

Efficient intra-household allocations and distribution factors: implications and identification.*

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Abstract

This paper provides an exhaustive characterization of testability and identifiability issues in the collective framework in the absence of price variation; it thus provides a theoretical underpinning for a number of empirical works that have been developed recently. We first provide a simple and general test of the Pareto efficiency hypothesis, which is consistent with all possible assumptions on the private or public nature of goods, all possible consumption externalities between household members, and all types of interdependent individual preferences and domestic production technology; moreover, the test is proved to be necessary and sufficient. We then provide a complete analysis of the identification problem; we show under which assumptions it is possible to identify, from the observation of the household consumption of private goods, the allocation of these goods within the household as well as the Engel curves of individual household members.

JEL classification:

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1 Introduction

That a household comprising several adult members with specific preferences does not necessarily behave as a single rational agent should not be an object of debate. We know, at least since Arrow's famous impossibility theorem, that groups do not usually behave as individuals. Yet, for decades most theoretical and applied micro-economic work on household consumption, labor supply, savings or fertility behavior has been based on the assumption that indeed household decisions could be analyzed as stemming from a unique, well behaved utility function; this sometimes known as the unitary assumption.

The natural explanation for such a practice in the demand literature is that only household expenditures are usually observed. In general, there is practically no information on the way these expenditures are allocated among the various members of the household - possibly for public use - nor on the way that allocation may depend on aggregate savings or fertility decisions. The household thus appears as a black box which is conveniently modeled using the single rational agent hypothesis. It is quite revealing that the first attempts away from that practice were made on one hand in relation with the few cases where it was possible to observe the allocation of some good within the household or some consequences of that allocation, and on the other hand when endogenizing the formation (marriage) or the breakdown of households. The first 'bargaining' models of household behavior have thus developed in connection with models of joint labor supply within the household as well as divorce theory; see Manser and Brown (1980), McElroy and Horney (1981) and Becker (1991). Likewise, some literature has built up on the issue of food allocation within families in developing countries based on the observation that in some countries boys seem to be better nourished than girls; see Behrman (2000) for a review of this literature.

It remains that the unitary approach is questionable from a theoretical perspective. An additional problem is that it seems to be contradicted by the data. Perhaps the most convincing falsification has been provided by the literature on 'distribution factors'. Distribution factors are defined as variables that can influence household behavior, if at all, only through their impact on the decision process; that is, they affect neither preferences nor the budget constraint, and should thus be irrelevant in a unitary context. A first wave of models, starting with Thomas (1990), Schultz (1990), Bourguignon *et al* (1993) and Browning *et al* (1994), test the 'income pooling' hypothesis, according to which, controlling for total household income, its distribution between members should not affect behavior. All these papers reject the income pooling assumption.

A common concern raised by these models is that labor as well as non labor income may be endogenous, which could bias the results. For that reason, recent contributions have explored the impact of other, arguably more exogenous distribution factors. Thomas *et al.* (1997), using an Indonesian survey, have shown that the distribution of wealth by gender at marriage has a significant impact on children health in those areas where wealth remains under the contributor's control, even when current wealth and income were controlled

for.¹ Duflo (1999) has derived similar conclusions from a careful analysis of a reform of the South African social pension program that extended the benefits to a large, previously not covered black population; she shows that the impact of this windfall gain on the health of children crucially depends on the gender of the recipient - a typical distribution factor². Chiappori, Fortin and Lacroix (2002) use the state of the marriage market, as proxied by the sex ratio by age, race and state, and the legislation on divorce as particular distribution factors to study household labor supply; they find that an environment more favorable to women is associated with a significantly lower (resp. higher) level of female (resp. male) labor supply. In a similar context, Rubalcava and Thomas (2000) refer to the generosity of single parent benefits and reach identical conclusions.

Recognizing that households might not behave according to the single rational agent model does not mean that there cannot be any restriction on their aggregate consumption or joint labor supply behavior. Rationality may still be present under one form or another at the household level. The problem is precisely to know under what form. The research program in that area thus consists of investigating alternative hypothesis about decision making in the household and testing them against each other on the basis of the restrictions they may imply for the household demand and labor supply functions. If some of these hypotheses appear to hold against empirical evidence, it may be expected that the corresponding restrictions on household demand behavior will permit the identification, at least partially, of the intra-household allocation mechanism and then the welfare of individual household members.

Various contributions have tried to introduce such alternative frameworks. Manser and Brown (1980) and McElroy and Horney (1981) have proposed models based on cooperative game theory and bargaining. A more general approach has been proposed by Chiappori (1988, 1992), Bourguignon *et al.* (1993), Browning and Chiappori (1998) and Chiappori and Ekeland (2001a), who have developed a “collective” framework. In its most general version, the collective approach relies on the sole assumption that household decisions are Pareto efficient; no additional requirement is made upon the choice of the particular outcome on the Pareto frontier as in the bargaining models. The obvious advantage of this approach is its generality; it is clear, for instance, that all previous bargaining approaches are nested within the collective framework.

The question, of course, is whether this approach is not simply too general; i.e., is it able to generate testable restrictions at all? Surprisingly enough, several contributions have shown that, indeed, the collective model, even in its most general version, could generate strong testable restrictions on observed behavior. Two families of tests can be distinguished, depending on whether price variations can be observed in the data. Tests based on price effects have been

¹See also Galasso (1999) for a similar investigation.

²Specifically, Duflo finds that the consequences of this windfall gain on child nutrition dramatically depends on the gender of the recipient. Using the same data base, Bertrand *et al.* (2000) study the impact on labor supply of younger women within the household, and find again that the new benefits result in a much larger reduction of labor supply when they are received by a woman.

first introduced by Chiappori (1992) in a very simple model of labor supply with two egotistic agents and no public good. He shows that the Pareto efficiency assumption indeed imposes restrictions on joint labor supply functions. These conditions have been tested by Fortin and Lacroix (1997), Blundell *et al* (2000) and Chiappori, Fortin and Lacroix (2002). All three papers find that, while Slutsky symmetry is rejected by the data, the collective conditions are not. These tests have been later extended to a general frameworks by Browning and Chiappori (1998) and Chiappori and Ekeland (2002a,b). The conditions these contributions derive take the form of partial differential equations that generalize the standard Slutsky conditions of the unitary model; specifically, they state that the Slutsky matrix is the sum of a symmetric matrix and a matrix of rank at most $S-1$, where S is the number of members. This property has been tested, and not rejected, on consumption data by Browning and Chiappori (1998).

Alternatively, one can consider the test problem in the absence of price variation; then the approach must rely exclusively upon the effect of income or distribution factors. This is typically the case for the standard cross-sectional analysis of consumption patterns, where it is assumed that individuals over the sample face identical prices.³ In this context, a second family of tests can be used, that generalize the 'income pooling' family discussed above. Although simple versions of such tests have been used in various contexts (Bourguignon *et al*, 1993; Browning *et al.*, 1994; Thomas *et al.*, 1997), no comprehensive theoretical analysis have been provided so far.

The first goal of the present paper is to investigate more carefully the theoretical properties of the empirical procedures used in the preceding papers. We find that they are surprisingly general and powerful. First, a simple general test of the Pareto efficiency hypothesis is presented which is consistent with all possible assumptions on the private or public nature of goods, all possible consumption externalities between household members, and all types of interdependent individual preferences and domestic production technology.⁴ Moreover, the test is proved to be necessary and sufficient: if it is satisfied, then it is always possible to interpret observed behavior as if it was stemming from a collective framework with well-chosen preferences. Second, a test is provided of some separability properties in the preceding framework which are equivalent to considering private goods and egotistic or 'caring' agents.

A second issue is the identification problem: when and to what extent is it possible to recover the underlying structure - preferences and the decision process - from observed behavior? With price variations, an identification result was first derived in the labor supply case by Chiappori (1992), then extended by Blundell *et al* (2000), Chiappori, Fortin and Lacroix (2002) and Chiappori and Ekeland (2002b). When only income and distribution factors vary, Browning *et al* (1994) have shown how it was possible, using a parametric approach,

³The term 'cross-sectional' is slightly ambiguous in this context, since cross sectional analysis of labor supply often uses (possibly instrumented) wage differences between agents as price variations. Throughout the paper, our interest is in demand analysis, and we identify cross-sectional with the absence of price variation.

