}Z$ vertex takes the form

$$\Gamma_{Z_\mu}^{(i)} = \frac{1}{(4\pi)^2} \frac{g^3}{\cos\theta_W} U_{js}^* U_{jd} \bar{s}_L \gamma_\mu d_L \Gamma^{(i)}, \tag{2·6}$$

where $i=a, b, \cdots, h$. $\Gamma^{(i)}$ are given in Appendix A. The sum of all $\Gamma^{(i)}$ yields in the R_ξ gauge

$$\Gamma_Z \equiv \sum_{i=1}^{h} \Gamma^{(i)} = \frac{1}{4} x_j - \frac{3}{8} \frac{1}{x_j - 1}$$
$$+ \frac{3}{8} \frac{2x_j^2 - x_j}{(x_j-1)^2} \ln x_j + \gamma(x_j, \xi) - (x_j \to x_1). \tag{2·7}$$



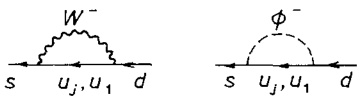


Fig. 4. The one-loop contributing to the self-energy part of the $d \leftrightarrow s$ transition, the blob in Fig. 3.

Fig. 3. The one-loop diagrams contributing to the induced $d\bar{s}Z$ vertex. The same diagrams with Z boson replaced by photon give the induced $d\bar{s}\gamma$ vertex.

(The singularities at $x_j=1$ in the second and third terms are superficial and are absent in their sum.) The ξ-dependent term in (2·7) is given by

$$\gamma(x,\xi)=\frac{1}{\xi x-1}\left(\frac{3}{4}\frac{1}{x-1}+\frac{1}{8}\frac{1}{\xi x-1}\right)x\ln x-\frac{1}{8}\frac{1}{\xi}\frac{1}{\xi x-1}$$
$$\times\left[\left(\frac{5\xi+1}{\xi-1}-\frac{1}{\xi x-1}\right)\ln\xi+1\right]. \qquad (2\cdot 8)$$

For $x_1 \ll x_j \ll 1$, the above results (2·7) and (2·8) coincide with those of Refs. 17) and 18) up to order of x_j. We note in passing that the term $x_j/4$ becomes dominant for large x_j in Eq. (2·7). This term arises, as seen from Appendix A, from diagrams of unphysical scalar exchange (diagrams, $a+b$, d and h of Fig. 3).

The contribution of the Z exchange diagram to $\mathcal{L}_{\mathrm{eff}}$ can immediately be found from the induced $d\bar{s}Z$ coupling (2·3). The result is

$$\bar{C}_Z=\bar{D}_Z=\Gamma_Z. \qquad (2\cdot 9)$$

The computation of the photon exchange contribution is somewhat more involved and requires the calculation of the induced $d\bar{s}\gamma$ coupling up to second order in the external momenta. The resulting coefficients $\tilde{H}_1$ and $\tilde{H}_2$ are given in

Appendix B.

2.2. *Box-diagrams*

There are four types of box-diagrams, as shown in Fig. 2. Their contributions to $\mathcal{L}_{\rm eff}$ are given in Appendix C. The sum of the four contributions yields for $d\bar{s}\to\mu\bar{\mu}$ and $d\bar{s}\to\nu\bar{\nu}$

$$\bar{C}_{\square}=-\frac{3}{8}\frac{x_j}{(x_j-1)^2}\ln x_j-\frac{3}{8}\frac{1}{x_j-1}-c(x_j,\xi)-(x_j\to x_1), \tag{2·10}$$

$$\begin{aligned}\bar{D}_{\square}=&-\frac{1}{8}\frac{1}{y_i-x_j}\left(\frac{y_i-4}{y_i-1}\right)^2 y_i\ln y_i\\&+\frac{1}{8}\left[\frac{x_j}{y_i-x_j}\left(\frac{x_j-4}{x_j-1}\right)^2+\frac{x_j-7}{x_j-1}\right]x_j\ln x_j\\&-\frac{9}{8}\frac{1}{y_i-1}\frac{1}{x_j-1}-d(x_j,\xi)-(x_j\to x_1),\end{aligned} \tag{2·11}$$

where $y_i\equiv m_{Li}^2/m_W{}^2$ with L_i denoting the lepton associated with the neutrino ν_i. The ξ-dependent terms $c(x,\xi)$ and $d(x,\xi)$ are found to be equal to $\gamma(x,\xi)$, as they should be.

As for $d\bar{s}\to\bar{s}d$, only box-diagrams contribute to this process(Fig. 2). The sum of their contributions to $\mathcal{L}_{\rm eff}$(see Appendix C) yields

$$\begin{aligned}\bar{E}(x_j,x_k)=-x_jx_k\Bigg\{&\frac{1}{x_j-x_k}\left[\frac{1}{4}-\frac{3}{2}\frac{1}{x_j-1}-\frac{3}{4}\frac{1}{(x_j-1)^2}\right]\ln x_j+(x_j\leftrightarrow x_k)\\&-\frac{3}{4}\frac{1}{(x_j-1)(x_k-1)}\Bigg\},\end{aligned} \tag{2·12}$$

$$\bar{E}(x_j,x_j)\equiv\bar{E}(x_j)=-\frac{3}{2}\left(\frac{x_j}{x_j-1}\right)^3\ln x_j-x_j\left[\frac{1}{4}-\frac{9}{4}\frac{1}{x_j-1}-\frac{3}{2}\frac{1}{(x_j-1)^2}\right]. \tag{2·13}$$

This result is of course ξ-independent. The result (2·12) differs from the expression used in Ref. 9), in which only the diagram of two-W-boson exchange was calculated in $\xi=1$ gauge.

