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EFFICIENT METHODS FOR FINDING TRANSFER FUNCTION ZEROS OF POWER SYSTEMS

Nelson Martins
Member IEEE

Herminio J.C.P. Pinto

CEPEL – Centro de Pesquisas de Energia Eletrica
Caixa Postal 2754
20.001, Rio de Janeiro, RJ, Brazil

Leonardo T.G. Lima
Member IEEE

MARTE Engenharia
Av. Pres. Vargas 542/2202
20.000, Rio de Janeiro, RJ, Brazil

Abstract – This paper is probably the first to describe algorithms suited to the efficient calculation of both proper and non-proper transfer function zeros of linearized dynamic models for large interconnected power systems. The paper also describes an improvement to the well known AESOPS algorithm, formulating it as an exact transfer function zero finding problem which is efficiently solved by a Newton-Raphson iterative scheme. Large power system results are presented in the paper.

Keywords: Power System Stability, Low Damped Oscillations, Additional Feedback, Excitation Control, Transfer Function Zeros, Large Scale Systems, Sparse Eigenanalysis.

I. INTRODUCTION

The location of the zeros of the open-loop transfer function of a feedback system is closely related to the ease or difficulty with which the system is controlled. The movement of zeros following system changes is a rather complex subject and little work has been done in association with the power system problem [1,2].

The use of the *augmented system equations* (see Appendix) for the small-signal stability problem has already allowed the efficient calculation of eigenvalues, eigenvectors, frequency response plots, transfer function residues, participation factors and step response plots [2,3,4,5,6,7,8,9] for large scale systems.

This paper comes in response to the need for efficient algorithms for the calculation of transfer function zeros of large power system dynamic models [4]. Newton-Raphson, Inverse Iteration and Simultaneous Iteration algorithms [3,5,9], applied to the *augmented system equation*, are described. With such algorithms an optional facility may be added to comprehensive packages for small-signal stability analysis enabling engineers to carry out controller design with extra valuable information.

The EPRI software for the analysis of small-signal stability of large scale power systems uses two alternative techniques to compute eigenvalues which complement each other [4]:

1. the AESOPS algorithm which is a successful heuristically based algorithm and computes one eigenvalue at a time;
2. the Modified Arnoldi method, which has a sound mathematical basis and can find typically up to five eigenvalues simultaneously.

A recent paper [10] has explained the AESOPS algorithm in terms of traditional eigenvalue analysis. The results presented in [10] for the 10-machine New England system did not, however, clearly attested the superiority of the algorithm proposed by its authors over the original AESOPS in the eigensolution of large practical power systems.

The power system eigenvalues were shown in [4] to be equal to the zeros of a special transfer function. This fact were not used to advantage in [4] due to the lack of an exact analytical expression for such transfer function and of an adequate transfer function zero finding method for large scale systems. These two obstacles were obviated in the work reported in this paper, leading to the improved AESOPS algorithm of sections V and VI.

All the algorithms of this paper have been implemented exploiting the *augmented system equations* sparse structure. The notations adopted in the paper are defined as used.

II. TRANSFER FUNCTION ZEROS

Consider the dynamic system equations:

$$\dot{\underline{x}} = A\,\underline{x} + \underline{b}\,u \qquad y = \underline{c}^t\,\underline{x} \tag{1}$$

where A is a state matrix of order n, $\underline{x}$ is the state vector, u is a single input and y is a single output which have been specified.

The objective here is to find the zeros of the open-loop transfer function $y(s)/u(s) = \underline{c}^t\,(sI{-}A)^{-1}\,\underline{b}$. From Root Locus theory it is known that the closed-loop transfer function poles tend to the open-loop transfer function zeros as the feedback loop gain tends to infinity [11]. This concept was used to derive the basic algorithm of this paper which is similar to that described in [12].

The closed-loop system will be defined here as having a control signal u proportional to the output variable y:

$$u(s) = K\,y(s) \tag{2}$$

The poles of the closed-loop system are then the eigenvalues of the state equation:

$$\dot{\underline{x}} = (A + \underline{b}K\underline{c}^t)\,\underline{x} = A_{cl}\,\underline{x} \tag{3}$$

The eigenvalues of matrix A_{cl} will coincide with the open-loop transfer function zeros when the feedback gain K approaches infinity. In this case, matrix A_{cl} differs from A by the introduction of very large elements in the locations defined by the product $\underline{b}\,\underline{c}^t$. Matrix A_{cl} is of the same order of the whole system, is real and unsymmetric and its eigenvalues can be obtained by a standard QR routine [13]. As a transfer function normally has less zeros than poles, the QR eigensolution will contain extraneous

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zeros which assume larger values as the feedback gain K is increased. These extraneous zeros should theoretically go to infinity with the feedback gain K, but this does not happen due to rounding errors.

