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Research Article

Efficiency Bounds for Two-Stage Production Systems

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Traditional data envelopment analysis (DEA) models find the most desirable weights for each decision-making unit (DMU) in order to estimate the highest efficiency score as possible. These efficiency scores are then used for ranking the DMUs. The main drawback is that the efficiency scores based on weights obtained from the standard DEA models ignore other feasible weights; this is due to the fact that DEA may have multiple solutions for each DMU. To overcome this problem, Salo and Punkka (2011) deemed each DMU as a "Black Box" and developed models to obtain the efficiency bounds for each DMU over sets of all its feasible weights. In many real world applications, there are DMUs that have a two-stage production system. In this paper, we extend the Salo and Punkka's (2011) model to a more common and practical case considering the two-stage production structure. The proposed approach calculates each DMU's efficiency bounds for the overall system as well as efficiency bounds for each subsystem/substage. An application for nonlife insurance companies has been discussed to illustrate the applicability of the proposed approach and show the usefulness of this method.

1. Introduction

Data Envelopment Analysis (DEA), first developed by Charnes et al. [1], has been proven as an effective tool for performance evaluation and benchmarking. This technique makes no assumptions on the production function and imposes no subjective weights on multiple inputs and multiple outputs. DEA has been widely applied in many areas [2]. The DEA technique allows a DMU to choose the most favorable weights to achieve the best possible relative efficiency. However, the standard efficiency scores do not consider all the possible weights as they only consider the weights most favorable to each DMU. The main issue that has been ignored in the past literature is that the efficiency score of a DMU relative to other DMUs can change over different weights when applying the DEA models. Hence, it is important to consider all possible weights to evaluate each DMU.

To overcome this problem, Salo and Punkka [3] have proposed a procedure to obtain the efficiency bounds by taking into account all possible weights (see also [4]). That is to say, the efficiency bounds show how the DMUs' efficiency ratios relate to each other for all feasible weights, rather than for those weights only for which the data envelopment analysis (DEA) efficiency score of some DMU is maximized. They have introduced an efficiency bound for all possible efficiency scores that is determined by the lower and upper bounds of the efficiency scores. The efficiency bounds show how much more efficient a given DMU can be relative to some other DMU or a subset of other DMUs. For this purpose, Green et al. [5] developed a new model by putting the CCR model into a mixed-binary linear programming framework to obtain the efficiency bounds in data envelopment analysis. Entani and Tanaka [6] proposed the interval DEA model to obtain an efficiency interval consisting of evaluations from both the optimistic and pessimistic viewpoints. Wang and Yang [7] proposed a pair of bounded DEA models to measure the overall performances of a group of decision-making units (DMUs), which were characterized by interval efficiencies. To overcome the problem of these models incapable of determining an efficiency interval for any DMU when there is a zero value for each output, Azizi and Wang [8] proposed a pair of improved bounded DEA models to overcome the drawback. All these methods for obtaining the efficiency bounds treated each DMU as a "Black Box". Thus, they ignored the internal structure of the production system.

However, as discussed in many DEA studies, in many real application, DMUs have a two-stage structure, i.e., outputs from the first stage become the inputs to the second stage. Outputs from the first stage are referred to as intermediate measures. Seiford and Zhu [9] use the standard DEA approach to measure the profitability and marketability of US commercial banks which does not address potential conflicts between the two stages arising from the intermediate measures. For example, the second stage may have to reduce its inputs (intermediate measures) in order to achieve an 'efficient' status, which imply a reduction in the first stage outputs. In a survey by Cook et al. [10], they pointed out that the approaches of modeling DMUs with a two-stage production process can be categorized as four types, i.e., standard DEA methodology, efficiency decomposition methodology, network DEA, and game-theoretic approaches. The standard DEA methodology simply uses the standard DEA model, i.e., two separate DEA models to calculate the efficiencies of two stages (e.g., Seiford and Zhu [9]; Zhu [11]; and Sexton and Lewis [12]); the efficiency decomposition methodology is that given the efficiency scores of stage 1 and stage 2, the overall efficiency could be defined as the product or the arithmetic mean of two substages' efficiencies (e.g., Kao and Hwang [13]; Chen et al. [14], and Chen et al. [15]); the network DEA approach extends the two-stage process to more general situation (e.g., Tone and Tsutsui [16]; Tone and Tsutsui [17]; Izadikhah et al. [18]); game-theoretic approaches introduce game theory to the efficiency evaluation of two-stage structure (e.g., Liang et al. [19]; Zha and Liang [20]; Li et al. [21]; Guo and Zhu [22]; and Izadikhah et al. [18]). Except for the standard DEA approach, all other approaches attempt to correct for the above-referenced conflict issue. And, two-stage DEA has been extensively applied to many areas, such as hotels ([23, 24]; Huang et al. [25]), R&D departments (Li et al. [21]; Liu and Lu, [26]), information technology (Shao and Lin [27]; Chen and Zhu [28]; and Kao and Hwang [29]), insurance companies (Yang [30]; Kao and Hwang [13]), industry (Wu et al. [31]; Chen et al. [32]; and Li et al. [33]), and banks (Paradi et al. [34]; Huang et al. [23, 24]; Wang et al. [35]; and Zhu et al. [36]).

