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EFFICIENT CALCULATION OF ELEMENTARY PARAMETERS OF TRANSFORMERS

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Abstract. - Very efficient procedures for computing elementary parameters (turn leakage inductances and capacitances) in a transformer are presented. The turns are used as a calculation base to permit modeling at very high frequencies. Turn-to-turn (or loop) leakage inductances are obtained by an image method. The charge simulation method is used for finding the capacitances between turns and from turns to ground. The new methods are very efficient compared with the use of the technique of finite elements and are also remarkably accurate. Thus, the short circuit (or test) leakage inductance can be obtained from turn-to-turn information. Examples of calculated parameters are given for illustration. For validation, the results are compared with the parameters obtained using finite elements and tests. The elementary parameters can be used to create reduced order computational models for the calculation of transient phenomena.

Keywords: Electromagnetic transients, Transformer modeling, Leakage inductance of transformers, Capacitance of transformers.

INTRODUCTION

For the study of electromagnetic transients in power systems, component models which are valid for a wide frequency range are needed. Synchronous machines and transmission lines have adequate models that are almost universally accepted. However, no power transformer model, appropriate for a wide range of frequencies, is yet available. One unsolved related problem is the accurate and efficient calculation of the model parameters (inductances and capacitances, resistances and conductances). The present paper intends to contribute in this direction.

The purpose of this paper is the calculation of the parameters that are necessary for the construction of a transient model for transformers. It presents methodologies for computing the leakage inductances and capacitances of transformers. The approach is based on the representation of the winding by its individual turns, in contrast to existing methods, used by transformer designers, which take a global geometrical approach for windings or sections. The new method is simple and general, and particularly appropriate for the resolution needed for the calculation of transients. The complete model, which includes the windings as well as the iron core magnetization, will be presented in a subsequent paper.

We compute the parameters on a turn-to-turn basis. The resulting model is thus adequate for high frequency transients, but it can be reduced to lower order for studies of slower transients. We assume axisymmetrical geometry and infinite iron core permeability. In this way, we separate the physical phenomena occurring inside the core from the phenomena in the window of the transformer (air, insulation and conductors).

A widely used procedure for estimating parameters is the technique of finite elements. For a high frequency model, this would require lengthy computations, mainly due to postprocessing, and tedious manual work to enter the geometric data. We propose alternative methods for the computation of the parameters.

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It has been recognized for a long time that the use of self and mutual inductances for the calculation of low frequency transients may lead to computational difficulties. The main reason is:

For computing transients using an approach based on self and mutual inductances we will have to solve a very ill-conditioned set of equations. The equations are ill-conditioned because the coils (or turns) are very tightly coupled due to the presence of the (unsaturated or lightly saturated) iron-core and, as a consequence, the elements in the inductance matrix **L** are almost identical.

To surmount this difficulty, we deal with the leakage inductances between turns.

One could, theoretically, compute the leakage inductances between turns by subtracting mutual inductances from self inductances, but taking the difference of two nearly equal numbers leads to results of low accuracy. Recently [1],[2], there have been significant advances in the analytical calculation of self and mutual impedances in transformers. Although the analytical expressions obtained require numerical evaluation, the self and mutual impedances are calculated with accuracy. However, quantities related to leakage impedances (differences between self and mutual impedances) are not calculated with the same degree of accuracy, in accordance with the remark made above. To get accurate leakage related quantities, the authors of [1] and [2] rely on tests to compute some adjustment parameters. There is, thus, a need to calculate leakage inductances accurately and efficiently using a direct approach. To this end, and with the aim of avoiding the use of finite elements for computing leakage inductances, a very efficient and sufficiently accurate image method is proposed in this paper.

For the first time in the case of transformers, to our knowledge, the charge simulation approach for computing the capacitances is used. The purpose of using this method is to give a less time-consuming alternative to the use of finite elements. This technique has been employed before for air-cored reactors [3] and other electrode configurations [4],[5]. The method is very efficient compared with the method of finite elements when we compute elementary capacitances, and its accuracy is impressive.

PARAMETER CALCULATION

Leakage Inductances

In this section we will first show how to construct a loop inductance matrix based on leakage turn-to-turn information. Later in this section, we will show how to compute efficiently the turn-to-turn leakage inductances using the image method.

The behavior of an N-turn transformer can be described in terms of self and mutual inductances by

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} & \cdots & L_{1N} \\ L_{21} & L_{22} & \cdots & L_{2N} \\ \vdots & \vdots & & \vdots \\ L_{N1} & L_{N2} & \cdots & L_{NN} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} \tag{1}$$

We assume $\mu \to \infty$. Consequently, all the elements in matrix **L** tend to infinity. It is clear, therefore, that we cannot use a circuit model with self and mutual inductances. Note, however, that with $\mu = \infty$ the magnetizing current must be zero. Thus, we have

$$\sum_{k=1}^{N} i_k = 0 \tag{2}$$

In an actual transformer, the iron permeability is not infinity nor is the magnetizing current zero, but the turns are so tightly coupled that the matrix **L** is ill-conditioned (since its elements are almost equal). Therefore, one is compelled to work with leakage inductances in order to have a well behaved model. To derive a model based on leakage inductances, a procedure similar to using a slack node in power system studies is followed. In this case, we take a turn as a reference (turn N) and use it to assure that condition (2) is met. Then

$$i_N = -\sum_{k=1}^{N-1} i_k = -i_1 - i_2 - \cdots - i_{N-1} \tag{2a}$$

The Loop Inductance Matrix L′

Subtracting the last row in equation (1) from the first $N-1$ rows, we have

$$\begin{bmatrix} v_1 - v_N \\ v_2 - v_N \\ \vdots \\ v_{N-1} - v_N \end{bmatrix} = \begin{bmatrix} L_{11} - L_{N1} & L_{12} - L_{N2} & \cdots & L_{1,N-1} - L_{N-1,N} \\ L_{21} - L_{N1} & L_{22} - L_{N2} & \cdots & L_{2,N-1} - L_{N-1,N} \\ \vdots & \vdots & & \vdots \\ L_{N-1,1} - L_{N1} & L_{N-1,2} - L_{N2} & \cdots & L_{N-1,N-1} - L_{N-1,N} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_{N-1} \end{bmatrix} +$$

$$+ \begin{bmatrix} L_{1N} - L_{NN} \\ L_{2N} - L_{NN} \\ \vdots \\ L_{N-1,N} - L_{NN} \end{bmatrix} \frac{d}{dt} i_N$$