⁴For a related work, see Dauphin and Fortin (2001)

to identify the intra-household allocation process under the Pareto efficiency hypothesis when the consumption by one household member of at least one good is observed. The second purpose of the present paper is to extend these results. Specifically, we provide a complete analysis of the identification problem; we show under which assumptions it is possible to identify, from the observation of the household consumption of private goods, the allocation of these goods within the household as well as the Engel curves of individual household members.

The paper is organized around the preceding sequence of results. A first section describes the general structure of the model used to represent consumption decisions in a 2-person household. The following three sections are devoted to the three basic results above.

2 The basic framework

We consider a two adult household in which the two people are denoted A and B . We assume for the moment that there are n consumption goods and that they all are market goods which may be consumed either privately or publicly by the two agents. We thus denote $q^i \in \mathbb{R}_+^n$ the vector of private consumption of agent i ($= A, B$), and by $Q \in \mathbb{R}_+^n$ the vector of public consumption. The household consumption vector of private goods ($q^A + q^B$) is denoted by q , and that of total consumption ($q + Q$) by C . In line with the previous assumption, all prices are normalized to unity so that the budget constraint is:

$$e' \cdot (q^A + q^B + Q) = e' \cdot C = x$$

where e is a vector of ones ($\in \mathbb{R}_+^n$), Here, x can be considered either as total income or, as in standard cross-sectional analysis of consumption patterns, as exogenous total household expenditure.

Each person has preferences represented by $u^A(q^A, q^B, Q; a)$ and $u^B(q^A, q^B, Q; a)$ respectively, where a is a vector of characteristics that affects preferences directly. We refer to the a variables as *preference factors*. Thus a might include age, race, education of the two agents, number and age of children, etc.. We refer to this preference structure as ‘altruistic preferences’ because the private consumption of each member enters the preferences of the other. Note though that this might simply reflect positive or negative consumption externalities rather than a true altruistic behavior. Also, this general formulation does not exclude the possibility that one person does not care about the other. In summary, no restriction is placed on preferences, beyond assuming that they can be represented by a utility function for each adult in the household.

Since individual preferences u^A and u^B generally differ we need to specify how households make decisions about what to buy; that is, how they choose q^A , q^B and Q , and thus C . We assume that the choices depend not only on total expenditure x and preference factors a but also on a set of *distribution factors* z , disjoint from a . Specifically:

Assumption A1 : *There exist a set of K variables $z = (z_1, \dots, z_K)$ such that:*

- . individual preferences are independent of z*
- . the overall household budget constraint does not depend on z*
- . the value of z does influence the decision process.*

In other words, distribution factors are variables that affect the choices of (q^A, q^B, Q) but *do not enter* directly the utility functions or the budget constraint. As discussed in the introduction, several examples of distribution factors can be found in the literature. Distribution factors will play a key role in the following for three reasons. First, the mere fact that they can influence behavior will contradict the traditional, unitary framework - as recognized by the numerous works mentioned above. Secondly, rational models which allow for an influence of distribution factors upon behavior, necessarily restrict the form taken by this influence. This will generate tests of that extended rationality, as it will become clear in the following. Finally, distribution factors are extremely helpful in recovering some features of the intra-household decision process, a point that will be emphasized in the last sections of the paper.

Notation. In the next section and a few other parts of the paper, the household demand for good i is denoted $\xi_i(x, a, z)$ ($i = 1, \dots, n$). We use this notation when we do not want to distinguish between public goods (then $\xi_i \equiv X_i$), aggregate consumption of a private commodity (then $\xi_i = x_i^A + x_i^B$), or even individual consumption (then $\xi_i = x_i^A$ or $\xi_i = x_i^B$). In particular, the general tests described in the next section are valid whatever the particular interpretation. We assume all demand functions to be continuously differentiable.

In considering the restrictions implied by various assumptions below we have found it useful to use a novel type of ‘conditional’ demand function whereby the demand for one good is expressed as a function of the demand for another good as well as total expenditure and preference and distribution factors. *Conditional demand* functions are often used in demand analysis where we assume a single utility function. In that framework, the demand for one set of goods (the ‘goods of interest’) are conditioned on the price of these goods, total expenditures on these goods and the quantities of another set of goods (the ‘conditioning goods’); see Browning and Meghir (1991) for further discussion. Another version of demands that are conditioned on another demand is the concept of an *m-demand* introduced in Browning (2003). In this setting, one normal good is denoted the reference good. The demand curve for the reference good is then inverted to give total expenditure as a function of prices and the quantity of the reference good. This is then substituted into the Marshallian demands for other goods to give m-demands that depend on prices and the demand for the reference good. M-demands are useful in empirical modelling when we do not observe all quantities (and hence total expenditure is not observed) but we do observe all prices.

In the extended rational setting considered here, we define a somewhat different type of conditional demand function that turns out to be useful. Consider

the demand for good j , $C_j = \xi_j(x, a, z)$. Assume that ξ_j is strictly monotone in one distribution factor at least (say, y_1). Then it can be inverted on this factor:

$$z_1 = \zeta(x, a, z_{-1}, \xi_j)$$

where z_{-1} is the vector of distribution factors without the first element. Now substitute this into the demand for good i :

$$C_i = \xi_i(x, a, z_1, z_{-1}) = \xi_i[x, a, \zeta(x, a, z_{-1}, \xi_j), z_{-1}] = \theta_i^j(x, a, z_{-1}, \xi_j).$$

Thus the demand for good i can be written as a function of total expenditure, preference factors, all distribution factors but the first and the demand for good j . To distinguish this conditioning from the more conventional conditional demands discussed above, we shall refer to them as *z-conditional* demands. Note that, in the unitary setting, there are no distribution factors, so that *z-conditional* demands are not defined in this case. Finally, and unless otherwise stated, in the following we take good 1 as the conditioning good. In this case, we drop the (upper) index in the notation of *z-conditional* demand:

$$\theta_i^1(m, z, y_{-1}, \xi_j) \equiv \theta_i(m, z, y_{-1}, \xi_j) \text{ for } i = 2, \dots, n.$$

Various contributions apply the conditional demand approach developed here to collective models; the reader is referred in particular to Dauphin and Fortin (2001) and Donni (2000).

3 Testing the unitary model and Pareto efficiency

3.1 The unitary model

In this section we investigate the restrictions imposed on the demand functions, $\xi_i(x, a, z)$, and their *z-conditional* counterparts, $\theta_i(x, a, z_{-1}, q_1)$, by the properties that one may be willing to assume about the intra-household decision process or its outcome. We shall essentially consider three hypotheses: the ‘unitary’ model; the ‘collective’ approach, as characterized by Pareto efficiency of the allocation of goods; and an additional, bargaining-type condition. We begin with the unitary model, where we assume that a unique utility function is maximized. Formally :

Definition 1 *Let (q^A, q^B, Q) be given, C^1 functions of (x, a, z) . These are compatible with unitary rationality if there exists a utility function $U(q^A, q^B, Q; a)$ such that, for every (x, a, z) , the vector (q^A, q^B, Q) maximizes U subject to the budget constraint.*

The restrictions implied by this framework are trivial. Indeed, it assumes that the household maximizes a single utility function, that represents the

'household preferences' in some sense. A consequence is that, by definition, the household demand functions should depend on total expenditure x and the preference factors a , but not on the distribution factors, z . Formally :

Proposition 2 *A given, C^1 (direct) demand function is compatible with unitary rationality if and only if it satisfies:*

$$\frac{\partial \xi_i(x, a, z)}{\partial z_k} = 0 \quad \forall i = 1, \dots, n; k = 1, \dots, K \quad (1)$$

This condition is an immediate generalization of the 'income pooling' hypothesis, which has been tested, for instance, by Schultz (1990), Thomas (1990), Bourguignon et al. (1993), Browning et al. (1994), Fortin and Lacroix (1997), Phipps and Burton (1992), Lundberg *et al* (1997) and others. Interestingly enough, it has been rejected by the data in all the above studies.