In this paper, we develop a method to obtain the efficiency bounds for the classic two-stage production systems as discussed by Seiford and Zhu [9], Chen and Zhu [28], and Kao and Hwang [13]. That is, the first subsystem uses inputs to produce outputs that then become the inputs to the second subsystem to produce the final outputs. The proposed model calculates each DMU's efficiency bounds for the overall system as well as two subsystems. Unlike conventional efficiency scores, the results show how the DMUs' efficiency ratios for the overall system and two subsystems relate to each other for all feasible weights. We believe that this process provides more accurate information for decision makers by identifying the best (and/or worst) DMUs in the overall system and both subsystems over all feasible weights. Besides, the proposed approach provides information regarding the sensitivity of the DMU's efficiency bounds for the overall system and both two subsystems over sets of all feasible weights.

The remainder of this paper is organized as follows. In the next section, the procedure of efficiency bounds by Salo and Punkka [3] has been reviewed briefly. Then, in Section 3, a method is developed to obtain the efficiency bounds considering the two-stage production systems. This is followed by illustration example in Section 4. An application is also given in this section to show the usefulness of the proposed procedure. Finally, conclusions and direction for future research are given in Section 5.

2. Efficiency Bounds by Salo and Punkka [3]

Assume that there are n DMUs denoted as $\text{DMU}_j (j = 1, \ldots, n)$. Each DMU uses inputs $x_{ij} (i = 1, \ldots, m)$ to produce outputs $y_{rj} (r = 1, \ldots, s)$. Based on the definition of Charnes et al. [1], the efficiency of DMU_k is calculated by the CCR multiplier form as follows:

$$E_k^{CCR}(u, v) = \max \quad \frac{\sum_{r=1}^{s} u_r y_{rk}}{\sum_{i=1}^{m} v_i x_{ik}} \tag{1}$$

$$\text{s.t.} \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad \forall j \tag{2}$$

$$\begin{aligned} & u_r \ge 0 \quad \forall r \\ & v_i \ge 0 \quad \forall i \end{aligned} \tag{3}$$

The model could be transformed into linear programming model by use of the Charnes-Cooper transformation [37]. $u_r^* (r = 1, \ldots, s)$ and $v_i^* (i = 1, \ldots, m)$ are the optimal solution of model (2) and the associated optimal output and input weights. DMU_k is termed efficient if and only if the optimal objective is equal to one and the optimal weight vectors are all larger than zero; i.e., $E_k^{CCR}(u^*, v^*) = 1$ and $u_r^* > 0, v_i^* > 0$. For any feasible weights $u_r (r = 1, \ldots, s)$ and $v_i (i = 1, \ldots, m)$, Salo and Punkka [3] defined efficiency dominance between DMUs based on the efficiency scores of model (2).

Definition 1. DMU_k dominates DMU_l (denoted by $DMU_k \succ DMU_l$) if and only if

$$\begin{aligned} & E_k(u, v) \ge E_l(u, v) \quad \forall (u, v) \in (S_u, S_v) \\ & E_k(u, v) > E_l(u, v) \quad \textit{for some } (u, v) \in (S_u, S_v) \end{aligned} \tag{4}$$

Thus, if $DMU_k \succ DMU_l$, the efficiency ratio of DMU_k is at least as high as that of DMU_l for all feasible weights, and moreover, there exist some weights for which its efficiency is strictly higher.

The dominance relation in Definition 1 could be calculated based on the following pairwise efficiency ratio:

$$D_{k,l}(u, v) = \frac{E_k(u, v)}{E_l(u, v)}. \tag{5}$$

However, the relative efficiency ratio (5) is nonlinear in weights (u, v). Salo and Punkka [3] proposed the following model to maximize and minimize the ratio through linear programming.

Theorem 2. *The optimum of the maximization (minimization problem)*

$$\begin{aligned} \max_{u,v} \quad & \left(\min_{u,v}\right) \sum_{r=1}^{s} u_r y_{rk} \\ \text{s.t.} \quad & \sum_{r=1}^{s} u_r y_{rl} \sum_{i=1}^{m} v_i x_{il} \\ & \sum_{i=1}^{m} v_i x_{ik} = 1 \\ & u_r, v_i \geq 0, \ \forall r, i \end{aligned} \tag{6}$$

is the maximum of (minimum) $D_{k,l}(u,v)$ for all weight scenarios, i.e., $\underline{D}_{k,l}$ and $\overline{D}_{k,l}$.