Using equation (2a) to incorporate the last term into the matrix, we obtain

$$\begin{bmatrix} v_1 - v_N \\ v_2 - v_N \\ \vdots \\ v_{N-1} - v_N \end{bmatrix} = \begin{bmatrix} L'_{11} & L'_{12} & \cdots & L'_{1,N-1} \\ L'_{21} & L'_{22} & \cdots & L'_{2,N-1} \\ \vdots & \vdots & & \vdots \\ L'_{N-1,1} & L'_{N-1,2} & \cdots & L'_{N-1,N-1} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_{N-1} \end{bmatrix} \tag{3}$$

The elements of the above loop inductance matrix **L′**, as functions of the elements of the matrix **L**, are:

$$L'_{ij} = L_{ij} - L_{iN} - L_{Nj} + L_{NN} \tag{4}$$

We note that the elements of **L′** can be obtained from leakage inductance tests. This is what permits us to use an alternative method of images to calculate the elements of the loop inductance matrix **L′**. Indeed, if $i = j$, then equation (4) becomes (since $L_{ij} = L_{ji}$)

$$L'_{ii} = L_{ii} + L_{NN} - 2\,L_{iN}$$

which is the definition of the leakage inductance between turns i and N. Thus

$$L'_{ii} = L_{leak\;iN} \tag{5}$$

For $i \neq j$, adding $\pm \frac{1}{2} L_{ii}$ and $\pm \frac{1}{2} L_{jj}$ to (4), we get

$$L'_{i,j} = \tfrac{1}{2}\,[(L_{ii} + L_{NN} - 2\,L_{iN}) + (L_{jj} + L_{NN} - 2\,L_{jN}) - (L_{ii} + L_{jj} - 2\,L_{ij})]$$

Thus, in terms of leakage inductances, we have

$$L'_{ij} = \tfrac{1}{2}(L_{leak\;iN} + L_{leak\;jN} - L_{leak\;ij}) \qquad i \neq j \tag{6}$$

The voltage difference $v_i - v_N$ between the two turn voltages will be called loop voltage. **L′** is called the loop inductance matrix because it is related to the loops formed by two turns connected in opposition. In this connection, the magnetic flux due to the current flowing through one of the turns and the flux produced by the current in the other turn are in opposite direction. For example, loop i is the path formed by turns i and N when they are connected in opposition (see Figure 1). The elements of **L′** can be obtained in two ways:

a) Following standard tests (this is an indirect approach): we perform $N(N-1)/2$ short circuit tests for all possible pairs of turns (i.e. loops) and in this way obtain the leakage fluxes (inductances). Then we use equations (5) and (6) to get the elements of the matrix. This is the approach that we will follow, since we can obtain the loop leakage inductances efficiently and accurately (see next subsection).

b) Using non-standard tests (this a direct method): based on equation (3) we perform $N-1$ short circuit tests, using turn N as a reference and measuring one leakage flux (inductance) for the self terms and $N-2$ linkage fluxes (inductances) for the mutual terms.

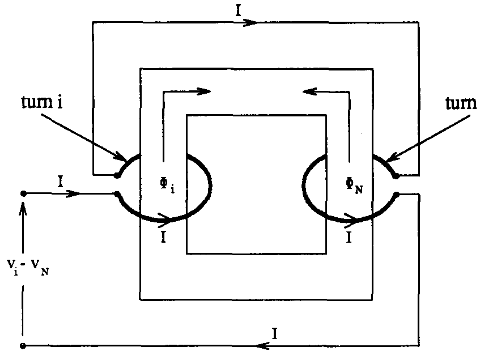


Figure 1. Loop i formed by turns i and N in opposition.

Image Conductors

Inspired by the successful use of complex depths (images) in the calculation of parameters for transmission lines [6] (which permits to avoid the relatively cumbersome procedure based on Carson's formulae) we have substituted the finite elements method in the calculation of leakage inductances for transformers, by a method based on images (see Figure 2). As the magnetic permeability of the iron is very high (infinity in our case) compared with the permeability of the air, the magnetic field is everywhere perpendicular to the iron. Thus the surface of the core is an equipotential for the magnetic scalar potential. In this way, the iron-core surface could be considered as a cylindrical mirror for the magnetic field.

To compute the $N(N-1)/2$ leakage inductances that we need to evaluate the elements of the loop inductance matrix **L′** (equations (5) and (6)), we will need to know the magnetic vector potential that has only a tangential component, A_ϕ. The calculation of A_ϕ requires the evaluation of elliptic integrals. These functions are readily available in many mathematical libraries (e.g. IMSL) or they can be programmed with a fraction of the effort needed for preparing a data file for finite elements.