2.3. *Coefficients $\bar{C}$ and $\bar{D}$*

Combining the results obtained in §§ 2.1 and 2.2 with x_1 being set to zero, we find for $d\bar{s}\to\mu\bar{\mu}$ and $d\bar{s}\to\nu\bar{\nu}$

$$\bar{C}=\bar{C}(x_j,x_1=0)=\frac{3}{4}\left(\frac{x_j}{x_j-1}\right)^2\ln x_j+\frac{1}{4}x_j-\frac{3}{4}\frac{x_j}{x_j-1}, \tag{2·14}$$

$$\begin{aligned}\bar{D}=&\bar{D}(x_j,x_1=0;y_i)\\=&-\frac{1}{8}\frac{y_ix_j}{y_i-x_j}\left(\frac{y_i-4}{y_i-1}\right)^2\ln y_i+\frac{1}{8}\left[\frac{x_j}{y_i-x_j}\left(\frac{x_j-4}{x_j-1}\right)^2\right.\end{aligned}$$

$$+1+3\frac{1}{(x_j-1)^2}\Big]x_j\ln x_j+\frac{1}{4}x_j-\frac{3}{8}\Big(1+3\frac{1}{y_i-1}\Big)\frac{x_j}{x_j-1}. \qquad (2\cdot15)$$

§ 3. Analysis of the processes $K_L\to\mu\bar{\mu}$, $K^-\to\pi^+\nu\bar{\nu}$ and the K_L-K_S mass difference

The effective Lagrangian (2·2) is of the usual $V-A$ current-current type, with the effect of heavy fermions contained in the coefficients $\tilde{C}$, $\tilde{D}$ and $\tilde{E}$. The matrix elements of these neutral-current products for $K_L\to\mu\bar{\mu}$ and $K^-\to\pi^+\nu\bar{\nu}$ can be related in a trivial manner to those of the charged-current products for $K^+\to\bar{\mu}\nu_\mu$ and $K^+\to\pi^0\bar{e}\nu_e$, respectively.[16] As for the matrix element for $K^0\leftrightarrow\bar{K}^0$, some complication arises from the fact that a hadron is present in both initial and final states.[11),7)] We will not discuss this problem but take the vacuum dominance approximation. We will first spell out the general expressions relating the coefficients $\tilde{C}$ and $\tilde{D}_i$ to the decay rates and $\tilde{E}$ to the K_L-K_S mass difference.

3.1. *General formulae*

a. $K_L\to\mu\bar{\mu}$

This process is *CP* conserving, so that only the real part of $U^*_{js}U_{jd}$ contributes to the decay rate. We find from Eqs. (2·2) and (2·4)

$$\frac{\tau(K^+)}{\tau(K_L)}\frac{B(K_L\to\mu\bar{\mu})_{sd}}{B(K^+\to\bar{\mu}\nu_\mu)}\simeq 4\chi^2\frac{[\mathrm{Re}\tilde{C}(\{x_j\})]^2}{U^2_{us}}, \qquad (3\cdot1)$$

where we have set $(1-4\,m_\mu{}^2/m_K{}^2)^{1/2}/(1-m_\mu{}^2/m_K{}^2)^2\simeq1$. The subscript *sd* stands for short-distance contribution, i.e., the one which is induced by the effective local interaction (2·2). χ is defined by Eq. (2·3). We take $\chi=0.0025$, which corresponds to $\sin^2\theta_W=0.23$.

$$\frac{B(K_L\to\mu\bar{\mu})_{sd}}{B(K^+\to\bar{\mu}\nu_\mu)}\simeq1.05\cdot10^{-4}\frac{[\mathrm{Re}\tilde{C}(\{x_j\})]^2}{U^2_{us}} \qquad (3\cdot2)$$

The short-distance contribution to $B(K_L\to\mu\bar{\mu})$ can be extracted from the measured value of $B(K_L\to\mu\bar{\mu})$ by taking account of other contributions including that from the two-photon intermediate state. Here we quote the result from previous analyses:[8),18),19)]

$$B(K_L\to\mu\bar{\mu})_{sd}\lesssim6\cdot10^{-9}. \qquad (3\cdot3)$$

Substitution of this empirical result into Eq. (3·2) yields the bound

$$[\sum_{j=2}^{N}\mathrm{Re}(U^*_{js}U_{jd})\bar{C}(x_j,0)]^2/U^2_{us}\lesssim0.8\cdot10^{-4}. \qquad (3\cdot4)$$

b. $K^+ \to \pi^+ \nu \bar{\nu}$

Since the neutrinos are assumed to be massless, they all contribute to this decay rate. Neglecting the electron mass, we have

$$\frac{B(K^+ \to \pi^+ \nu \bar{\nu})}{B(K^+ \to \pi^0 \bar{e} \nu_e)} = 2\varkappa^2 \sum_{i=1}^{N} \frac{|\tilde{D}_i(\{x_j\}, y_i)|^2}{U_{us}^2}. \tag{3·5}$$