The closed-loop system poles, i.e., the eigenvalues of $\mathbf{A}_{cl}$ can also be found by solving the generalized eigenvalue problem $\mathbf{A}_g \underline{\mathbf{x}}_g = \lambda\, \mathbf{B}\, \underline{\mathbf{x}}_g$:

$$\left[\begin{array}{c|c} \mathbf{A} & \underline{\mathbf{b}} \\ \hline \underline{\mathbf{c}}^t & -1/K \end{array}\right] \left[\begin{array}{c} \underline{\mathbf{x}} \\ \hline u \end{array}\right] = \lambda \left[\begin{array}{c|c} \mathbf{I} & \underline{\mathbf{0}} \\ \hline \underline{\mathbf{0}}^t & 0 \end{array}\right] \left[\begin{array}{c} \underline{\mathbf{x}} \\ \hline u \end{array}\right] \tag{4}$$

where $\mathbf{I}$ is the identity matrix, $\underline{\mathbf{0}}^t$ is a row vector with all elements equal to zero and $\underline{\mathbf{x}}_g$ comprises both $\underline{\mathbf{x}}$ and scalar input u. Note that as the value of K tends to infinity, the matrix element $1/K$ tends to zero. A QZ routine [13] for solving the generalized eigenproblem of equation (4) directly deals with the case where the matrix element $1/K$ is identical to zero, and therefore the extraneous zeros assume such large magnitudes that can easily be identified and discarded. The solution of the generalized eigenvalue problem of equation (4) should therefore be preferred to the method of [12] for finding all the transfer function zeros of a moderate size system.

III. CALCULATION OF ZEROS FOR LARGE SCALE SYSTEMS

The use of a QZ routine to solve for all the zeros of the specified transfer function is a prohibitively expensive task in large scale systems. The only alternative in large system problems is to solve for one zero at a time or for several zeros at a time located around a fixed point which can be placed at will in various parts of the complex plane. Efficient algorithms can be developed to exploit the sparse structure of the *augmented system equations* which are described in the Appendix.

The generalized eigenvalue problem of (4) can be solved, one zero at a time, by the inverse iteration algorithm [3], whose basic scheme is shown below for the case where the matrix element $1/K$ is equal to zero:

a. Solve for $\underline{\mathbf{w}}_{k+1}$:

$$\left[\begin{array}{c|c} \mathbf{A}-q\mathbf{I} & \underline{\mathbf{b}} \\ \hline \underline{\mathbf{c}}^t & 0 \end{array}\right] \left[\begin{array}{c} \underline{\mathbf{w}}_{k+1} \\ \hline u \end{array}\right] = \left[\begin{array}{c} \underline{\mathbf{z}}_k \\ \hline 0 \end{array}\right] \tag{5}$$

b. Compute the vector $\underline{\mathbf{z}}_{k+1}$ for the next iteration:

$$\underline{\mathbf{z}}_{k+1} = \frac{\underline{\mathbf{w}}_{k+1}}{max(\underline{\mathbf{w}}_{k+1})}$$

Convergence occurs when the change in $\underline{\mathbf{z}}$ at any iteration is less than some specified tolerance. In this algorithm the subscript k is the iteration number, q the specified approximate value of the desired transfer function zero z_i and $max(\underline{\mathbf{w}}_{k+1})$ is the element of largest magnitude in this vector. The vector $\underline{\mathbf{z}}_k$ has arbitrary initial value and corresponds to the zero direction vector at convergence. After convergence, the factor $1/(z_i-q)$ will be dominant in the element $max(\underline{\mathbf{w}}_{k+1})$ and the correct zero z_i is given by:

$$z_i = q + 1/max(\underline{\mathbf{w}}_{k+1})$$

Note that the transfer function zero and zero direction vector of the matrix in equation (5) form a pair which is analogous to the eigenvalue-eigenvector pair of the state matrix $\mathbf{A}$.

