

# Efficient cubic spline interpolation implemented with FIR filters

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**Abstract:** Classical Cubic spline interpolation needs to solve a set of equations of high dimension. In this work we show how to compute the interpolant using a FIR digital filter, with a reduced number of operations per interpolated point and high accuracy. Additionally, the computation can be made on real time as the signal samples are acquired. Following this approach, we show how to obtain easily the derivatives of the interpolant in a similar way, and also signal approximations to reduce the oscillations that appear when using high order splines. These techniques are very well suited to compute continuous representations of image contours on closed shapes and to find its curvature and singularities.

**Keywords:** representation of contours; cubic spline interpolation, splines; least squares filter; B-splines; filter approximation.

## I. Introduction

Features from images, like contours [1] or lines, often provide enough information to characterize and classify them. Examples are images from fingerprints, hand signatures, liquid level and palm print, [2], [3], [4], [5]. Other examples are images of fish otoliths, which can be used to accurately separate different species. Otoliths are calcified structures of the inner ear of teleost fish, and have auditory and sensory functions. The database AFORO is an example of online classification based on contours, it stores over 2000 images of otoliths and can be used to carry out morphometric analyses and online classification [6], [7], [8], [9] (<http://www.cmima.csic.es/aforo/>).

The classifiers used in AFORO apply a discrete polar representation of the contour radius, but this can lead to ambiguities in curves that are quite convex or concave. In these cases, using the contour curvature instead of the radius could solve this problem. However, the curvature is calculated with first and second order derivatives, which are not well defined in the discrete world. In addition they are very susceptible to noise, such as the quantization noise that is always present in digital signals.

To solve these problems we propose using a continuous representation of the contour with splines, which results in a very suitable method for this type of operation.

The easiest option would be to use linear splines, but this does not guarantee the continuity of the derivatives at the connection points. In contrast, higher order, quadratic or cubic splines do not show this problem until the second or third derivative respectively.

The splines are calculated by solving a system of equations with dimensions equal to the number of points to interpolate, multiplied by a number equal to or greater than the order of the splines. This is not viable in web applications that need to compute this representation in runtime.

Unser [10], [11] developed a mathematical theory for implementing splines as digital filters for uniformly equispaced samples. Although his solution is very elegant it implies applying causal and anticausal IIR filters. Based on this solution there is an efficient implementation of these splines, using approaches with FIR filters obtained by windowing the impulse response [12], [13].

In this paper we show another way of implementing these splines with FIR filters. The main advantages of using FIR filters instead of anticausal IIR filters is that they can be implemented online, that is, as they receive the data, although they do have a delay because they are not causal filters. In [14] there is a basic definition and hardware implementation of FIR filters.

If we interpolate the original contour points with splines directly, it is very possible that oscillations will appear in the continuous contour obtained. One cause could be the quantization noise. To avoid this we can first filter the data to obtain smooth contours that are still faithful to the original contour. We propose applying the least squares filter, defined in [11], which is a good compromise between computational needs and the desired result.

In this paper we develop these points in the following sections: First, in section II, we show a way to implement FIR filters based on splines, such as mathematical operators for calculating first and second order derivatives. Section III

shows the connection between this work and Unser's solution based on B-splines. We do this based on the Z Transform of the spline equation system, and demonstrate the connection between the determinant of this equation system and the anticausal IIR filter he proposes for moving a signal to the B-spline domain. In the section IV we present an approximation of one FIR operator to do cubic spline interpolation with only 5 multiplications and ten additions, with an error less than 0.1%. In section V there are some examples of cubic spline interpolation. Section VI details the application for obtaining the best continuous approximation to a discrete contour, interpolating with cubic splines but first applying the FIR approximation and invariant to the translations of the least squares filter. The way of implementing this filter does not involve downsampling, unlike the solution given in [11].

Finally, we analyze the results, give some conclusions and outline possible future applications of the work.

## II. Polynomial splines

Interpolating with splines implies defining polynomials between samples [15], so that at the junction points the polynomials coincide but there is also continuity in their derivatives up to one order immediately below the level of the splines. In the case of cubic splines, the polynomials that join the samples show continuity until the second derivative and have the following form (1):

$$P_n(x) = Y_n + b_n \cdot (x - x_n) + c_n \cdot (x - x_n)^2 + d_n \cdot (x - x_n)^3 \quad (1)$$

In the case of equispaced samples and normalized period ( $T=1$ ), each  $n$  polynomial exists between  $x=x_n$  and  $x=x_n+1$ . Each polynomial has 3 unknowns:  $b_n$ ,  $c_n$  and  $d_n$ .  $Y_n$  is the value of the left sample from where the  $n$  polynomial starts. The equations (2), (3) and (4) are taken from the conditions of continuity of the polynomials and from the first and second derivatives in the knots; however, there are always two degrees of freedom, which are usually fixed by imposing that the first or the second derivative at the beginning and ending sample are zero or imposing that the interpolation is periodic so that the last sample continues from the first.