The optimization problems in model (6) obtain the upper and lower bounds on how efficient DMU_k can be relative to DMU_l across feasible weights. It is worth noting that if the benchmark set L contains all DMUs, then $\underline{D}_{k,l}$(or $\overline{D}_{k,l}$) is equal to the CCR-DEA score. If DMU_k is not contained in the benchmark set L, $\underline{D}_{k,l}$(or $\overline{D}_{k,l}$) is the super efficiency of DMU_k relative to this set of DMUs (see, e.g., [38]). For example, if $\underline{D}_{k,l} = 1.2$, the efficiency score of DMU_k is at least 20% higher than that of DMU_l. And, if $\overline{D}_{k,l} = 1.5$, the efficiency score of DMU_k can be at most 50% higher than that of DMU_l. If the minimum $\underline{D}_{k,l}$ is greater than one, DMU_k dominates DMU_l. If the minimum $\underline{D}_{k,l}$ is less than one, the dominance does not hold. If the minimum is equal to one, we could judge the dominance relation by further maximizing the linear program of model (6). If the resulting maximum $\overline{D}_{k,l}$ is greater than one, the dominance holds, but if not, then DMU_k and DMU_l have the same efficiency score (2) for all feasible weights.

Based on model (6), the lower efficiency bound of DMU_k, which is the efficiency of DMU_k relative to the most efficient DMUs in the benchmark group for different inputs/outputs weights. Thus, the lower bound of DMU_k' efficiency scores denoted by $\underline{D}_{k,\overline{L}}$ is $\min_{l\in L}\underline{D}_{k,l}$, i.e., $\underline{D}_{k,\overline{L}} = \min_{l\in L}\underline{D}_{k,l}$.

The following proposition shows how to find the upper bound of efficiency scores, denoted as $\overline{D}_{k,\overline{L}}$, which is relative to the most efficient DMUs in the benchmark group for DMU_k.

Theorem 3. *$\overline{D}_{k,\overline{L}}$ is the optimum of the maximization problem*

$$\begin{aligned} \max_{u,v} \quad & \sum_{r=1}^{s} u_r y_{rk} \\ \text{s.t.} \quad & \sum_{r=1}^{s} u_r y_{rl} \leq \sum_{i=1}^{m} v_i x_{il}, \quad l \in L \\ & \sum_{i=1}^{m} v_i x_{ik} = 1 \\ & u_r, v_i \geq 0, \ \forall r, i. \end{aligned} \tag{7}$$

The optimal value of model (7) is the upper bound of DMU_k' efficiency scores over sets of all feasible weights. The upper bound of DMU_k' efficiency scores defined as the maximum of values of how efficient DMU_k is relative to the most efficient DMUs in the benchmark group for different input/output weights. Based on models (6) and (7), the efficiency bounds $[\underline{D}_{k,\overline{L}}, \overline{D}_{k,\overline{L}}]$ of DMU_k can be computed. These models treat the production systems as "Black Box".

The approach can be generalized to systems composed of two subsystems connected in series. In the next section, we will discuss how to calculate the efficiency bounds for each DMU with a two-stage production system.

3. Efficiency Bounds for Two-Stage Production Systems

Suppose the operation of a DMU can be divided into two subsystems or processes, as depicted in Figure 1. For DMU_k, subsystem 1 applies inputs x_{ik} $(i = 1, \ldots, m)$ to produce the intermediate products z_{dk} $(d = 1, \ldots, D)$. All these intermediate products are then used by subsystem 2 to produce the final outputs y_{rk} $(r = 1, \ldots, s)$. Based on the definition of Kao and Huang [13], DMU_k's efficiency scores for the overall system and two subsystems are defined as

$$E_k = \frac{\sum_r u_r y_{rk}}{\sum_i v_i x_{ik}} \tag{8}$$

$$E_k^1 = \frac{\sum_d w_d^1 z_{dk}}{\sum_i v_i x_{ik}} \tag{9}$$

$$E_k^2 = \frac{\sum_r u_r y_{rk}}{\sum_d w_d^2 z_{dk}} \tag{10}$$

where $u_r (r = 1, \ldots, s)$ and $v_i (i = 1, \ldots, m)$ are the output weights and input weights, respectively. Accordingly, w_d^1 $(d = 1, \ldots, D)$ and w_d^2 $(d = 1, \ldots, D)$ are the weights attached to the intermediate measures for subsystem 1 and subsystem 2, respectively. Similar to Kao and Hwang [13] and Liang et al. [19], we assume that the weights attached to the intermediate outputs in both subsystem 1 and subsystem 2 are the same, i.e., $w_d^1 = w_d^2$. This assumption represents the serial relationship between the two subsystems [14]. If we solve the two-stage DEA without this assumption, then our method is identical to independently employing the model for each subsystem. Therefore, this paper assumes $w_d^1 = w_d^2 = w_d$.