Equation (5) is now expressed in terms of the *augmented system equations*:

$$\left[\begin{array}{c|c|c} \mathbf{J}_1-q\mathbf{I} & \mathbf{J}_2 & \underline{\mathbf{b}}^a \\ \cline{1-2} \mathbf{J}_3 & \mathbf{J}_4 & \\ \hline \underline{\mathbf{c}}_x^t & \underline{\mathbf{c}}_r^t & 0 \end{array}\right] \left[\begin{array}{c} \underline{\mathbf{w}}_{k+1} \\ \hline \underline{\mathbf{r}}_{k+1} \\ \hline u_{k+1} \end{array}\right] = \left[\begin{array}{c} \underline{\mathbf{z}}_k \\ \hline \underline{\mathbf{0}} \\ \hline 0 \end{array}\right] \tag{6}$$

The use of equation (6) allows the efficient calculation of a single transfer function zero for large scale systems. The simultaneous iteration algorithm [14] has already been applied to the *augmented system equations* for the calculation of a group of eigenvalues which are nearest to a given estimate q [9]. A similar algorithm was here applied to the matrix problem depicted in equation (6) to obtain a group of transfer function zeros and large power system results are presented in section VIII.

IV. FINDING INVARIANT ZEROS IN THE MULTI-INPUT-MULTI-OUTPUT CASE

The zero finding algorithms described in sections II and III can readily be extended to the multi-input-multi-output case. When m inputs and m outputs are simultaneously considered, vectors $\underline{\mathbf{b}}$ and $\underline{\mathbf{c}}^t$ of equation (4) become matrices B and C of appropriate dimensions. The invariant zeros [15] of a large scale system matrix can be calculated by the inverse iteration and simultaneous iteration algorithms. The transmission zeros of a transfer function matrix are a subset of the system matrix invariant zeros [15].

A brief result on a 5-machine system is presented in section VIII, but further research is needed into this area.

V. THE AESOPS ALGORITHM FORMULATED AS A ZERO FINDING PROBLEM

The AESOPS algorithm [4] is a heuristically based one-at-a-time eigenvalue method designed to compute the electromechanical modes of oscillation for large power systems. The AESOPS algorithm is derived from the linearized equation of motion of a chosen generator, to which a complex frequency disturbance in the mechanical torque is applied. At every iteration, a corrected value for this complex frequency disturbance is applied until the system becomes resonant. This iterative process is almost always convergent and the converged complex frequency value corresponds to an electromechanical eigenvalue which is dominant at the disturbed generator.

An interesting paper [10] has suggested improvements to the basic AESOPS algorithm, but lacked large scale system results to substantiate its claims. In this section, an improved AESOPS algorithm is proposed which requires the calculation of the zeros of a specially tailored transfer function [4].

Consider the block diagram of Figure 1 which describes the torque-angle loop dynamics of the disturbed j-th generator in a large power system. The mechanical damping constant D_j is here assumed to be zero for brevity, but was fully considered in the computer algorithm implementation. The variables $\Delta\delta(s)$, $\Delta\omega(s)$, $\Delta T_{el}(s)$ and $\Delta T_m(s)$ of this section should rigorously all have a subscript j to relate them to the j-th generator, but this subscript was omitted for simplicity. The inertia constant of the j-th generator is denoted by H_j.

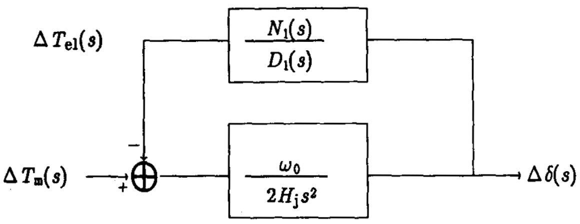

Figure 1. Torque-Angle Loop of Disturbed j-th Generator

From the inspection of the Figure 1 one can write:

$$\Delta\delta(s) = \frac{\omega_0}{2H_j s^2}\left(\Delta T_m(s) - \Delta T_{el}(s)\right) \qquad (7)$$

By choosing the mechanical torque and rotor angle as output and input variables respectively, one gets:

$$\Delta T_m(s) = \left(\frac{2H_j s^2}{\omega_0} + \frac{N_l(s)}{D_l(s)}\right)\Delta\delta(s) \qquad (8)$$

It can readily be seen that the zeros of the transfer function $\Delta T_m(s)/\Delta\delta(s)$ of (8) are equal to the poles of the closed loop system of Figure 1. The desired power system eigenvalues are therefore given by the zeros of the transfer function $\Delta T_m(s)/\Delta\delta(s)$.