$$P_n(x_{n+1}) = Y_n + b_n + c_n + d_n = P_{n+1}(x_{n+1}) = Y_{n+1} \quad (2)$$

$$b_n + 2 \cdot c_n + 3 \cdot d_n = b_{n+1} \quad (3)$$

$$2 \cdot c_n + 6 \cdot d_n = 2 \cdot c_{n+1} \quad (4)$$

As an example of magnitude, interpolating 16 samples will involve solving a linear system of 48 equations, and a system of 100 samples implies solving a system of 300 equations. In this paper we have solved this equation system for different numbers of samples, and experimentally, by simple observation, we have concluded that the polynomial coefficients can be calculated using the discrete convolution between the samples and coefficients that can be represented perfectly by an anticausal FIR filter. For example, if there are 5 samples to interpolate, coefficient  $b$ , which coincides with the first derivative at the point where the sample is, can be determined with the following expression (5):

$$b(n) = 0.80 \cdot (y(n+1) - y(n-1)) - 0.21 \cdot (y(n+2) - y(n-2)) \quad (5)$$

$b(n)$  = the coefficient  $b$  of the polynomial  $n$ .

$y(n)$  = the  $n$ -th sample of the sequence.

The inverse matrix method was used to solve the equation system. To find the stability of the solution the equation system has been solved for different numbers of samples. Table I shows the result of the filter coefficients for calculating the first derivative with cubic splines in periodic form, interpolating between 16 and 32 samples respectively

|         |     | with 16 Knots | with 32 Knots |
|---------|-----|---------------|---------------|
| k=0     |     | 0             | 0             |
| k=+/-1  | +/- | 0,80384757    | 0,803847577   |
| k=+/-2  | -/+ | -0,21539028   | -0,215390309  |
| k=+/-3  | +/- | 0,057713549   | 0,057713659   |
| k=+/-4  | -/+ | -0,015463918  | -0,015464328  |
| k=+/-5  | +/- | 0,004142121   | 0,004143654   |
| k=+/-6  | -/+ | -0,001104566  | -0,001110289  |
| k=+/-7  | +/- | 0,000276141   | 0,000297501   |
| k=+/-8  | -/+ | 0,00E+00      | -7,97E-05     |
| k=+/-9  | +/- |               | 2,14E-05      |
| k=+/-10 | -/+ |               | -5,72E-06     |
| k=+/-11 | +/- |               | 1,53E-06      |
| k=+/-12 | -/+ |               | -4,11E-07     |
| k=+/-13 | +/- |               | 1,10E-07      |
| k=+/-14 | -/+ |               | -2,94E-08     |
| k=+/-15 | +/- |               | 7,34E-09      |
| k=+/-16 | -/+ |               | -1,22E-09     |

Table 1. Coefficients of the first derivative operator based on cubic splines

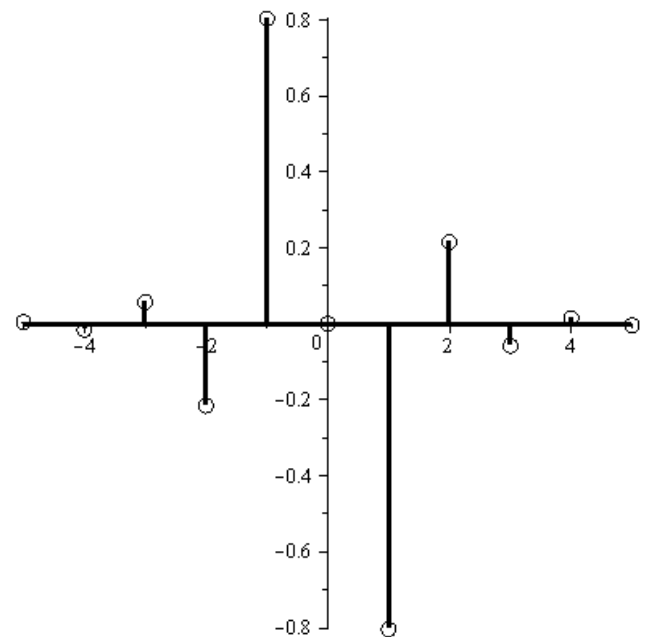
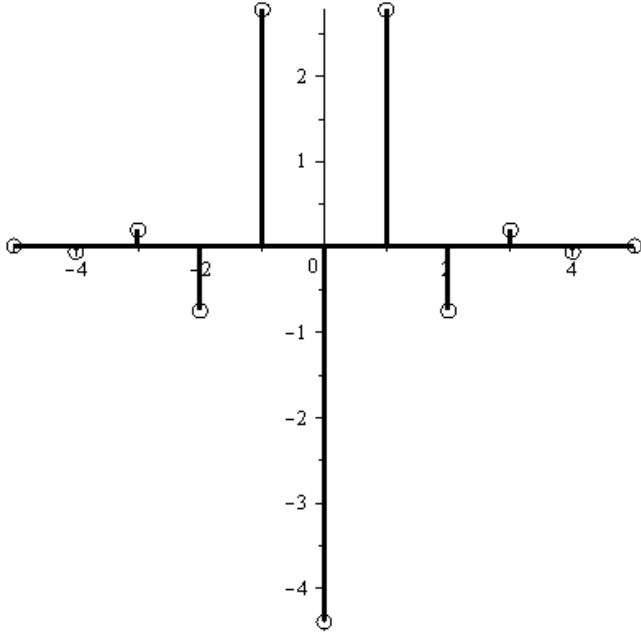