The magnetic vector potential for a circular filament with unit current is [7]

$$A_\phi(r,z) = \frac{\mu_0}{\pi k}\sqrt{\frac{a}{r}}\left[\left(1 - \frac{k^2}{2}\right)K(k) - E(k)\right] \tag{7}$$

where

$$k = \sqrt{\frac{4\,a\,r}{z^2 + (a+r)^2}} \tag{8}$$

$E(k)$ = elliptic integral of second kind and argument k
$K(k)$ = elliptic integral of first kind and argument k
r = radius of observation filament
a = radius of excited filament
z = vertical separation between the two filaments

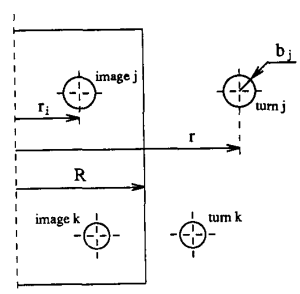


Figure 2. Image method for the loops formed by turns j and k around a core of radius R.

We compute the linkage flux (self and mutual) for the system formed by the turns (in air) and their images (without core) by calculating the vector potential and then integrating around the circular contour (Neumann's formula); see Figure 2. Then, the self-inductance is

$$L_{ii}^{image} = L_{int} + 2\pi\,(a_i - b_i)\,[A_\phi(a_i - b_i, 0) + A_\phi^{image}(a_i - b_i, 0)] \tag{9}$$

where

L_{int} = internal inductance = $\mu_0/8\pi$
a_i = radius of turn i
b_i = conductor radius of turn i
$A_\phi(a_i - b_i, 0)$ = vector potential due to the current in the actual turn
$A_\phi^{image}(a_i - b_i, 0)$ = vector potential due to the current in the image conductor

The mutual inductance is

$$L_{ij}^{image} = (2\pi a_i)\,[A_\phi(a_j, z) + A_\phi^{image}(a_j, z)] \tag{10a}$$

$$= (2\pi a_j)\,[A_\phi(a_i, z) + A_\phi^{image}(a_i, z)] \tag{10b}$$

where

$$z = z_i - z_j$$

To obtain the leakage inductances we use

$$L_{leak\,ij} = L_{ii}^{image} + L_{jj}^{image} - 2\,L_{ij}^{image} \tag{11}$$

Note that, in this case, it is possible to obtain the leakage inductances by subtracting the mutual inductances from the self-inductances, since the former are at least one order of magnitude smaller than the latter. This is not the case when we have the mutual and self-inductances with the iron core and actual geometry. The self- and mutual inductances from the image method given in equations (9) and (10) are not related to the ones in the transformer (equation (1)). However, their differences (in the leakage inductance sense: $L_{ii} + L_{jj} - 2\,L_{ij}$) are very close, as demonstrated later.

We have two parameters for adjusting the leakage inductance value:

a) the radius of the image conductor, and
b) the current flowing through it.

By analogy between the iron leg surface and a curved mirror, the radius of the image conductor can be obtained using the location of an optical image (see Figure 2)

$$r_i = \frac{1}{\dfrac{2}{R} - \dfrac{1}{r}} \tag{12}$$

To adjust the current flowing in the image conductor, we have computed the leakage inductance of various conductor configurations. As a conclusion, when we are interested in the terminal behavior of the transformer, the best value for this current is estimated to be 2.5 times the current flowing in the actual conductor. The current in the image conductors should flow in the same direction as the current in the turns. A current of 1.0 gives good results when the conductors are not close to the leg or to the yokes, or when the conductors are close to each other. For extreme cases, the best current in the image conductors is 4.0. However, using a current of 2.5 for all the image conductors, we get maximum errors of ± 17.7% for a standard design. Moreover, when we compute the test (or total) leakage inductance, the errors compensate and the resulting leakage inductance is very accurate. Two geometrical arrangements were used for comparison (see Figure 3):

A) leg only (no yoke: open geometry), and
B) window (closed geometry).

All comparisons were made against a 2-D finite element program (axisymmetric). We did not have a 3-D field calculation package for comparison with the real geometry. However, the two extreme cases (leg only and window) gave close limits, thus confirming the accuracy of the method. An example of the calculation can be found in the results section.

Turn-to-Turn Capacitances

As a more efficient alternative to the method of finite elements, the capacitances were calculated by a charge simulation approach. The basic principle behind it is to assume that a potential difference ($v = 1$, for convenience) is applied between one turn and all the others connected to each other and to the core. Since all metallic surfaces are equipotentials, we can evaluate a number of simulated charges that will produce the boundary conditions to be met. The charges are on circular filaments and are assumed to be placed inside the metal (see Figure 4). We have to use a number of rings equal to the total number of charges, n, to serve as reference points with specified potential. We can apply a potential difference v (=1) between the core and a conductor, but the potential v_0 of the core (potential with respect to infinity) is not yet known; see Figure 4. Thus, for the boundary points on the core and on the conductors with no excitation, we have $v_i = v_0$; for the boundary points on the excited conductor, we have $v_i = v_0 + 1$. In this way, for each boundary point i we can write an equation as follows:

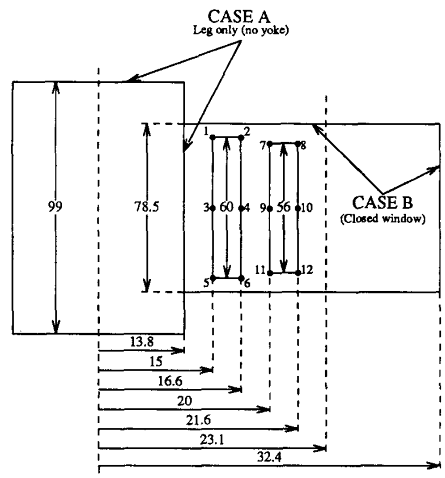
$$\sum_{j=1}^{n} P_{ij}\,q_j \;-\; v_0 = \begin{cases} 0 & \text{for non-excited conductors} \\ 1 & \text{for excited conductor} \end{cases} \tag{13}$$

Since the total charge has to be zero, we have

$$\sum_{j=1}^{n} q_j = 0 \tag{14}$$

Arranging the equation in matrix form for all boundaries, we have a system of $n+1$ equations

$$\begin{bmatrix} \mathbf{P} & -\mathbf{1}^T \\ \mathbf{1} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ v_0 \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ 0 \end{bmatrix} \tag{15}$$



Not to scale - Dimensions in cm

Figure 3. Example: 12-turn transformer.