Using $B(K^+ \to \pi^0 \bar{e} \nu_e) = 0.048$,[19] we have

$$B(K^+ \to \pi^+ \nu \bar{\nu}) = 0.60 \cdot 10^{-6} \sum_{i=1}^{N} |\tilde{D}_i(\{x_j\}, y_i)|^2 / U_{us}^2. \tag{3·6}$$

At present, experimentally, a very weak upper bound on the ratio $B(K^+ \to \pi^+ \nu \bar{\nu})$ is known:[20]

$$B(K^+ \to \pi^+ \nu \bar{\nu}) < 0.6 \cdot 10^{-6}. \tag{3·7}$$

Substitution of this upper bound into Eq. (3·6) yields

$$\sum_{i=1}^{N} | \sum_{j=2}^{N} U_{js}^* U_{jd} \bar{D}(x_j, 0; y_i)|^2 / U_{us}^2 < 1.0. \tag{3·8}$$

c. K_L-K_S mass difference

The K_L-K_S mass difference is given by

$$\Delta m_K = m_L - m_S \simeq -\frac{1}{m_K} \mathrm{Re} \langle \bar{K}^0 | \mathcal{L}_{\mathrm{eff}} | K^0 \rangle. \tag{3·9}$$

Assuming the vacuum dominance in the intermediate states, we obtain

$$\Delta m_K / m_K = (2/3) \varkappa (G_F / \sqrt{2}) f_K{}^2 \,\mathrm{Re}\, \tilde{E} \simeq 3.8 \cdot 10^{-2} \,\mathrm{Re}\, \tilde{E} \tag{3·10}$$

Let us see how the coefficients $\tilde{C}$, $\tilde{D}$ and $\tilde{E}$ behave for large x_j and y_i. As expected, superheavy fermions do give non-vanishing contributions to $\tilde{C}$, $\tilde{D}$ and $\tilde{E}$ in the limit of large x_j and y_i, unless the mixing factors are inhibitingly small. What is rather striking is that their importance increases linearly with $x_j \equiv m_{uj}^2 / m_W{}^2$, as seen from the following behaviours of $\bar{C}$, $\bar{D}$ and $\bar{E}$ for large x_j and y_i.

$$\bar{C}(x_j, 0) \sim \frac{1}{4} x_j, \qquad (1 \ll x_j) \tag{3·11}$$

$$\bar{D}(x_j, 0; y_i)$$

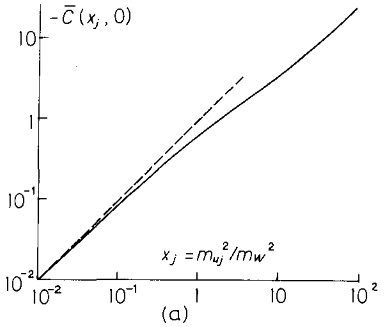


Fig. 5. (a) The coefficient $\bar{C}(x_j)$ as a function of x. The small x approximation is also shown (dashed line) for comparison.

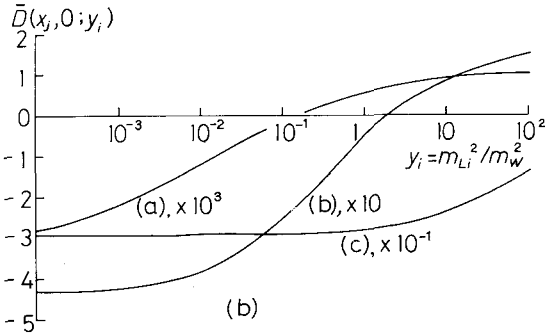


Fig. 5. (b) The coefficient $\bar{D}(x_j, y_i)$ as a function of y_i for typical values of x; (a) $m_{uj}=1.3$ GeV, (b) $m_{uj}=30$ GeV and (c) $m_{uj}=800$ GeV.

$$\sim\begin{cases} \frac{1}{4}x_j-\left(\frac{1}{8}y_j-\frac{3}{4}\right)\ln x_j-\frac{1}{8}y_j\left(1-\frac{3}{y_i-1}\right)^2\ln y_i \\ \qquad -\frac{3}{8}-\frac{9}{8}\frac{1}{y_i-1}, \qquad (1,\ y_i\ll x_j) \\ -x_j\left\{\frac{1}{8}\ln y_i-\frac{1}{8}\left[1+\frac{3}{(x_j-1)^2}\right]\ln x_j-\frac{1}{4}+\frac{3}{8}\frac{1}{x_j-1}\right\}, \qquad (1,\ x_j\ll y_i) \end{cases} \tag{3·12}$$

$$\bar{E}(x_j, x_k)\sim\begin{cases} -\frac{1}{4}x_j\ln(x_k/x_j), & (1\ll x_j\ll x_k) \\ -x_j\ln(x_k^{1/4}/x_j), & (x_j\ll 1\ll x_k) \end{cases} \tag{3·13}$$

$$\bar{E}(x_j)\sim-\frac{1}{4}x_j+\frac{3}{2}\ln x_j. \qquad (1\ll x_j) \tag{3·14}$$