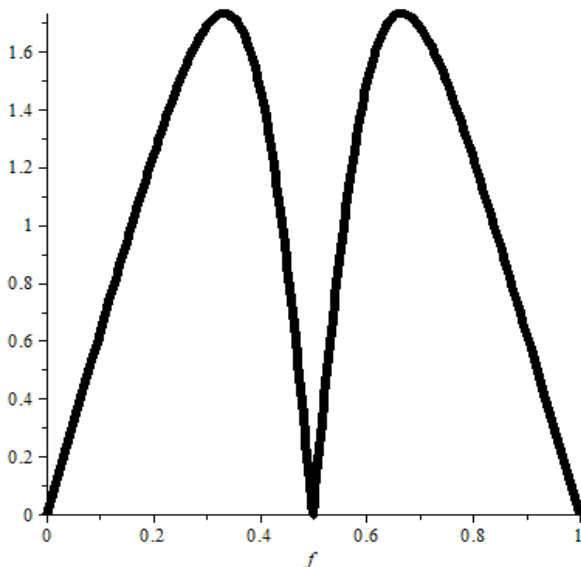
Figure 1. Time response of the first derivative operator, based on cubic splines.

In the same way, similar results are obtained for the second derivative calculation, which at the point of the sample is equal to  $2 \cdot c_n$ . It can be concluded that calculating the first and second derivative of a discrete signal, which has become continuous with splines, is equivalent to carrying out the convolution of the samples with the discrete operators shown in Fig. 1 and Fig. 2.



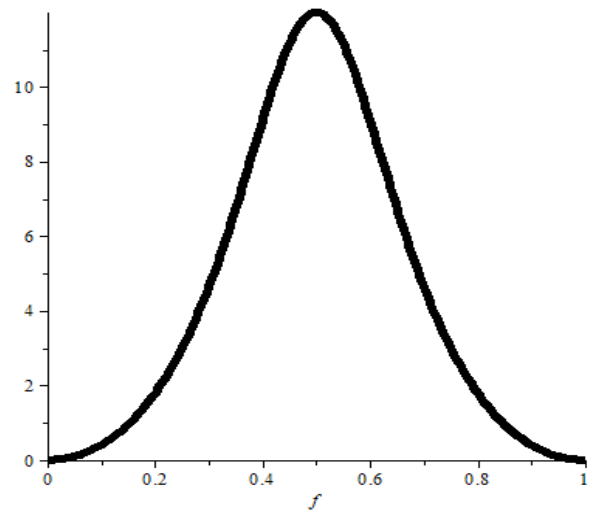
**Figure 2.** Time response of the second derivative operator, based on cubic splines.

In Fig. 3 and Fig. 4 are shown the magnitude of the frequency response of the first and second derivative operators.



**Figure 3.** Magnitude of the frequency response of the first derivative operator based on cubic splines. Band pass filter.

We can see that the first derivative operator is a band pass filter and the second derivative operator is a high pass filter.



**Figure 4.** Magnitude of the frequency response of the second derivative operator based on cubic splines. High pass filter.

### III. Connection between polynomial interpolation with splines and B-splines

Next we determine the connection between the interpolation method presented in the previous section and that based on B-splines proposed by Unser in [10] and [11].

The first thing we will do is to consider that with the interpolation with splines we have a sequence of polynomials and we also have a sequence of coefficients  $b_n$ ,  $c_n$  and  $d_n$  of the polynomials, therefore we can calculate the Z Transform of these sequences. We can also find the Z Transform of the equation system (2), (3) and (4) implied by the interpolation with splines:

$$B(z) + C(z) + D(z) = (z-1) \cdot Y(z) \quad (6)$$

$$B(z) + 2 \cdot C(z) + 3 \cdot D(z) = z \cdot B(z) \quad (7)$$

$$2 \cdot C(z) + 6 \cdot D(z) = z \cdot 2 \cdot C(z) \quad (8)$$

$B(z)$  = Z Transform of the coefficients  $b_n$  of the spline polynomials.

