Efficient Deployment of Connected Sensing Devices Using Circle Packing Algorithms

Rabie A. Ramadan
Computer Engineering Department
Cairo University
Cairo, Egypt
rabie@rabieramadan.org

Salah Abdel-Mageid
Systems and Computers Department
Al-Azhar University
Cairo, Egypt
smageid@azhar.edu.eg

***Abstract*— In this paper, we explore different sensor deployment problems and how these problems can be solved optimally using the current packing approaches in terms of small-scale problems. In addition, we consider the deployment of either homogenous or heterogeneous sensing devices. The deployment objectives are to maximize the coverage of the monitored field and use the best of the sensing devices characteristics as well as developing a connected deployment scheme. We propose a novel algorithm named Sequential Packing-based Deployment Algorithm (SPDA) for the deployment of heterogeneous sensors in order to maximize the coverage of the monitored field and connectivity of the deployed sensors. The algorithm is inspired from the packing theories in computational geometry where it benefits from many of the observations properties that are captured from the optimal packing solutions. The algorithm efficiency is examined using different case studies.**

***Keywords*—sensor networks, packing, deployment , coverage, connectivity.**

I. Introduction

Recent advances of sensing technology enable the use of wireless sensor networks (WSN) in different applications such as border security, habitat monitoring, traffic surveillance, and heath applications. These networks contain many ad hoc sensors that collaborate to collect, analyze, and aggregate the sensed data as well as routing it to the sink node (base station). The problem with these sensors is that they are usually suffer from very limited battery, restricted memory size, small sensing and communication ranges, and limited computational capabilities. In addition, the deployed sensors could be homogenous or heterogeneous; homogenous sensors share the same characteristics while heterogeneous sensors may differ in one or more of their characteristics. Obviously, the optimal deployment of such nodes will affect different aspects of the sensor network operations such as coverage, connectivity, routing, and lifetime. However, due to the complexity and the number of parameters that need to be considered, optimal solutions have been introduced for very constrained and small-scale problems.

In this paper, we show that different versions of the deployment problem including using homogenous and heterogeneous sensors that could be solved optimally by the analogy to circle packing problem. In addition, we present some of the learned lessons from the properties and structures that guarantee optimal solutions for the packing problem. We explain how these properties and structures can benefit the deployment of a large number of sensors. The final contribution of this paper is the sequential packing-based algorithm (SPDA) for the deployment of connected heterogeneous sensors.

The first contribution to the deployment problem was by Chvatal [19] in 1975 who introduced the art gallery problem. This problem is a well known NP-hard computational geometry question, where the aim is to find the minimum number of observers required for full coverage of an art gallery. Since then, coverage in WSN usually is the main objective in most of the deployment algorithms. It is considered as one of the measures to the quality of the final deployment scheme. For example, Howard et. al. in [1] introduced an incremental deployment algorithm in which the new sensor placement is based on the sensed information from the deployed ones. Similarly, unmanned vehicle and a flying robot [14] have been used to deploy nodes incrementally. The unmanned vehicle/robot sensor is used to help in collecting information about the deployed sensors. Although these techniques demonstrated good performance, they are costly compared to the cost of the tiny deployed sensors. In addition, having a powerful sensor or unmanned vehicle might not be available for each application. Moreover, [7][6][23] studied the deployment of sensors for the purpose of coverage and connectivity. The authors considered only cases where there is a binary relationship between sensors communication and sensing ranges. The most related research to our work in this paper is the algorithm developed by Miu-ling at. el in [13] where the authors utilize the concept of circle packing in sensor deployment; whoever, the purpose of the algorithm is to find the most suitable sensing range that maximizes the monitored field coverage. The authors tend to neglect the connectivity issues as well as the other types of the deployment problems.

This paper is organized as follows: the circle packing problem is overviewed in section II; section III introduces different versions of the deployment problem and their equivalent packing solutions; section IV presents the deployment of a connected sensors problem; section V shows the details of the sequential packing-based deployment algorithm; the simulation results are illustrated in section VI; Finally, the paper concludes in section I.

II. OVERVIEW ON THE CIRCLE PACKING PROBLEM

Circle packing problem is a distinguished NP-Hard problem in the field of computational geometry [11]. It studies the arrangement of non-overlapping equal/non-equal size circles into a given plan. The objective is to minimize the wasted plan's areas. There are several well developed theories that handle the packing in different shapes such as rectangle, circle, triangle, and square. Because of the large body of the literature in this field, our work in this paper focuses only on packing circles into either square or rectangle plan. The problem is introduced by William Thurston in 1985 [22]. Since that time, it has been mapped to different fields including brain mapping [12], brain random walks [4], tilings [16], numerical analysis [3], and complex analysis [17] . Consequently, we believe that the mapping of this problem to the field of WSN will enrich its research in many aspects such as the deployment.