An important remark is that a model with individual utility functions and a weighted sum of these as the household utility function *is formally a unitary model so long as the weights do not depend on distribution factors*. This fact has two consequences. First, the key insight of collective models is not that the household does not maximize some common index, but rather that this common index, if it exists, will in general depend *directly* on distribution factors (as well as prices and incomes). It is well known, for instance, that the Nash bargaining solution can be expressed as maximizing the product of individual surpluses. The crucial point, however, is that each agent's surplus (and therefore the index that is maximized by the household) cannot be seen as a 'household utility' in the unitary sense because it involves the agent's status quo point, which typically varies with prices, income and distribution factors.

A second and more surprising implication of this result is the following. Consider a model of collective decision making in which the household maximizes a weighted sum of individual utilities, the weights being functions of household income but *not* of distribution factors. Although this model does *not* belong to the unitary class (since the index maximized by the household is income-dependent), it is *observationally equivalent* to a unitary setting, in the sense that any demand function $\xi(x, a, z)$ it generates could alternatively be generated by a unitary framework. This paradoxical conclusion is due to the specific nature of the problem, and more precisely to the absence of price variations.⁵ This stresses the fact that on cross-sectional data without price variations, distribution factors are indispensable to distinguish between the unitary and the collective setting.

3.2 The collective approach

We now consider the more general framework, in which we explicitly recognize that the household consists of two members with potentially different preferences. Our *only* assumption, at this stage, is that the intra-household decision

⁵On the contrary, it can readily be checked that when price variations are available, the demand stemming from a model entailing income-dependent weights will not in general satisfy Slutsky symmetry.

process, whatever its particular features, always leads to a Pareto efficient outcome (PE). This hypothesis characterizes what we call ‘collective rationality’. Let us state it formally :

Definition 3 Let (q^A, q^B, Q) be given, C^1 functions of (x, a, z) . These are compatible with collective rationality if there exists two utility functions $u^A(q^A, q^B, Q; a)$ and $u^B(q^A, q^B, Q; a)$ such that, for every (x, a, z) , the vector (q^A, q^B, Q) is Pareto-efficient. That is, for any other bundle (q^{oA}, q^{oB}, Q^o) such that

$$u^i(q^{oA}, q^{oB}, Q^o; a) \geq u^i(q^A, q^B, Q; a)$$

for $i = A, B$ (with at least one strict inequality), then

$$e'(q^{oA} + q^{oB} + Q^o) > e'(q^A + q^B + Q)$$

This definition is quite general since no assumption whatsoever is made upon the form of individual preferences, the public or private nature of consumption goods or particular features of the intra-household decision process (beyond efficiency). Yet, strong restrictions on household demand functions obtain.

Our first crucial result is stated in the following Proposition, which provides a *necessary and sufficient* characterization of collective demand in the cross-sectional context:

Proposition 4 Consider a point $P = (x, a, z)$ at which $\frac{\partial \xi^i}{\partial z_1} \neq 0$ for all i . Without a priori restrictions on individual preferences $u^i(q^A, q^B, Q; a)$, $i = A, B$, a given system of C^1 demand functions is compatible with collective rationality in some open neighbourhood of P if and only if either $K = 1$ or it satisfies any of the following, equivalent conditions :

i) there exist real valued functions Ξ_1, \dots, Ξ_n and μ such that :

$$\xi_i(x, a, z) = \Xi_i[x, a, \mu(x, a, z)] \quad \forall i = 1, \dots, n \quad (2)$$

ii) household demand functions satisfy:

$$\frac{\partial \xi^i / \partial z_k}{\partial \xi^i / \partial z_1} = \frac{\partial \xi^j / \partial z_k}{\partial \xi^j / \partial z_1} \quad \forall i = 1, \dots, n; j = 1, \dots, n; k = 2, \dots, K \quad (3)$$

iii) there exists at least one good j such that:

$$\frac{\partial \theta_i^j(x, a, z_{-1}, q_j)}{\partial z_k} = 0 \quad \forall i \neq j, i = 1, \dots, n; k = 2, \dots, K \quad (4)$$

Proof. Let us first consider the case where there are at least two distribution factors, i.e. $K > 1$. From the Pareto-efficiency assumption, demands should be solutions of the following program :

$$\max_{q^A, q^B, Q} u^A(q^A, q^B, Q; a) + \mu \cdot u^B(q^A, q^B, Q; a)$$

$$\text{s.t.} \quad e' \cdot (q^A + q^B + Q) = x$$

Here, the set of PE allocations is fully described when μ varies within \mathbb{R}_+^n . The particular location of the solution on the Pareto frontier should of course be allowed to depend on all relevant parameters: i.e., μ will in general be a function of (x, a, z) . Household demand functions can thus be written:

$$\xi_i(x, a, z) = \Xi_i[x, a, \mu(x, a, z)] \quad \forall i = 1, \dots, n$$

as stated in condition (2). Then (3) comes from the fact that:

$$\frac{\partial \xi^i / \partial z_k}{\partial \xi^i / \partial z_1} = \frac{\partial \mu / \partial z_k}{\partial \mu / \partial z_1} \quad \forall i, k \quad (5)$$

Finally, in the neighborhood of any point where a z -conditional demand can be defined, (2) allows us to (locally) express μ as a function of q_j , x and a . Replacing in the direct demand function for good i leads to (4). Hence (2), (3) and (4) are equivalent necessary conditions for observed demand functions to be consistent with collective rationality.

For sufficiency, note that according to (2) there exists some function $\nu(x, a, z)$ such that $\xi(x, a, z)$ can be expressed as a function $\Xi(x, a, \nu)$ of x, a and ν alone. Take some arbitrary function $G(\xi_1, \xi_2, \dots, \xi_n; a)$ that is positive, increasing and quasi-concave with respect to the variables ξ_1 . Define then:

$$M(x, a, \nu) = G[\Xi_1(x, a, \nu), \dots, \Xi_n(x, a, \nu)]$$

We will now show that there exist two increasing and quasi-concave utility functions $v^A(Q, a)$ and $v^B(Q, a)$ such that the observed demand functions are solutions of (P) for $\mu(x, a, z) = M[x, a, \nu(x, a, z)]$. Clearly, these utility functions $v^i(X, z)$ are particular cases of the general utility functions $u^i(\cdot)$ appearing in (P) because they depend only on public goods.

The necessary and sufficient first order conditions implied by (P) are:

$$\forall(x, a, z) \quad D_\xi v^A(\xi, a) + \mu \cdot D_\xi v^B(\xi, a) = \lambda \cdot e$$

where $D_\xi v^i$ is the gradient of v^i and λ is an arbitrary scalar function of (x, a, z) .

Define then:

$$\begin{aligned} \nu^B(\xi, a) &= A(\xi_1 + \xi_2 + \dots + \xi_n) + B[(G(\xi, a))] \\ \nu^A(\xi, a) &= C[(G(\xi, a))] \end{aligned}$$

where A, C are arbitrary increasing scalar functions and B is a scalar function defined by:

$$B'(q) = -q \cdot C'(q)$$

A is taken to be large enough with respect to B so that ν^B is increasing. These functions ν^A and ν^B are thus increasing and quasi-concave. Moreover, it can easily be checked that they satisfy (A1). It follows that the solution of (P)

with these functions ν^i is the set of observed demand functions $\xi(x, a, z)$ which satisfy the equivalent conditions (2)-(4) in Proposition 2.

It remains to show that Proposition 2 remains valid in the case where there is only one distribution factor, i.e. $K = 1$. On one hand, condition (2) is trivially satisfied since it corresponds to a mere change of variable of z , whereas conditions (3) and (4) become irrelevant. On the other hand, the above sufficiency argument in the case $K > 1$ remains valid when $K = 1$ since it is solely based on condition (2). This shows that in the case of a single distribution factor and without a priori restrictions on individual preferences all observed demand functions are consistent with collective rationality. ■

When all consumptions are private, condition (3) has been known to be necessary for quite a long time.⁶ Proposition 4 complements existing knowledge in three directions. First, it shows that the condition is necessary even in the most general case (entailing public consumption, externalities, etc.); second, it provides various equivalent versions of the conditions; third and more importantly, it shows that these conditions are also sufficient, in the sense that any demand function satisfying them is compatible with collective rationality.

