

# Efficient Algorithms for Arbitrary Sample Rate Conversion with Application to Wave Field Synthesis

Andreas Franck



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