How the functions $\bar{C}$ and $\bar{D}$ behave for smaller values of x_j and y_i can be found in Fig. 5, together with the approximate expressions for small x_j and y_i, which were used in earlier analyses.[7),8)] The difference between the exact expression and the approximate ones becomes significant, e.g., about 20% for $\bar{C}$, for $m_{uj}\gtrsim 30$ GeV.

We observe an interesting feature in Fig. 5(b) regarding the role of heavy leptons in the decay $K^+\to\pi^+\nu\bar{\nu}$. Though superheavy leptons are not decoupled from this low-energy process, as remarked before, the contribution of heavy leptons with masses $y_i\gg x_j$ to $\bar{D}(x_j, 0; y_i)$ is suppressed considerably; $\bar{D}(x_j, 0; y_i)$ even changes sign as y_i increases. Therefore, the number of massless neutrinos cannot be estimated by dividing the width $\Gamma(K^+\to\pi^+\nu\bar{\nu})$ by the electron-neutrino contribution to the width, as argued usually.[21)]

3.2. *Kobayashi-Maskawa model*

To derive more quantitative results we have to fix the number of generations.

Here we will analyse the three processes taking the simplest scheme, i.e., the six-quark model of Kobayashi and Maskawa.[14] Shrock et al.[7),8)] have made quite detailed analyses of the decay rate of $K_L \to \mu\bar{\mu}$ and the K_L-K_S mass difference, assuming $m_t \ll m_W$. We can now extend their analyses to the case $m_t \gtrsim m_W$ just by replacing their small quark-mass approximations of $\bar{C}$ and $\bar{E}$ by our exact expressions.

In the Kobayashi-Maskawa model, the mixing matrix elements U_{jk} are given in terms of three mixing angles θ_i and a CP violating phase δ. Define

$$\lambda_c \equiv U_{cs}^* U_{cd} = -(c_1 c_2 + s_2 t_3 e^{i\delta}) s_1 c_2 c_3,$$
$$\lambda_t \equiv U_{ts}^* U_{td} = -(c_1 s_2 - c_2 t_3 e^{i\delta}) s_1 s_2 c_3, \qquad (3\cdot15)$$

where $c_i = \cos\theta_i$, $s_i = \sin\theta_i$ and $t_i = \tan\theta_i$. The coefficients for the three processes are given by

$$\tilde{C} = \lambda_c \bar{C}(x_c, 0) + \lambda_t \bar{C}(x_t, 0), \qquad (3\cdot16)$$
$$\tilde{D}_e = \tilde{D}_\mu = \lambda_c \bar{D}(x_c, 0; 0) + \lambda_t \bar{D}(x_t, 0; 0),$$
$$\tilde{D}_\tau = \lambda_c \bar{D}(x_c, 0; y_\tau) + \lambda_t \bar{D}(x_t, 0; 0), \qquad (3\cdot17)$$
$$\tilde{E} = \lambda_c^2 \bar{E}(x_c) + 2\lambda_c \lambda_t \bar{E}(x_c, x_t) + \lambda_t^2 \bar{E}(x_t). \qquad (3\cdot18)$$

a. $K_L \to \mu\bar{\mu}$

The charmed quark contribution to (3·4) amounts to less than a few percent of the bound on the right-hand side, and hence it can be safely ignored. The experimental bound then yields

$$|(c_1 s_2 + c_2 t_3 c_\delta) s_2| \cdot |\bar{C}(x_t, 0)| \lesssim 0.9 \cdot 10^{-2}, \qquad (3\cdot19)$$

where $t_3 = \tan\theta_3$. Since c_1 is known precisely, the above inequality may be used to derive a bound on $|s_2|$ as a function of s_3 for given values of m_t. Figure 6

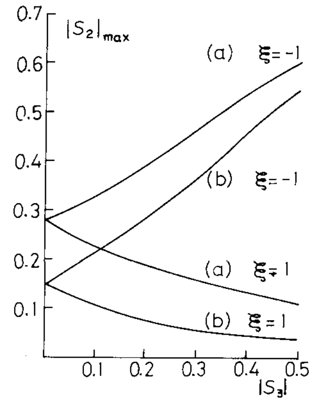


Fig. 6. Upper bounds on $|s_2|$ as a function of $|s_3|$ for given values of m_t; $m_t = 30$ GeV (a) and 60 GeV (b). The bounds are obtained by analyzing the $K_L \to \mu\bar{\mu}$ decay in the Kobayashi-Maskawa model. $\xi \equiv \mathrm{sgn}(t_2 t_3 c_\delta c_1^{-1})$.