$C(z)$  = Z Transform of the coefficients  $c_n$  of the spline polynomials.

$D(z)$  = Z Transform of the coefficients  $d_n$  of the spline polynomials.

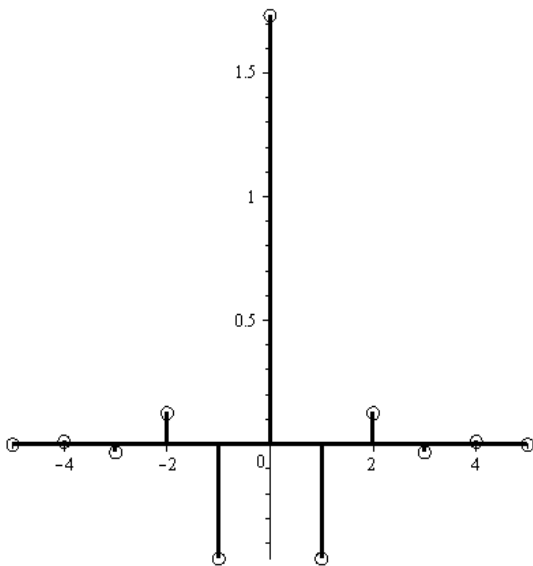
$Y(z)$  = Z Transform of the samples (the input signal).

The first equation (6) is the continuity of the polynomial. The second equation (7) represents the continuity of the first derivative in the knots, and the third (8) is due to the continuity of the second derivative.

This equation system written in matrix is the following (9):

$$\begin{pmatrix} 1 & 1 & 1 \\ (1-z) & 2 & 3 \\ 0 & (2-2 \cdot z) & 6 \end{pmatrix} \cdot \begin{pmatrix} B(z) \\ C(z) \\ D(z) \end{pmatrix} = \begin{pmatrix} (z-1) \cdot Y(z) \\ 0 \\ 0 \end{pmatrix} \quad (9)$$

The solutions are the expressions (10), (11) and (12)



**Figure 5.** Impulse response of the operator for moving a signal to the cubic B-spline domain,  $(B^3(z))^{-1}$ .

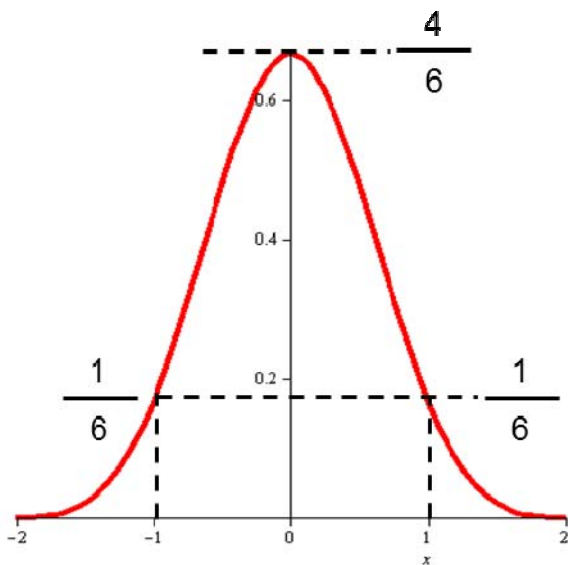
$$B(z) = \frac{3 \cdot (z - z^{-1})}{z^{-1} + 4 + z} \cdot Y(z) \tag{10}$$

$$C(z) = \frac{3 \cdot (z - 2 + z^{-1})}{z^{-1} + 4 + z} \cdot Y(z) \tag{11}$$

$$D(z) = \frac{(z - 1) \cdot (z - 2 + z^{-1})}{z^{-1} + 4 + z} \cdot Y(z) \tag{12}$$

The determinant of this equation system is equation (13):

$$\begin{aligned} (2 + 8 \cdot z + 2 \cdot z^2) &= 2 \cdot z \cdot (z^{-1} + 4 + z) = \\ &= 12 \cdot z \cdot \frac{(z^{-1} + 4 + z)}{6} \end{aligned} \tag{13}$$



**Figure 6.** Continuous cubic B-spline, and discrete cubic B-spline  $(z^{-1}+4+z)/6$ .

It can be seen that this determinant (13) coincides with the Z Transform of the discrete symmetric and shifted cubic B-spline  $B^3(z)$  [10], multiplied by a constant factor equal to 12, figure 6.

