

# Efficient Method for ECG Compression Using Two Dimensional Multiwavelet Transform

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**Abstract**—In this paper we introduce an effective ECG compression algorithm based on two dimensional multi-wavelet transform. Multi-wavelets offer simultaneous orthogonality, symmetry and short support, which is not possible with scalar two-channel wavelet systems. These features are known to be important in signal processing. Thus multiwavelet offers the possibility of superior performance for image processing applications. The SPIHT algorithm has achieved notable success in still image coding. We suggested applying SPIHT algorithm to 2-D multi-wavelet transform of 2-D arranged ECG signals. Experiments on selected records of ECG from MIT-BIH arrhythmia database revealed that the proposed algorithm is significantly more efficient in comparison with previously proposed ECG compression schemes.

**Keywords**— ECG signal compression, multi-rate processing, 2-D Multiwavelet, Prefiltering.

## I. INTRODUCTION

Biological signal compression, especially ECG compression has an important role in diagnosis, taking care of patients and signal transfer through communication lines. Normally, a 24 hours recording is desirable to detect heart abnormalities or disorders. This long term ECG monitoring is called Holter monitoring in automated ECG analysis. As an example, with the sampling rate of 360 Hz, 11 bit/sample data resolution, a 24 hours record requires about 43 MBytes per channel. Therefore, efficient coding of the ECG is an important problem in biological signal processing. In the past, many schemes have been presented for compression of ECG data. These techniques can be classified in two categories: (1) direct compression such as Amplitude-Zone-Time Epoch Coding (AZTEC), turning point (TP), coordinate reduction time coding system (CORTES), Fan algorithm, Scan-Along Polygonal Approximation (SAPA), and the long-term prediction (LTP). (2) Transformational methods such as Fourier transform, Walsh Transform, Karhunen-Loeve Transform (KLT), and Wavelet Transform (WT). In most cases, direct methods are superior to transform methods with

respect to system simplicity and error. However, transform methods usually achieve higher compression ratios and are insensitive to noise existing in original ECG signal [2].

Among the methods mentioned above, wavelet transform is an efficient tool in signal processing aimed at compressing ECG signals. Multiwavelet is well known for its good approximation and data-compression properties. Recently, much interest has been generated in the study of multiwavelet. Multiwavelet, due to a larger flexibility in constructing smooth, compactly supported and symmetric scaling functions, have even better approximation and data-compression properties; see the discussion in [3], [4] and [5]. Also applying multiwavelets in signal processing [6,7,8,9], compression [8,10,11] and noise elimination [8,11,12] indicates the superiority of multiwavelet to wavelet.

Since the reasonable results in image compression have been achieved by SPIHT algorithm and subjects that mentioned above, in this paper we suggest to apply SPIHT algorithm to 2-D multiwavelet transform of ECG signals.

## II. MULTIWAVELET

### A. A Short History of Multiwavelet

Multiwavelets constitute a new chapter which has been added to wavelet theory in recent years. In multiwavelets, more than one scaling functions and mother wavelet are used to represent a given signal.

The first construction for polynomial multiwavelet was given by Alpert, who used them as a basis for the representation of certain operators. Later, Geronimo, Hardin and Massopust constructed a multi-scaling function with 2 components using fractal interpolation.

In [5], multiwavelets based on Cardinal Hermite splines were constructed. In spite of the many theoretical result on multiwavelet, their successful applications to various problems in signal processing are still limited.

Unlike scalar wavelets in which Mallat's pyramid algorithm have provided a solution for good signal decomposition and reconstruction, a good framework for the application of the multiwavelet is still not available. Nevertheless, several researchers have proposed method of how to apply a given multiwavelet filter to signal and image decomposition. For example, Xia *et al* [14, 15] have proposed new algorithm to compute multiwavelet transform coefficients by using appropriate pre- and post-filtering filters, and have indicated that the energy compaction for discrete multiwavelet transform may be better than that obtained using conventional

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discrete scalar wavelet transforms.