presents such bounds on $|s_2|$ for $m_t=30$ GeV and 60 GeV.

b. K_L-K_S mass difference

Equation(3·10) gives

$$\mathrm{Re}(\lambda_c^2)\bar{E}(x_c)+2\mathrm{Re}(\lambda_c\lambda_t)\bar{E}(x_c,\ x_t)+\mathrm{Re}(\lambda_t^2)\bar{E}(x_t)\simeq 1.9\cdot 10^{-5}. \qquad (3\cdot 20)$$

Combined with another constraint equation for CP violation effect in the K^0-$\bar{K}^0$ transition, this equation can be used to derive conditional bounds on $|s_2|$ and $|\delta|$ as functions of $|s_3|$ for given values of m_t. The procedure of such an analysis is described in Ref. 7). Interested readers may repeat a similar exercise using $\tilde{E}$ given in the previous section.

c. $K^+\to\pi^+\nu\bar{\nu}$

The presently available bound (3·8) is too weak to derive meaningful constraints on the mixing angles. We will instead try to estimate the decay rate of $K^+\to\pi^+\nu\bar{\nu}$. This is possible if either the charmed quark or the top quark contribution dominates the effective Lagrangian; the two cases occur for $|s_2|\ll 10^{-1}$ and $|s_2|\gg 10^{-1}$ respectively.

The case of charmed quark dominance is reduced to the calculation in the four-quark scheme.[16] We then have

$$B(K^+\to\pi^+\nu\bar{\nu})\simeq 6.0\cdot 10^{-7}x_c^2\left[2\left(\frac{3}{2}\ln x_c+\frac{1}{2}\right)^2 + \left(2\ln y_\tau-\frac{1}{2}\ln x_c+\frac{1}{2}\right)^2\right]. \qquad (3\cdot 21)$$

For $m_\tau=1.81$ GeV and $m_c\simeq 1.3$ GeV, we have[22]

$$B(K^+\to\pi^+\nu\bar{\nu})\simeq 1.6\cdot 10^{-11}, \qquad (3\cdot 22)$$

which appears to be too tiny to be accessible experimentally.

As for the case of top quark dominance, we have from (3·5), (3·17) and (3·15)

$$B(K^+\to\pi^+\nu\bar{\nu})\simeq 6.0\cdot 10^{-7}|(c_1s_2-c_2t_3e^{i\delta})s_2|^2 \times\frac{3}{16}x_t^2\left[3\frac{x_t-2}{(x_t-1)^2}\ln x_t+\frac{x_t+2}{x_t-1}\right]^2. \qquad (3\cdot 23)$$

The ratio $B(K^+\to\pi^+\nu\bar{\nu})/B(K_L\to\mu\bar{\mu})$ can be predicted with less uncertainty,

$$\frac{B(K^+\to\pi^+\nu\bar{\nu})}{B(K_L\to\mu\bar{\mu})_{sd}}=0.009\frac{3|\bar{D}(x_t,\ 0;\ 0)|^2}{[\mathrm{Re}\ C(x_t,\ 0)]^2}. \qquad (3\cdot 24)$$

Figure 7 gives the ratio as a function of x_t. To make an order-of-magnitude estimate, suppose that the dispersive part of $B(K_L\to\mu\bar{\mu})$, $(3.2\pm 2.4)\cdot 10^{-9}$, obtained by use of unitarity[8),22)] is due to the short-distance contribution. Then, $B(K^+\to\pi^+\nu\bar{\nu})$ is predicted to be about 10^{-9} for $m_t\simeq 30$ GeV, a number which is smaller

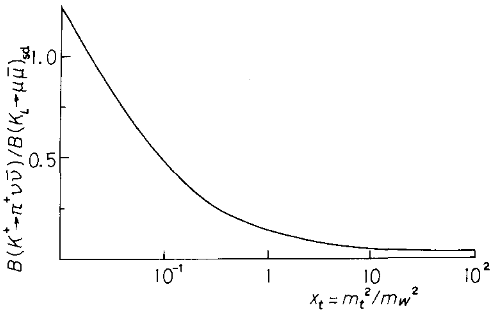


Fig. 7. The predicted ratio of $B(K^+\to\pi^+\nu\bar{\nu})/[B(K^-\to\mu\bar{\mu})]_{sd}$ as a function of the top quark mass ($x_t=m_t^2/m_W^2$) in the Kobayashi-Maskawa model.

than the presently available upper limit[20] by three orders.

§ 4. Light-heavy quark mixing

Some time ago, Glashow and Weinberg proposed a view that flavour-changing neutral currents are "naturally" suppressed to order αG_F.[23] Such view is valid if all quarks are much lighter than the weak-bosons. On the contrary, if there exist superheavy quarks, with masses comparable to the weak-boson mass, the absence of flavour changing neutral-currents can no longer be imposed naturally. The experimental fact that strangeness-changing neutral-currents are absent to αG_F has to explained by some dynamical mechanism rather than by the naturality argument.

