
Efficient Dominating and Edge Dominating Sets for Graphs and Hypergraphs

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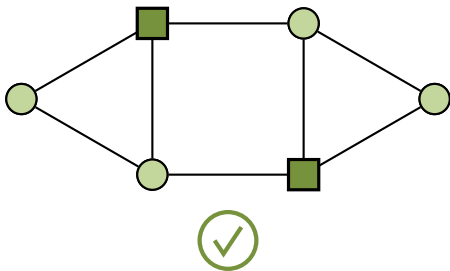
The Problem

Domination

$$D \subseteq V \text{ with } \bigcup_{d \in D} N[d] = V$$

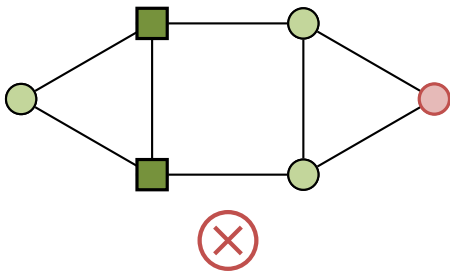
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Efficient Domination

Exactly one $d \in D$ for each $v \in V$

$$\forall v \in V : \exists! d \in D : v \in N[d]$$

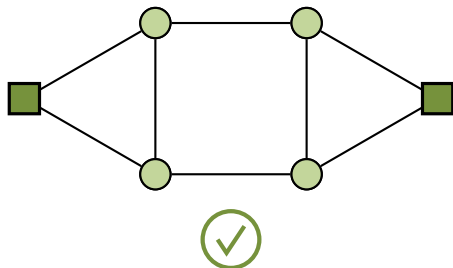
Exact cover of the closed neighbourhoods

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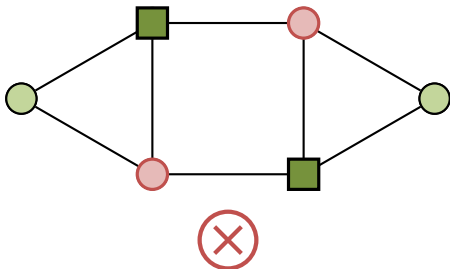


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Exact cover of the closed neighbourhoods



Efficient Domination

packing and covering problem

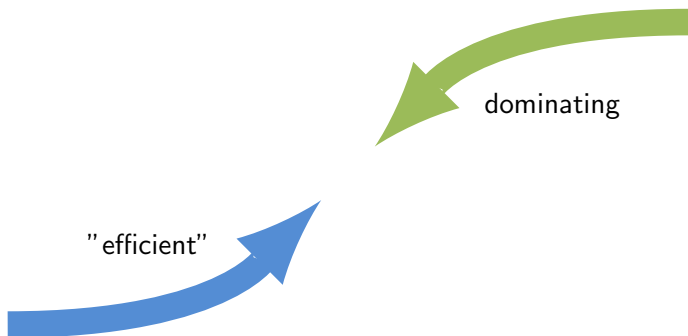
Efficient Domination

packing and covering problem



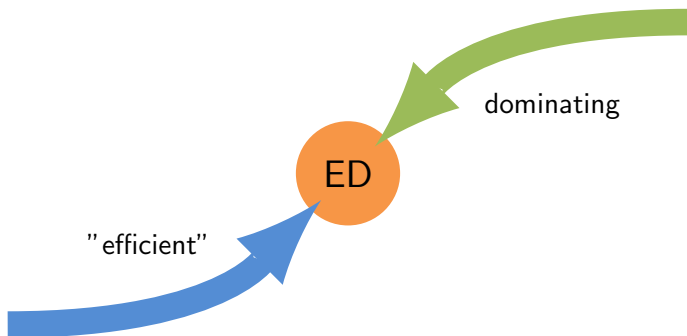
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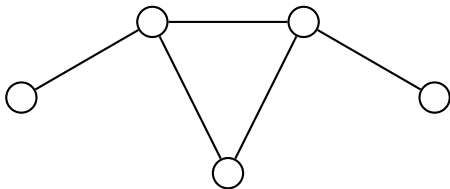
Efficient Domination

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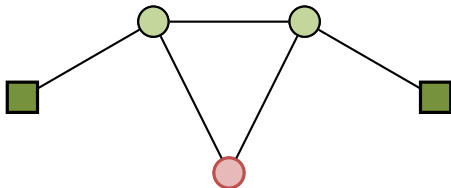
Efficient Domination