How should Proposition 4 be interpreted? The basic idea is that, by definition, distribution factors do not influence the Pareto set. They may affect consumption, but only through their effect upon the *location* of the final outcome on the Pareto frontier - or, equivalently, upon the *respective weighting* of each member's utility that is implicit in this location. The key point is that this effect is one-dimensional. This explains why restrictions appear only in the case where there are more than one distribution factors. Whatever the number of such factors, they can only influence consumption through a single, real-valued function μ . This is what is expressed by conditions i) and ii).

This simple idea has two important consequences. First, let us compute q_i as a z^1 -conditional function of (x, a, q_j, z_{-1}) . Then collective rationality implies that it should not depend on z_{-1} . The reason is that, for given values of x and a , whenever distribution factors (z_1, z_{-1}) contain some information that is relevant for intra-household allocation (hence for household behavior), this information, which is one-dimensional (as we have seen above), is *fully* summarized by the value of q_j . Once we condition on q_j, z_{-1} then becomes irrelevant. This is the meaning of condition iii).

A second, very important consequence relates to the question of the number of distribution factors to be taken into account. At the level of generality considered here, Proposition 4 says that at least two distribution factors are needed to test the hypothesis of collective rationality. Thus, in full generality, collective rationality imposes *no* restriction on household demand functions in the case where there is only one distribution factor. Now, this does not mean that no other restrictions can possibly be found, but rather that such restrictions require some *additional assumptions* to be made upon the form of individual preferences $u^i(q^A, q^B, Q; a)$, $i = A, B$. In particular, we shall see below that

⁶See for instance Bourguignon et al. (1993); Browning et al., (1994); Thomas *et al.*, (1997) for the direct form and Dauphin and Fortin (2001) for the z -conditional demand version.

further restrictions appear in the case of a single distribution factor - and come in addition to those in Proposition 4 in the case of more than one distribution factor - when some goods are private and/or consumed exclusively by one member of the household.

Finally, two additional remarks should be made.

Remark 1 Proposition 4 provides two distinct ways of testing for efficiency. The first - condition (3) - relies on testing for cross-equation restrictions in a system of unconditional demand equations. The other method (4) tests for exclusion restrictions in a conditional demand framework. Empirically, the latter is likely to be more powerful for at least two reasons. First we can employ single equation methods (or even non-parametric methods). Second, single equation exclusion tests are more robust than tests of the equality of parameters across equations.

As an illustration, assume that the household has three sources of *exogenous* income - say, w^A , w^B and w^o , with $x = w^A + w^B + w^o$. Then, while x does enter the budget constraint, any two income sources - say, w^A and w^B - do not, and can be taken as distribution factors. Hence Proposition 4 applies. In the present case, the partial derivatives in (3) and (4) may be interpreted as the household 'marginal propensity to consume' a given good with respect to the various components of household income. The unitary model would require that these propensities be equal for all goods. Through condition (3), collective rationality requires that these marginal propensities to consume must be proportional *across all goods*, whereas condition (4) requires them to be zero, conditionally on the demand for another good.

Remark 2 Proposition 4 generalizes easily to the Beckerian framework where domestic goods produced by the household are taken into account. Adding a domestic production function to go from the market inputs to the goods actually consumed by household members and taking into account the allocation of domestic labor do not modify the above tests on household demands for market goods.

3.3 Bargaining

Many papers that have analyzed intra-household decision processes have assumed a bargaining framework. If we take an axiomatic approach and include efficiency as one of our axioms then necessarily the bargained outcome will satisfy the conditions in Proposition 4. Of course, the bargaining framework should be expected to impose *additional* restrictions. Although a general assessment of the extra testable restrictions that bargaining implies is a difficult and still largely unsolved problem, an easy minimal test can be described as follows. Assume that some distribution factors are known to be positively correlated with member B 's (resp. A 's) threat point. Then, in program (P), μ should be increasing (resp. decreasing) in that distribution factor. But equation (5) provides an easy test of this property. Formally :

Proposition 5 Assume that μ is known to be increasing in z_1 and decreasing in z_2 . Then the demand functions consistent with any bargaining model are such that:

$$\frac{\partial \xi^i / \partial z_1}{\partial \xi^i / \partial z_2} = \frac{\partial \xi^j / \partial z_1}{\partial \xi^j / \partial z_2} \leq 0 \quad \forall i = 1, \dots, n; j = 1, \dots, n$$

Thus if we assume a priori that two distribution factors have these properties then we have a further testable restriction. The obvious factors to take are the incomes of the two partners. Indeed, if we are willing to go further and assume that it is only the relative value of these incomes, z_1/z_2 that matters then we have in addition:

$$\frac{\partial \xi^i}{\partial \text{Log}(z_1)} + \frac{\partial \xi^i}{\partial \text{Log}(z_2)} = 0 \quad \forall i = 1, \dots, n$$

This is simple to test and easy to interpret. As an illustration, Browning *et al* (1994) test the above restrictions on Canadian data, and find they are not rejected.

3.4 Examples

To round off this section we present two parametric examples. To simplify the exposition we shall assume that there are no preference factors a and that there are exactly two distribution factors, z_1 and z_2 . We first model the unrestricted household demands as a quadratic in (x, z) :

$$\xi_i = a_i + b_i x + c_i x^2 + d_i z_1 + e_i z_2 + f_i z_1^2 + g_i z_2^2 + h_i x z_1 + k_i x z_2 + l_i z_1 z_2 \quad (6)$$

The restrictions implied by the unitary model are simply $d_i = e_i = \dots = l_i = 0$. The restrictions implied by collective rationality - conditions (3) above - are a little more difficult to determine. We can show that either one of the following proportionality relationships must hold:

$$\xi_i = a_i + b_i x + c_i x^2 + \lambda_i (d_i z_1 + e_i z_2 + f_i z_1^2 + g_i z_2^2 + h_i x z_1 + k_i x z_2 + l_i z_1 z_2) \quad (7a)$$

or:

$$\xi_i = a_i + b_i x + c_i x^2 + \lambda_i (z_1 + \alpha z_2) + \mu_i (z_1 + \alpha z_2)^2 + \omega_i x (z_1 + \alpha z_2) \quad (7b)$$

Thus, either all the terms involving the distribution factors z_1 and z_2 must be proportional across all demand functions, or all the demand functions must be quadratic in the same linear function $(z_1 + \alpha z_2)$ of these factors. It is also easily shown that the z -conditional demands consistent with (7b) have the following expression under collective rationality:

$$\theta_i = \alpha_i + \beta_i x + \gamma_i x^2 + \delta_i q_1 + (\phi_i + \psi_i x) \cdot \sqrt{1 + \beta x + \gamma x^2 + \delta q_1} \quad (8)$$

(where conditioning is made on q_1). If (7a) holds we have in addition that $\phi_i = \psi_i = 0$. Note that in the absence of theoretical restrictions, z -conditional demands derived from the quadratic demand functions (6) would also involve terms in $z_2, z_2^2, x.z_2$ and $q_1.z_2$ both in the first part of the RHS of (8) and under the square root sign.

As a second example, consider the case where the household demand function take the following extended Working-Leser form:

$$\xi_i = a_i + b_i x + c_i x \text{Log} x + d_i \text{Log} z_1 + e_i \text{Log} z_2 \quad (9)$$

The associated z -conditional demand functions, conditioning on q_1 , are given by:

$$\theta_i = \alpha_i + \beta_i x + \gamma_i x \text{Log} x + \delta_i \text{Log} z_2 + \eta_i q_1 \quad (10)$$

It is then easily shown that collective rationality implies that d_i/e_i be the same for all $i=1, \dots, n$, or, equivalently, that $\delta_i = 0$ for $i = 2, \dots, n$.

4 Private goods and caring agents: testing for collective rationality and recovering the sharing rule

4.1 The sharing rule

In the previous section we did not impose any restrictions on preferences - beyond assuming them representable by a utility function - or on the public or private nature of the goods which are consumed. In this section we concentrate on the allocation of private goods across the members of the household. To do so, we impose the following restriction on individual preferences:

Assumption A2 : $u^i(q^A, q^B, Q) = \psi^i[\phi^A(q^A, Q; a), \phi^B(q^B, Q; a), a], i = A, B$

Here A and B have ‘egotistic’ preferences represented by the functions ϕ^A and ϕ^B respectively, defined over their own consumptions of private and public goods. Both egotistic preferences enter person i ’s over-all utility function ψ^i . These functions are a particular case of altruism. Following Becker (1981) we refer to these utility functions as *caring*. In comparison with the general formulation in the preceding sections, we see that this hypothesis is equivalent to some type of separability in the preferences of the two household members. Of course, caring utility functions include the special case of egotistic preferences for which $\psi^i(\phi^A, \phi^B) = \phi^i$.