3.1. Efficiency Bounds of a DMU for the Overall System. As discussed in Section 2, choosing different weights may lead to different efficiency scores for a "Black Box" DMU. Similarly, choosing different weights may result in different efficiency scores for a DMU with a two-stage production system.

Proposition 4. *The optimum of the maximization (or minimization) problem*

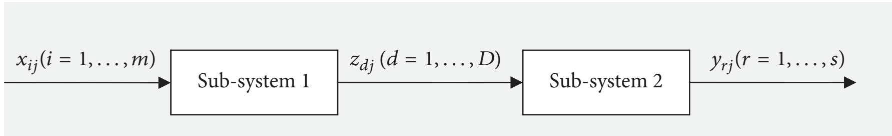

Figure 1: Two-stage production system.

$$\begin{aligned}
\max_{u,v,w} \quad (\min_{u,v,w}) & \sum_{r=1}^{s} u_r y_{rk} \\
\text{s.t.} \quad & \sum_{r=1}^{s} u_r y_{rl} = \sum_{d=1}^{D} w_d z_{dl} \\
& \sum_{d=1}^{D} w_d z_{dl} = \sum_{i=1}^{m} v_i x_{il} \\
& \sum_{i=1}^{m} v_i x_{ik} = 1 \\
& u_r, v_i, w_d \geq 0, \ \forall r, i, d
\end{aligned} \tag{11}$$

is the maximum (or minimum) of $D_{k,l}(u, w, v)$ for all weight scenarios, i.e., $\overline{D}_{k,l}(u, w, v)$ (or $\underline{D}_{k,l}(u, w, v)$). The proof of this proposition is given in the Appendix.

In the following model, the optimum of the maximization problem, i.e., $\overline{D}_{k,\overline{L}}$, is the upper efficiency bound of DMU_k considering each DMU' two-stage production structure.

Proposition 5. *$\overline{D}_{k,\overline{L}}$ is the optimum of the maximization problem*

$$\begin{aligned}
\max_{u,v} \quad & \sum_{r=1}^{s} u_r y_{rk} \\
\text{s.t.} \quad & \sum_{r=1}^{s} u_r y_{rl} \leq \sum_{d=1}^{D} w_d z_{dl}, \quad l \in L \\
& \sum_{d=1}^{D} w_d z_{dj} \leq \sum_{i=1}^{m} v_i x_{il}, \quad l \in L \\
& \sum_{i=1}^{m} v_i x_{ik} = 1 \\
& u_r, v_i \geq 0, \ \forall r, i.
\end{aligned} \tag{12}$$

By solving model (12), the upper efficiency bound $\overline{D}_{k,\overline{L}}$ could be obtained. The proof of this proposition is given in the Appendix.

3.2. Efficiency Bounds of a DMU for Both Subsystems. In this section, we discuss the efficiency bounds of DMUs for two subsystems.

Proposition 6. *The optimum of the maximization (minimization problem)*

$$\begin{aligned}
\min_{v,w} \quad & \sum_{d=1}^{D} w_d z_{dk} \\
\text{s.t.} \quad & \sum_{d=1}^{D} w_d z_{dj} = \sum_{i=1}^{m} v_i x_{il} \\
& \sum_{i=1}^{m} v_i x_{ik} = 1 \\
& w_d, v_i \geq 0, \ \forall d, i
\end{aligned} \tag{13}$$

is the minimum of $D^1_{k,l}(w, v)$ for subsystem 1 for all weight scenarios.

Based on model (13), the lower bound of DMU_k's efficiency scores for subsystem 1, which is the efficiency of DMU_k relative to the most efficient DMUs in the benchmark group for different inputs/outputs weights. Thus, the lower bound of DMU_k's efficiency scores for subsystem 1 denoted by $\underline{D}^1_{k,\overline{L}}$ is $\min_{l \in L} \underline{D}^1_{k,l}$, i.e., $\underline{D}^1_{k,\overline{L}} = \min_{l \in L} \underline{D}^1_{k,l}$. The proof of this proposition is similar to the proof of Theorem 3 in Salo and Punkka [3].

In the following model, the optimum of the maximization problem, i.e., $\overline{D}^1_{k,\overline{L}}$, is the upper bound of DMU_k' efficiency score for subsystem one.

