EFFECTS OF THE KELVIN-HELMHOLTZ SURFACE INSTABILITY ON SUPERSONIC JETS

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Abstract

We have performed an exact numerical calculation of the linear growth and phase velocity of Kelvin-Helmholtz unstable wave modes on a supersonic jet of cylindrical cross section. An expression for the maximally unstable wavenumber of each wave mode is found. Provided a sharp velocity discontinuity exists all wave modes are unstable. A combination of rapid jet expansion and velocity shear across a jet can effectively stabilize all wave modes. The more likely case of slow jet expansion and of velocity shear at the jet surface allows wave modes with maximally unstable wavelength longer than or on the order of the jet radius to grow. The relative energy in different wave modes and effect on the jet is investigated. Energy input into a jet resulting from surface instability is discussed.

I. Introduction

It has been suggested that surface instabilities such as the Kelvin-Helmholtz instability generate internal turbulence in jets and radio sources which can serve to reaccelerate particles (Pacholczyk and Scott 1976) and Dobrowolny (1972) has shown that the Kelvin-Helmholtz instability generates MHD waves in a collisionless plasma. Several authors have assumed the existence of turbulence in the form of small amplitude MHD waves and used this to investigate particle acceleration. The wave spectrum was assumed to be a power law (Lacombe 1977; Ferrari, Trussoni and Zaninetti 1979) or related to the growth rate of surface instabilities (Eilek 1979). The MHD waves were found to heat the fluid and accelerate particles but did not lead to an appropriate particle energy distribution or emission spectrum if radiative losses were included. These calculations also assumed a uniform input of wave energy throughout a source but energy input at the surface implies that wave energy is not uniformly distributed. This last problem has been addressed by Eilek (1982) who finds that energy input restricted to a thin surface layer produces a turbulent edge in MHD waves and strong limb brightening can occur. However, observed radio jets do not appear to be strongly limb brightened. A partial resolution of these problems may be found through investigation of the behavior of perturbations to a jet surface which is Kelvin-Helmholtz unstable.

Supersonic jets in pressure balance with an external medium are unstable to surface perturbations analogous to the Kelvin-Helmholtz instability of a vortex sheet (Ferrari, Trussoni and Zaninetti 1978; Hardee 1979; Ray 1981). Growing perturbations of long maximally unstable wavelength are pinching and helical wave modes and previous work has provided estimates of wave phase velocity and maximum rate of growth (Ray 1981; Hardee 1982). In addition to these wave modes a cylindrical jet with sharp velocity discontinuity is unstable to an infinite number of other wave modes with shorter maximally unstable wavelength and more rapid rate of growth. These growing waves convert directed flow energy into wave energy in the jet and external medium. Because initial growth of surface perturbations is rapid several authors have investigated ways of slowing the rate of growth and stabilizing a jet. The maximum rate of growth is decreased as the amplitude of a helical wave increases (Benford 1981) and growth of perturbations to the jet surface can be linear rather than exponential if a jet expands rapidly enough (Hardee 1982). Short wavelength perturbations to a jet surface can be stabilized by the presence of a velocity shear between jet and external medium (Ray 1982). While it may be possible to completely stabilize a jet by a combination of jet expansion and velocity shear, true jet conditions are not yet well known and it seems likely that some instability will be present. For this reason we must consider the behavior of long and short wavelength perturbations to a jet and consider the effect of the resulting growing wave modes on a jet.

In this paper we will proceed by calculating exactly the wave phase velocity, growth rate and maximally unstable wavelength of Kelvin-Helmholtz unstable eigenmodes on a jet of cylindrical cross section. Both jet and external medium are assumed to be isothermal (see Hardee 1982, hereafter H II). The estimates made in H II forpinching and helical wave modes are confirmed and the work is extended to the wave modes describing perturbations of shorter wavelength. Stabilization of these wave modes is investigated. The depth interior to the jet surface affected by a wave mode is derived and relative energy input into a jet is estimated. The consequences for particle acceleration models are discussed.

II. Dispersion Relation and Eigenmodes

It was shown in HII that the wave modes which propagate along the surface of a jet of cylindrical cross section are eigenmodes of the dispersion relation

$$\frac{J_n'(k\Psi\xi_n)\; H_n^{(1)}(k\Psi\zeta_n)}{J_n(k\Psi\xi_n)\; H_n^{(1)\prime}(k\Psi\zeta_n)}\; \frac{\xi_n}{\zeta_n} = \gamma^2 \eta\; \frac{(\Phi - M_{in})^2}{\Phi^2} \; . \tag{1}$$

In equation (1) J_n is the Bessel function and $H_n^{(1)}$ is the Hankel function describing an outward propagating disturbance. The primes indicate derivatives with respect to the arguments of the Bessel and Hankel functions and the meaning of the symbols is

$$\xi_n \equiv \gamma[(\Phi - M_{in})^2 - (1 - \frac{u^2/c^2}{M_{in}}\Phi)^2 - \frac{n^2}{\gamma^2 k^2} + 2i(1 - \frac{u^2/c^2}{M_{in}}\Phi)/\gamma k]^{1/2}$$

and

$$\zeta_n \equiv [\Phi^2 - (1 + n^2/k^2 - 2i/k)\nu]^{1/2}/\nu^{1/2}$$

where

$$\Phi = \omega/ka_{in} \qquad M_{in} = u/a_{in} \qquad \gamma = (1-u^2/c^2)^{-1/2}$$

$$\eta = \frac{\rho_{in} + P_o/c^2}{\rho_{ex} + P_o/c^2} \qquad \nu = a_{ex}^2/a_{in}^2 \; .$$

The subscript in or ex refers to quantities interior or exterior to the jet respectively. This dispersion relation is valid for jets of constant radius or expanding jets of instantaneous radius $R = r\Psi$ where r is the distance from the origin and $\Psi \equiv \sin\Theta$ where Θ is one half the jet opening angle. In this notation $k\Psi = k_{ex}R$ is the wavenumber normalized to the instantaneous jet radius and $k_{ex} = 2\pi/\lambda$ is the wavenumber. Eigenmodes of the dispersion relation are wave modes with $n = 0, 1, 2 \ldots$. A wave mode propagates along the jet surface with form $A = A_n \exp[i(k_{ex}r + n\phi - \omega t)]$.