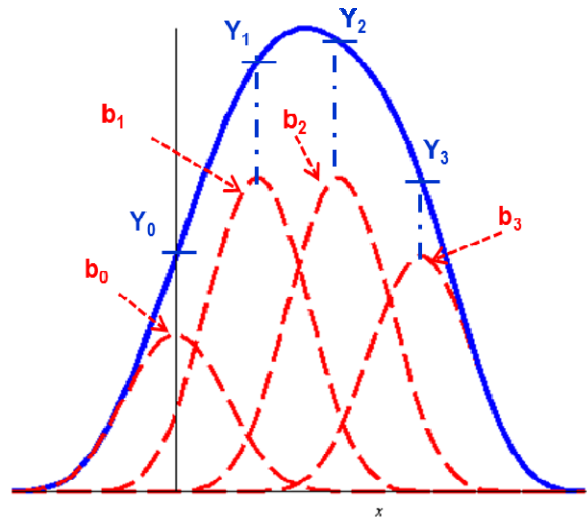
Similar results are obtained for splines of any order. This proves the connection between the interpolation with splines polynomials presented in section II and the B-splines functions proposed by Unser [10], [11].

$$(B^3(z))^{-1} = \frac{6}{(z^{-1} + 4 + z)} \tag{14}$$

The inverse of the discrete cubic B-spline  $B^3(z)$  is the Direct B-spline operator  $(B^3(z))^{-1}$ , (14), proposed in [11] to move samples to the B-spline domain; however, it has the inconvenience of being an anticausal IIR filter, even though Unser proposed a system for solving it, it cannot be applied in real time as it is necessary to have the entire complete signal from the beginning.

The advantage of the system we propose in section II is that we can obtain an approximation of this splines filter with a finite impulse response. The impulse response of the operator  $(B^3(z))^{-1}$  is shown in Fig. 5., and its Z transform is (15).

$$\begin{aligned} (B^3(z))^{-1} &= 1.732 - 0.464 \cdot (z + z^{-1}) + \\ &+ 0.124 \cdot (z^2 + z^{-2}) - 0.033 \cdot (z^3 + z^{-3}) \end{aligned} \tag{15}$$



**Figure 7.** Example of cubic B-splines interpolation.

In figure 7 is shown an example of interpolation like a lineal combination of shifted and weighted B-splines. The  $b_n$  coefficients are calculated with the operator  $(B^3(z))^{-1}$ , figure 5. The way of calculating the first derivative proposed by Unser [10] in the case of cubic splines is:

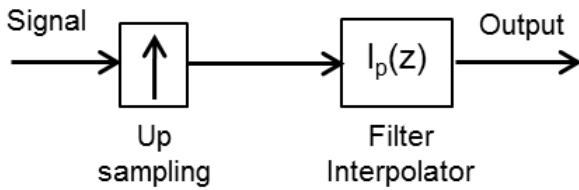
- With the  $(B^3(z))^{-1}$  filter move samples to the cubic B-spline domain.
- Apply the operator  $(1-z^{-1})$ .
- Apply the operator  $(z+1)/2$

Equation (16) evidences that the method proposed by Unser and the method that we propose (10) for calculating the first derivative are the same.

$$First\_D(z) = \frac{6}{z^{-1} + 4 + z} \cdot (1 - z^{-1}) \cdot \frac{(z+1)}{2} = \frac{3 \cdot (z - z^{-1})}{z^{-1} + 4 + z} \quad (16)$$

#### IV. Cubic spline interpolator

Spline interpolators allow to compute all the points between each two samples, although the most common is to find the medium point, at twice the original sampling rate. Figure 8 shows the steps required to compute it. First we need to upsample the signal, insert zeros between each two samples. Then we apply the filter  $I_p$  and obtain the medium point of the cubic spline polynomial. These steps can be repeated iteratively, and at each iteration we'll obtain a version of the signal with double sampling rate



**Figure 8.** Diagram to interpolate.

Next, we show how to compute the interpolating filter  $I_p(z)$ . First we must to calculate with equation 17 the medium point of the each spline polynomial,  $x=0.5$

$$I_{0.5}(z) = 1 + B(z) \cdot 0.5 + C(z) \cdot 0.5^2 + D(z) \cdot 0.5^3 \quad (17)$$

Use for  $B(z)$ ,  $C(z)$  and  $D(z)$  the expressions obtained in (10), (11) and (12), and the result is the equation (18).

$$I_{0.5}(z) = 1 + \frac{3 \cdot (z - z^{-1})}{z^{-1} + 4 + z} \cdot 0.5 + \frac{3 \cdot (z - 2 + z^{-1})}{z^{-1} + 4 + z} \cdot 0.5^2 + \frac{(z - 1) \cdot (z - 2 + z^{-1})}{z^{-1} + 4 + z} \cdot 0.5^3 \quad (18)$$