*B. Multi-Scaling Functions and Multiwavelets*

The concept of multi-resolution analysis can be extended from the scalar case to general dimension  $r \in N$ . A vector valued function  $\Phi = [\phi_1 \ \phi_2 \ \dots \ \phi_r]^T$  belonging to  $L^2(\mathbb{R})^r$ ,  $r \in N$  is called a multi-scaling function if the sequence of closed spaces:

$$V_j = \overline{\text{span} \left\{ 2^{j/2} \phi_i(2^j \cdot -k) : 1 \leq i \leq r, k \in \mathbb{Z} \right\}}, \quad j \in \mathbb{Z}$$

constitute a multi-resolution analysis of multiplicity  $r$  (MRA) for  $L^2(\mathbb{R})$ . The multiscaling function must satisfy the two-scale dilation equation

$$\Phi(t) = \sqrt{2} \sum_k \mathbf{G}_k \Phi(2t - k) \tag{1}$$

Where  $\mathbf{G}_{k \in \mathbb{Z}} \in L^2(\mathbb{Z})^{r \times r}$  are  $r \times r$  scaling coefficients.

Now let  $W_j$  denote a complementary space of  $V_j$  in  $V_{j+1}$ . The

vector valued function  $\Psi = [\psi_1 \ \psi_2 \ \dots \ \psi_r]^T$  such that:

$$W_j = \overline{\text{span} \left\{ 2^{j/2} \psi_i(2^j \cdot -k) : 1 \leq i \leq r, k \in \mathbb{Z} \right\}}, \quad j \in \mathbb{Z}$$

called a multiwavelet. multiwavelet functions must satisfy the two-scale equation:

$$\Psi(t) = \sqrt{2} \sum_k \mathbf{H}_k \Phi(2t - k) \tag{2}$$

Where  $\mathbf{H}_{k \in \mathbb{Z}} \in L^2(\mathbb{Z})^{r \times r}$  is a  $r \times r$  matrix of coefficients [4, 9]. The two-scale equation (3) and (4) can be realized as a Multi-filter bank operating on  $r$  input data streams and filtering them in two  $2r$  output data stream, each of which is downsampled by a factor two. If we denote by  $x(t)$  a given signal and assume that  $x(t) \in V_0$ , then

$$x(t) = \sqrt{2} \sum_k \mathbf{V}_{0,k}^T \Phi(t - k) \tag{3}$$

And the scaling coefficient  $\mathbf{V}_{1,k}$  of the first level can be consider as a result of low-pass multifiltering and downsampling :

$$\mathbf{V}_{1,k} = \sum_m \mathbf{G}_{m-2k} \mathbf{V}_{0,m} \tag{4}$$

Analogously, the first level multiwavelet coefficients  $\mathbf{W}_{1,k}$  are obtained using high-pass multifiltering and down-sampling:

$$\mathbf{W}_{1,k} = \sum_m \mathbf{H}_{m-2k} \mathbf{V}_{0,m} \tag{5}$$

Full multiwavelet decomposition of the signal  $x(t)$  can be found by iterative filtering of the scaling coefficient:

$$\mathbf{V}_{j,k} = \sum_m \mathbf{G}_{m-2k} \mathbf{V}_{j-1,m} \tag{6}$$

$$\mathbf{W}_{j,k} = \sum_m \mathbf{H}_{m-2k} \mathbf{V}_{j-1,m} \tag{7}$$

Note that  $\mathbf{V}_{j,k}$  and  $\mathbf{W}_{j,k}$  are  $r \times 1$  column vectors.

*C. Multiwavelet in Comparison with Wavelet*

The multiwavelet idea originates from the generalization of scalar wavelets; instead of one scaling function and one wavelet, multiple scaling functions and wavelets are used. This leads to more degree of freedom in constructing wavelets. Therefore opposed to scalar wavelets, properties such as compact support, orthogonality, symmetry, vanishing moments, short support can be gathered simultaneously in multiwavelets, which are fundamental in signal processing [8,9].

The increase in degree of freedom in multiwavelets is obtained at the expense of replacing scalars with matrices, scalar functions with vector functions and single matrices with block of matrices. Also, prefiltering is an essential task which should be performed for any use of multiwavelet in the signal processing [8, 14].