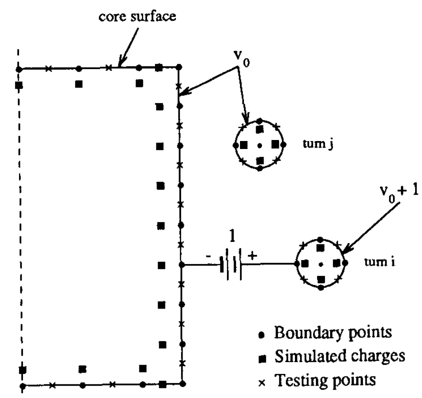


Figure 4. Charge simulation method.

where

$$\mathbf{1} = [\,1\ 1\ \cdots\ 1\,]$$

$$P_{i,j} = \frac{K(k_{i,j})}{2\,\pi\,\varepsilon\,\sqrt{z_{i,j}^2 + (a_i + r_j)^2}} \tag{16}$$

and,

$P_{i,j}$ = potential coefficients (see [7])
$K(k_{i,j})$ = elliptic integral of first kind and argument k
$k_{i,j}$ = argument defined in equation (8)
r_i = radius of observation ring i
a_j = radius of charged ring j
$z_{i,j}$ = vertical separation between the two rings i,j
n = total number of charges
N = number of turns

The elements of the excitation vector $\mathbf{v}$ in equation (15) are zero, according to equation (13), with the exception of the corresponding entries for the excited conductor that are 1. Note that we can use a different number of charges in each conductor. From the definition of self- and mutual capacitances we can see that:

a) The total charge computed inside each non-excited conductor is the capacitance of that conductor with respect to the excited one (mutual capacitance).

b) The charge inside the excited conductor is the self-capacitance. It is the sum of all mutual capacitances and the capacitance from this conductor to ground (core).

In this way, a column of the turn capacitance matrix $\mathbf{C}^{turn}$ is computed from each solution of the equation.

A new set of points (filament rings), different from those used to compute the charges, are taken to test the uniformity of the potential at the surface of the conductors (see Figure 4). There is no systematic technique to find a suitable charge-boundary arrangement. However, even with little prior experience, one can get highly accurate results by trial and error in a reasonable amount of time. Care should be taken to avoid placing a charge in the location of a boundary or test point, since doing so would lead to singularity of the elliptic integral $K(k)$ as $k \to 1$. Also, two charges should not occupy the same point in order to prevent the potential coefficients matrix from becoming singular.

When we have several insulating materials (e.g. paper and oil or air), the dielectric constant ε in equation (16) must be the equivalent dielectric constant for the arrangement. Its proper choice has to be based on tests and experience.

TEST LEAKAGE INDUCTANCE

Previously, we have calculated the leakage inductances for simple loops formed by two turns. In this section we will show how the total or test leakage inductance for a two winding transformer is calculated from the simple loops information. This test leakage inductance corresponds to the leakage inductance that we would measure in a bucking test (when the ampere-turns of the two windings are equal and of opposite direction). This test can be easily performed only when the two windings being tested have an equal number of turns. It should be mentioned that a short circuit test gives almost identical results since the magnetizing current in this test is very small.

Note that the leakage inductance is usually defined for a pair of windings (two windings at a time). For a transformer with more than two windings, a matrix similar to our loop inductance matrix $\mathbf{L}'$ has to be constructed as shown in references [8], [9], [10], [11], [12].

The voltage-current equation for the simple loops of equation (3) is, in matrix form

$$\mathbf{v}_{loop} = \mathbf{L}' \frac{d}{dt}\,\mathbf{i}_{loop} \tag{17}$$

Recall that the above equation has dimension $N-1$.

Each winding consists of a number of turns connected in series, say N_P and N_S. Thus, equation (2) becomes

$$N_P\, i_P + N_S\, i_S = 0 \tag{18}$$

Equation (18) expresses the bucking test condition

$$N_P\, i_P = -N_S\, i_S \tag{18a}$$

Based on this relation, we can introduce a distribution vector $\mathbf{a}$, of dimension N, showing the relative current magnitudes in the turns during the bucking test. The first N_P elements of $\mathbf{a}$ are normalized to 1 while the remaining N_S are equal to: $\alpha = -N_P\,/\,N_S$. Thus, we have

$$\mathbf{a}^T = [\,1, 1, \cdots, 1, \alpha, \alpha, \cdots, \alpha\,] \tag{19}$$

We note that the $N-1$ loop currents coincide with the first $N-1$ turn currents. Therefore, the corresponding loop-test distribution vector $\mathbf{a}'$ is of order $N-1$, and its elements equal to the first $N-1$ elements of $\mathbf{a}$.

We obtain the test leakage inductance from equation (17) by applying the power-invariant transformation

$$\mathbf{i}_{loop} = \mathbf{a}'\ i_{test} \tag{20}$$

$$v_{test} = \mathbf{a}'^T\ \mathbf{v}_{loop} \tag{21}$$

This yields

$$v_{test} = L_{test}\,\frac{d}{dt}\,i_{test} \tag{22}$$

where

$$L_{test} = \mathbf{a}'^{\,T}\ \mathbf{L}'\ \mathbf{a}' \tag{23}$$

Developing this product we obtain

$$L_{test} = \sum_{i=1}^{N_P}\sum_{j=1}^{N_P} L'_{i,j} + \left(\frac{N_P}{N_s}\right)^2 \sum_{i=N_P+1}^{N-1}\ \sum_{j=N_P+1}^{N-1} L'_{i,j} - 2\left(\frac{N_P}{N_s}\right)\sum_{i=1}^{N_P}\ \sum_{j=N_P+1}^{N-1} L'_{i,j} \tag{24}$$

We note that equation (24) is similar to the equation used for computing leakage inductances ($L_{ii} + L_{jj} - 2\,L_{ij}$). The test voltage, obtained from equation (21), is

$$v_{test} = \sum_{i=1}^{N_P} v_i \ -\ \frac{N_P}{N_S}\sum_{i=N_P+1}^{N} v_i \tag{25}$$

Two concrete examples of this calculation are presented in Appendix 1. In the next section, the results are compared with those obtained with the technique of finite elements and by tests.