Simplify in (19):

$$I_{0.5}(z) = \frac{0.125 \cdot (z^{-1} + 23 + 23 \cdot z + z^2)}{z^{-1} + 4 + z} \quad (19)$$

Upsample and delay, equation (20):

$$I_{0.5}(z) = z^{-1} \cdot \left( \frac{0.125 \cdot (z^{-2} + 23 + 23 \cdot z^2 + z^4)}{z^{-2} + 4 + z^2} \right) \quad (20)$$

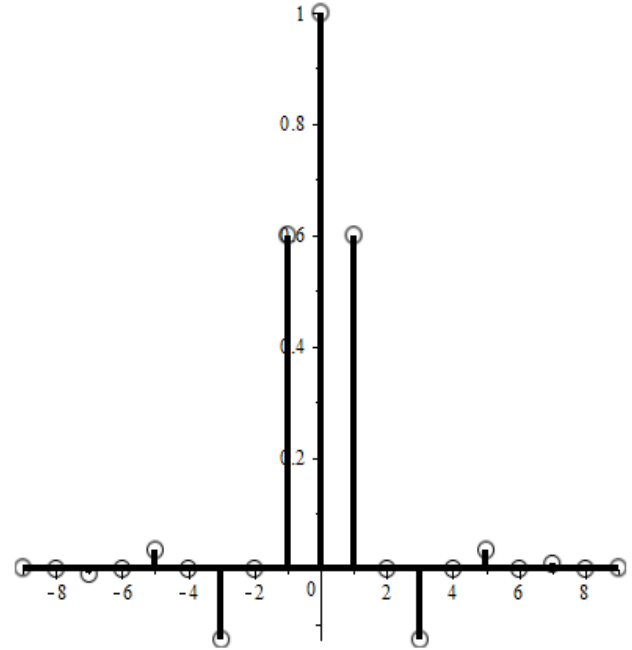
Add 1 to the filter in order to keep the original signal values, equation (21)

$$I_p(z) = 1 + \frac{6}{z^{-2} + 4 + z^2} \cdot \frac{(z^{-3} + 23 \cdot z^{-1} + 23 \cdot z + z^3)}{48} \quad (21)$$

Finally, use a FIR approximation of  $(B^3(z))^{-1}$  to obtain the final equation (22) of the interpolator. The accuracy obtained will depend on the FIR order. With 10 coefficients (10 additions, 5 multiplications), the differences between our

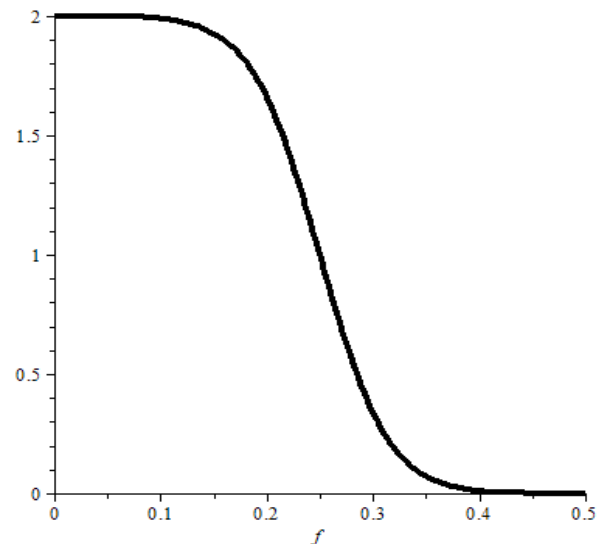
approximation and the cubic spline interpolant, computed with the system of equations is less than 0.1 %.

$$I_p(z) = 1 + 0.6 \cdot (z + z^{-1}) - 0.127 \cdot (z^3 + z^{-3}) + 0.034 \cdot (z^5 + z^{-5}) - 0.009 \cdot (z^7 + z^{-7}) + 0.002 \cdot (z^9 + z^{-9}) \quad (22)$$



**Figure 9.** Time response of the cubic B-spline interpolator.

Figure 9 shows the impulse response of the cubic spline interpolator and figure 10 shows the magnitude of its frequency response. We can see it's a low pass filter.