Does not always exist



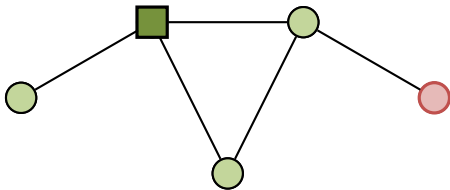
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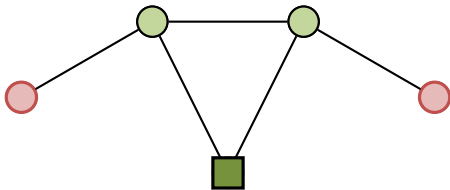
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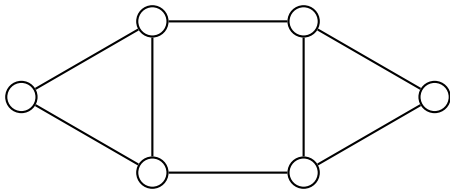
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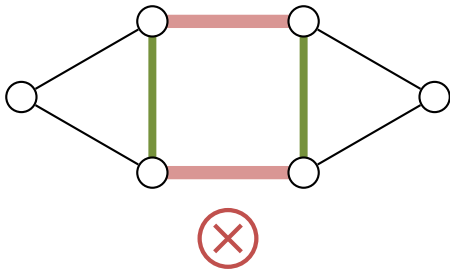
Efficient Edge Domination

the same with edges



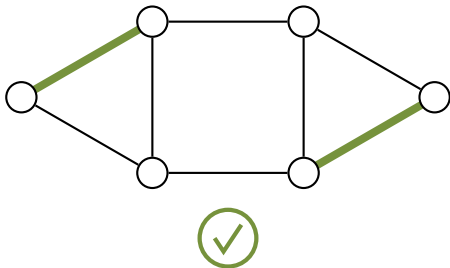
Efficient Edge Domination

the same with edges



Efficient Edge Domination

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Other Names

Efficient Domination

- ▶ Independent Perfect Domination

Efficient Edge Domination

- ▶ Dominating Induced Matching

Existing Results

ED

co-comparability	linear	Chang et al 1995
planar bipartite, chordal bipartite	NP-c.	Lu, Tang 2002
chordal	NP-c.	Yenn, Lee 1996

EED

P_7 -free	linear	ISAAC 2011
planar bipartite	NP-c.	Lu, Ko, Tang 2002
chordal	linear	Lu, Ko, Tang 2002
and more...		

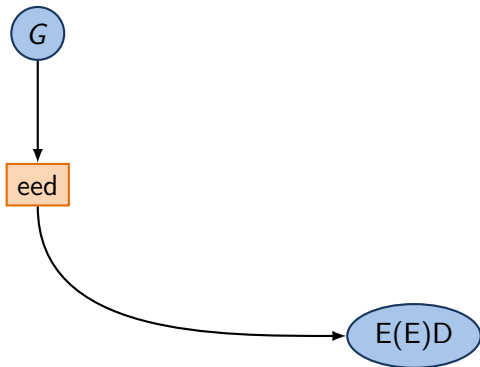
Our Solution

By Definition

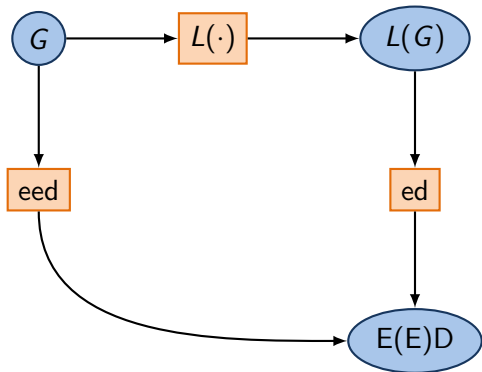
The following are equivalent for any D :

- (i) D is EED in G
- (ii) D is ED in $L(G)$
- (iii) D is dominating set in $L(G)$ and independent set in $L(G)^2$

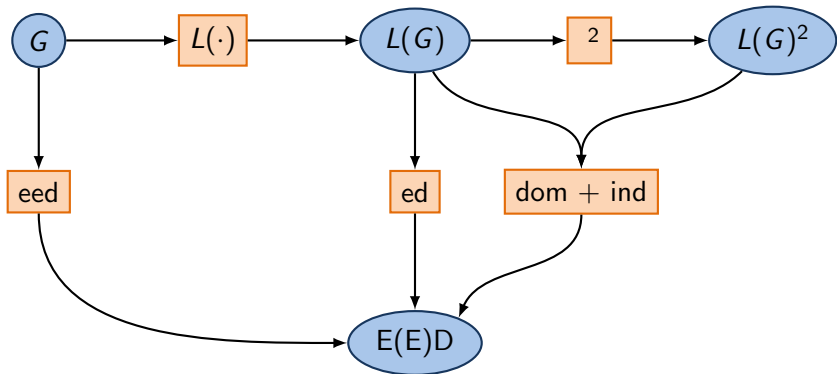
What is known



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Our Approach

Vertex weight function $\omega : V \rightarrow \mathbb{N}$ with $\omega(v) = |N[v]|$