Equation (8) can be expressed in the form:

$$\Delta T_m(s) = \underline{c}^t\,\underline{x}(s) + d(s)\,\Delta\delta(s) \qquad (9)$$

in which the output variable $\Delta T_m(s)$ depends not only on the vector $\underline{x}(s)$ but also on the system input $\Delta\delta(s)$ and its derivatives.

Let A be the (nxn) state matrix of the global multimachine power system. The AESOPS algorithm requires the opening of the torque-angle loop of the disturbed j-th generator. The opening of this torque-angle loop implies making zero the $\Delta\delta$ and $\Delta\omega$ states of the j-th generator and letting the column of the $\Delta\delta$ state become the input vector $\underline{b}_\delta$ to the system.

The zeros of (9) can therefore be found by solving the generalized eigenvalue problem:

$$\left(\begin{array}{c|c} A' & \underline{b}_\delta \\ \hline \underline{c}^t & d(s) \end{array}\right)\left(\begin{array}{c} \underline{x} \\ \hline \delta \end{array}\right) = \lambda \left(\begin{array}{c|c} I & \underline{0} \\ \hline \underline{0}^t & 0 \end{array}\right)\left(\begin{array}{c} \underline{x} \\ \hline \delta \end{array}\right) \qquad (10)$$

where A' is a matrix of order $(n\text{–}2)$ due to the elimination of states $\Delta\delta$ and $\Delta\omega$ of the j-th generator. The vector $\underline{x}(s)$ used in this section and the next is also of order $(n\text{–}2)$. The term $d(s)$ is given by:

$$d(s) = c_\delta + \frac{2H_j}{\omega_0}s^2 \qquad (11)$$

where c_δ is a real constant which depends on the system operating point.

The generalized eigenvalue problem described by equation (10) cannot be adequately solved by the inverse iteration algorithm since the matrix on the left part of the equation is a functional of the Laplace variable s. A more convenient way to solve this problem would be by using the Newton-Raphson method, as described in the next section.

VI. A NEWTON-RAPHSON SOLUTION SCHEME FOR THE IMPROVED AESOPS ALGORITHM

When s is a zero of the transfer function $\Delta T_m(s)/\Delta\delta(s)$ described in equation (8) it satisfies:

$$\underline{c}^t(sI-A')^{-1}\underline{b}_\delta + d(s) = 0 \qquad (12)$$

Transfer function zeros can be found, one at a time, through use of an iterative algorithm such as Newton-Raphson. Solving equation (12) is equivalent to solving:

$$\begin{aligned} (sI-A')\,\underline{x}(s) - \underline{b}_\delta &= \underline{0} \\ \underline{c}^t\,\underline{x}(s) + d(s) &= 0 \end{aligned} \qquad (13)$$

which is a non-linear system with $(n\text{–}1)$ equations in $(n\text{–}1)$ unknowns. The unknowns are the Laplace variable s and the vector $\underline{x}(s)$ which is of order $(n\text{–}2)$ since the states $\Delta\delta$ and $\Delta\omega$ of the j-th disturbed generator were removed.

The Newton-Raphson algorithm for solving (13) is given by:

a. provide initial estimates $\underline{x}_0$, s_0

b. calculate the vector of residues $\underline{f}(s_k,\underline{x}_k)$

$$\left(\begin{array}{c|c} s_kI-A' & -\underline{b}_\delta \\ \hline \underline{c}^t & d(s_k) \end{array}\right)\left(\begin{array}{c} \underline{x}_k \\ \hline 1 \end{array}\right) = \left(\begin{array}{c} \underline{t}_k \\ \hline y_k \end{array}\right) = \underline{f}(s_k,\underline{x}_k) \qquad (14)$$

where the value of the input variable $\delta(s)$ is set to unity.

c. Stop process if change in $\underline{f}(s_k,\underline{x}_k)$ is below the specified tolerance.