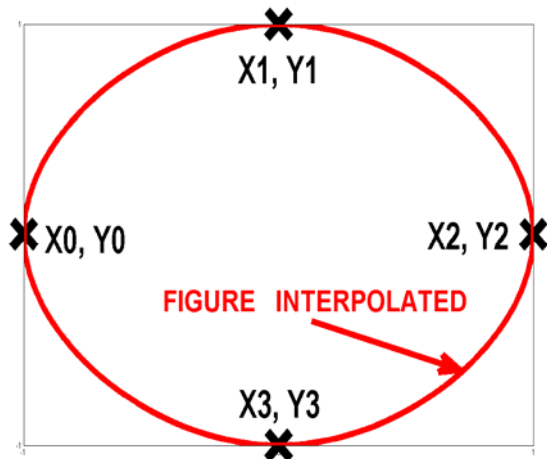
**Figure 10.** Magnitude of the frequency response of the cubic B-spline interpolator.

The number of operations depends on the required accuracy and is independent of the total number of samples of the signal, while the classical resolution of the cubic interpolant needs to solve a system with 3 equations per sample

## V. Interpolating examples

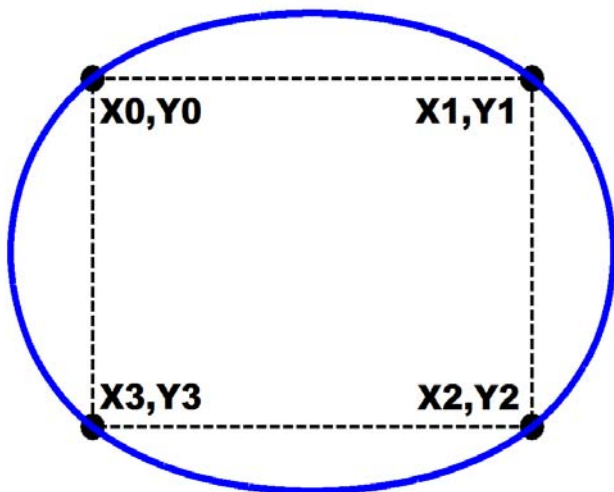
In this section we show three examples of interpolation of shapes defined by their contour (a circle, a square and a contour of an otolith). We use the interpolator from section IV recursively to obtain a signal with a sampling rate 1024 times the original one.

The circle has initially only 4 samples. In Fig. 11 is shown the four initial samples and the figure interpolated.



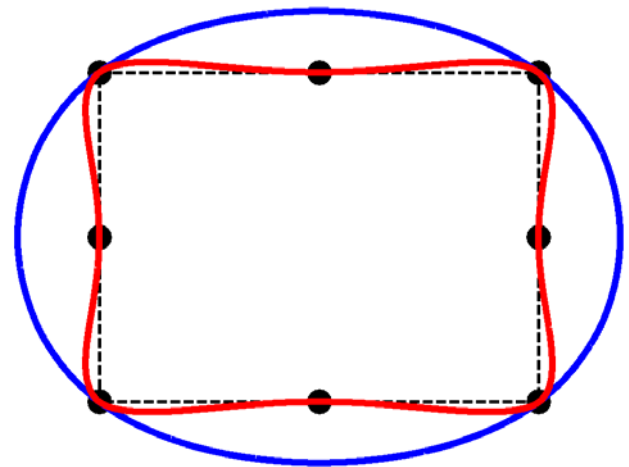
**Figure 11.** Example of interpolating a circumference from only 4 samples.

The cubic spline interpolation of the square shape, Fig. 12, from only four samples does not allow to represent it without ambiguities. We need more samples to represent it accurately.



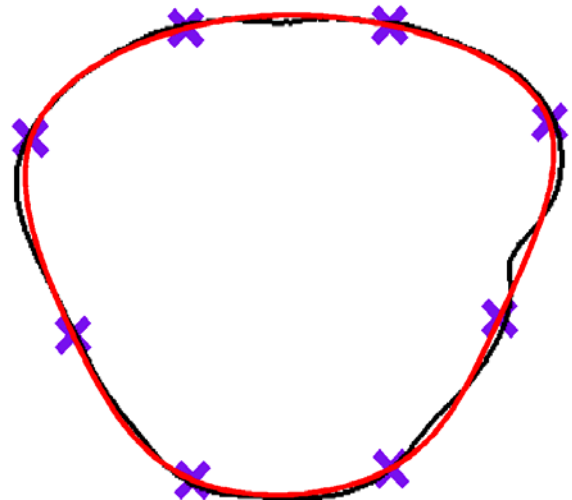
**Figure 12.** Example of interpolating a square from only 4 samples.

The best interpolant for this shape would be a linear spline, although cubic splines can give good results if we increment the number of original samples. In Fig. 13 the interpolation of 4 samples square appears in blue, while the results for the 8 samples versions appear in red.



**Figure 13.** Example of cubic spline interpolation of a square. In blue, exterior continuous line, from 4 samples. In red, interior continuous line, from 8 samples.