RESULTS

Several turn configurations were used to test the alternative methods described here for the computation of parameters. As an example consider the transformer data given in Figure 3, where we show the conductors in the extreme positions (1,2,...,12). All other conductors have inductances and capacitances in between those of the conductors shown in this figure.

Leakage Inductances

In the next table 11 leakage inductances (in henry) out of the total of 66 are presented and compared against the results from those obtained by the method of finite elements. The following table was obtained using a current of 2.5 in all image conductors.

LEAKAGE INDUCTANCE	IMAGE METHOD	FINITE ELEMENTS			
		(A)	% Error	(B)	% Error
L(1, 2)	0.83390e-06	0.74017e-06	-12.7	0.74052e-06	-12.6
L(1, 3)	0.33448e-05	0.31906e-05	-4.8	0.41133e-05	18.7
L(1, 4)	0.30403e-05	0.30822e-05	1.4	0.40111e-05	24.2
L(1, 5)	0.34491e-05	0.38188e-05	9.7	0.67759e-05	49.1
L(1, 6)	0.31610e-05	0.37053e-05	14.7	0.66787e-05	52.7
L(1, 7)	0.18691e-05	0.15878e-05	-17.7	0.16030e-05	-16.6
L(1, 8)	0.21073e-05	0.18007e-05	-17.0	0.18230e-05	-15.6
L(1, 9)	0.32321e-05	0.32889e-05	1.7	0.42389e-05	23.8
L(1,10)	0.32922e-05	0.33104e-05	0.6	0.42830e-05	23.1
L(1,11)	0.33815e-05	0.38761e-05	12.8	0.66948e-05	49.5
L(1,12)	0.34545e-05	0.39059e-05	11.6	0.67650e-05	48.9

We are presenting one of the worst cases. The other 55 leakage inductances have errors in the same range or smaller. We adjusted the leakage inductances calculated with the image method to be closer to the geometry of case A (leg only) than to the geometry of case B (closed window) because the actual geometry of a transformer in the region at the top of the windings shows close to 90% air and 10% iron for the leakage flux path (see Figure 5), as the yokes cover only a fraction of the top of the windings. Some of the values obtained with the image method lie in the region between the two geometries but, in general, the values are slightly smaller than those of geometry A.

We can adjust the image currents to treat independently:

a) the turns near the leg or yoke, which requires large image currents ($i_{image} \simeq 4.0$)

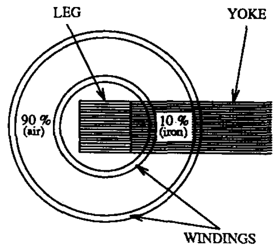


Figure 5. Region at the top of the windings.

b) the turns close to each other, which requires small image currents ($i_{image} \approx 1.0$)

c) the turns far from each other and from the iron, which requires no image current ($i_{image} \approx 0.0$)

The results obtained by adjusting the image currents as described above are shown in the next table.

LEAKAGE INDUCTANCE	IMAGE CURRENT	% Error
L(1, 2)	1.0	2.5
L(1, 3)	2.5	-4.8
L(1, 4)	2.5	1.4
L(1, 5)	3.0	-0.3
L(1, 6)	3.5	-1.7
L(1, 7)	0.0	0.4
L(1, 8)	0.0	1.4
L(1, 9)	2.5	1.7
L(1,10)	2.5	0.6
L(1,11)	3.5	-0.1
L(1,12)	3.5	-0.7

In this way we have obtained very accurate turn-to-turn leakage inductances. This adjustment requires some extra computational effort, mainly for evaluating the distances between conductors and distances from conductors to the leg and yokes. This process has to be followed if we want accurate local responses (e.g. internal voltage distribution during a transient). If we are interested only in the terminal behavior we believe, however, that this sophistication is not necessary. As we will show next, the errors introduced by using a common rule, of a multiplier equal to 2.5, for the current in the image conductors, compensate each other when we calculate the test leakage inductance.

We computed the test leakage inductance using the arrangement shown in Figure 6. The step by step process is described in Appendix 1. We used for comparison the two configurations with finite elements (A and B) and with a familiar approximate equation which assumes that the field is axial [13]:

$$L_{leak} = \frac{\mu_0 N^2}{l_S} \left(l\,\delta + \frac{a_1 l_1 + a_2 l_2}{3} \right) \tag{26}$$

The results are shown in the following table (with geometry A as reference).

Method	Leakage Inductance [H]	% Difference
Finite Elements (A)	1.43677E-5	---
Finite Elements (B)	1.45932E-5	- 1.57
Image Method	1.43600E-5	0.05
Equation (26)	1.18409E-5	17.59

We performed the bucking test (windings connected in series opposition) and the short circuit test on a 2 kVA transformer, 110/110 V, with two windings of 118 turns in two layers. In Appendix 2 the geometrical data are presented. The leakage inductance obtained from tests is 4.5×10^{-4} H. The same result was obtained in both tests (bucking and short circuit). Using the method described above (and in Appendix 1) to simulate the bucking test (equation (24)) we got 4.3×10^{-4} H. The error is less that 5%.