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Theorem

The following are equivalent for any $D \subseteq V$:

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- (ii) D is min. w. dominating set in G with $\omega(D) = |V|$
- (iii) D is max. w. independent set in G^2 with $\omega(D) = |V|$

Our Approach

Vertex weight function $\omega : V \rightarrow \mathbb{N}$ with $\omega(v) = |N[v]|$

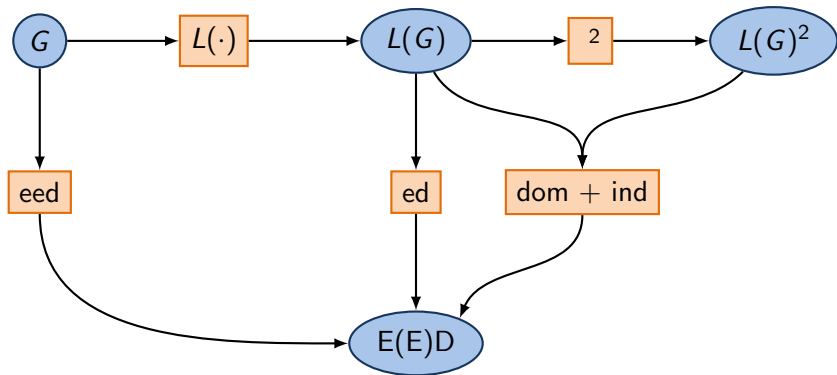
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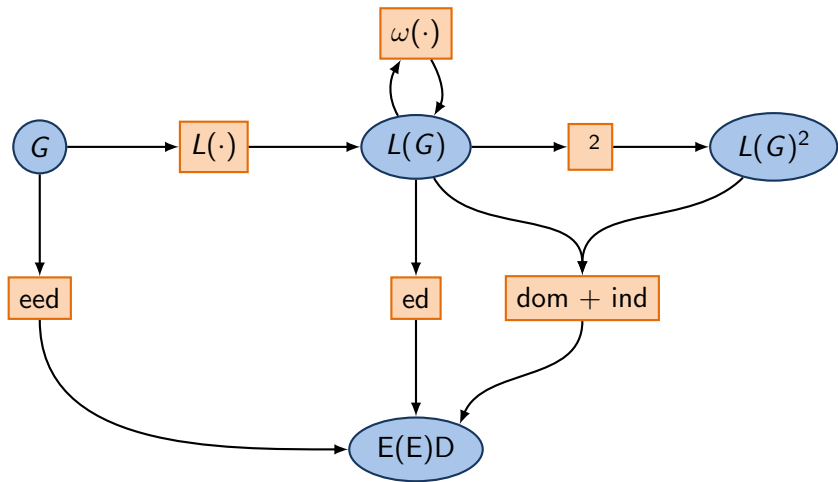
- (i) D is ED in G
- (ii) D is min. w. dominating set in G with $\omega(D) = |V|$
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(i) \Leftrightarrow (iii) was also found by MARTIN MILANIČ.