The $n = 0$ wave mode pinches a jet and the $n = 1$ wave mode helically distorts a jet. In HII it was shown that these two wave modes were maximally unstable at wavelengths $\lambda_o^{(p)} \sim 0.6(M_{in}/\eta^{1/3})R$ and $\lambda_1^{(p)} \sim 1.6(M_{in}/\eta^{1/3})R$. The wave mode characterized by $n = 2$ distorts a jet in an elliptical shape; the wave mode characterized by $n = 3$ distorts a jet in a triangular shape, etc.. For convenience we will call all wave modes with $n > 1$, fluting wave modes. These waves will not alter the jet cross section appreciably like the $n = 0$ wave mode or move the jet axis transversely like the $n = 1$ wave mode.

We expect fluting wave modes to be maximally unstable at some wavelength $\lambda_n^{(p)}$ just as we found for the $n = 0$ and $n = 1$ wave modes. To see that this must be the case we take equation 1 and write it in the following form (Appendix B of HII)

$$\gamma^2 \eta \frac{(\Phi - M_{in})^2}{\Phi^2} = \frac{\xi_n}{\zeta_n} \frac{\left[\frac{n}{k\Psi\xi_n} - J_{n+1}(k\Psi\xi_n)/J_n(k\Psi\xi_n)\right]}{\left[-\frac{n}{k\Psi\zeta_n} + H_{n-1}^{(1)}(k\Psi\zeta_n)/H_n^{(1)}(k\Psi\zeta_n)\right]} \qquad (2)$$

When $n/k\Psi\xi_n >> 1$ and $n/k\Psi\zeta_n >> 1$ equation (2) is easily seen to become

$$\gamma^2 \eta \frac{(\Phi - M_{in})^2}{\Phi^2} \sim -1 \qquad (3)$$

This expression is just the usual expression for Kelvin-Helmholtz instability of a vortex sheet:

$$\omega \sim \left[\frac{\gamma^2\eta}{1+\gamma^2\eta} \pm i \frac{\gamma\eta^{1/2}}{1+\gamma^2\eta}\right] ku. \qquad (4)$$

We can see that the small wavenumber approximation used to obtain equation (4) remains valid to larger wavenumbers as n is increased. Thus we expect equation (4) to adequately describe the growth and propagation of waves with higher n to larger wavenumber, and we expect faster growth at shorter wavelength as n is increased. We can show that a maximally unstable wavenumber must exist by examining equation (1) in the limit $n/k\Psi\xi_n << 1$ and $n/k\Psi\zeta_n << 1$. In this limit equation (1) can be written (Appendix B of HII) in the following form

$$e^{2i\chi_n} \sim \frac{[\Phi^2\xi_n+\gamma^2\eta(\Phi-M_{in})^2\zeta_n+i\frac{n}{k\Psi}[\gamma^2\eta(\Phi-M_{in})^2 + \Phi^2]}{[\Phi^2\xi_n-\gamma^2\eta(\Phi-M_{in})^2\zeta_n-i\frac{n}{k\Psi}[\gamma^2\eta(\Phi-M_{in})^2 + \Phi^2]} \tag{5}$$

where $\chi_n \equiv k\Psi\xi_n - (\frac{2n+1}{4})\pi$.

If we take the natural logarithm of equation (5) then

$$\xi_n \sim [\frac{2n+1}{4}] \frac{\pi}{k\Psi} - \frac{i}{2k\Psi} A^{(n)} \tag{6}$$

where $A^{(n)}$ is the logarithm of the right hand side of equation (5). If equation (6) is squared and we use $\xi_n \sim [(\Phi-M_{in})^2-1]^{1/2}$ and assume that $A_R^{(n)} >> A_I^{(n)}$ where $A_R^{(n)}$ and $A_I^{(n)}$ are the real and imaginary points of $A^{(n)}$, respectively, it follows that

$$\Phi \sim (M_{in}-1) - i\frac{(2n+1)\pi}{8k^2\Psi^2} A_R^{(n)} \tag{7}$$

We need to point out an error in equations (14) and (22) in HII in which the imaginary part needs to be multiplied by a factor of 2. For $\Phi_R \lesssim M_{in} - 1$ we find that $A_R^{(n)} < 0$ and all wave modes are unstable but with rapidly decreasing growth rate as the wavenumber becomes larger. Thus a maximally unstable wavenumber must exist. At a given wavenumber we see that the growth rate in equation (7) becomes larger as n increases.

The maximally unstable wavenumber, maximum growth rate and wave phase velocity of the different wave modes cannot be obtained analytically. We have found exact solutions to equation (1) numerically. In figure 1 we plot the growth rate $\omega_I R$ in cm $\sec^{-1}$, where R is instantaneous jet radius as a function of the normalized wavenumber $k\Psi = 2\pi R/\lambda$ for wave modes n equal 0 through 5 and 7 and 9. The calculations assume a jet 10 times denser than the surrounding medium, $\eta = 10$, that the jet velocity $u = 9 \times 10^8$ cm s^{-1}, that $M_{in} \equiv u/a_{in} = 9.9$, and that the jet is expanding with an opening angle $2\Theta = 0.08$ radian. We find that the maximally unstable wavenumber is not sensitive to jet opening angle. These exact results when combined with the density and Mach number dependence of the maximally unstable wavenumber from HII which is valid when $\eta > 1$ and $M_{in} >> 1$ show that

$$k_n^{(p)} \Psi \sim 1.9 \left(n + \frac{1.1}{n^{3/2} + 0.2}\right)(\eta^{1/3}/M_{in}) \tag{8}$$

At large n, $k_n^{(p)}\Psi \sim 1.9n(\eta^{1/3}/M_{in})$. The asymptotic expression is accurate to 10% when n=3.