We concentrate here on private goods and we ignore the decision concerning public goods Q . One way to proceed would be to condition everywhere on public goods. For the sake of simplicity, we prefer to assume the following separability property between private goods and public goods in individual preferences:

Assumption A3 : $\phi^i(q^i, Q; a) = f^i[\gamma^i(q^i; a), Q; a]$

Also, from now on, x denotes total expenditure on private goods: $x = e'(q^A + q^B)$. It must be stressed that all the preceding assumptions make sense only if it is possible to distinguish *a priori* public and private goods. In that case, the consumption vectors q^A and q^B , on one hand, and the vector Q , on the other are defined on disjoint sets of goods. Such a requirement was not necessary in the preceding section.

We restrict our attention in this section to the case of a *single* distribution factor z . There is no loss of generality in doing so, since we have seen in Proposition 4 that collective rationality implies that various distribution factors affect the intra-household allocation of goods through the one-dimensional factor μ . If demand functions satisfy conditions (2)-(4), the effects of all distribution factors may be summarized into those of a single one. In this case, Proposition 4 has shown that collective rationality was not imposing any restriction to demand functions. Our objective in this section is precisely to show that this is not the case when one restricts individual preferences through assumptions A2 and A3 to the case of private goods and caring agents. Before doing so we introduce the fundamental notion of a *sharing rule*:

Proposition 6 (existence of a sharing rule) *Let (q^A, q^B) be functions of (x, a, z) compatible with collective rationality. Assume, in addition, that the corresponding individual utilities satisfy Assumptions A2 and A3 above. Then there exists a function $\rho(x, a, z)$ such that q^i is a solution to:*

$$\max \gamma^i(q^i; a) \text{ subject to } e'q^i = x^i$$

with $i = A, B$, $x^A = \rho(x, a, z)$ and $x^B = x - \rho(x, a, z)$.

This proposition is a particular case of the general equivalence between a Pareto optimum and a decentralized equilibrium if there are no externalities or public goods. It thus requires no formal proof. The function $\rho(x, a, z)$, which denotes the part of total expenditure on private goods that person A receives is the 'sharing rule'. It describes the rule of budget sharing that the two agents implicitly apply among themselves when choosing a particular Pareto efficient allocation. Of course, we are not assuming that households of caring agents explicitly use such a sharing rule. Proposition 6 simply states that the outcome of the household allocation process can be characterized in this way.

4.2 Collective rationality, private goods and caring: a first characterization

In the preceding section, we have shown that all demand functions (for public or private goods) were consistent with collective rationality if there was only one distribution factor. In this section we are restricting attention to the case of caring and separable preferences. The natural question arises of whether there are additional restrictions stemming from these hypotheses which would permit us to test collective rationality, in the case where observed demands depend on

only one distribution factor, or which would come in addition of those included in Proposition 4 in the case of two or more distribution factors.

The answer is positive. There are additional restrictions that must be satisfied by joint demand functions in the case of private goods and collectively rational caring agents. These can be expressed at different levels of generality. At a basic level, the restriction is equivalent to taking explicitly into account the sharing rule either in direct, or in z -conditional demands. At a higher level of generality, we shall then see that it is in fact possible to recover the sharing rule between caring agents from the observation of their joint demand for private goods, provided these demands satisfy some restrictions. In turn these restrictions provide a general test of the joint hypothesis of collective rationality, private goods and caring agents.

The basic restrictions that must be satisfied by joint demand functions is expressed in the following Lemma (preference factors a are dropped for convenience).

Lemma 7 *Assume collective rationality, A2, A3. Then :*

1. *Direct demands must satisfy the following : there exists a real-valued function ρ and $2.n$ real-valued functions α_i and β_i such that:*

$$q_i(z, x) = \alpha_i [\rho(z, x)] + \beta_i [x - \rho(z, x)] \text{ for } i = 1, \dots, n \quad (11a)$$

2. *z -conditional demands must satisfy the following : there exist two real-valued functions F and G such that:*

$$\theta_i[s+t, F(t)+G(s)] = \theta_i [t, F(t)+G(0)] + \theta_i[s, F(0)+G(s)] - \theta_i[0, F(0)+G(0)] \quad (11b)$$

for all t, s in \mathbb{R}_+ and for $i = 2, \dots, n$.

In (11a), α_i and β_i are A and B 's respective demands for good i -i.e. their Engel curves; (11a) is restrictive because it must be fulfilled across goods for the same function ρ . In (11b), t and s represent the total expenditures of A and B respectively, that is ρ and $(x - \rho)$, and F and G are the demands for the conditioning good by A and B respectively. Again, the testable restriction in (11b) is that the functions F and G must be the same across all goods but the conditioning one. Note that this condition does not put any restriction on the individual demands for the conditioning good. An equivalent but more direct set of restrictions will be given in the next subsection.

Example

Although the conditions given in (11a) and (11b) may appear somewhat involved, they are not too difficult to work with for particular functional forms for θ_i . As an illustration, we may consider again the case of Working-Leser demand equation (9) above:

$$\begin{aligned} \xi_i &= a_i + b_i x + c_i x \text{Log} x + d_i \text{Log} z_1 + e_i \text{Log} z_2 \\ \theta_i &= \alpha_i + \beta_i x + \gamma_i x \text{Log} x + \delta_i \text{Log} z_2 + \eta_i q_1 \end{aligned}$$

As we have seen, collective rationality imposes $\delta_i = 0$, which is equivalent to the d_i 's and e_i 's being proportional across goods. Now, let us consider (11b). We have that

$$\begin{aligned} & \alpha_i + \beta_i.(t + s) + \gamma_i.(t + s).Log(t + s) + \eta_i.[F(t) + G(s)] \\ = & \alpha_i + \beta_i.t + \gamma_i.t.Logt + \eta_i[F(t) + G(0)] + \alpha_i + \beta_i.s \\ & + \gamma_i.s.Logs + \eta_i.[F(0) + G(s)] - \alpha_i - \eta_i.[F(0) + G(0)] \end{aligned}$$

which gives :

$$\gamma_i.(t + s).Log(t + s) = \gamma_i.t.Log(t) + \gamma_i.s.Log(s)$$

This imposes that $\gamma_i = 0$ for $i > 1$, so that the three sets of coefficients c_i, d_i and e_i must now be proportional. Then direct demands become :

$$\xi_i = a_i + b_i.x + r_i.\pi, \quad \text{where } \pi = x.Logx + d.Logz_1 + e.Logz_2$$

We now consider the identification problem. In its more general version, the problem can be stated as follows: when is it possible to recover the underlying structure from observed behavior? Obviously, in the absence of price variations, individual preferences cannot be recovered. The 'structure' at stake here is the sharing rule and individual Engel curves; that is, we ask whether it is possible, from the observation of household aggregate demand, to identify the splitting of private consumption between members and the demand function of each member.

The identification problem may be approached from a parametric or a non parametric perspective. In the parametric approach, a particular functional form is chosen for the structural model, and a reduced form for the demand function is derived. In particular, the derivation emphasizes the links between the parameters of the structural model and the coefficient of the demand function that will be taken to data. Identification, in this context, is equivalent to the uniqueness of the set of parameters of the structural models corresponding to any given specific values of the (estimated) coefficients of the reduced form. Note that uniqueness, hence identification, is *conditional on the functional form*; that is, it obtains (at best) within a specific and narrow set, defined by the functional form chosen at the outset. Nonparametric identification obviously implies parametric identification for any possible functional form. Conversely, however, it may be the case that parametric identification obtains for a particular class of (possibly flexible) functional forms, whereas nonparametric identification does not hold. This means simply that only one 'structure' (at most) is compatible with observed behavior *within the class under consideration*, whereas *functionally different* preferences and sharing rules do generate the same demand function. It is important to note, in contrast, that our approach to the identification problem is explicitly *nonparametric*: we try to derive uniqueness within the general class of (smooth) demand functions.