Proposition 7. *$\overline{D}^1_{k,\overline{L}}$ is the optimum of the maximization problem*

$$\begin{aligned}
\max_{u,v} \quad & \sum_{\text{d}=1}^{D} w_d z_{dk} \\
\text{s.t.} \quad & \sum_{d=1}^{D} w_d z_{dj} \leq \sum_{i=1}^{m} v_i x_{il}, \quad l \in L \\
& \sum_{i=1}^{m} v_i x_{ik} = 1 \\
& w_d, v_i \geq 0, \ \forall d, i.
\end{aligned} \tag{14}$$

The optimal value of model (14) may be greater than 1 or less than one or equal to 1. If the optimal value is greater than 1, then DMU_k dominates DMU_l. The proof of this proposition is similar to the proof of Theorem 4 in Salo and Punkka [3].

Similarly, the lower and upper bound of DMU_k' efficiency scores for subsystem 2 are calculated by the following two linear programs.

Table 1: Ranking intervals for each nonlife insurance company.

DMU	NO.	Our approach		Salo & Punkka [3]'s model	
		$\underline{D}_{k,\overline{L}}(u,w,v)$	$\overline{D}_{k,\overline{L}}(u,w,v)$	$\underline{D}_{k,\overline{L}}(u,v)$	$\overline{D}_{k,\overline{L}}(u,v)$
Taiwan Fire	1	0.1253	0.6992	0.1626	0.9840
Chung Kuo	2	0.1565	0.6248	0.1161	1.0000
Tai Ping	3	0.0374	0.6900	0.0485	0.9884
China Mariners	4	0.0317	0.3042	0.0535	0.4882
Fubon	5	0.6035	0.7670	0.2282	1.0000
Zurich	6	0.1057	0.3897	0.127	0.5938
Taian	7	0.1118	0.2766	0.1984	0.4698
Ming Tai	8	0.1587	0.2752	0.2004	0.4148
Central	9	0.0673	0.2233	0.1296	0.3270
The First	10	0.1413	0.4660	0.1672	0.7807
Kuo Hua	11	0.0047	0.1639	0.0154	0.2826
Union	12	0.2005	0.7596	0.1182	1.0000
Shingkong	13	0.0568	0.2078	0.1167	0.3527
South China	14	0.0847	0.2886	0.1812	0.4696
Cathay Century	15	0.1415	0.6138	0.299	0.9793
Allianz President	16	0.0504	0.3202	0.1372	0.4717
Newa	17	0.0948	0.3600	0.2454	0.6349
AIU	18	0.0418	0.2588	0.1616	0.4271
North America	19	0.0119	0.4112	0.184	0.8220
Federal	20	0.0068	0.5465	0.302	0.9351
Royal&Sun Alliance	21	0.0017	0.2008	0.1202	0.3328
Asia	22	0.0011	0.5895	0.285	1.0000
AXA	23	0.0008	0.4203	0.0011	0.5990
Mitsui Sumitomo	24	0.0043	0.1348	0.0517	0.2571

Proposition 8. *The optimum of the minimization problem*

$$\begin{aligned} \min_{u,w} \quad & \sum_{r=1}^{s} u_r y_{rk} \\ s.t. \quad & \sum_{r=1}^{s} u_r y_{rl} = \sum_{d=1}^{D} w_d z_{dl} \\ & \sum_{d=1}^{D} w_d z_{dk} = 1 \\ & u_r, v_i, w_d \geq 0, \ \forall r, i, d \end{aligned} \tag{15}$$

is the minimum of $D_{k,l}^2(u,v)$ *for all weight scenarios.*

Based on model (15), the lower bound of DMU_k's efficiency scores for subsystem 2, which is the efficiency of DMU_k relative to the most efficient DMUs in the benchmark group for different inputs/outputs weights. Thus, the lower bound of DMU_k's efficiency scores for subsystem 2 denoted by $\underline{D}_{k,\overline{L}}^2$ is $\min_{l\in L}\underline{D}_{k,l}^2$, i.e., $\underline{D}_{k,\overline{L}}^2 = \min_{l\in L}\underline{D}_{k,l}^2$. The proof of this proposition is similar to the proof of Theorem 3 in Salo and Punkka [3].

Proposition 9. $\underline{D}_{k,\overline{L}}^2$ *is the optimum of the maximization problem*

$$\begin{aligned} \max_{u,v} \quad & \sum_{r=1}^{s} u_r y_{rk} \\ s.t. \quad & \sum_{r=1}^{s} u_r y_{rl} \leq \sum_{d=1}^{D} w_d z_{dl}, \quad l \in L \\ & \sum_{d=1}^{D} w_d z_{dk} = 1 \\ & u_r, v_i \geq 0, \ \forall r, i. \end{aligned} \tag{16}$$

Thus, the efficiency bounds $\lfloor \underline{D}_{k,\overline{L}}^2, \overline{D}_{k,\overline{L}}^2 \rfloor$ *of* DMU_k *for subsystem 2 could be obtained by model (15) and model (16).*