The corresponding relationship for the wavelength is

$$\lambda_n^{(p)} \sim 3.3 \left[\frac{M_{in}/\eta^{1/3}}{n+1.1/(n^{3/2}+0.2)} \right] R \tag{9}$$

For n less than 3 we have that $\lambda_o^{(p)} \sim 0.6\ (M_{in}/\eta^{1/3})R$, $\lambda_1^{(p)} \sim 1.7\ (M_{in}/\eta^{1/3})R$ and $\lambda_2^{(p)} \sim 1.4(M_{in}/\eta^{1/3})R$. For larger values of n, $\lambda_n^{(p)} \sim 3.3n^{-1}(M_{in}/\eta^{1/3})R$ is accurate to better than 10%. The values of $\lambda_o^{(p)}$ and $\lambda_1^{(p)}$ found here are in excellent agreement with the estimates given in HII.

The phase velocity of the different wave modes normalized to the jet velocity u as a function of the normalized wavenumber is shown in figure 2. At wavenumbers much less than the maximally unstable wavenumber the phase velocity, v_{ph}, approaches u or $[\gamma^2\eta/(1+\gamma^2\eta]u$ for the n = 0 and n > 0 wave modes, respectively. At wavenumbers much larger than the maximally unstable wavenumber the phase velocity approaches $u - a_{in}$ for all $n \gtrsim 0$. For the n = 0 wave mode the minimum phase velocity occurs at wavenumber larger than the maximally unstable wavenumber. The phase velocity at the maximally unstable wavenumber is $v_{ph} \sim u-a_{in}$. This is the result for a small amplitude wave driven by the Bernoulli effect. Larger amplitudes will develop only for smaller wavenumbers and higher phase velocities and any large scale jet pinching should develop with wavelength greater than $\lambda_o^{(p)}$. For all other wave modes the minimum phase velocity occurs near the maximally unstable wavenumber. In general, when $n \geq 1$ we may assume that the dominant wavenumber and wavelength associated with a particular wave mode is within 10% of the value given by equations (8) and (9). We note that the reduction in phase velocity seen at the dominant wavenumber implies more rapid spatial growth than if the phase velocity remained between $[\gamma^2\eta/(1+\gamma^2\eta]$ u and $u-a_{in}$ as assumed in HII. For n = 1 the minimum phase velocity is about 10% less than the value predicted using equation (4). This suggests that spatial growth of the helical wave is 10% faster than was estimated in HII. For other wave modes as n becomes larger the maximum growth rate approaches that predicted using equation (4) with $k = k_n^{(p)}$ and v_{ph} is reduced significantly below that predicted using equation (4) with result that spatial growth is somewhat more rapid than would be predicted using equation (4).

While this result depends on M_{in} and η we find no large change as M_{in} and η are varied. As a consequence when n is sufficiently large, we expect initial wave mode growth at small wave amplitude to be approximated by equation (4) until non-linear effects at larger amplitude become important. Of course, if jet expansion is rapid enough the wave remains in the small amplitude limit and spatial growth is linear as the maximally unstable wavelength increases proportional to $R = r\Psi$ (see HII).

III. Jet Application

Jet expansion can serve to stabilize the wave modes which are maximally unstable at longer wavelength. To see that this cannot be the case for wave modes that are maximally unstable at shorter wavelength recall that $v_{ph} \lesssim [\gamma^2\eta/(1+\gamma^2\eta)]u$ and $\omega_I \lesssim [\gamma\eta^{1/2}/(1+\gamma^2\eta)]2\pi u/\lambda_n^{(p)}$ for all wave modes with $n \geq 1$. We define a spatial growth length as $\ell_e = v_{ph}\tau_e$ where $\tau_e = \omega_I^{-1}$. If we use equation (4) exactly to obtain v_{ph} and ω_I and use $\lambda_n^{(p)} \sim 3.3\ n^{-1}(M_{in}/\eta^{1/3})R$ then $\ell_e \sim 0.52\ n^{-1}\ \eta^{1/6}\ M_{in}R$. This value of the spatial growth length is sufficiently accurate for our purposes and exact calculation shows the numerical factor is only a weak function of n. Wave amplitude grows as $A = A_o e^{(r/\ell_e)}$ where r is distance along the jet surface. For an expanding jet $R = r\Psi$ and $(r/\ell_e) \sim 1.9\ n/(\eta^{1/6}M_{in}\Psi)$. While the growth rate and phase velocity of growing perturbations will change as wave amplitude increases it is certain that a large value of (r/ℓ_e) implies that large wave amplitudes can occur. For our particular choice of parameters $(\eta^{1/6}M_{in}\Psi) \sim 0.85$ and $(r/\ell_e) \sim 2.2n$. It is clear that the helical wave mode $(n = 1)$ can be restricted to only a few e-folding lengths but this is not the case for $n > 1$. While somewhat more rapid jet expansion is of some help we point out that $M_{in}\Psi < 1$ for a jet in pressure balance with the external medium. A jet in free expansion with $M_{in}\Psi \sim 1$ is stabilized by virtue of its expansion at the sound speed which restricts surface instability to a thin surface region.