III. THE DEPLOYMENT PROBLEM AND THE CORRESPONDING SOLUTIONS FROM CIRCLE PACKING THEORIES

In this section, different versions of the sensor deployment problem are introduced. Our modeling to the problem includes the deployment of homogenous and heterogeneous sensors. Sensors' sensing and communication ranges are assumed to be disk-based (circle). In addition, sensors' communication ranges are considered large enough to build a connected network among the deployed sensors. Later in the next sections, we show how we can deal with different communication ranges during the deployment process. The objective of the deployment is to maximize the coverage of the monitored field. The coverage is maximized by minimizing the sensors overlapping areas and the uncovered spots in the monitored field.

A. Deployment of Homogenous Sensors

$D(r, F) \in D_r$ is a sensor deployment of sensing devices with sensing range r in a square monitored field F, where

$$D_r = \{(x_1, y_1), \ldots\ldots (x_s, y_s)\} \mid$$
$$\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \geq 2r_s \in [0, F]$$

$, 1 \leq i, j \leq s$, and $x_i y_i \in F$. The objective is to optimally deploy the number of sensors s where $s \geq 2$ in the field F without overlapping and minimizing F uncovered areas.

The daughter of this deployment problem is the packing of equal size non-overlapping circles in a plan. Sensors and monitored fields in the deployment problem are represented by circles and packing's plan respectively. Different forms and solutions for optimal and near optimal solutions are described in the following paragraphs.

Packing of equal size problem comes in different forms that have been proved equal. Some of these forms are stated as follows:

1) *Find the maximum circle radius r_n, such that n equal no-overlapping circles fit in a given square.*
2) *Locate n points in a given square, such that the minimum distance m_n between any two points is maximal.*
3) *What is the smallest square of side ρ_n that can fit n non-overlapping and equal circles?*

From the first glance, these forms seem computational geometry related problems. However, the problem could be formulated as a MaxMin problem [2]. The formulation deals with scattering *n* points in a square such that their minimal distance is maximized.

$$d_{ij} = Max \min_{i \neq j} \|s^i - s^j\|$$

$$\|s^i - s^j\| = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$
$$s^i \in [0, F]^2$$

, where $\|s^i - s^j\|$ is the Euclidian distance between the points i and j.

Moreover, many attempts have been done to prove the optimal packing of small-scale problems as in [8][9]. By studying such solutions to the circle packing and its similarity to the deployment of homogenous sensors, we can apply the same techniques of equal circle packing solutions to the sensor deployment. However, the most important thing that studied the circle packing problems is the following conclusions that will be used later in our proposed deployment algorithm:

Property 1: At each vertex of the square, one of the following conditions hold:

1) At least one point of the optimal solution coincides with the vertex of the square.
2) Two points of the optimal solution belong to the edge determined by the vertices and have a distance of m_n, where m_n is the minimal distance between the points in the optimal solution.

Property 2: There exists always optimal solution to the packing problem such that along each edge of the square there is no portion of the edge of width greater than or equal to twice the optimal distance m_n which does not contain any point of the optimal solution.

These properties conclude that an optimal solution will always have as many points as possible located along the boundary of the square. They also confirm our observation on the importance of number of contacts for an optimal packing. Nevertheless, we can benefit from such properties in solving large-scale problems where the best effort heuristics are the only feasible solutions

B. Deployment of Heterogeneous Sensors

$D(r_i, F) \in D_{r_S}$ is a sensor deployment with sensors' sensing ranges $r_i \in \{r_1, r_2, r_3 \ldots\ldots r_S\}$ in a monitored field F, where

$$D_{r_S} = \{(x_1, y_1), \ldots\ldots (x_S, y_S)\} \mid \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \geq 2r_S \in [0, F]$$,

$1 \leq i, j \leq s$, and $x_i y_i \in F$. The objective is to optimally deploy the number of sensors s where $s \geq 2$ in the field F without overlapping and with minimizing F uncovered areas.