A simple explanation of this suppression of $\Delta S\neq 0$ neutral-currents to αG_F is provided by assuming a certain correlation between heavy quark masses and light-heavy quark mixing. To see how this works, we demand that $\Delta S\neq 0$ neutral-current interactions are absent to order αG_F. This restriction means for $d\bar{s}\to\mu\bar{\mu}$ and $d\bar{s}\to\nu_i\bar{\nu}_i$

$$[\sum_{j=2}^{N}\mathrm{Re}(U_{js}^* U_{jd})\bar{C}(x_j,0)]^2/U_{us}^2\ll 1,$$
$$|\sum_{j=2}^{N} U_{js}^* U_{jd}\bar{D}(x_j,0;y_i)|^2/U_{us}^2\ll 1. \qquad (4\cdot1)$$

We further demand that these conditions are to be satisfied without invoking accidentally large cancelation between several terms of different generations. The constraint (4·1) then implies, by use of of Eqs. (3·11) and (3·12), that light-superheavy quark mixing satisfies

$$|U_{jq}|\ll x_j^{-1/2}=m_W/m_{uj}, \qquad (4\cdot2)$$

where q stands for light quarks d and s. Namely, $|U_{jq}|$ has to be less than m_W/m_{uj} times a factor much smaller than unity.

The above argument may be extended to flavour-changing neutral-current processes at low energies. The suppression of such neutral-currents to αG_F is now guaranteed if the inequality (4·2) is satisfied for all "light"-superheavy quark mixing. Here, "light" should perhaps mean lighter than the weak-bosons. "Light quarks" then include charmed quark and perhaps bottom quark.

Fröggatt and Nielsen[24] have recently put forward an argument that quark mixing angles are given order-of-magnitude wise in terms of quark masses by

$$|U_{jq}| \sim (m_q/m_j)^{1/2} \qquad \text{for} \quad m_q \ll m_j. \tag{4·3}$$

The two relations (4·2) and (4·3) are compatible provided that superheavy quark masses are bounded by

$$m_j/m_W \lesssim m_W/m_q. \tag{4·4}$$

The bound is most restrictive for $q=b$ quark and gives

$$m_j/m_W \lesssim 10. \tag{4·5}$$

It is curious that this constraint on heavy quark masses is consistent with the bound derived from the consideration of the one-loop corrections to the weak-bosons masses.[2]

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Appendix

In this appendix we give some details of our calculation of the effective $d\bar{s}Z$ and $d\bar{s}\gamma$ couplings and the box-diagrams. We cope with divergences which appear in the calculation of the $d\bar{s}Z$ couplings in the dimensional method. The results are presented in the $\xi=1$ gauge.

A. *Induced $d\bar{s}Z$ couplings*

The one-loop contribution to the $d\bar{s}Z$ coupling is calculated by a direct summation of Feynman graphs.[18] $\Gamma^{(i)}$, defined by Eq. (2·6), for each of the diagrams in Fig. 3 are given by

$$\Gamma^{(a+b)} = -\left[\frac{1}{2}(Q-1)\sin^2\theta_W + \frac{1}{4}\right]$$

$$\times\left[\frac{x_j^2}{(x_j-1)^2}\ln x_j-\frac{x_j}{x_j-1}-x_jf_1(x_j)\right]-(x_j\to x_1),$$

$$\Gamma^{(c)}=\left(\frac{1}{2}Q\sin^2\theta_W-\frac{1}{4}\right)\frac{x_j^2}{(x_j-1)^2}\ln x_j+\frac{1}{2}\frac{x_j}{(x_j-1)^2}\ln x_j$$

$$-\left(\frac{1}{2}Q\sin^2\theta_W+\frac{1}{4}\right)\frac{x_j}{x_j-1}-(x_j\to x_1),$$

$$\Gamma^{(d)}=-\frac{1}{2}Q\sin^2\theta_W\left(1-\frac{2}{n}\right)x_jf_2(x_j)+\frac{1}{4}(Q\sin^2\theta_W-1)$$

$$\times\left[\frac{x_j^2}{(x_j-1)^2}\ln x_j-x_j-\frac{x_j}{x_j-1}\right]-(x_j\to x_1),$$

$$\Gamma^{(e)}=\frac{3}{2}(1-\sin^2\theta_W)\left[\frac{x_j^2}{(x_j-1)^2}\ln x_j-\frac{1}{x_j-1}\right]-(x_j\to x_1),$$

$$\Gamma^{(f+g)}=\sin^2\theta_W\left[\frac{x_j^2}{(x_j-1)^2}\ln x_j-\frac{x_j}{x_j-1}\right]-(x_j\to x_1),$$

$$\Gamma^{(h)}=\left(\frac{1}{2}-\sin^2\theta_W\right)\left\{\frac{1}{4}\left[\frac{x_j^2}{(x_j-1)^2}\ln x_j-\frac{x_j}{x_j-1}\right]\right.$$

$$\left.-\frac{1}{n}x_jf_2(x_j)\right\}-(x_j\to x_1), \qquad (A\cdot 1)$$

where $x_j=m_j^2/m_W^2$ and $Q=2/3$, the charge of up-quarks, $f_1(x)$ and $f_2(x)$ are given by

$$f_1(x)=-\frac{1}{n-4}+\frac{1}{2}\left[-\gamma_E+\ln(4\pi)-\ln m_W^2\right]$$

$$+\frac{3}{4}-\frac{1}{2}\left[\frac{x^2}{(x-1)^2}\ln x-\frac{1}{x-1}\right],$$

$$f_2(x)=-2\frac{1}{n-4}-\gamma_E+\ln(4\pi)-\ln m_W^2+1-\frac{x}{x-1}\ln x. \qquad (A\cdot 2)$$