4.3 The case of exclusive and assignable goods

4.3.1 A general statement

We start with two particular cases where some information is available about *individual* consumption of household members; then new tests and new ways of recovering the sharing rule may be found. While in principle rather specific, these cases are empirically very important; most existing empirical works rely on assumptions of this type.

We may, in some cases, observe how much of a particular good each person consumes; this good is then said to be ‘assignable’. For instance, we may observe independently male and female clothing expenditures, or individual food consumptions. Alternatively, some goods may be consumed by one person only. This is the ‘exclusive’ case. One example would be information on the smoking or drinking patterns of one household member, provided that the same commodity is not consumed by the spouse - an idea reminiscent of Rothbarth’s ‘adult goods’ assumption (see Deaton (1987)). Note that, in the present cross-sectional framework, an assignable good is equivalent to a couple of exclusive goods, one being consumed by A and the other by B .

Before considering successively these two cases, we may stress their common feature: whenever one good is known to be exclusively consumed by one member - say, member A - this provides some information on the sharing rule, as described in the following Proposition:

Proposition 8 (*One exclusive good*) *Assume collective rationality and A2, A3. If the consumption of exactly one exclusive good (consumed by member A) is observed, and if the demand function of member A for this good is monotone, then we can recover the sharing rule $\rho(z, x)$ up to a transformation - i.e., if $\rho(z, x)$ is one solution, then any solution is of the form $F[\rho(z, x)]$, where F is strictly monotone- without any restriction on the observed demand function.*

Proof. >From (11a), an exclusive good consumed by member A is such that:

$$q(z, x) = \alpha[\rho(z, x)]$$

The function ρ is thus some transformation of the observed demand function $q(z, x)$. ■

Note that if the implicit individual demand function α is not (globally) monotone, then the result holds on any subset of income and distribution factors over which α is monotone. In particular, the result holds locally almost everywhere.

The next step, of course, is to identify the transformation F . This is what is done in the remainder of this section. Notice, however, that, except in the case where all goods are assignable, and therefore the total (private) consumption of both members can be observed, the sharing rule can only be identified up to an additive constant. In all the other cases, we can only observe how the sharing rule changes with total expenditure, x , and the distribution factor, z , but not total individual expenditures (see Chiappori (1992) for a precise statement).

The preceding proposition suggests that it is more convenient to use z -conditional rather than direct demand functions wherever a good may be safely assumed to be exclusive. Indeed, conditioning on that good is equivalent to considering combinations of z and x such that the sharing rule is constant and should permit to identify easily the individual Engel curves for non-exclusive or non-assignable goods. This explains why many of the following propositions are expressed in terms of conditional demands.

4.3.2 The case of an assignable good

We begin with the simplest case, where we observe both members' respective consumptions of an assignable good (or, equivalently, of an exclusive good for member A and an exclusive good for member B). Then the following restrictions on the two observed demand functions must hold.

Proposition 9 (*One assignable good*) *Assume collective rationality, A2 and A3. Assume in addition that good 1 is an exclusive good consumed by member A, and that good 2 is an exclusive good consumed by member B. Consider an open set on which the demand for good 2, conditional on that for good 1 is such that: $\frac{\partial \theta_2}{\partial x} \neq 0$ and $\frac{\partial \theta_2}{\partial q} \neq 0$. Then the following, equivalent properties hold true :*

i) there exists a function $F(t)$ satisfying:

$$\theta_2[t + s, F(t)] = \theta_2[s, F(0)] \quad (1)$$

for all non-negative s and t

ii) there exists two functions β and g such that:

$$\theta_2(x, q_1) = \beta[x - g(q_1)] \quad (2a)$$

iii) θ_2 satisfies:

$$\frac{\partial}{\partial x} \left[\frac{\partial \theta_2 / \partial q_1}{\partial \theta_2 / \partial x} \right] = 0 \quad (3)$$

Proof. (1) is directly obtained from (11b) and the exclusivity condition on good 2. (2a) expresses the fact that the demand for good 2 is that of member B and thus depends only on the share of private expenditure going to him/her. The function $g(q_1)$ in that expression is the share going to member A and thus the inverse of his/her own demand function (as in proposition 9), which is in fact the function $F(\cdot)$ appearing in (1). Finally, (3) is a translation of (2a) into a partial differential equation. Differentiating (2a) with respect to x and q_1 yields:

$$\partial \theta_2 / \partial x = \beta' [x - g(q_1)]$$

and

$$\partial \theta_2 / \partial q_1 = -g'(q_1) \cdot \beta' [x - g(q_1)]$$

Assuming that Θ is non linear in x , we have that:

$$g'(q_1) = -\frac{\partial\theta_2/\partial q_1}{\partial\theta_2/\partial x} \quad (4)$$

This must be a function of q_1 alone, which generates condition (3). Reciprocally, (4) implies that $\theta_2(\cdot)$ is a transformation of a function that is additively separable in x and q_1 . ■

The preceding proposition provides a way of testing collective rationality, private goods and caring in the case where the consumption of an exclusive good is observed for each household member. The test is presented in terms of conditional demand. Transposing it to direct demand can be made by the change of variables $(x, q_1) \rightarrow (x, z)$ based on the observation of the direct demand function for good 1, $q_1(x, z)$. Likewise, the sharing rule is easily recovered through that same change of variable. The function $g(q_1)$ in the Proposition above is the share of private expenditures going to member A. This function is obtained, up to an additive constant, by integrating the differential equation (4) above. Then replacing q_1 by its direct demand expression $q_1(x, z)$ yields the sharing rule:

$$\rho(x, z) = g[q_1(x, z)]$$

It is also possible to use direct demand functions throughout, as shown in the following.

Proposition 10 (Recovering the sharing rule with one assignable good).

Assume collective rationality, A2 and A3, and that q_1 and q_2 are consumed exclusively respectively by members A and B. Assume that the direct demand for both goods (as functions of x and z) are observed and that the corresponding conditional demand for good 2, $\theta_2(x, q_1)$ fulfills the conditions of proposition 10. Then, the sharing rule is given, up to an additive constant, by the following equivalent differential equations:

i)

$$\begin{aligned} g'(q_1) &= -\frac{\partial\theta_2/\partial q_1}{\partial\theta_2/\partial x} \\ \rho(x, z) &= g[q_1(x, z)] \end{aligned} \quad (5)$$

ii)

$$\begin{aligned} \frac{\partial\rho}{\partial x} &= \frac{\frac{\partial q_1/\partial x}{\partial q_1/\partial z}}{\frac{\partial q_1/\partial x}{\partial q_1/\partial z} - \frac{\partial q_2/\partial x}{\partial q_2/\partial z}} \\ \frac{\partial\rho}{\partial z} &= \frac{1}{\frac{\partial q_1/\partial x}{\partial q_1/\partial z} - \frac{\partial q_2/\partial x}{\partial q_2/\partial z}} \end{aligned} \quad (6)$$

Proof. Only a proof of (ii) is needed at this stage. From (11a) for exclusive goods we have:

$$q_1(z, x) = \alpha[\rho(z, x)]; \quad q_2(z, x) = \beta[x - \rho(z, x)]$$

Differentiating the observed demand functions with respect to z and x yields:

$$\begin{aligned} \frac{\partial q_1}{\partial z} &= \alpha' \cdot \frac{\partial \rho}{\partial z} \\ \frac{\partial q_1}{\partial x} &= \alpha' \cdot \frac{\partial \rho}{\partial x} \end{aligned}$$

$$\begin{aligned} \frac{\partial q_2}{\partial z} &= -\beta' \cdot \frac{\partial \rho}{\partial z} \\ \frac{\partial q_2}{\partial x} &= \beta' \cdot (1 - \frac{\partial \rho}{\partial x}) \end{aligned}$$