In this problem, a set of heterogeneous sensors are used to monitor a given field *F*. sensors are assumed to have different sensing capabilities. Some of them might have larger sensing ranges than others. Obviously, this problem is similar to the packing of non-equal size non-overlapping circles in a plan. A little has been done to solve this problem. For example, Zhang in [5] proposed an algorithm that combines the simulated annealing and the energy-based heuristic. A similar algorithm is proposed by WenQi [20] in which elastic forces is used to move the circles to their best placement. Finally, a greedy algorithm is proposed by Huang [21]in which circles are added sequentially and the distances between the add circles and the new circle are minimized. Again, these algorithms are typical solutions to the deployment problem if the connectivity among the sensors is not a major concern. However, they can be adapted to guarantee sensors connectivity. In section V, we propose an algorithm that adapts the solution introduced by Huang for a connected WSN.

C. Full Coverage Deployment

In this section, we are trying to answer the following question:

"Given a monitored field F with side σ and a number of sensors s, what is least number of sensors that can fully cover a monitored field F with minimum overlapping?"

By analogy to the packing theory, the answer to this question is equivalent to finding the least number of circles that fully cover a plan. It is also called "tiling problem" which has a large number of practical applications such as placing service centers, locating and dimensioning telecommunications centers. Figure 1 shows an example on covering a plan with four equal circles.

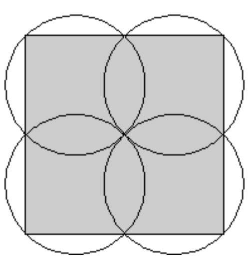


Figure 1: Covering a square with four equal circles

Covering a plan by circles is another NP-Hard problem. The best know results for this problem is up to 30 circles that covers a unit square [10]. The algorithm is based on two nested procedures. The inner procedure assumes constant circles radius and moves the circles to the uncovered areas in the square. The outer procedure adjusting the circles' radius based on whether the coverage is found by the inner procedure or not. Again, we can benefit from such algorithm in optimally deploying up to 30 sensors in a monitored field to fully cover it.

IV. Deployment of Connected Sensing Devices

In the previous sections, we showed that packing algorithms could be used to solve different versions of the sensor deployment problem. These solutions assume sensors with large communication ranges such that the communication among the deployed sensors is taken care of which is the case in most of the current WSNs applications. On the other hand, in some other networks, connectivity is a major concern that might affect the network performance; especially, when heterogeneous sensors are used. In this section, we introduce packing-based algorithm for deployment of heterogeneous sensing devices. However, deployment of homogenous sensors, is a sub-problem of the problem in hand. Sensors are assumed to have different communication and sensing ranges. The main deployment objectives are to maximize the coverage of the monitored field and produce a connected deployment scheme. The coverage is maximized by covering as much as possible from the monitored field and minimizing sensors' overlapping areas. In addition, sensors in the final deployment scheme have to be connected. The following is the formal definition to the problem.

Suppose a bounded 2D plan (square or rectangle) *F* of a given width *w* and length *l*, and a finite set of sensors *S = {1,2,3,....s}* that are not necessary having the same sensing ranges $r_1, \ldots, r_n$ or the same communication range $c_1, \ldots, c_n$. Any sensor placed in the given plan is denoted by $p_i = (i, x_i, y_i, r_i, c_i)$, where x_i and y_i are the coordinates of the sensor. r_i and c_i are added to p_i for the coverage and connectivity purposes. Therefore, the deployment scheme *P* including all sensors of a subset $S' \subseteq S$ is given by $P = \{(i, x_i, y_i, r_i, c_i) \mid i \in S' \subseteq S\}$. The problem is:

$$Max\left(\sum_{i \in S'} \frac{\pi r_i^2}{l.w} - \frac{1}{2}\sum_{ij \in S'} \Delta_{ij}\right)$$, where Δ_{ij} is the overlapping area between sensors *i* and *j*. (1)

Such that:

$$\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \geq min(c_i, c_j) \quad i \neq j, \forall i, j \in S' \quad (2)$$

$$x_i - r_i \geq 0 \quad \forall i \in S' \quad (3)$$

$$l - x_i - r_i \geq 0 \quad \forall i \in S' \quad (4)$$

$$y_i - r_i \geq 0 \quad \forall i \in S' \quad (5)$$

$$w - y_i - r_i \geq 0 \quad \forall i \in S' \quad (6)$$

$$\Delta_{ij} = \begin{cases} \succ 0 \text{ if there is an intersection between sensors i and j} \\ 0 \quad \text{Otherwise} \end{cases} \quad (7)$$

Equation (1) represents the objective of the deployment problem. Constraint (2) guarantees the connectivity among the deployed sensors. Constraints (3) to (6) restrict the sensors' deployment to the boundary of the monitored field. Constraint in (7) is used to limit the area of the intersection to no less than zero value.