d. Evaluate the Jacobian of (14) and solve for the new increments $\Delta\underline{x}_k$ and Δs_k:

$$\left(\begin{array}{c|c} s_kI-A' & \underline{x}_k \\ \hline \underline{c}^t & \frac{4H_j s_k}{\omega_0} \end{array}\right)\left(\begin{array}{c} \Delta\underline{x}_{k+1} \\ \hline \Delta s_{k+1} \end{array}\right) = -\left(\begin{array}{c} \underline{t}_k \\ \hline y_k \end{array}\right) \qquad (15)$$

e. Obtain $\underline{x}_{k+1} = \underline{x}_k + \Delta\underline{x}_{k+1}$, $s_{k+1} = s_k + \Delta s_{k+1}$ and return to step "b".

For the solution of large scale problems, equations (14) and (15) must be expressed in terms of the *augmented system equations*, described in Appendix. Equation (15) is expressed below in the desired form:

$$\begin{bmatrix} s_k\mathbf{I}-\mathbf{J}_1' & -\mathbf{J}_2' & \underline{\mathbf{x}}_k \\ -\mathbf{J}_3' & -\mathbf{J}_4' & \underline{0} \\ \underline{\mathbf{c}}^t_{\mathbf{x}} & \underline{\mathbf{c}}^t_{\mathbf{r}} & \frac{4H_i s_k}{\omega_0} \end{bmatrix} \begin{bmatrix} \Delta\underline{\mathbf{x}}_{k+1} \\ \Delta\underline{\mathbf{r}}_{k+1} \\ \Delta s_{k+1} \end{bmatrix} = - \begin{bmatrix} \underline{\mathbf{f}}^x_k \\ \underline{\mathbf{f}}^r_k \\ y_k \end{bmatrix} \quad (16)$$

where $\mathbf{J}_1' - \mathbf{J}_2'(\mathbf{J}_4')^{-1}\mathbf{J}_3' = \mathbf{A}'$.

The original AESOPS algorithm has the good characteristic of converging to the dominant electromechanical modes of the disturbed generator in spite of bad initial values for $\underline{\mathbf{x}}_0$ and s_0. The improved AESOPS algorithm described in this section also has the same characteristics. This desired robustness was obtained by using the augmented initial vector:

$$\begin{bmatrix} \underline{\mathbf{x}}_0 \\ \underline{\mathbf{r}}_0 \\ s_0 \end{bmatrix} = \begin{bmatrix} \underline{0} \\ \underline{\mathbf{b}}_\delta \\ s_0 \end{bmatrix}$$

VII. EXTENDING THE AESOPS ALGORITHM CONCEPT TO OTHER ACTIVE SYSTEM COMPONENTS

The algorithm described in this section is an extension of the AESOPS concept (see section V) to other active components in a large system. The objective is to find modes which are dominant at the bus terminals of a static VAr compensator (SVC) or a HVDC link. These modes could be either electromechanical modes or be mainly associated with the control loop dynamics of the active components considered. The algorithm should converge to one or the other type of mode depending on the initial conditions provided. Figure 2 shows a block diagram which describes the complete power system dynamics through the disturbed static compensator voltage control loop. The block $SVC(s)$ denotes the transfer function of the static compensator while $N_2(s)/D_2(s)$ relates the compensator shunt admittance (ΔB_v) with the deviations in the regulated bus voltage (ΔV). The additional signal loop of the static compensator [2] can easily be considered but was here ignored for brevity.

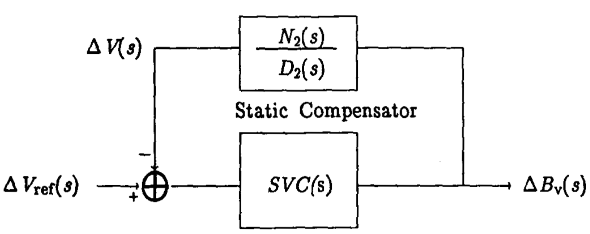

Figure 2. Power System Representation the Through SVC Voltage Control Loop

From inspection of Figure 2, one can write:

$$\Delta B_v(s) = SVC(s)\left(\Delta V_{ref}(s) - \frac{N_2(s)}{D_2(s)}\Delta B_v(s)\right) \quad (17)$$

By choosing the compensator shunt admittance and the bus voltage reference (ΔV_{ref}) as the input and output variables respectively, one gets:

$$\Delta V_{ref}(s) = \left(\frac{N_2(s)}{D_2(s)} + \frac{1}{SVC(s)}\right)\Delta B_v(s) \quad (18)$$

Again one can note that the zeros of the transfer function $\Delta V_{ref}(s)/\Delta B_v(s)$ of (18) are equal to the poles of the closed loop system of Figure 2. The remaining considerations are similar to the material contained in section V of this paper. As the Newton-Raphson method is very sensitive to the initial values given, there is a need for an initialization vector in order to make this algorithm converge to the desired dominant modes of the static compensator.