Solving for ρ_x and ρ_z yields (6). It may be shown that the condition under which that resolution is possible -i.e. $\frac{\partial q^A}{\partial z} \neq 0$ $\frac{\partial q^B}{\partial z} \neq 0$ $\frac{\partial q^A/\partial x}{\partial q^A/\partial z} \neq \frac{\partial q^B/\partial x}{\partial q^B/\partial z}$ - is equivalent to the conditional demand $\theta_2(x, q_1)$ being well defined -i.e. $\partial\theta_2/\partial x \neq 0$; $\partial\theta_2/\partial q \neq 0$ - as in Proposition 8. It may also be shown that the integrability condition of (6), that is the cross-derivative restriction:

$$\frac{\partial}{\partial z} \left(\frac{\frac{\partial q^A/\partial x}{\partial q^A/\partial z}}{\frac{\partial q^A/\partial x}{\partial q^A/\partial z} - \frac{\partial q^B/\partial x}{\partial q^B/\partial z}} \right) = \frac{\partial}{\partial x} \left(\frac{1}{\frac{\partial q^A/\partial x}{\partial q^A/\partial z} - \frac{\partial q^B/\partial x}{\partial q^B/\partial z}} \right)$$

is equivalent to condition (3) above after a change of variables. ■

Several remarks are in order. First, it is possible in the present case to recover not only the sharing rule, but also the Engel curves for each individual, up to an additive constant. Note that this identification result still holds when, say, the preferences are identical for the two household members, or when they are linear. With an assignable good, it is therefore possible to identify the sharing rule, and the Engel curves, up to a constant with no restriction at all on preferences; as we will see later, this is not possible in the general case. Secondly, the identification of the sharing rule and individual Engel curves can be performed using only the observed marginal propensities to consume out of the total budget and out of the distribution factor. In other words, identification requires to use only the first derivatives of the observed demand functions and does not rely upon non-linearities. This is important, since identification based upon non linearities is generally less robust.

4.3.3 The case of one exclusive good and one private good

A less demanding assumption is that one good only is known to be exclusive. This may be particularly adequate whenever the private nature of some consumption is debatable. For instance, Browning *et al.* (1994) assume that female

clothing is indeed an exclusive consumption, whereas they allow for a public good component in male clothing. We thus consider a situation in which the (individual) consumption of an exclusive good and the aggregate consumption of a private non-assignable good are observed.

Restrictions implied by collective rationality turn out to be easier to express (and to test) in terms of z -conditional demands. Specifically, the demand for good 2 conditional on good 1 are summarized in the following.

Proposition 11 *(One private and one exclusive good) Assume collective rationality, A2 and A3. Assume in addition that good 1 is an exclusive good consumed by member A, and that good 2 is a private joint consumption good. Consider an open set on which the z -conditional demand $\theta_2(x, q_1)$ is such that $\partial^2\theta_2/\partial x^2 \neq 0$ and $\partial^2\theta_2/\partial x\partial q \neq 0$. Then the following, equivalent properties hold :*

i) there exists a function $F(t)$ satisfying:

$$\theta_2[t + s, F(t)] = \theta_2[t, F(t)] + \theta_2[s, F(0)] - \theta_2[0, F(0)] \quad (7)$$

for all positive s and t .

ii) equivalently, there exist three functions α, β and g such that:

$$\theta_2(x, q_1) = \alpha[g(q_1)] + \beta[x - g(q_1)] \quad (8a)$$

iii) equivalently, θ_2 is such that

$$\frac{\partial}{\partial x} \left[\frac{\partial^2\theta_2/\partial x\partial q}{\partial^2\theta_2/\partial x^2} \right] = 0 \quad (9)$$

Proof. Proof. (i) is simply (11b). ii) is a restatement of (11a) where $g(q_1)$ is the share of total expenditures going to member A, given that q_1 is exclusively consumed by him/her. Finally (9) is the partial differential equation expression of (8a). The equivalent of relationship (4) above is obtained now by differentiating (8a) twice:

$$g'(q_1) = -\frac{\partial^2\theta_2/\partial q_1\partial x}{\partial^2\theta_2/\partial x^2} \quad (10)$$

which leads to (9). Reciprocally (9) implies that θ_{2x} is the transformation of a function that is additively separable in x and q_1 . Hence (8a). ■

As in the preceding case, the sharing rule may be easily recovered from the preceding differential equation in x_1 and the direct demand function $q_1(x, z)$ through $\rho(x, z) = g[q_1(x, z)]$. As before, it is thus defined up to an additive constant. Things are a little more complex in the present case when one uses direct demand functions, although, as in the preceding case, all properties on conditional demands have their counterpart on direct demand functions. We leave these derivations to the interested reader. ■

The basic difference between the present case and that of an assignable good is essentially that both the identification of the sharing rule and the test for collective rationality, private goods and caring agents now rely on second (rather than first) derivatives of the observed demand functions. They may thus be less robust. For the same reason identification now requires demand functions to be non-linear.

One could also consider other cases where more than a private good, or more than one or two exclusive goods would be observed. As in the general case, these additional observations do not give more information on the sharing rule, but they provide further tests of the joint hypothesis of collective rationality, private goods (and, possibly, exclusiveness of the goods assumed to be so).

4.4 Examples.

To illustrate the preceding properties consider the case where good 1 is exclusive and the observed demand for it is linear in x and z , and where the observed demand for good 2 is quadratic.

$$\begin{aligned} q_1 &= a_0 + a_1x + a_2z \\ q_2 &= \alpha_0 + \alpha_1x + \alpha_2x^2 + \beta_1z + \beta_2z^2 + \gamma xz \end{aligned}$$

If only the demand for good 1 is observed then the sharing rule is of the type:

$$\rho(x, z) = F(a_1x + a_2z)$$

and identification can only be obtained through an additional arbitrary restriction. If both goods 1 and 2 are observed, then it is possible to derive the conditional demand for good 2. It is also quadratic in x and q_1 :

$$\theta_2 = A_0 + A_1x + A_2x^2 + B_1q_1 + B_2q_1^2 + Cxq_1$$

If good 2 is exclusive to member B then condition (3) implies that $B_2 = C = 0$ and Proposition 2 yields:

$$g'(q_1) = -[B_1 + 2B_2q_1]/A_1$$

and, after integration:

$$g(q_1) = k - [B_1q_1 + B_2q_1^2]/A_1$$

where k is some constant. The corresponding sharing rule thus is:

$$\rho(x, z) = k - [B_1(a_1x + a_2z) + B_2(a_1x + a_2z)^2]/A_1$$

If good 2 corresponds to the joint consumption of both members, then Proposition 2 applies. Condition (9) does not impose any restriction because the conditional demand is quadratic on x and q_1 . The sharing rule is given by:

$$g'(q_1) = -C/(2A_2); \rho(x, z) = k - [C/(2A_2)](a_1x + a_2z)$$

It is thus linear in x and z . Indeed, this is a particular case of the example analyzed in section 4 of a linear sharing rule consistent with two private goods and quadratic demand functions.

4.5 Estimation and test from joint demands : the general case

The previous results suggest that whenever information is available about individual consumptions, then it is in general possible to recover the sharing rule and individual Engel curves (up to an additive constant). We now show a much more surprising result - namely that, generically on preferences, identification obtains *even without information on private consumptions*.

The general argument Let us start with a single consumption good. According to (11a), collective rationality implies that aggregate demand by the two household members is of the form:

$$q_i(z, x) = \alpha_i[\rho(z, x)] + \beta_i[x - \rho(z, x)]$$

This leads to the following partial derivatives:

$$\begin{aligned} \frac{\partial q_i}{\partial z} &= (\alpha'_i - \beta'_i) \frac{\partial \rho}{\partial z} \\ \frac{\partial q_i}{\partial x} &= (\alpha'_i - \beta'_i) \frac{\partial \rho}{\partial x} + \beta'_i \end{aligned} \quad (11)$$

where it is assumed that q_i does indeed depend on z . Then from (11), we can compute α'_i and β'_i :

$$\begin{aligned} \alpha'_i &= \frac{\rho_z \cdot q_{i,x} + (1 - \rho_x) \cdot q_{i,z}}{\rho_z} \\ \beta'_i &= \frac{\rho_z \cdot q_{i,x} - \rho_x \cdot q_{i,z}}{\rho_z} \end{aligned} \quad (12)$$

where $q_{i,a}$ stands for $\frac{\partial q_i}{\partial a}$.