If a jet is in pressure balance with an external medium the only way to stabilize the jet surface to wave modes with shorter wavelengths is by velocity and density shear. For a linear velocity shear layer Ray (1982) finds that waves with wavenumber $k \geq 1.28\ h^{-1}$ where h is the scale height of the shear layer are stabilized. To apply this result to a cylindrical jet we need to determine the true wavelength of a wave mode at maximum growth. The true wavelength is always less than the wavelength λ at constant angle ϕ (cylindrical or spherical geometry). Figure 3 illustrates the propagation geometry of wave modes on a cylindrical jet of constant cross section. The true wavelength between wave fronts is given by $\Lambda_n = \{[2\pi R/n]/[(2\pi R/n)^2 + \lambda_n^2]^{1/2}\}\lambda_n$. The true maximally growing wavelength for a wave mode and therefore the true characteristic wavelength of a wave mode is

$$\Lambda_n^{(p)} = \{[2\pi R/n]/[(2\pi R/n)^2 + \lambda_n^{(p)2}]^{1/2}\}\lambda_n^{(p)} . \tag{10}$$

This relation is modified only slightly by jet expansion (see figure 1 in HII) and may be used with confidence with R being the instantaneous jet radius. The stability criterion can be written $\Lambda \leq 4.91\,h$. If, for example, the depth of a shear layer were R/4 then $\Lambda \leq 1.2\,R$ would be stabilized. It is informative to rewrite the stability condition in terms of λ and we find that wavelengths

$$\lambda_n > \lambda_n^{min} = 2\pi n^{-1}\ [(1.28R/nh)^2 - 1]^{-1/2} R \tag{11}$$

are unstable and shorter wavelengths are stabilized. Immediately we see that wave modes with $n > 1.28\ R/h$ must be completely stabilized. This occurs because no matter the wavelength λ_n for these wave modes, the true wavelength Λ_n is less than the minimum unstable wavelength. Since the maximum depth of the shear layer is h = R for a cylindrical jet it would not appear possible to stabilize the n = 1 helical wave mode by velocity shear. However, it is likely that only wave modes with small n, say n < 10, will be unstable. For example, using the parameters $\eta = 10$, $M_{in} = 9.9$ and h = R/4 we find $\lambda_1^{(p)} \sim 7.8R > \lambda_1^{min} = 3.1R$, through $\lambda_4^{(p)} \sim 3.8R > \lambda_4^{min} = 3.0R$ but $\lambda_5^{(p)} \sim 3.0R < \lambda_5^{min} = 8.1R$. The maximum rate of growth of the wave mode n = 5 is reduced to about that of the n = 2 wave mode (see figure 1) and will actually be reduced to less than this if full effects of a shear layer are included (see Ray 1982). Wave modes with n > 5 would be absolutely stable. We conclude that only a few wave modes will be unstable on an expanding jet with significant velocity shear.

The observational consequences of the n = 1 helical wave mode have already been discussed in detail in HII. It is now necessary to consider the effect of the remaining unstable wave modes on the jet material. The effect of shorter wavelength wave modes on the jet material will be limited by the portion of a jet affected by a surface wave. For a jet of cylindrical cross section the displacement amplitude of jet material interior to the jet surface is given by (HII equation A18)

$$\eta(\theta) = \frac{1}{r}\left[\frac{\rho'_{in}\, a_{in}^2}{\omega_{in}^2 (\rho_{in} + P_o/c^2)}\right]\beta_n^{in} \cos\theta \quad \frac{J'_n(\beta_n^{in}\psi)}{J_n(\beta_n^{in}\psi)} \tag{12}$$

In equation 12 $\beta_n^{in} \equiv k\xi_n$ and spherical geometry is assumed. At the jet surface $\theta = \Theta$ and $\psi = \Psi$. To obtain the amplitude as a function of jet radius we form the ratio

$$\eta(\theta)/\eta(\Theta) = \frac{\cos\theta}{\cos\Theta} \quad \frac{J'_n(\beta_n^{in}\psi)}{J'_n(\beta_n^{in}\Psi)} \tag{13}$$

where $\rho_{in}(\theta)/\rho_{in}(\Theta) = J_n(\beta_n^{in}\psi)/J_n(\beta_n^{in}\Psi)$. Equation (13) can be used to obtain an estimate of the depth affected by each wave mode n.

Provided $n \geq 2$ the characteristic wavenumber for each wave mode can be approximated by $k_n^{(p)}\Psi \sim 1.9n\,(\eta^{1/3}/M_{in})$ with acceptable accuracy. The arguments of the Bessel functions in equation 13 are proportional to n and we may use the following approximation for the Bessel function:

$$J_n(nz) \sim \left(\frac{4\chi}{1-z^2}\right)^{1/4} \mathrm{Ai}\,(n^{2/3}\chi)/n^{1/3} \tag{14}$$

where Ai is the Airy function and

$$\frac{2}{3}\chi^{3/2} = \ln \frac{1+\sqrt{1-z^2}}{z} - \sqrt{1-z^2}$$

or

$$\frac{2}{3}(-\chi)^{3/2} = \sqrt{z^2-1} - \arccos(1/z)$$

so χ is real when z is positive (Abramowitz and Stegun 1964). In equation (14) $z = 1.9\,(\eta^{1/3}/M_{in})(\psi/\Psi)\,\xi_n$. If we use equation (4) for $\Phi = \frac{\omega}{ka_{in}}$ at $k_n^{(p)}$ and the fact that $M_{in} >> 1$ for a supersonic jet it follows that

$$\xi_n \sim \begin{cases} M_{in} & \eta << 1 \\ iM_{in}/\eta^{1/2} & \eta >> 1 \end{cases}$$