VIII. RESULTS ON TRANSFER FUNCTION ZEROS

The open loop transfer function zeros of a plant are not altered by the addition of a feedback controller. Consider the case where the system has an unstable pair of poles and that feedback stabilization is attempted through an input–output pair whose transfer function exhibits an unstable pair of zeros in the neighborhood of the poles to be damped. A root locus branch [11] will exist between these neighboring pairs of poles and zeros, irrespective of the feedback controller transfer function. Therefore, it is not possible in practice to stabilize this system through this feedback control loop.

The knowledge on the location of transfer function zeros enables control engineers to carry out controller design more effectively. The results presented in this section are intended to show the potential of the algorithms developed and estimulate power system control engineers to further investigate the practical application of this extra facility.

Results on the 5-Machine System

The 5-machine system is shown in Figure 3 and full data is provided in [2] together with the considerations on its stabilization difficulties. The machines are referred to as G_1, G_2, G_3, G_4 and G_7 according to the buses where they are connected. Machine G_7 is a dynamic equivalent for a power-importing area which is represented as a large synchronous motor.

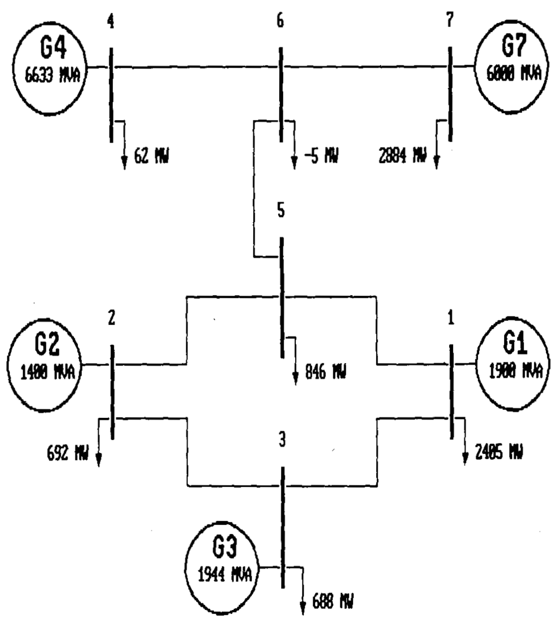

Figure 3. 5-Machine System

This system has a pair of unstable eigenvalues $\lambda = +0.646 + j\,5.391$ and any attempt to stabilize it through excitation control on G_4 is bound to fail. Reference [2] shows the root locus of the critical eigenvalues as the gain of a rotor speed-derived stabilizer at the G_4 generator is varied. The critical electromechanical mode is seen to always remain unstable due to the presence of an unstable pair of zeros in the $\Delta w^4(s)/\Delta V_r^4(s)$ transfer function ($z = +0.049 + j\,5.908$). The upperscript 4 denotes a variable of generator G_4 and ΔV_r the deviations in the exciter reference voltage. In the small signal stability area the complex zeros and poles always occur in complex conjugate pairs. A complex conjugate pair ($a \pm j\,b$) is here typed as ($a + j\,b$) for better readability.

The 5-machine system has 28 eigenvalues (poles) and the $\Delta w^4(s)/\Delta V_r^4(s)$ has 25 finite zeros which were obtained by the QZ eigenroutine. The three extraneous zeros had magnitudes larger than 10^5. The critical zeros for different transfer functions are presented in Table 1 and discussed in the following lines:

1,2. The critical pair of zeros are identical for $\Delta w^4(s)/\Delta V_r^4(s)$ and $\Delta P_t^4(s)/\Delta V_r^4(s)$, where P_t is the generator terminal power.

3. The symbol $R_{4\text{-}6}$ denotes the apparent resistance of the transmission line between buses 4 and 6. This signal, for this particular system, is worse then the two previous signals since its critical zeros are more unstable.