V. SEQUENTIAL PACKING-BASED DEPLOYMENT ALGORITHMS (SPDA)

Now, we propose SPDA algorithm as a solution to the deployment of connected sensing devices. SPDA adapts the algorithm introduced in [21] which shows significant progress in packing non-equal circles. In addition, SPDA takes into consideration our observations and properties of the previous circle packing algorithms mentioned in the previous sections. In our algorithm, sensors are deployed sequentially in which at each step a potential sensor has to be selected. The method of selecting a potential sensor is based on the type of sensors to be deployed. For example, using homogenous sensors, sensors might be selected randomly or based on their identifiers; while, using heterogeneous sensing devices, sensors might be sorted based on their sensing or communication ranges. For each potential sensor, one or more potential placement points are identified. A point is considered potential if and only if satisfies the following rules.

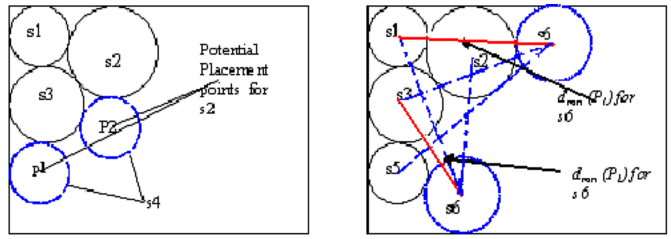
1) The potential sensor s_i has to touch at least two other items such as two deployed sensors (circles), one of the field's borders and a deployed sensor, or two borders.
2) If a potential sensor s_i is not the first sensor to be deployed, it must be placed within the communication range of at least one of the deployed sensors.
3) The potential placement point coordinates must be within the borders of the monitored field

Figure 2a shows an example on the potential placement points in a field that has three deployed sensors *s1*, *s2*, and *s3*. The potential placement points of *s4* (dashed line circles) are limited to two points, *p1* and *p2*, assuming that it cannot communicate to *s1* and *s2* while it is able to connect to *s3*.

As shown, this may lead to more than one potential placement points. To select a final placement point for a sensor s_i, each point is evaluated and the point with the minimum value is selected. First, the Euclidian distance d_{ij} is computed from each point P_i to the untouched sensors (circles) s_j. Then, the minimum distance $d_{min}(P_i)$ is assigned to P_i ; this value represents how much the point is far from the untouched circles . A point with the minimum value is selected to be the final deployment position for this sensor. However, we benefited from the two properties of the optimal packing introduced in section III by prioritizing the potential points that touches one of the borders over other potential placement points.

Figure 2b shows an example on the selection of a final potential point. As shown, two potential points P_1 and *P2* for *s6* are identified. The distances, dished lines, between each point and the untouched sensors' disks are computed. The minimum distance from each point is selected (solid lines in the figure). The point associated with the shortest distance is chosen for the final deployment position of *s6* which is, in this case, *P2*.

If no potential point was identified because of the shortage in the communication range (rule number 2), the current sensor has to intersect with one or more of the deployed sensors. The current sensor has to move toward a deployed sensor that gives the minimum overlapping.



(a) (b)

Figure 2 : (a) Example on the potential placement points, (b) example on the potential points evaluation

Figure 3 shows the details of the deployment algorithm in which the first sensor is deployed at one of the field's corners (lines 2 and 3) such that the sensor's disk touches at least two of the field's borders. For a potential sensor to be deployed, it has to touch at least one of the deployed sensors and one of the borders, two of the deployed sensors, and/or two of the field's borders. A potential placement point is identified while the connectivity between the current sensor and at least one the deployed sensors is considered (line 5). If no potential point satisfies these conditions, the sensor has to be deployed at a point that gives minimum overlapping (lines 8). Otherwise, potential placement points are evaluated and the current sensor is placed at a point that has the minimum value (lines 10, 11, and 12). This process continues until there is no sensor available or the field is totally covered (line 14). Then, the contribution of each sensor is computed (line 18).