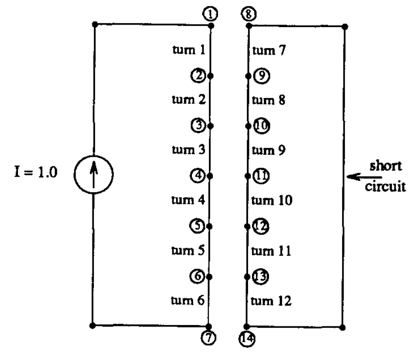


Figure 6. Test leakage inductance for the 12-turn transformer.

As we can see the errors obtained in the calculation of leakage inductances for turns compensate each other when we lump the turns to form a winding. Note also that there is not much difference between the geometries (A) and (B) using finite elements to calculate the parameters. This means that the effect of the yokes in the leakage inductance is very small. Therefore, we do not introduce large errors when we neglect the yokes in the image method. The assumption that the magnetic field is axial (equation (26)) leads to a larger error.

Capacitances

For the 12-turn transformer (Figure 3) no more than 30 simulated charges in the core and 4 charges per conductor are needed to obtain very good potential profiles along the surfaces. The maximum error found was around 1% of the specified potential in the testing points. The first column of the capacitance matrix for this example is presented in the next table. The assumed geometry is leg only, no yoke (case A). The values with closed window (case B) are 20% to 25% greater than those shown in the table. But, as mentioned before, the real geometry is closer to case A than to case B (see Figure 5).

TURN TO TURN CAPACITANCES [F]	
TURNS (i,j)	CHARGE SIMULATION
C(1,1)	0.1251667e-09
C(2,1)	-0.1965621e-10
C(3,1)	-0.9477953e-11
C(4,1)	-0.1966649e-10
C(5,1)	-0.9264550e-11
C(6,1)	-0.1965621e-10
C(7,1)	-0.9477953e-11
C(8,1)	-0.5964382e-11
C(9,1)	-0.6836441e-11
C(10,1)	-0.5550385e-11
C(11,1)	-0.6815330e-11
C(12,1)	-0.5964382e-11

These values were calculated with $\varepsilon = 1$. For a transformer with different insulating materials we should multiply the values of the capacitance by the equivalent dielectric constant of the insulation.

Another test for the capacitances was to use a transformer with a very tall leg and the conductors were placed (radially) close to the surface and axially far apart from each other. In this way, the geometry resembles a multi-conductor transmission line over a flat (perfect conductor) plane. Then we compared the capacitances obtained for this transformer with the capacitances calculated with equations for a transmission line. As the height over the ground in the transmission line formulae, we used the radius of the turn minus the radius of the core of the transformer ($h_i = a_i - R$), and as the length of the conductor, we used the length of the turn ($l_i = 2\pi a_i$). The errors were found to be less than 1%.

Frequency Response

The short circuit frequency response for the transformer shown in Appendix 2 was obtained with a model derived from the parameters calculated in this paper. In Figure 7 the simulated frequency response is compared with field tests. We present the input admittance in the primary when the secondary is short circuited. It can be seen that the results are in good agreement up to a frequency of 700 kHz. The differences beyond this frequency, especially at the resonance point, may be attributed to the losses (in the conductor and core), neglected in the simulation. The insulation of the transformer consists of paper ($\varepsilon_r = 3$, for impregnated paper) and air; we used an equivalent dielectric constant $\varepsilon_r = 2.5$. Details concerning the construction of the model will be presented in a future paper.

CONCLUSIONS

Very efficient methods have been described for the computation of the turn-to-turn parameters of transformers. The leakage inductances are obtained in a simple way and with reasonable accuracy by an approach based on images, analogous to the methods used in the calculation of transmission line parameters. The validity of the method has been confirmed by comparison with short circuit inductances computed with the method of finite elements and a classical design formula. The capacitances are obtained very accurately by a charge simulation approach.

The turn-to-turn parameters are intended to be used in a reduced order winding model, in conjunction with the iron core magnetization model, for the complete modeling of transformers for calculation of transients. The details of the development of such a model will be reported in a sequel to this paper.

ACKNOWLEDGEMENTS

Financial support by the Natural Sciences and Engineering Research Council of Canada is gratefully acknowledged. The first author wishes to express his sincere gratitude to the National Council of Science and Technology of Mexico (CONACYT) and to the National Polytechnic Institute of Mexico for the financial support of his studies at the University of Toronto.

REFERENCES

[1]. D.J. Wilcox, M. Conlon, and W.G. Hurley, "Calculation of Self and Mutual Impedances for Coils on Ferromagnetic Cores", IEE Proceedings, Vol. 135, Pt. A, No. 7, September 1988, pp. 470-476.

[2]. D.J. Wilcox, W.G. Hurley, and M. Conlon, "Calculation of Self and Mutual Impedances Between Sections of Transformer Windings", IEE Proceeding, Vol. 136, Pt. C, No. 5, September 1989, pp. 308-314.

[3]. A. Dahab, "Analysis of Voltage Transients in Air-Cored Reactors", Ph.D. Thesis, University of Toronto, November 1987.

[4]. D.R. Cruise, "A Numerical Method for the Determination of an Electric Field About a Complicated Boundary", Journal of Apllied Physics, Vol. 34, No. 12, December 1963, pp. 3477-3479.

[5]. H. Singer, H. Steinbigler, and P. Weiss, "A Charge Simulation Method for the Calculation of High Voltage Fields", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-94, 1974, pp 1660-1668.

[6]. A. Deri, G. Tevan, A. Semlyen, and A. Castanheira, "The complex Ground Plane - A Simplified Model for Homogeneous and Multi-Layer Earth Return", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-100, No. 8, August 1981, pp. 3686-3693.