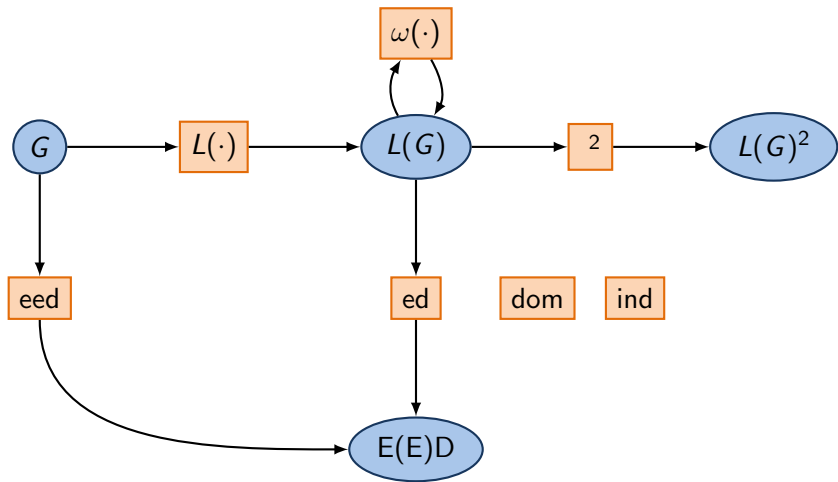
Our Approach



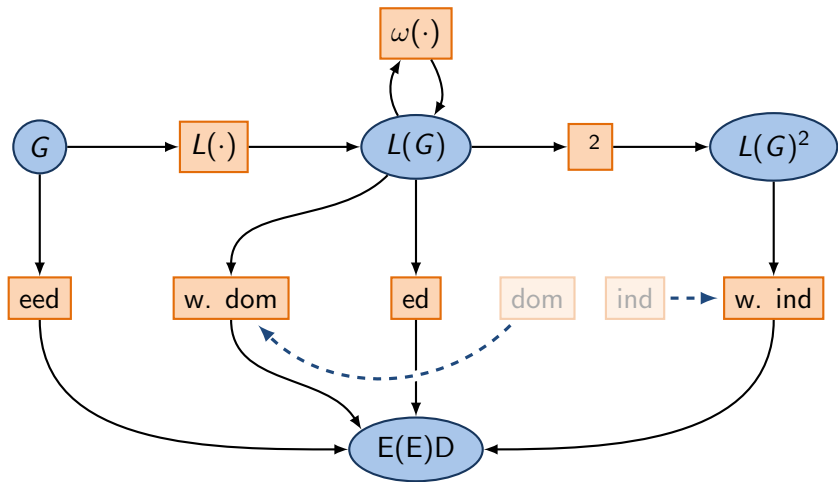
Our Approach



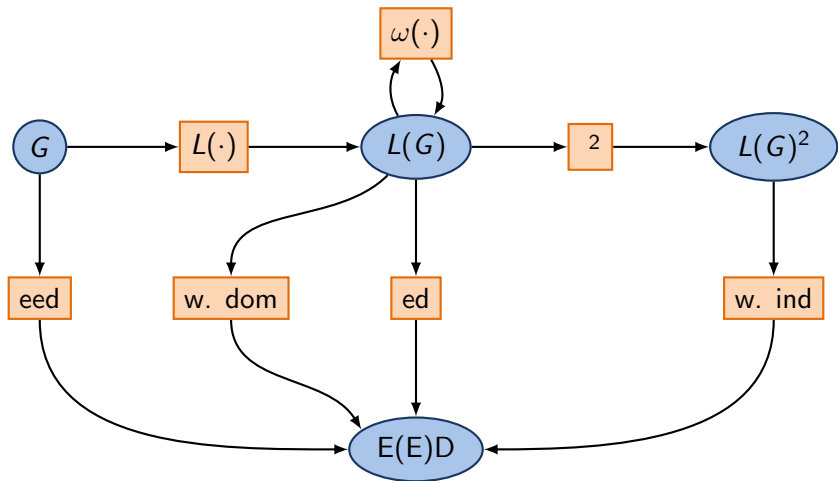
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Our Approach



Our Approach



This includes ...

... for EED

- ▶ min. w. (perfect, independent) edge domination
- ▶ min. w. dominating matching
- ▶ max. w. induced matching

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... for EED

- ▶ min. w. (perfect, independent) edge domination
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... for ED

- ▶ min. w. perfect domination
- ▶ min. w. independent domination

dually chordal

- ▶ ED: linear
using independent set for G^2
- ▶ EED: linear
 G has EED \Rightarrow dually chordal \leftrightarrow chordal

This includes strongly chordal graphs.

AT-free

- ▶ ED: polynomial
using independent set for G^2
(G^2 is AT-free)
- ▶ EED: polynomial
using independent set for $L(G)^2$
($L(G)^2$ is AT-free)

interval bigraphs

- ▶ ED: polynomial using domination for G

interval-filament

- ▶ EED: polynomial
using independent set for $L(G)^2$
($L(G)^2$ is interval-filament)

weakly chordal

- ▶ EED: polynomial
using independent set for $L(G)^2$
($L(G)^2$ is weakly chordal)

Results

	ED	EED
dually chordal	linear	linear
AT-free	polynomial	polynomial
interval bigraphs	polynomial	
interval-filament		polynomial
weakly chordal		polynomial