In SPDA , we evaluate the performance of the algorithm according to the following definitions:

Definition 1: [Sensors coverage-contribution] Sensors contribute to the coverage of the monitoring field by the area of its sensing disks. However, if two or more sensors compete (overlap) fully or partially on the same zone, only the overlapped area of the most reliable sensor will be considered as a contribution to the overall coverage. We realize that multiple-coverage (k-coverage) could be required in some applications. However, this problem is beyond the scope of this paper.

Definition 2: [Weighted-coverage] The coverage, in this context, is measured by the sum over the zones' weight and the reliability of the sensor that monitors this zone. If more than one sensor is sharing the zones monitoring without overlapping, the coverage contribution by each sensor is computed and multiplied by the zones weight as well as the reliability of each sensor. Otherwise, only the contribution of the most reliable sensor is considered.

Since the number of potential placement points is limited by the borders of the deployment field and the communication range of the sensing devices, it is obvious that the algorithm has a bounded polynomial time. The worst case to find potential placement points requires $O(m\ (|S|-m))$ operations, where m is the deployed sensors. This occurs when there is no limit on the communication range of the given sensing devices. Therefore, each potential sensor forms a potential placement point with each deployed sensor and the field's border. Computing the distances between the untouched circles and the potential points requires $O(m^2(|S|-m))$ operations. Finding the point that has the minimum distance requires $O(m)$ operations, as is finding the touched circles with the largest communication range in case of overlapping is a must. Therefore, the estimated worst case complexity of algorithm 1 is $O(s^3)$, where s is the number of sensors.