But α_i (resp. β_i) must be a function of $\rho(z, x)$ (resp. $x - \rho(z, x)$). Writing that the derivative of α'_i along the locus $\rho(z, x) = \text{constant}$ must be equal to zero leads to the following, partial differential equation in $\rho(z, x)$:

$$\begin{aligned} &\frac{1}{q_{i,z}} \cdot [q_{i,xx} \rho_z + q_{i,xz} (1 - 2\rho_x) - q_{i,zz} \frac{\rho_x (1 - \rho_x)}{\rho_z}] \\ &= \frac{1}{\rho_z} \cdot [\rho_{xx} \rho_z + \rho_{xz} (1 - 2\rho_x) - \rho_{zz} \frac{\rho_x (1 - \rho_x)}{\rho_z}] \end{aligned} \quad (13)$$

This is a first information on the sharing rule $\rho(z, x)$. If one observes the aggregate demand function of the household for a given good, $q_i(z, x)$, then the sharing rule must satisfy the partial differential equation (13). Equivalently a test of collective rationality for an observed aggregate demand function $q_i(z, x)$ is that there exists a function $\rho(z, x)$ such that (13) hold. However, this equation is rather complex and does not say much on the way the sharing rule depends on the observed demand behavior for good i .

More can be obtained when the aggregate demand for two goods, rather than a single one, is observed. Without loss of generality, assume these are goods 1 and 2. Then (13) must be satisfied for $i = 1$ and 2. Equalizing the right hand-side of (13) for $i = 1, 2$ then yields:

$$Q_{xx}^{12} + Q_{xz}^{12} \frac{1 - 2\rho_x}{\rho_z} - Q_{zz}^{12} \frac{\rho_x(1 - \rho_x)}{\rho_z^2} = 0 \quad (14)$$

with:

$$Q_{at}^{ij} = \frac{q_{iat}}{q_{iz}} - \frac{q_{jat}}{q_{jz}}$$

The difference with (13) and the case of only one good is that the sharing rule must satisfy a first order, rather than a second order, partial difference equation, which is more restrictive. Two remarks can be made:

- In general, an equation like (14) defines ρ up to some boundary condition (say, to some function $f(z) = \rho(z, \cdot)$). Again in general, the equation (13) will be sufficient to pin down the function f . So one can expect that the function ρ will be identified from (13) and (14), although we do not attempt to give a formal proof.
- In any case, restrictions will in general be generated on the demand functions $q_1(z, x)$ and $q_2(z, x)$.

The following example substantiate this claim.

An example Though estimation of the sharing rule is always possible from three ("regular") demand functions, two of them may in some cases be sufficient. As an illustration, consider the case where the demand functions may be assumed to be quadratic:

$$q_i = a_i + b_i x + c_i x^2 + d_i z + e_i z^2 + f_i x \cdot z \quad i = 1, 2$$

An advantage of quadratic demands is that the fundamental differential equation (13) admits linear solutions. Indeed, if ρ_x and ρ_z are constant, then the RHS of (13) vanishes. Define then: $u = -\frac{\rho_x}{\rho_z}$ and $v = \frac{1 - \rho_x}{\rho_z}$. We have that:

$$2c_i + f_i(u + v) + 2e_i uv = 0, \quad i = 1, 2 \quad (15)$$

If the coefficients e and f are not proportional across goods, we can recover $u + v$ and uv . Moreover, if the expression $[(u + v)^2 - 4uv]$ is positive, one can recover u and v up to a permutation; then $\rho_x = \frac{u}{(u - v)}$ and $\rho_z = \frac{1}{(v - u)}$, which defines the sharing rule as a linear function of z and x up to an additive constant.

Three commodities Going a step further now means observing the demand functions of a household for three rather than two goods. Following the preceding logical sequence, this should lead to still more restrictions on the sharing rule

and therefore an easier identification than previously. This is actually the case, provided that demand functions satisfy some regularity conditions. Indeed, one can derive in that case two first order PDE, namely (14) and

$$Q_{xx}^{12} + Q_{xz}^{12} \frac{1 - 2\rho_x}{\rho_z} - Q_{zz}^{12} \frac{\rho_x(1 - \rho_x)}{\rho_z^2} = 0 \quad (16)$$

A result by Chiappori and Ekeland (2005) guarantees that, generically, (14) and (16) identify ρ up to a constant and a permutation of members. Also, it is clear that recovering the sharing rule, up to a constant, implies at the same time recovering the individual Engel curves. Indeed, equations (12) give the individual marginal propensity to consume each good i as a function of z and x . Integrating these equations yield the individual curves up to a constant, and of course up to a permutation of the two individuals.

It is clear, from the structure of the equations at stake, that identification obtains up to a permutation of members: it is possible to say that one individual in the household is getting $\rho(z, x)$ and has associated Engel curves $\alpha_1, \alpha_2, \dots$, but it is not possible to say whether that individual is A or B . In order to pin down this last issue, in the absence of assignable or exclusive commodities, a bargaining argument may be used. If the distribution factor is known to favor member A , then ρ represents member A 's allocation (instead of member B 's) if and only if ρ is increasing in z .

Needless to say, the determination of the sharing rule through (16) and of the individual Engel curves through (12) is extremely complex. We have not been able for instance to find analytical specifications of the aggregate demand functions which would permit to derive analytically the sharing rule. The simplest functional forms lead to rather intractable first-order differential equations on ρ . But, of course, solutions of these differential equations can be worked out numerically. The important and remarkable result here is that collective rationality implies enough restrictions on aggregate household demand functions so as to recover the sharing rule and individual Engel curves from the observation of aggregate marginal propensities to consume and the way they change as a function of both total expenditures and the distribution factor.

Finally, identification is only 'generic', in the sense that it relies on the non-linearities of the demand functions. Estimation and tests might then lack robustness. More precisely, the following proposition shows that the identification of the sharing rule and the test of collective rationality is not possible in the case of linear or 'quasi-linear' demand functions.

Proposition 12 (*linear and quasi-linear demand systems*) *Assume collective rationality, A2 and A3. The following two properties are equivalent :*

i) Direct demands are of the form

$$q_i = a_i + b_i x + c_i A(z, x) \quad (17)$$

ii) Conditional demands are linear :

$$\theta_i = \alpha_i + \beta_i x + \eta_i q_1 \quad (18)$$

Moreover, if these conditions are fulfilled, any function of the form $f[A(y, m)]$, and any function of the form $f[m - A(y, m)]$, where f is an arbitrary monotonic transformation, is a possible sharing rule.

Proof. That (17) and (18) are equivalent is obvious. Also, for any f , define α_i and β_i by :

$$\begin{aligned}\alpha_i(u) &= c_i f^{-1}(u) + b_i u \\ &\text{and} \\ \beta_i(v) &= a_i + b_i v\end{aligned}$$

Then (11) is obviously fulfilled. Note that, in this case, the conditions of Proposition 5 do not apply. Also, it is interesting to note that all equations (12) are proportional, so that considering several consumption goods does not bring additional information. As we shall see in the next section [??], the only way to identify the sharing rule in that case is to observe an assignable good. ■

5 Conclusion

In this paper, we have investigated the properties of the ‘collective’ approach to household behavior. This only relies upon one general assumption : that decisions taken within a household are ‘cooperative’ or ‘collectively rational’, that is, lead to Pareto efficient outcomes. What we have shown is that this very general setting has considerable empirical implications. It leads in particular to a sequence of tests which throw some light into the usual black box that is used to analyze the household consumption decisions. Remarkably enough, our techniques only require a distinction between those factors which may be behind the allocation process within the household - individual earnings in the first place, but not only them - and those that are likely to affect personal preferences. It does not require in particular any knowledge of the actual intra-household allocation of goods. The most general test of cooperation does not even require any assumption on the nature of the goods that are consumed or produced within the household.

Additional tests are available when one wants to go further and infer from the joint spending behavior on private goods by the household some information on who gets what. A general test is available in the case where the analysis is restricted to private goods only. It has even been shown that it is possible, if that test is satisfied, to recover from the observation of joint consumption behavior, information on the intra-household allocation of these goods and on individual preferences (Engel curves). More information and more restrictive tests may be obtained in the case where at least one individual consumption is observed. Whether those tests are robust and will actually provide more information on intra-household decision processes will be taken up in forthcoming empirical work.

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