As a result we find

$$z \sim \begin{cases} 1.9\,\eta^{1/3}\,(\psi/\Psi) & \eta << 1 \\ 1.9\,i\,\eta^{-1/6}(\psi/\Psi) & \eta >> 1 \end{cases}$$

and in general $|z|<1$. When $|z|$ is small $\chi \sim \left(-\frac{2}{3}\ln(2/z)-1\right)^{2/3}$, $\mathrm{Ai}(n^{2/3}\chi) \sim \frac{1}{2}\pi^{-1/2}(n^{2/3}\chi)^{-1/4}\exp[-\frac{2}{3}n\chi^{3/2}]$ and $4\chi/(1-z^2) \sim 4\chi(1+z^2)$ can be used with result that

$$J_n(nz) \sim \frac{1}{\sqrt{2\pi n}}\left(\frac{ez}{2}\right)^n \tag{15}$$

Because $J_n'(nz) = \frac{1}{z} J_n(nz) - J_{n+1}(nz) \sim J_n(nz)/z$ it follows that

$$J_n'(nz) \sim \frac{(e/2)^n}{\sqrt{2\pi n}} z^{n-1} \tag{16}$$

The approximate form of the derivative of the Bessel function inserted in equation (13) results in the following relation between the displacement amplitude at the jet surface and inside the jet:

$$\eta(\theta)/\eta(\Theta) \sim \frac{\cos\theta}{\cos\Theta} \left(\frac{\psi}{\Psi}\right)^{n-1} \tag{17}$$

Equation (17) implies that the wave amplitude interior to the jet surface is

$$A_n \sim A_n^R \quad (b/R)^{n-1} \tag{18}$$

where A_n^R is the surface amplitude and $0 \le b \le R$ where b is the radial distance inside the jet. The lack of radial dependence of the n = 1 helical wave mode results because the entire jet is displaced.

If conditions are such that $(r/\ell_e) >> 1$ for a wave mode then it is reasonable to assume that the wave amplitude at the jet surface has grown to its maximum. While maximum amplitudes and associated fluid motions need to be determined by numerical simulation, ordered displacement of the jet surface is limited to motion of the fluid at the sound speed transverse to the flow velocity. Wave modes with $n \ge 1$ appear as sinusoidal displacement of the jet surface $\eta(r) = A_n^R \sin(2\pi r/\lambda_n^{(p)})$ at fixed angle ϕ. The maximum amplitude of such a sinusoidal oscillation is $A_n^R \sim \frac{1}{2\pi} \lambda_n^{(p)} [\frac{a_{in}}{u - v_{ph}}]$ where $u - v_{ph}$ is the fluid velocity in the wave frame. Because v_{ph} at $\lambda_n^{(p)}$ is only a weak function of n, $A_n^R \propto n^{-1}$. The energy in each wave mode is proportional to $\Delta M(\Delta u)^2$ where ΔM is the jet material participating in motion at velocity Δu. If we continue to assume that $\Delta u \sim a_{in}$ at the jet surface then $\Delta u \propto (b/R)^{n-1}$ independent of $\lambda_n^{(p)}$. To the extent that wave modes are independent and all of the jet material is involved, the energy in a wave mode interior to the jet surface is $E_n \propto (b/R)^{2(n - 1)}$. The total wave energy interior to the jet surface is of the form

$$E \sim E_o \sum_n \alpha_n (b/R)^{2(n - 1)} \tag{19}$$

A fundamental scale size of $\Lambda_n^{(p)}$ exists for each term in the summation and α_n is a weak function of $\Lambda_n^{(p)}$. An upper limit to the summation is set by the depth of the shear layer and a lower limit may be set by jet expansion. Near the upper and lower limits α_n must decrease rapidly. Near the upper limit wave growth is slowed because the fundamental scale size is shifted to longer wavelength and at the lower limit wave growth is slowed and maximum wave amplitude is not reached.

IV. Discussion

Each unstable wave mode can be regarded as driving the jet surface at a length scale $\Lambda_n^{(p)}$ and a frequency $V_{ph}/\Lambda_n^{(p)} = (u - v_{ph})/\lambda_n^{(p)}$. If $\Lambda_n^{(p)}$ is less than the jet diameter it is likely that eddies of size $\Lambda_n^{(p)}$ can be created in the jet fluid. Dissipation in the jet fluid leads to a cascade to eddies of smaller size with a Kolmogorov energy spectrum $W_k \propto k^{-5/3}$ and with the smallest eddy size governed by dissipation processes in the fluid (Landau and Lifschitz 1959). Wave modes with $\Lambda_n^{(p)}$ larger than the jet diameter cannot generate eddies in the jet and must affect the flow in a more global sense. If we consider $\eta = 10$ and $M_{in} = 9.9$ we find that $\Lambda_1^{(p)} = 4.9$ R, $\Lambda_2^{(p)} = 2.8$ R and $\Lambda_{n\geq 3}^{(p)} \approx 5.8\ n^{-1}R$. In this case the helical and elliptical wave modes cannot produce eddies in the jet material without disrupting the flow. These wave modes must dissipate jet energy in the external medium and jet by heating of the fluid (Benford 1981). However, each unstable wave mode with $n \geq 3$ can produce eddies in the jet material. Other values of η and M_{in} can reduce $\Lambda_1^{(p)}$ and $\Lambda_2^{(p)}$ to less than a jet diameter. To the extent that each wave mode is independent, each drives a fundamental eddy size with total energy associated with each wave mode given by the appropriate term in equation (19). Wave modes which produce eddies in the jet dissipate energy at a rate $\sim (\Delta u)^3/\Lambda_n^{(p)}$ (Landau and Lifschitz 1959). The dissipation rate increases as the fundamental scale size decreases.

If we continue to assume that $\Delta u \sim a_{in}$ at the jet surface independent of $\Lambda_n^{(p)}$, then energy input into the jet is of the form

$$\dot{\varepsilon} \sim \dot{\varepsilon}_o \sum_n \beta_n (b/R)^{3(n-1)} \qquad (20)$$

where $\beta_n \sim n\,\alpha_n$.