Algorithm 1

```
1:  Select the potential sensor s_i to be deployed
2:  IF (i = 0) then
3:  Deploy the sensor at one of the field's corners such that it touches its
    two borders.
4:  ELSE
5:  Compute the potential placement points such that the current sensor
    is connected to one of the deployed sensors and ;
6:  The sensor has to touch  at least two of items (two sensors, two
    borders, one border and one sensor).
7:     IF no potential point satisfies the steps 5 and 6
8:       Place the sensor at the point that gives minimum overlapping
9:     ELSE
10:      Compute dij between each potential point for the current sensor
    and the untouched deployed  sensors.
11:        Compute d_min for each point.
12:        Place the sensor at the point that has the minimum value.
13:    End IF
14:         IF there is no more sensors and/or or the field is totally covered
    got to 16,
15:    ELSE go to 1.
16:         End IF
17: End IF
18: Compute the coverage contribution of each sensor
19: Stop.
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Figure 3: Deployment of homogenous sensors

VI. SIMULATION RESULTS

In this section, different case studies are used to show the correctness and the performance of the packing-based algorithm (SPDA). In the first case study, we investigate the correctness of the algorithms with different problem settings. In the second case study, the effect of sensors characteristics on the coverage performance is discussed. In the final case study, the deployment of special networks where there is a binary relationship between the sensors communication and sensing ranges is explained. All of the experiments introduced in this section are conducted on a Dell machine with 2.2 GHZ processor and 1 GB memory. The algorithm is implemented using C sharp in a dot net environment.

Case Study 1: Successful Deployment of the Packing-Based Algorithms

In this case study, we show the correctness of the deployment algorithm. We start by deploying homogenous sensors. As shown in figure 4a, 25 sensors that are used to monitor 600m by 600m field. Sensors are assumed to be homogenous in which their communication ranges are at least double the sensing ranges (120m). The algorithm was able to deploy the sensors successfully with maximizing the covered areas and avoiding overlapping. The coverage percentage in this case is 81% which is the maximum coverage that can be achieved without overlapping. In the second experiment shown in figure 4b, 12 sensors with communication range (140m) less than the sensing range (80m) are used. The algorithm deployed the sensors with minimum overlapped areas and satisfied the connectivity constraints. Adding more sensors lead to more overlapping between the sensors areas as shown in figure 4c.

Using heterogeneous sensors**,** figure 4d presents a deployment scheme for 20 sensors that differ in their sensing and communication ranges in 600m x 600m field. Sensors are sorted based on their sensing ranges and sequentially deployed into the field. As can be seen, sensors are packed efficiently with minimum overlapping due to the shortage in the sensors communication ranges. Sensors with small sensing range are also accommodated properly in the field.

Case Study 2: Effect of Sensors Characteristics on the Coverage Performance

In this set of experiments, a 300 meters field is monitored by a set of 40 sensors. Sensors characteristics are randomly generated based on uniform distribution function. Before the deployment, sensors are sorted either based on their sensing ranges (r_s) or communication ranges (c_s). The average results over 20 runs are summarized in figure 5. The results conclude that using heterogeneous sensors, sensors' characteristics (communication and sensing ranges) are equally important and almost have the same coverage performance. Multi-criteria sorting techniques might be needed for better performance. However, figure 5b shows that the running time is slightly higher by a few seconds when the communication range is used as sorting base. This difference comes from the variance of the communication ranges that the algorithm has to take care of.

Case Study 3: Effect of the Binary Relationship between the Communication and Sensing Ranges on the Coverage Performance

A field *F* is configured, as mentioned above, to be monitored by different number of sensors that range from 5 to 40. 10 curves are generated to show the coverage performance with different communication range values. For each sensor, the communication range is represented as a percentage of the sensing range. The average results illustrated in figure 6 show that increasing the communication range increases the coverage performance of the monitored field. Nevertheless, increasing the communication range to more than double the sensing range is greatly affected the coverage performance. For example, a communication range that doubles the sensing range gives 90,000 units of coverage while increasing the communication range to 225% adds 30000 units of coverage. This value is increased to 88,000 coverage units when sensors communication range is increased to 250% of the sensing range. Therefore, based on these set of experiments, it is recommended to use sensors with communication ranges larger than double the sensing ranges. However, further investigation is required for the level of interference among the sensors in this case.

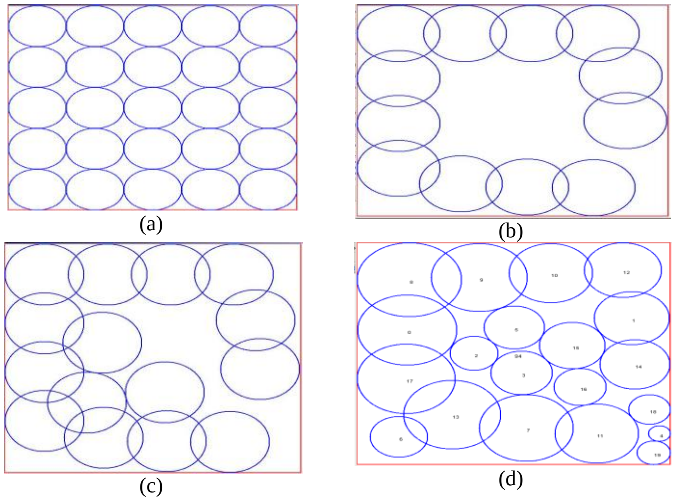

Figure 4 : SPDA successful deployment with different settings

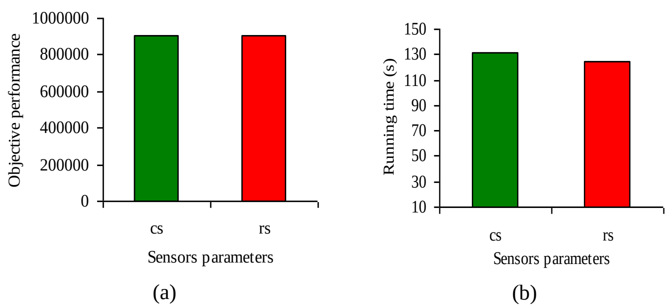

Figure 5: : Effect of Sensors Characteristics on the Algorithms Performance

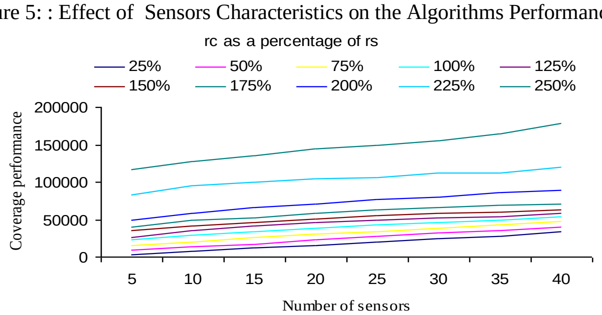

Figure 6: The relationship between the Communication and Sensing Ranges

VII. CONCLUSION

In this paper, different versions of the sensor deployment problem are studied. We explained how these versions of the problem could be mapped to packing problems. Using the theories, properties, and the structures as well as the heuristics proposed for the packing problem, the deployment problem, in most cases, could be solved optimally. In addition, we introduced a packing-based deployment algorithm for a connected WSN. The algorithm adapts one of the recent algorithms that show good performance in packing non-equal size circles in a plan. The conducted experiments illustrated the performance of the proposed algorithm as well as the effect of the binary relationship between sensors communication and sensing ranges.

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