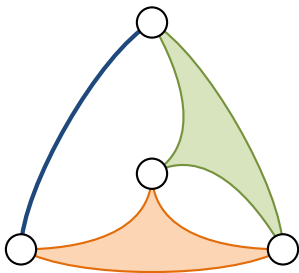
A View on Hypergraphs

Hypergraphs

Hypergraph: $H = (V, \mathcal{E})$ with $\mathcal{E} \subseteq \wp(V) \setminus \emptyset$

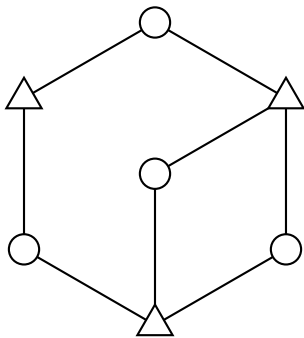
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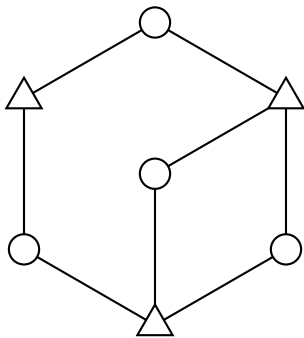
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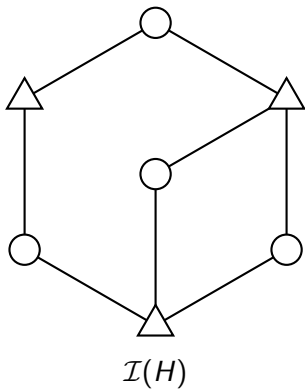


incidence graph of H — $\mathcal{I}(H)$

Duality

Hypergraph

$$H = (\circ, \triangle)$$



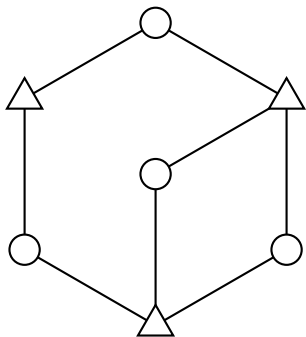
Duality

Hypergraph

$$H = (\bigcirc, \triangle)$$

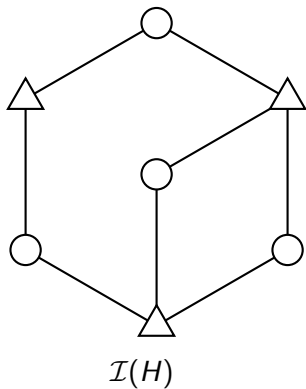
Dual Hypergraph

$$H^* = (\triangle, \bigcirc)$$

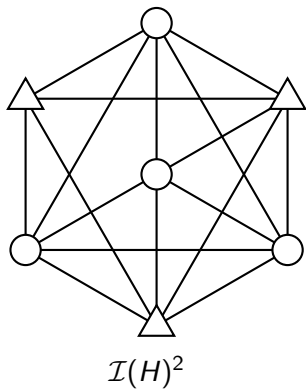


$$\mathcal{I}(H) = \mathcal{I}(H^*)$$

Line / 2-Section graph

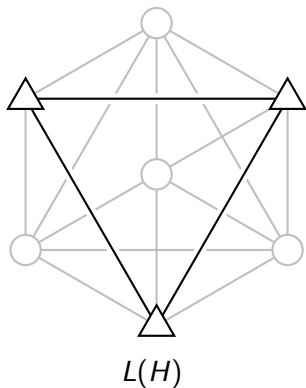


Line / 2-Section graph



line graph

$$L(H) = \mathcal{I}(H)^2[\mathcal{E}]$$

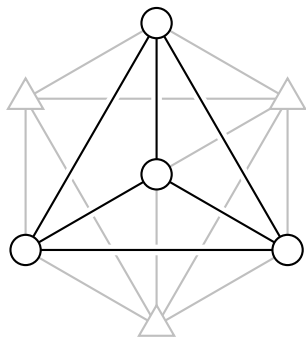


line graph

$$L(H) = \mathcal{I}(H)^2[\mathcal{E}]$$

2-Section graph

$$2Sec(H) = \mathcal{I}(H)^2[V]$$



$2Sec(H)$

Definition on Hypergraphs

D is ED in $H \Leftrightarrow D$ is ED in $2\text{Sec}(H)$.

D is EED in $H \Leftrightarrow D$ is ED in $L(H)$.

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Theorem

H has an ED $\Leftrightarrow H^*$ has an EED.

Results on hypergraphs

	ED	EED
α -acyclic	NP-complete	polynomial
hypertree	polynomial	NP-complete

Thank you for your attention!