In equation (20) a lower limit to n is set when $\Lambda_n^{(p)}$ is greater than the jet diameter as dissipation is no longer through fluid turbulence driven at scale size $\Lambda_n^{(p)}$. Modifications to this simple relation will occur if wave modes strongly interact and if fluid turbulence generated at shorter $\Lambda_n^{(p)}$ propagates into the jet faster than perturbations at longer $\Lambda_n^{(p)}$ grow to large amplitude.

The effect of Kelvin-Helmholtz unstable surface waves on the jet interior and external medium is of fundamental importance to an understanding of observed jets. We have found that the presence of velocity shear may stabilize all but one or two of the long wavelength wave modes and that these long wavelength wave modes can be resticted to only a few e-folding lengths and small amplitude by jet expansion. If these conditions are met then we might expect that little of the flow energy would be dissipated in the external medium or internally in the jet because turbulence would be unlikely to develop. However, if the jet edge is relatively sharp then we expect a number of wave modes with $\Lambda_n^{(p)}$ less than the jet diameter and $(r/\ell_e) >> 1$. The likely case would be one in which some wave modes with say $2 < n \leq 10$ grow rapidly and those of larger n are surpressed by velocity shear. It is clear that restrictions on wave mode stability are likely to result in fundamental eddy sizes on the order of the jet radius or suppress the development of short wavelength wave modes and jet turbulence entirely. If wave amplitude is limited by motion of the jet surface at the sound speed then wave energy is proportional to $(\Delta u)^2 \sim u^2/M_{in}^2$ and can be a large or small fraction of the jet flow energy. Interestingly, if $M_{in} >> 1$, velocity fluctuation in the jet material at the sound speed is a small fraction of the flow speed and the flow can appear well ordered in the observer frame. Even so, the flow would be characterized as strongly turbulent if 10% of the flow energy were converted to eddies in the jet frame.

Our present results suggest that surface instability can affect more than a thin surface layer and probably produces a Kolmogorov energy spectrum in fluid turbulence in a jet. Some of the wave energy converted into turbulence will simply heat the fluid through dissipation but the presence of magnetic field implies the generation of MHD waves (Kato 1968; Dobrowolny 1972). If the fluid is driven turbulent at a single wavenumber or only a narrow range of wavenumbers the MHD waves produced will have an energy spectrum $W_k \propto k^{-3/2}$ (Kraichnan 1965; DeYoung 1980). A Kolmogorov energy spectrum in fluid turbulence allows direct drive to MHD waves at many wavenumbers. The resulting MHD wave energy spectrum is not well determined but is probably as steep or steeper than Kolmogorov (Kato 1968; Henriksen and Eilek 1982). Whatever the exact spectrum, the MHD waves produced in this fashion no longer are confined to a thin surface region (see Eilek 1982) but can exist in a significant fraction of the jet volume with MHD wave energy proportional to the wave energy in fluid turbulence (see equation 19). This MHD turbulence can supply energy to an existing population of relativistic electrons and may produce the observed jet emission (Eilek and Henriksen 1981). Since MHD waves of the appropriate wavelengths for particle acceleration are strongly damped and do not travel far (Eilek 1982) the jet emissivity may be of the same form as equation (20). Such a jet emissivity should result in jets with intensity profiles flatter than if the emissivity were not a function of the jet radius.

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References

Abramowitz, M., and Stegun, I.A. 1965, Handbook of Mathematical Functions (New York: Dover).

Benford, G. 1981, Ap.J., 247, 792.

DeYoung, D.S. 1980, Ap.J., 241, 81.

Dobrowolny, M. 1972, Phys. Fluids, 15, 2263.

Eilek, J.A. 1979, Ap.J., 230, 373.

Eilek, J.A. 1982, Ap.J., 254, 472.

Eilek, J.A., and Henriksen, R.N. 1981, IAU Symposium 97, Extragalactic Radio Sources, ed. D. Heeschen and C. Wade (Dorurecht: Reidel).

Ferrari, A., Trussoni, E., and Zaninetti, L. 1978, Astr. Ap., 64, 43.

Ferrari, A., Trussoni, E., and Zaninetti, L. 1979, Astr. Ap., 79, 190.

Hardee, P.E. 1979, Ap.J., 234, 47.

Hardee, P.E. 1982, Ap.J., 257, 509.

Henriksen, R.N., and Eilek, J.A. 1982, preprint.

Kato, S. 1968, P.A.S.J., 20, 59.

Kraichnan, R.H. 1965, Phys. Fluids, 8, 1385.

Lacombe, C. 1977, Astr. Ap., 54, 1.

Landau, L.D., and Lifschitz, E.M. 1959, Fluid Mechanics (New York: Pergamon).

Pacholczyk, A.G., and Scott, J.S. 1976, Ap.J., 203, 313.

Ray, T.P. 1981, M.N.R.A.S., 196, 195.

Ray, T.P. 1982, M.N.R.A.S., 198, 617.