[7]. W.R. Smythe, "Static and Dynamic Electricity", McGraw-Hill, 1968.

[8]. H. W. Dommel, "Transformers Models in the Simulation of Electromagnetic Transients", Proc. 5th Power Systems Computation Conference, 1975, Sept. 1-5, paper 3.1/4.

[9]. J.H. McWhirter, C.D. Fahrnkopf, and J.H. Steele, "Determination of Impulse Stresses within Transformer Windings by Computers", AIEE Transactions on Power Apparatus and Systems, February 1957, pp. 1267-1274.

[10]. L.F. Blume, et al., "Transformer Engineering", Second Edition, John Wiley and Sons, 1951.

[11]. R.B. Shipley, D. Coleman, and C.F. Watts, "Transformer Circuits for Digital Studies", AIEE Transactions Pt. III (Power Apparatus and Systems), Vol-81, February 1963, pp. 1028-1031.

[12]. R.C. Dugan, R. Gabrik, J.C. Wright, and K.W. Patten, "Validated Techniques for Modeling Shell-Form EHV Transformers", IEEE Transactions on Power Delivery, Vol-4, No. 2, April 1989, pp. 1070-1078.

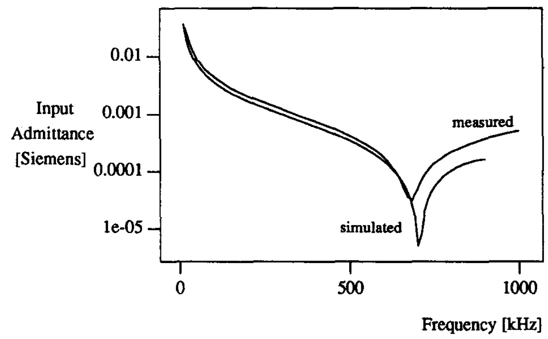


Figure 7. Frequency response

[13]. K. Karsai, D. Kerenyi, and L. Kiss, "Large Power Transformers", Elsevier, 1987.

APPENDICES

Appendix 1 - Test Leakage Inductance

We present a step by step calculation of the bucking test leakage inductance for the transformer shown in Figure 3 (case A) and connected as shown in Figure 6. We follow the procedure described under the heading Test Leakage Inductance.

To compute the equivalent leakage inductance, we start by constructing the corresponding loop inductance matrix $\mathbf{L}'$, equation (3) (of order 11 in our example). The turn leakage inductances are calculated with equations (5) and (6). The $L_{leak\,ij}$ terms are calculated with the image method by combining equations (9), (10), and (11). We have then

$$\begin{bmatrix} v_1 - v_{12} \\ v_2 - v_{12} \\ \vdots \\ v_{11} - v_{12} \end{bmatrix} = \begin{bmatrix} L'_{1,1} & L'_{1,2} & \cdots & L'_{1,11} \\ L'_{2,1} & L'_{2,2} & \cdots & L'_{2,11} \\ \vdots & \vdots & & \vdots \\ L'_{11,1} & L'_{11,2} & \cdots & L'_{11,11} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_{11} \end{bmatrix} \tag{A-1}$$

As a first example, we assume $N_P = N_S = 6$ so that $\alpha = -1$. The transpose of the distribution vector $\mathbf{a}'$ for our 12-turn (or 11-loop) transformer is:

$$\mathbf{a}'^T = [1,1,1,1,1,1,-1,-1,-1,-1,-1] \tag{A-2}$$

Thus, from equation (24) we obtain

$$L_{test} = \sum_{i=1}^{6}\sum_{j=1}^{6} L'_{i,j} + \sum_{i=7}^{11}\sum_{j=7}^{11} L'_{i,j} - 2\sum_{i=1}^{6}\sum_{j=7}^{11} L'_{i,j} \tag{A-3}$$

We obtain the test voltage from equation (21) as:

$$v_{test} = [1,1,1,1,1,1,-1,-1,-1,-1,-1] \begin{bmatrix} v_1 - v_{12} \\ v_2 - v_{12} \\ \vdots \\ v_{11} - v_{12} \end{bmatrix} \tag{A-4}$$

which yields

$$v_{test} = (v_1 + \cdots + v_6) - (v_7 + \cdots + v_{12}) \tag{A-5}$$

or, in accordance with equation (25),

$$v_{test} = \sum_{i=1}^{6} v_i - \sum_{i=7}^{12} v_i \tag{A-6}$$

As a second example, we consider the same 12-turn transformer with $N_P = 8$ and $N_S = 4$. The loop-test distribution vector is now

$$\mathbf{a}'^T = [1,1,1,1,1,1,1,1,-2,-2,-2] \tag{A-7}$$

The bucking test leakage inductance is:

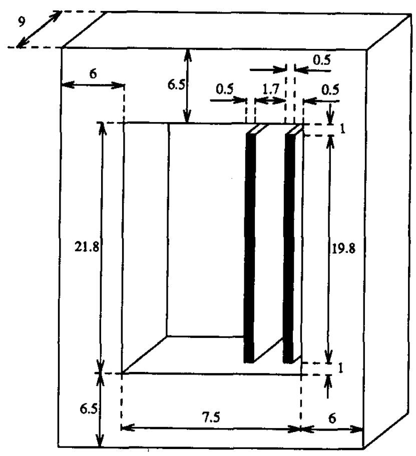
$$L_{test} = \sum_{i=1}^{8}\sum_{j=1}^{8} L'_{i,j} + (2)^2 \sum_{i=9}^{11}\sum_{j=9}^{11} L'_{i,j} - 2\,(2) \sum_{i=1}^{8}\sum_{j=9}^{11} L'_{i,j} \tag{A-8}$$

and the test voltage becomes

$$v_{test} = (v_1 + \cdots + v_8) - 2\,(v_9 + \cdots + v_{12}) = \sum_{i=1}^{8} v_i - 2\sum_{i=9}^{12} v_i \tag{A-9}$$

Appendix 2. Transformer Data

Single phase, 2 kVA, 2 windings, 110 V / 110 V, 2 layers per winding, 59 turns per layer, square conductor (a=3.5mm insulated). The physical dimensions are shown in Figure 8.