We note that no divergent integrals are involved in $\Gamma^{(f+g)}$, while divergent integrals become convergent because of the GIM mechanism in $\Gamma^{(c)}$ and $\Gamma^{(e)}$. As for $\Gamma^{(a+b)}$, $\Gamma^{(d)}$ and $\Gamma^{(h)}$, the terms $x_jf_1(x_j)-x_1f_1(x_1)$ and $x_jf_2(x_j)-x_1f_2(x_1)$ have poles at $n=4$. These poles disappear from the sum of the three terms, leaving a finite term

$$\Gamma^{(a+b)}+\Gamma^{(d)}+\Gamma^{(h)}=-\frac{1}{2}[(Q-1)\sin^2\theta_W+1]$$

$$\times\left[\frac{x_j^2}{(x_j-1)^2}\ln x_j-\frac{x_j}{x_j-1}\right]+\frac{1}{4}x_j-(x_j\to x_1). \qquad (A\cdot 3)$$

The sum of all terms in (A·1) thus yields Eqs. (2·7) and (2·8). Note also that the sum of the terms with Z coupled to the quark line is independent of the charge Q.

B. *Induced $d\bar{s}\gamma$ coupling*

The diagrams to be computed are those of Fig. 3 with Z being replaced by γ. The induced $d\bar{s}\gamma$ vertex takes the form

$$\Gamma_{\gamma\mu}=\frac{1}{(4\pi)^2}e\frac{g^2}{2M_W{}^2}U_{js}^* U_{jd}\,\bar{s}\Big[F_1(q^2\gamma_\mu-q_\mu \not{q})\frac{1-\gamma_5}{2}$$
$$+F_2\sigma_{\mu\nu}iq^\nu\Big(m_s\frac{1-\gamma_5}{2}+m_d\frac{1+\gamma_5}{2}\Big)\Big]d. \qquad (\text{B}\cdot 1)$$

Computing the one-loop diagrams to second order in the external momenta and masses, we find

$$F_1=Q\Big\{\Big[\frac{1}{12}\frac{1}{x_j-1}+\frac{13}{12}\frac{1}{(x_j-1)^2}-\frac{1}{2}\frac{1}{(x_j-1)^3}\Big]x_j$$
$$+\Big[\frac{2}{3}\frac{1}{x_j-1}+\Big(\frac{2}{3}\frac{1}{(x_j-1)^2}-\frac{5}{6}\frac{1}{(x_j-1)^3}+\frac{1}{2}\frac{1}{(x_j-1)^4}\Big)x_j\Big]\ln x_j\Big\}$$
$$-\Big[\frac{7}{3}\frac{1}{x_j-1}+\frac{13}{12}\frac{1}{(x_j-1)^2}-\frac{1}{2}\frac{1}{(x_j-1)^3}\Big]x_j$$
$$-\Big[\frac{1}{6}\frac{1}{x_j-1}-\frac{35}{12}\frac{1}{(x_j-1)^2}-\frac{5}{6}\frac{1}{(x_j-1)^3}+\frac{1}{2}\frac{1}{(x_j-1)^4}\Big]x_j\ln x_j$$
$$+f_1(\xi,x_j)-(x_j\to x_1) \qquad (\text{B}\cdot 2)$$

with $f_1(\xi,x)=-2\gamma(\xi,x)$ and

$$F_2=-Q\Big\{\Big[-\frac{1}{4}\frac{1}{x_j-1}+\frac{3}{4}\frac{1}{(x_j-1)^2}+\frac{3}{2}\frac{1}{(x_j-1)^3}\Big]x_j-\frac{3}{2}\frac{x_j{}^2}{(x_j-1)^4}\ln x_j\Big\}$$
$$+\Big[\frac{1}{2}\frac{1}{(x_j-1)}+\frac{9}{4}\frac{1}{(x_j-1)^2}+\frac{3}{2}\frac{1}{(x_j-1)^3}\Big]x_j-\frac{3}{2}\frac{x_j{}^3}{(x_j-1)^4}\ln x_j,$$
$$(\text{B}\cdot 3)$$

where the terms multiplied by Q arise from the diagrams (c)~(d). The coefficients H_1 and H_2 are given by $H_a=\sum_{j=2}^N U_{js}^* U_{jd}\ \bar{H}_a(x_j, x_1=0)$ $(a=1,2)$, where $\bar{H}_1=-4\bar{F}_1-8\bar{\Gamma}_z$ and $H_2=-4\bar{F}_2$.