4. Empirical Illustration

To illustrate the proposed approach of efficiency bounds for two-stage production systems, we use the following example. In this section, we take the data set of 24 nonlife insurance companies from [13]. These nonlife insurance companies' whole production system has a typical two-stage structure. The production system is divided into two subsystems: premium acquisition and profit generation. These companies are evaluated by using two inputs, two intermediates, and two outputs. Table 1 reports the efficiency bounds of each DMU based on our approach and Salo and Punkka [3]'s

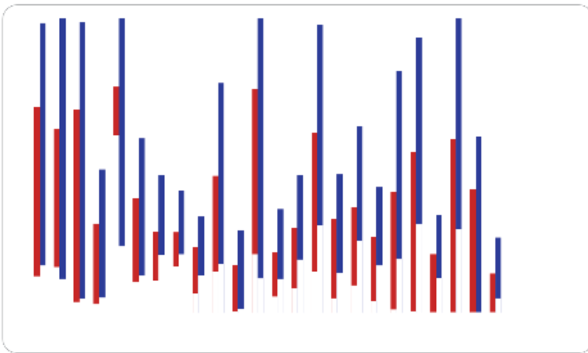

Figure 2: Efficiency bounds for each nonlife insurance company considering and without considering the two-stage production system.

approach. Column 5 and column 6 in Table 1 report the DMUs' efficiency bounds treating the production system as a "Black Box", which are represented graphically blue in Figure 2. Column 3 and column 4 in Table 1 report the DMUs' efficiency bounds when considering the inner production structure, which are represented graphically red in Figure 2.

From Figure 2, we can find that if we do not consider the two-stage production structure, the best DMUs are Chung Kuo (DMU 2), Fubon (DMU 5), Union (DMU 12), and Asia (DMU 22) as they have the best efficiency score of 1. Though these DMUs may have the best efficiency score of 1, they have wide efficiency bounds. Among these five DMUs, Asia (DMU 22) has the best performance as it has the narrowest efficiency bounds as well as the best efficiency score of 1. The worst DMUs are Tai Ping (DMU 3), China Mariners (DMU 4), Kuo Hua (DMU 11), AXA (DMU 23), and Mitsui Sumitomo (DMU 24) as their lower efficiency scores are less than 0.1. Among these five DMUs, the upper efficiency bound of Mitsui Sumitomo (DMU 24) is 0.2571, which is smaller than that of other four DMUs. Thus, Mitsui Sumitomo (DMU 24) is the worst DMU.

The best performers (or worst performers) based on our approach may not be the same as that of Salo and Punkka [3]'s approach. The red bar char in Figure 2 reports the DMUs' efficiency bounds when the two-stage structure is considered. Figure 2 shows that Taiwan Fire (DMU 1) may be the best DNU as its upper efficiency has the largest efficiency score of 0.6992. AXA (DMU 23) may be the worst DMU as its lower efficiency has the least efficiency score of 0.0008.

Besides, we can compare some DMUs over sets of all feasible weights. For example, Chung Kuo (DMU 2) has a best efficiency score of 0.6248 and a worst efficiency score of 0.1565, while Mitsui Sumitomo (DMU 24) has a best efficiency score of 0.1348 and a worst efficiency score of 0.0043. That is, $\underline{D}_{24,\overline{L}} < \underline{D}_{1,\overline{L}}$ and $\overline{D}_{24,\overline{L}} < \underline{D}_{2,\overline{L}} < \overline{D}_{2,\overline{L}}$. Hence, Chung Kuo (DMU 2) always performs better than Mitsui Sumitomo (DMU 24) regardless of the choice of the weights.

When the two-stage structure is considered, the efficiency bounds may be narrower based on our approach than that of Salo and Punkka [3]'s approach. For example, as shown in Column 3 and 4 in Table 1, it could be found that Taian (DMU 7) has the best efficiency score of 0.2766 and the worst efficiency of 0.1118. However, as shown in Column 5 and 6 in Table 1, it has the best efficiency score of 0.4698 and the worst efficiency score of 0.1984 over all feasible weights. Thus, the efficiency bounds based on Salo and Punkka [3]'s approach are wider.

As for two subsystems, Table 2 shows the subsystem's efficiency bounds of each insurance company. The third and fourth columns in Table 2 are the lower and upper efficiency scores for subsystem 1. The fifth and sixth columns in Table 2 are the lower and upper efficiency scores for subsystem 2.