Other application of the interpolator is approximate a contour of an otolith with a few samples. In the figure 14 it's shown the original contour and the interpolated with the cubic spline operator of equation (22) from only eight samples. The error of this approximation is due to the little number of samples, not from the cubic interpolator.



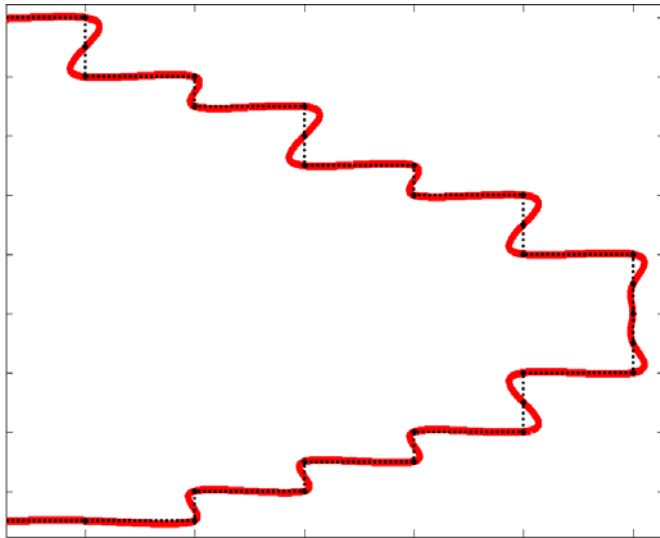
**Figure 14.** Interpolation of otolith contour from only 8 samples.

## VI. Obtaining an optimum continuous contour from its discrete representation with pixels

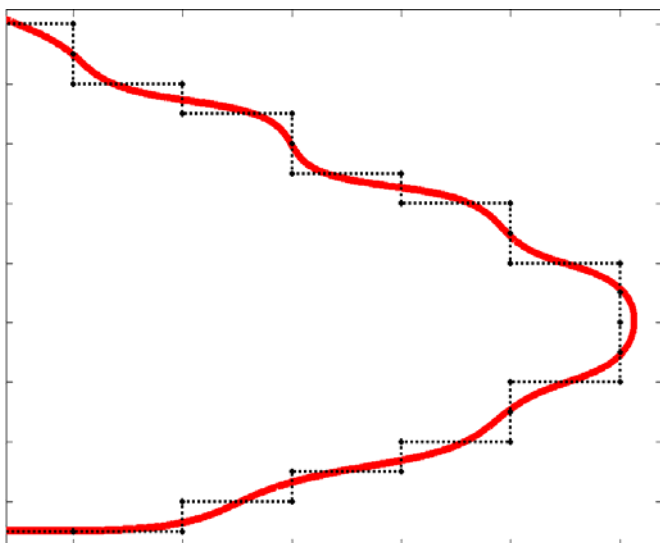
The contours of the figures defined by pixels are a discrete representation with a quantization error. In the continuous representation of the contour interpolated with splines (cubic splines in our case) there are oscillations that not fit the original contour. This effect can be observed in Fig. 15, and happen when interpolate all the pixels of the contour. When interpolate from only a few samples of the contour, like in figure 14 section V, this effect doesn't happen.

The solution of first filtering the signal is not simple. The problem is that if the signal is filtered with a very narrow filter the contour will be smooth but will not fit the original contour,

and if the filter is not so discriminant then the oscillations will not be removed completely. There are many types of low-pass filters that can be used, but in this work we propose a new FIR approximation of the least squares filter based on cubic B-splines [11]. This filter obtains an approximation with splines without making these go through the samples exactly, and therefore the curve obtained is smoother. The result is shown in Fig. 16.



**Figure 15.** Oscillations produced by interpolating a discrete contour with cubic splines.



**Figure 16.** Interpolated contour with cubic splines after filtering with a least squares filter.

Another proposal is to apply this filter iteratively, as long as the error between the original signal and the filtered signal does not exceed a threshold.

Obtaining the FIR version of this filter is similar to the procedure proposed in Section II.

## Conclusions

In this work we found a direct connection between the determinant of the equation system of the spline interpolation and the Z Transform of the discrete B-spline described in [10]. Based on this connection we proposed another way of

obtaining the FIR approximation of the cubic spline interpolator. By using only 10 coefficients (10 additions and 5 multiplications), the accuracy is as high as 99.9 %.

We also proposed applying a FIR approximation and invariant to the translations of the least squares filter based on cubic B-splines, to obtain a very good continuous representation of the contour of a figure from its image in pixels.

Another contribution of this work is a procedure for obtaining a temporal and frequency representation of the operators that are applied to calculate the derivatives of different orders of discrete signals.

## Acknowledgment

This work was supported by the project 3D Otolith shape analysis (AFORO 3D), ref: CTM2010-19701, from Spanish Government.

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