C. *Box-diagrams*

There are four types of box-diagrams, as shown in Fig.2.

The three diagrams with unphysical scalar exchange do not contribute to the process $d\bar{s}\to\mu\bar{\mu}$ when the muon mass is neglected. The remaining W-exchange diagram 4(a) yields the contribution given by (2·10).

The contributions of the four diagrams to the process $d\bar{s}\to\nu_i\bar{\nu}_i$ are

$$D_{\square}^{(a)}=2[g_1(x_j,\ y_i)-g_1(x_1,\ y_i)],$$

$$D_{\square}^{(b)}=D_{\square}^{(c)}=-\frac{1}{2}y_i[x_j g_0(x_j,\ y_i)-x_1 g_0(x_1,\ y_i)],$$

$$D_{\square}^{(d)}=\frac{1}{8}y_j[x_j g_1(x_j,\ y_i)-x_1 g_1(x_1,y_i)],\qquad (C.1)$$

where $x_j\equiv m_{uj}^2/m_W{}^2$ and $y_i\equiv m_{Li}^2/m_W{}^2$, and

$$g_1(x,\ y)=-\frac{1}{y-x}\left[\left(\frac{y}{y-1}\right)^2\ln y-\left(\frac{x}{x-1}\right)^2\ln x-\frac{1}{y-1}+\frac{1}{x-1}\right],$$

$$g_0(x,\ y)=-\frac{1}{y-x}\left[\frac{y}{(y-1)^2}\ln y-\frac{x}{(x-1)^2}\ln x-\frac{1}{y-1}+\frac{1}{x-1}\right].\qquad (C.2)$$

The sum of the four terms in (C.1) yields (2.11).

For the transition $d\bar{s}\to s\bar{d}$, we give the result which can be used in the general sequential scheme. Note that the box-diagram amplitude F is given by the N^2 terms,

$$F=\sum_{j=1}^{N}\sum_{k=1}^{N}\lambda_j\lambda_k F(m_{uj},\ m_{uk}),\qquad (C\cdot 3)$$

where N is the number of generations and λ_j is defined as a product of mixing matrix elements, i.e., $\lambda_j\equiv U_{js}^* U_{jd}$(U_{jk} is introduced in § 2 (II.d.). After a rearrangement using the unitarity relation for U_{jk}, $\lambda_u=-\sum_{j=2}^N\lambda_j$, the right-hand side of Eq. (C·3) can be written as

$$F=\sum_{j=2}^{N}\sum_{k=2}^{N}\lambda_j\lambda_k[F(m_{uj},\ m_{uk})$$
$$-F(m_{u1},\ m_{uk})-F(m_{uj},\ m_{u1})+F(m_{u1},\ m_{u1})].\qquad (C\cdot 4)$$

Correspondingly, the contributions to $\mathcal{L}_{\rm eff}$ from the four diagrams with exchange of j-th and k-th up-quarks can be written as

$$\bar{E}=2\sum_{i=a}^{d}[E_{\square}^{(i)}(x_j,\ x_k)$$
$$-E_{\square}^{(i)}(x_1,x_k)-E_{\square}^{(i)}(x_j,\ x_1)-E_{\square}^{(i)}(x_1,\ x_1)],\qquad (C\cdot 5)$$

where $i=a$, b, c and d. The calculation of $E^{(i)}$ $(x_j,\ x_k)$ is similar to that of $D^{(i)}$ for $d\bar{s}\to\nu\bar{\nu}$. The result is

$$E_{\square}^{(a)}(x_j,\ x_k)=-\frac{1}{2}g_1(x_j,\ x_k),$$

$$E_{\square}^{(b)}(x_j, x_k)=E_{\square}^{(c)}(x_j,x_k)=\frac{1}{2}x_jx_kg_0(x_j, x_k),$$

$$E_{\square}^{(d)}(x_j,x_k)=-\frac{1}{8}x_j,x_kg_1(x_j, x_k). \qquad (\mathrm{C}\cdot 6)$$

The sum of the four graphs yields

$$\begin{aligned}E_{\square}(x_j,x_k)&\equiv 2\sum_{i=a}^{d} E_{\square}^{(i)}(x_j, x_k)\\&=\frac{3}{4}g_1(x_j, x_k)-\left[\frac{x_jx_k}{x_j-x_k}\left(\frac{1}{4}-\frac{3}{2}\frac{1}{x_j-1}\right)+\frac{7}{4}\frac{x_j}{(x_j-1)^2}\right]\\&\qquad\times\ln x_j+\frac{7}{8}+\frac{7}{4}\frac{1}{x_j-1}+(x_j\leftrightarrow x_k). \qquad (\mathrm{C}\cdot 7)\end{aligned}$$

$\bar{E}(x_j, x_k)$ is then given by Eq. (C·5) with x_u being set to zero. Substituting Eq. (C·7) into Eq. (C·5) gives Eq. (2·12). $\bar{E}(x_j)$ is then obtained by taking the limit $x_k\to x_j$ in Eq. (2·12).

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