It shows that Central (DMU 9), Union (DMU 12), Cathay Century (DMU 15), North America (DMU 19), and Mitsui Sumitomo (DMU 24) are the best performers in subsystem 1. As Union (DMU 12) has the narrowest efficiency bound, it is the best DMU for subsystem 1. The source of inefficiency could also be identified; for example, Fubon (DMU 5) has the efficiency bound of $[0.6035, 0.7670]$. It can be seen that it performs well in subsystem 2 as the efficiency bound of subsystem 2 is $[1, 1]$, while it does not perform well in subsystem 1 as the efficiency bound of subsystem 1 is $[0.4468, 0.8375]$. Therefore, the reason why Fubon (DMU 5)'s overall efficiency is so low is its bad performance in subsystem 1. As for the subsystem 2, Tai Ping (DMU 3), Fubon (DMU 5), Newa (DMU 17), and Asia (DMU 22) are the best DMUs as their best efficiency scores are all 1. AXA (DMU 23) and Mitsui Sumitomo (DMU 24) may be the worst DMU as its worst efficiency score is the smallest.

5. Conclusions and Direction for Future Research

In previous DEA literature, each DMU is evaluated by using the most favorable weights. However, it ignores other feasible weights. To overcome this problem, Salo and Punkka [3] deemed each DMU as a "Black Box" and developed a series of models to obtain the efficiency bounds over sets of all feasible weights. In this paper, we expand their method by considering the internal structure of the DMUs. We extend their method to compute efficiency bounds for a two-stage production system and illustrate the method by revisiting reported DEA studies. Thus, the "Black Box" is opened, and more accurate information on the efficiency bounds for the overall system and both subsystems is provided to the decision maker. Unlike conventional efficiency scores, the results show how the DMUs' efficiency ratios for the overall system and two subsystems relate to each other for all feasible weights. The efficiency measure used in this paper is radial; some nonradial measures have also been proposed in the literature, such as the slack-based measure [16, 17]. Obtaining the efficiency bounds for two-stage production systems based on nonradial DEA is another interesting avenue to explore in the future.

Appendix

Proof of Proposition 4. Choose the weight (u^*, w^*, v^*) satisfying $D_{k,l}(u^*, w^*, v^*) \geq D_{k,l}(u, w, v)$, (u, w, v) are the

Table 2: Efficiency bounds for two subsystems.

DMU	NO.	Efficiency bounds for sub-system 1		Efficiency bounds for sub-system 2	
		$\underline{D}^1_{k,\overline{L}}$	$\overline{D}^1_{k,\overline{L}}$	$\underline{D}^2_{k,\overline{L}}$	$\overline{D}^2_{k,\overline{L}}$
Taiwan Fire	1	0.2183	0.9926	0.1253	0.7134
Chung Kuo	2	0.4619	0.9985	0.1565	0.6275
Tai Ping	3	0.1427	0.6900	0.0374	1.0000
China Mariners	4	0.0947	0.7243	0.0317	0.4323
Fubon	5	0.4468	0.8375	1.0000	1.0000
Zurich	6	0.2426	0.9637	0.1057	0.4057
Taian	7	0.1639	0.7521	0.1118	0.5378
Ming Tai	8	0.2891	0.7256	0.1587	0.5113
Central	9	0.1392	1.0000	0.0673	0.2920
The First	10	0.1285	0.8615	0.1413	0.6736
Kuo Hua	11	0.1639	0.7405	0.0047	0.3267
Union	12	0.2849	1.0000	0.2005	0.7596
Shingkong	13	0.2067	0.8107	0.0568	0.5435
South China	14	0.1186	0.7246	0.0847	0.5178
Cathay Century	15	0.1910	1.0000	0.1415	0.7047
Allianz President	16	0.1026	0.9072	0.0504	0.3847
Newa	17	0.0873	0.7233	0.0948	1.0000
AIU	18	0.2536	0.7935	0.0418	0.3737
North America	19	0.1231	1.0000	0.0119	0.4158
Federal	20	0.0336	0.9332	0.0068	0.9014
Royal&Sun Alliance	21	0.0103	0.7505	0.0017	0.2795
Asia	22	0.0037	0.5895	0.0011	1.0000
AXA	23	0.0127	0.8501	0.0003	0.5599
Mitsui Sumitomo	24	0.0607	1.0000	0.0043	0.3351

arbitrary weights. We define $v'_i = v^*_i / \sum_{i=1}^m v^*_i x_{ik}$. So, $\sum_{i=1}^m v'_i x_{ik} = 1$. We then define $w'_d = w^*_d \sum_{i=1}^m v'_i x_{il} / \sum_{d=1}^D w^*_d z_{dl}$. Thus, $\sum_{d=1}^D w'_d z_{dl} = \sum_{i=1}^m v'_i x_{il}$. Define $u'_r = u^*_r \sum_{i=1}^m w'_d z_{dl} / \sum_{r=1}^s u^*_r y_{rl}$; thus, $\sum_{r=1}^s u'_r y_{rl} = \sum_{d=1}^D w'_d z_{dl}$. The weights (u', w', v') satisfy five constraints in model (11). And by Lemma 1 in Salo and Punkka [3], $D_{k,l}(u^*, w^*, v^*) = D_{k,l}(u', w', v') = \sum_{r=1}^s u'_r y_{rk}$. So the maximum of model (11) over the five constraints in model (11) is at least as high as $D_{k,l}(u^*, w^*, v^*)$.