*D. Prefiltering the Data*

One of the challenges in realizing multiwavelets is the efficient prefiltering. In the case of scalar wavelets, the given signal data are usually assumed to be the scaling coefficients that are sampled at a certain resolution, and hence, we directly apply multiresolution decomposition on the given signal. But the same technique can not be employed directly in the multiwavelet setting and some prefiltering has to be performed on the input signal prior to multiwavelet decomposition. The type of the prefiltering employed is critical for the success of the results obtained in application.

As mentioned above, multifilter banks require a vector-valued input signal. There is a number of ways to produce such a signal from 2-D image data. Perhaps the most obvious method is to use adjacent rows and columns of the image data; this has already been attempted.

There could be infinitely many ways to do such prefiltering. There exist well known prefilters in literature [14, 15, and 16]. The most obvious way to get second input row is just to repeat the first on end use two identical rows of length  $n$ .

A different way to get the input rows for the multiwavelet filterbank is to preprocess the given scalar signal  $f(n)$ . In our implementation, first we refer to repeated row (RR) and second we refer to approximation prefilter (App). RR representations have proven to be useful in feature extraction; however, it requires more calculation than App representations. Furthermore, in data compression applications, one is seeking to remove redundancy, not increase it. Thus in this paper we apply multiwavelet with approximation prefilter. Experimental results shows that in this case, SA4 multiwavelet based on APP prefiltering method, slightly outperforms the other multiwavelets used in this study that were CL, GHM, BiGHM6, BiGHM2, BiH34n, BiH54n, Cardbal2, Cardbal3 and Cardbal4 multiwavelets.

III. TWO DIMENSIONAL ECG ARRAY

*A. Constructing 2-D ECG Array*

We used the technique reported in [13] for delineating cycles, period and amplitude normalization. The period of each beat

is normalized using multi-rate techniques and set to a constant number, i.e. 128 samples. This produces beats with a constant period, eliminating the effect of heart rate variability. First interpolating by a factor  $L$ , which is the constant number chosen to be the fixed period and then by down sampling with the appropriate factor for each cycle the length of each cycle becomes uniform hence period normalization is performed. The factor  $L$  is chosen to have a high value, so that there would be no error in down sampling.

Let  $x(n)$  be the input of an interpolation filter with an up sampling factor  $L$  and an impulse response  $h(n)$ . Then the output  $y(n)$  is given by:

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n - kL) \quad (8)$$

The up sampler just inserts  $L - 1$  zeros between successive samples. The filter  $h(n)$ , which operates at a rate  $L$  times higher than that of the input signal, replaces the inserted zeros with interpolated values. The polyphase implementation of this filter insures efficient interpolation. The output of a decimation filter  $y(n)$  with a down sampling factor  $M$ , is given by:

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(nM - k) \quad (9)$$

Since down sampling causes aliasing, a lowpass filter  $h(n)$  is used to remove it. If the signal does not contain frequencies above  $\pi/M$ , there is no need for the decimation filter and only down sampling is enough. Thus the change of sampling rate is a reversible process provided that the Nyquist condition is satisfied. The original sampling rate taken back by multi-rate techniques is recovered with no distortion. The output of the system is given by:

$$y_i(n) = \sum_{k=0}^{p_i-1} x_i(k)h(nM_i - kL) \quad (10)$$

where  $x_i(n)$  and  $y_i(n)$  are the  $n$ th samples of the  $i$ th input beat and output period and amplitude normalized (PAN) beat, respectively.  $p_i$  is the total number of samples in  $i$ th original beat,  $h(n)$  is the impulse response of the filter and  $L$  and  $M_i$  are the up-sampling and down-sampling factors, respectively for the  $i$ th beat vector [13].