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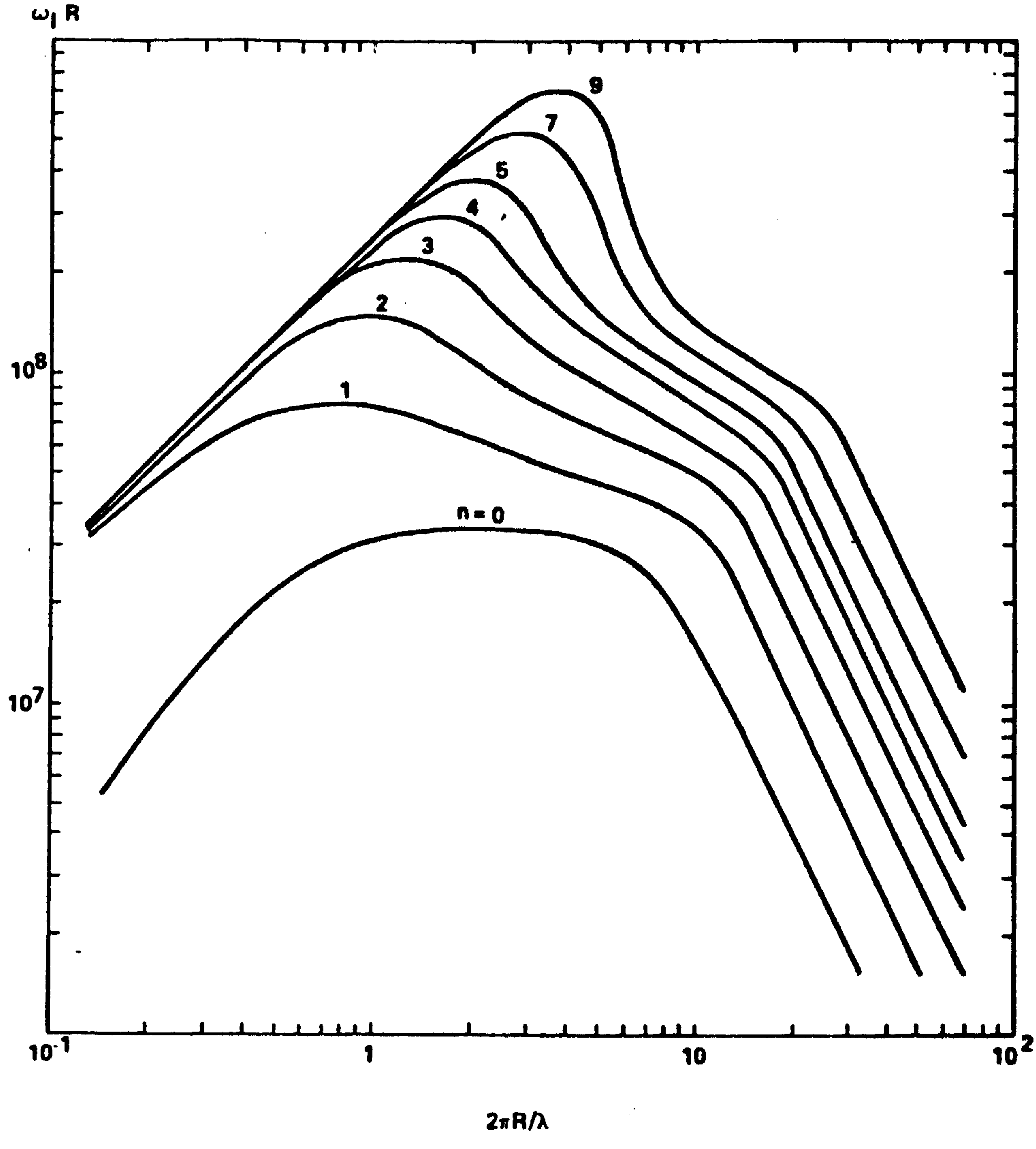

Figure 1: Wave mode growth rate $\omega_I R$ in cm sec^{-1} as a function of wavenumber normalized to the jet radius on a jet 10 times denser than the surrounding medium, Mach number of 9.9 and opening angle $2\Theta = 0.08$ radian.