Assume the maximum of model (6) is attained at (u^0, w^0, v^0). So, we have

$$D_{k,l}\left(u^0, w^0, v^0\right) = \frac{E_k\left(u^0, w^0, v^0\right)}{E_l\left(u^0, w^0, v^0\right)} = \frac{\sum_{r=1}^s u_r y_{rk}}{\sum_{d=1}^k w_d z_{dk}} \frac{\sum_{d=1}^k w_d z_{dk}}{\sum_{i=1}^m v_i x_{ik}} \frac{\sum_{d=1}^k w_d z_{dl}}{\sum_{r=1}^s u_r y_{rl}} \frac{\sum_{i=1}^m v_i x_{il}}{\sum_{d=1}^k w_d z_{dl}} = \sum_{r=1}^s u^0_r y_{rk}. \tag{A.1}$$

The weights (u^0, w^0, v^0) satisfy the five constraints in model (11). Thus, maximum of $D_{k,l}(u, w, v)$ over all the feasible weights would be larger or equal to the solution of the maximization problem in model (11). The proof of the minimization problem could be shown in the analogous way. □

Proof of Proposition 5 . $D_{k,\overline{L}}(u, w, v) = E_k(u, w, v) / \max_{l \in L} E_l(u, w, v)$. Let the maximum of $D_{k,\overline{L}}(u, w, v)$ be ζ^*. Thus, the optimum is attained at (u^*, w^*, v^*). There then exists some $l^* \in L$ such that $E_{l^*}(u^*, w^*, v^*) \geq E_l(u^*, w^*, v^*)$ $\forall l \in L$. Choose $v'_i = v^*_i / \sum_{i=1}^m v^*_i x_{ik}$, so that $\sum_{i=1}^m v'_i x_{ik} = 1$. Also, choose a constant $c_w > 0$ so that $\sum_{d=1}^D w'_d z_{dl^*} = \sum_{i=1}^m v'_i x_{il^*}$ for $w' = c_w w^*$. Also, choose a constant $c_u > 0$ so that $\sum_{r=1}^s u'_r y_{rl^*} = \sum_{d=1}^D w'_d z_{dl^*}$ for $u' = c_u u^*$. For any $l \in L$, we have

$$1 \geq D_{l,l^*}\left(u^*, v^*\right) = D_{l,l^*}\left(u', v'\right) = \frac{E_l\left(u', v'\right)}{E_{l^*}\left(u', v'\right)} = \frac{\sum_{r=1}^s u'_r y_{rl}}{\sum_{d=1}^D w'_d z_{dl}} \cdot \frac{\sum_{d=1}^D w'_d z_{dl}}{\sum_{i=1}^m v'_i x_{il}} \cdot \frac{\sum_{d=1}^D w'_d z_{dl^*}}{\sum_{r=1}^s u'_r y_{rl^*}} \cdot \frac{\sum_{i=1}^m v'_i x_{il^*}}{\sum_{d=1}^D w'_d z_{dl^*}} = \frac{\sum_{r=1}^s u'_r y_{rl}}{\sum_{d=1}^D w'_d z_{dl}} \cdot \frac{\sum_{d=1}^D w'_d z_{dl}}{\sum_{i=1}^m v'_i x_{il}}. \tag{A.2}$$

So, the constraints $\sum_{r=1}^s u_r y_{rl} \leq \sum_{d=1}^D w_d z_{dl}, l \in L$ and $\sum_{d=1}^D w_d z_{dj} \leq \sum_{i=1}^m v_i x_{il}, l \in L$ are satisfied by (u', w', v'). By construction, $\zeta^* = \max_{u,w,v} D_{k,\overline{L}}(u, w, v) = D_{k,l^*}(u', v') = \sum_{r=1}^s u'_r y_{rk}$, which shows the maximum of model (12) is at least as high as that of $D_{k,\overline{L}}(u, w, v)$.

Conversely, assume that the maximum of model (12), ζ', is attained at (u', w', v'), and choose $l' \in L$ so that the constraint in model (12) is binding (such that l' exists, for otherwise u'could be increased to improve the value of the objective function, which would be in violation of the optimality assumption). Now, $\max_{u,w,v} D_{k,\overline{L}}(u, w, v) \geq E_k(u', w', v')/E_{l'}(u', w', v') = \zeta'$, so that the maximum of $D_{k,\overline{L}}(u, w, v)$must be at least as high as that of model (12). □

Data Availability

The data is from the data set of 24 nonlife insurance companies from [13] or from the corresponding author upon request.

Conflicts of Interest

There are no conflicts of interest related to this paper.

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