Not to scale - Dimensions in cm

Figure 8. Transformer data.

Francisco de Leon was born in Mexico, in 1959. He received his B.Sc. degree and his M.Sc. degree (summa cum laude) from the National Polytechnic Institute of Mexico, in 1983 and 1986 respectively. From 1984 to 1987 he worked as a lecturer at the same institute. Currently, he is working towards his Ph.D. at the University of Toronto. His main interests are in the areas of transformer modeling and electromagnetic fields.

Adam Semlyen (F'88) was born and educated in Rumania where he obtained a Dipl. Ing. degree and his Ph.D. He started his career with an electric power utility and held an academic position at the Polytechnic Institute of Timisoara, Rumania. In 1969 he joined the University of Toronto where he is a professor in the Department of Electrical Engineering. His research interests include the steady state and dynamic analysis of power systems, electromagnetic transients, and power system optimization.

Discussion

C. M. Arturi, (Politecnico di Milano, Italy): I wish to congratulate Professor Semlyen and Dr. De Leon for their very interesting and well written paper on the subject of transformer modelling for an efficient simulation of current and voltage transients. They have given an appreciable contribution on a problem which is still partially unsolved.

I refer to the inductive parameters of the transformer models. In the paper, it has very clearly been shown how one can transit from eq. (1), which expresses the behaviour of a transformer, seen as a mutual inductor with N mutually coupled inductors, to eq. (3), which expresses the behaviour of the same system with the constraint that the permeance of the common magnetic circuit of any couple of inductors of the set must be infinite. In terms of current or M.M.F. this is stated by eq. (2).

In this way, instead of the self and mutual inductances of the matrix L, which are infinite, the system is identified by the reduced matrix inductances L′ of the eq. (3). The diagonal terms of matrix L′ coincide with some of the binary short-circuit (or leakage) inductances (eq.(5)) whereas the inductances out of the diagonal are obtained by linear combination of three of the binary short-circuit inductances (eq. (6)).

The physical meaning of the matrixes L and L′ is profoundly different. The self and mutual inductances matrix L represents the total magnetic energy of the system, which includes, in an indistinguishable way, both the energy related to the common flux, mainly located in the ferromagnetic core, and that related to the leakage fluxes, located in the window and in the space occupied by the windings and insulating materials. With the hypothesis that the permeance of the common magnetic circuit is infinite, namely, that the energy related to the common flux is zero, one obtains the reduced matrix L, which represents only the energy related with air-leakage fluxes.

The leakage inductances of the matrix L′ can also be expressed as a linear combination of some self and mutual inductances of the system. Prof. Semlyen and Dr. De Leon propose to compute the self and mutual inductances, necessary to obtain the leakage inductances, by means of the image method, replacing the actual closed configuration of the ferromagnetic core with an opened configuration, made of a cylindrical and rectilinear core with infinite length and permeability (eq. (11)).

I wonder if the computation method of the inductive parameters proposed by the authors, which gives finite values for self and mutual inductances between turns or windings, still takes into account the hypothesis on which the eq. (3) is based, namely, infinite permeance of the common magnetic circuit of any couple of turns or windings.

Could the authors please comment this point?

F. de Leon and A. Semlyen (University of Toronto): We appreciate the interest of professor Arturi in our paper and offer the following remarks regarding his question on the use of the image method in the computation of the leakage inductances.

The elements of the $\mathbf{L}'$ matrix are obtained directly rather than using the elements of $\mathbf{L}$. Equations (5) and (6) show which leakage inductances are necessary to compute the elements of $\mathbf{L}'$. These leakage inductances are calculated with equation (11) using the image method for obtaining the right hand side (self and mutual) inductances. The latter correspond to the geometry of Figure 2 and have no relation with the actual self and mutual inductances of the turns in the transformer as given in matrix $\mathbf{L}$. Since a pair of turns is considered at a time, connected in opposition as shown in Figure 1, equation (2) is satisfied.

In Figure A we show the physical interpretation of matrix $\mathbf{L}'$. We can see from equation (3) that the elements of $\mathbf{L}'$ represent voltages between loops or between turns in the presence of a short circuited turn.

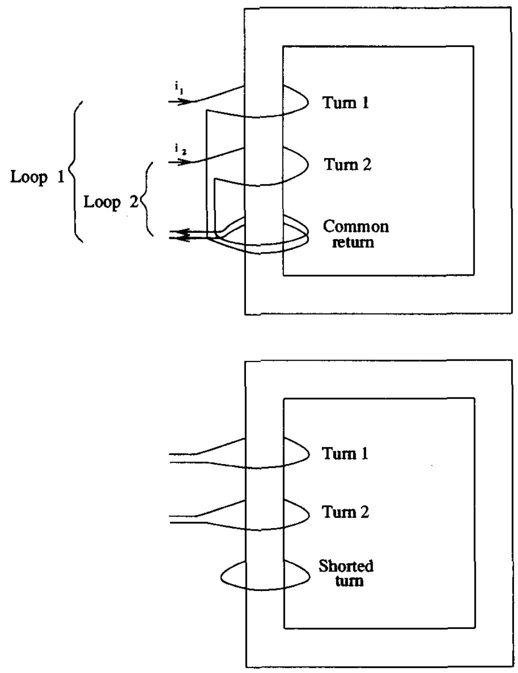


Figure A. Interpretation of of the elements of matrix $\mathbf{L}'$