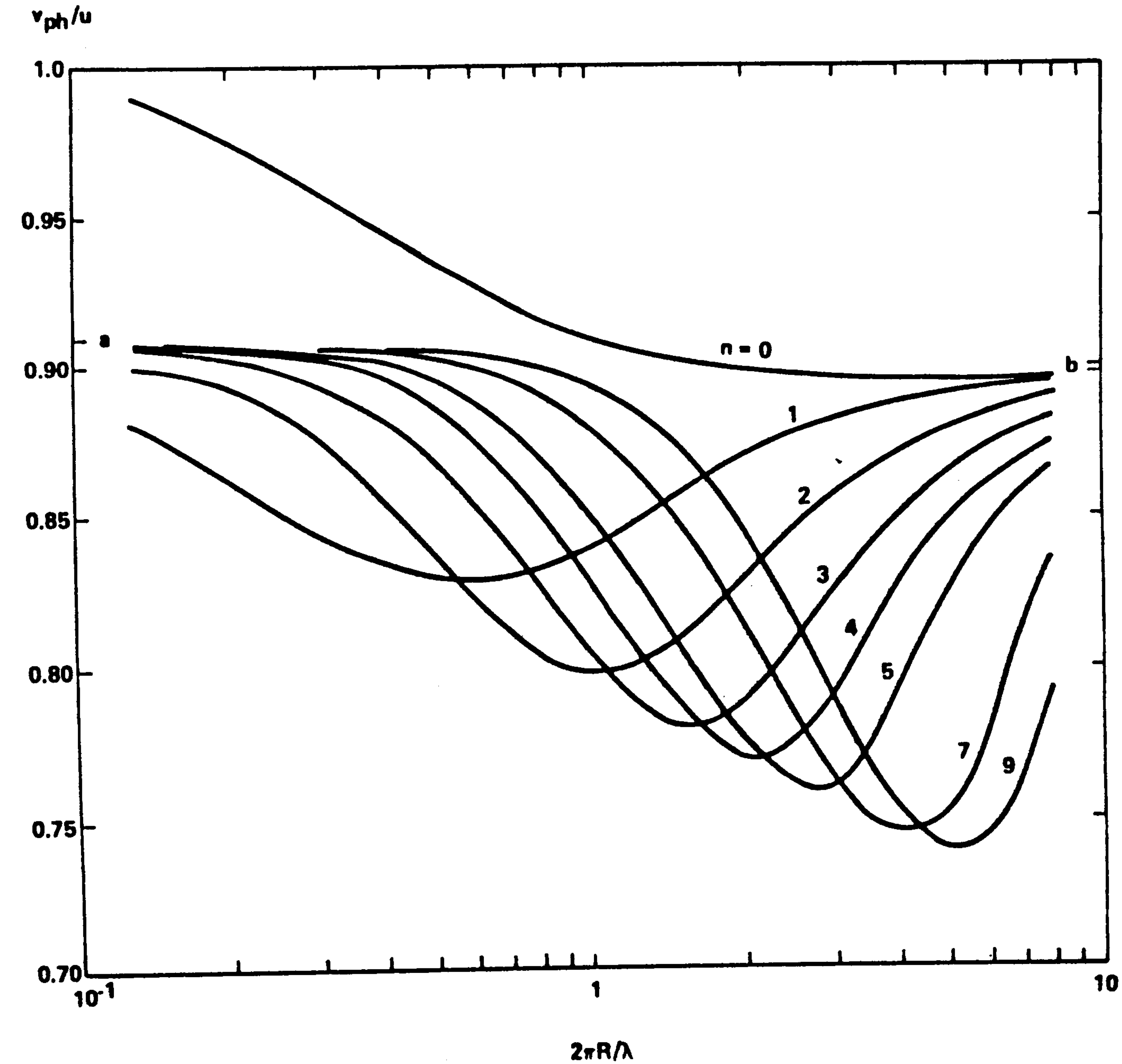

Figure 2: Wave mode phase velocity, v_{ph}, normalized to the jet velocity, u, as a function of the normalized wavenumber for the wave modes shown in Figure 1. The (a) indicates $v_{ph} = [\eta/(1 + \eta)]u$ and the (b) indicates $v_{ph} = u - a_{in}$.

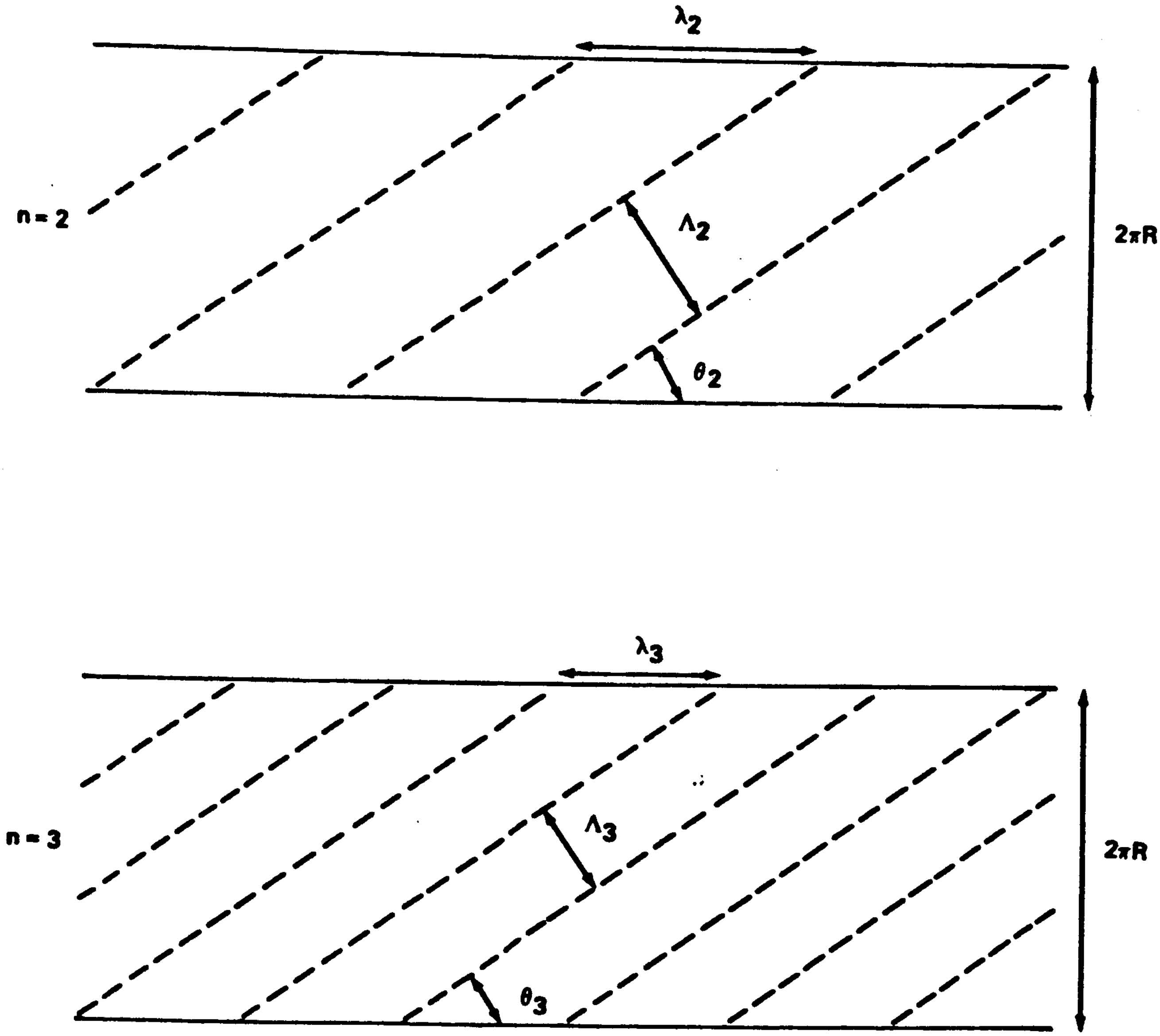

Figure 3. Propagation geometry of wave fronts (dashed lines) of wavemodes $n = 2$ and 3 around a cylindrical jet of circumference $2\pi R$ at equal propagation angle with respect to the flow velocity.