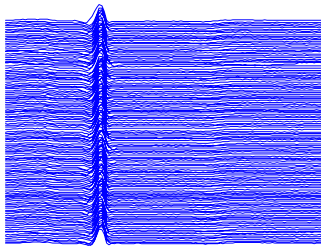


Fig. 1. 2-D ECG after period and amplitude normalization (PAN)

Amplitude normalization is performed in order to make the beats as similar as possible, and minimizing the variations between the magnitudes of the beats and setting the highest amplitude equal to one. After these normalizations, we put signals of 128 beats under together to construct an  $128 \times 128$  array of ECG signal that we treat in transformation as an image. A 2-D ECG array created using this approach is shown in Fig. 1.

### B. Proposed Compression Algorithm

To compress the 2-D array, there are many 2-D compression algorithms available, which are mostly used in image compression. In this paper, the 2-D multiwavelet transform by SPIHT (Set Partitioning in Hierarchical Trees) [1] coder is selected for implementing the 2-D transform. Fig. 2 shows the block diagram of proposed method.

## IV. RESULTS AND DISCUSSION

We used data in the MIT-BIH arrhythmia database to test the performance of our proposed algorithm. All ECG data used here are sampled at 360 Hz, 11 bits/sample.

We used PRD to measure distortion between the original signal and reconstructed signal. PRD can be defined as:

$$PRD = \sqrt{\frac{\sum (x_{or} - x_{re})^2}{\sum (x_{or})^2}} \times 100\% \quad (11)$$

where  $x_{or}$  and  $x_{re}$  are original and reconstructed signals of length  $N$ , respectively. Since the data used in the literatures are usually different in sampling frequency, and sample resolution, exact comparisons are inconclusive. Nonetheless, we compared the PRD result in similar compression ratio.

We used record numbers 100,101, 103, 105, 107, 117, 118, 119, 202, 205, 213, and 219 which consist of different rhythms, QRS complexes and morphologies and entopic beats. We compressed 1.4 minutes of data from each of these records. We report compression ratios from actual compressed file sizes and PRDs from decompressed files. Fig. 3 shows the PRD result value versus CR for each record of data and the average PRD values of this dataset are presented in Table I.

From Fig. 3 we see that the results for all data are approximately close to each other. It means the proposed algorithm is suitable for a variety of ECG data. For the sake of comparing our method with other methods in literature for different CRs and records, the algorithm was applied to records 117 and 119 from MIT-BIH database. Hilton presented a wavelet and wavelet packet based EZW encoder [17]. He reported the PRD value of 2.6% with compression ratio 8:1 for record 117 and compared it with the best previous results. The PRD value of the proposed method here is 1.83% for the same record and compression ratio which is significantly better than the encoders in [17] and [18]. In order to compare to ASEC [19], for record 119, they reported PRD result 5.5% at bitrate 183 bps, compared to our PRD of 4.87% at the same bitrate. The summary of this comparison appears in Table II. The simulation result for selected records indicate that the proposed method has good progressive reconstruction

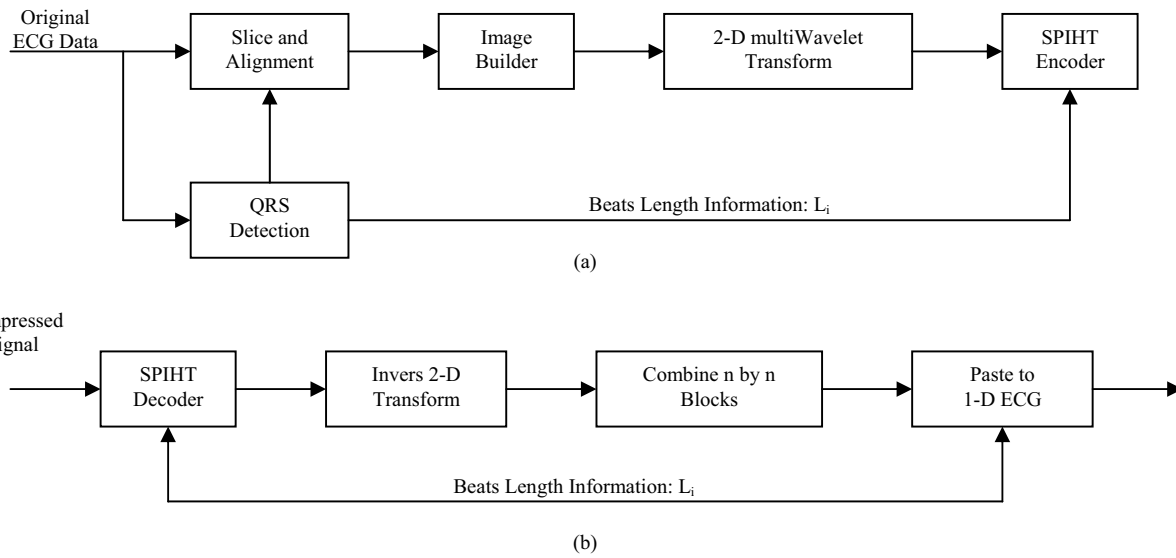


Fig. 2 Block diagram of the proposed algorithm. (a) Encoder (b) Decoder

quality, and that the reconstruction quality degrades gracefully all the way up to very high compression ratios, such as CR=90. Finally, to illustrate the progressive decompression quality of the presented method in order to investigate the effect of compressing ECG signals using proposed method from the clinical point of view, three waveforms including original, reconstructed waveforms and difference between original and reconstructed signal (error) of records 117, at the different CRs, are shown in Fig. 4. Note that reconstructed ECG signals are smoothed versions of the original signals.

V. CONCLUSION

In this paper, we proposed a new ECG compression scheme which combines the efficiency of multi-wavelet transform and SPIHT algorithm. It should be noted that a further improvement in results may be achieved with sophisticated implementation of multi-wavelet transform by considering computationally cost and effective prefiltering methods.

TABLE I. AVERAGE TEST RESULT FOR THE DATASET

<b>CR</b>	8	10	14	18	22	26	28	30
<b>PRD</b>	2.14	2.52	3.33	4.20	5.08	5.93	6.34	6.72

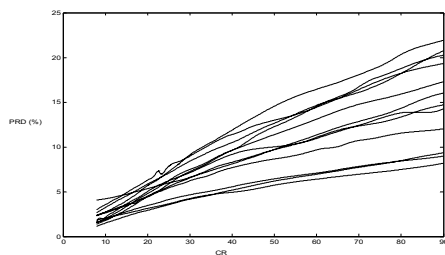
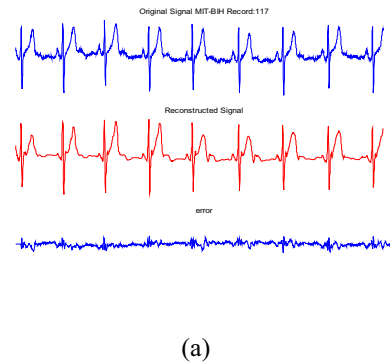


Fig. 3. The PRD results of MIT-BIH ECG data

TABLE II. PRD COMPARISON OF DIFFRENR ALGORITHM

Algorithm	Record	CR	PRD (%)
Hilton [17]	117	8:1	2.6
Djohan <i>et al.</i> [18]	117	8:1	3.9
<b>Proposed</b>	117	8:1	<b>1.83</b>
ASEC [19]	119	21.6:1	5.5
Lu <i>et.al</i> [20]	119	21.6:1	5
<b>Proposed</b>	119	21.6:1	<b>4.87</b>



(a)



(b)

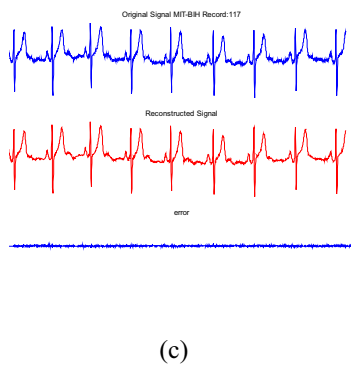


Fig. 4 – Compressing ECG using the sa4 with ap prefiltering method. The above figure shows the original signal, the middle shows reconstructed signal after compression and the bottom shows error between them. The first 2048 samples of MIT-BIH record 117 are showed. (a) CR=90, PRD=8.13% (b) CR=51.30, PRD=5.83% (c) CR=8, PRD=1.83%.

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