4,5,6. The critical pair of zeros for $\Delta w^1(s)/\Delta V_r^1(s)$, $\Delta w^2(s)/\Delta V_r^2(s)$, $\Delta w^3(s)/\Delta V_r^3(s)$ transfer functions are almost identical and very close to the unstable pair of eigenvalues ($\lambda = +0.646 + j\,5.391$). This unstable pair of eigenvalues is therefore not controllable from the excitation systems of G_1, G_2 and G_3.

7. There is no troublesome pair of complex zeros in the $\Delta w^7(s)/\Delta V_r^7(s)$ transfer function. Problem appears due to a real positive zero ($z = +7.012$), which informs in advance of the detrimental action that a stabilizer located at the synchronous motor G_7 would have on the system synchronizing torques.

8. The possibility of stabilizing the system through the function $\Delta w^4(s)/\Delta V_r^1(s)$ is discarded due to the existance of a higly unstable pair of zeros. The stabilizer to be added here would modulate the reference voltage of the G_1 exciter and be derived from the rotor speed signal of G_4.

Nº	Transfer Function Considered	Critical Zeros
1	$\Delta w^4(s)/\Delta V_r^4(s)$	+0.049 +j 5.908
2	$\Delta P_t^4(s)/\Delta V_r^4(s)$	+0.049 +j 5.908
3	$\Delta R_{4\text{-}6}(s)/\Delta V_r^4(s)$	+0.249 +j 6.404
4	$\Delta w^1(s)/\Delta V_r^1(s)$	+0.655 +j 5.379
5	$\Delta w^2(s)/\Delta V_r^2(s)$	+0.650 +j 5.376
6	$\Delta w^3(s)/\Delta V_r^3(s)$	+0.654 +j 5.380
7	$\Delta w^7(s)/\Delta V_r^7(s)$	−0.310 +j 5.748*
8	$\Delta w^4(s)/\Delta V_r^1(s)$	+0.899 +j 5.354

Table 1. Critical Pair of Zeros for Different Transfer Functions of the 5–Machine System

Note: * The function $\Delta w^7(s)/\Delta V_r^7(s)$ has another critical zero of value $z = +7.012$

Table 2 shows the critical pair of poles for the 5-machine system together with the critical pair of zeros for the $\Delta w^4(s)/\Delta V_r^4(s)$ transfer function. The various cases presented are described below:

1. Critical poles and zeros are presented for the base case condition described in [2].

2. A 10 percent reduction on the power interchange between G_4 and G_7 machines causes the troublesome pair of zeros to move slightly into the left-half plane, but the system continues to present basically the same stabilization problem.

3. The critical pair of zeros become stable when automatic excitation control is neglected on the generators G_1, G_2 and G_3. A single PSS at G_4 can now stabilize the system through modulation of the impedance loads at buses 1, 2, 3 and 5. The maximum damping achieved for the electromechanical eigenvalue is about 4% since it will coincide with $z = -0.242 + j\,5.660$ for infinite gain at the G_4 stabilizer.

4. The power system stabilizers (PSS) in G_1, G_2 and G_3 practically do not alter the unstable eigenvalue pair but have a strongly positive effect on the critical zeros: $z = -0.562 + j\,5.044$. The stabilizers at G_1, G_2 and G_3 are therefore needed not for being able to damp the unstable poles but for moving away the troublesome zeros.

5. The presence of a stabilizer only in G_1 also has a highly positive effect on $\Delta w^4(s)/\Delta V_r^4(s)$, since the critical zeros become very well damped.

Result Nº 5 of Table 2 informs in advance that a PSS in G_4 could stabilize the system if another PSS was already present at G_1. This result indicates that the transfer function matrix:

$$\begin{matrix} \Delta V_r^1(s) \longrightarrow \\ \Delta V_r^4(s) \longrightarrow \end{matrix} \boxed{\begin{matrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{matrix}} \begin{matrix} \longrightarrow \Delta w^1(s) \\ \longrightarrow \Delta w^4(s) \end{matrix}$$

has well damped transmission zeros (see Section IV). This is actually the case, since the least damped transmission zeros of the matrix, calculated by a QZ routine, are $z = -1.839 + j\,9.157$ and $z = -1.273 + j\,6.635$.

Cases Studied		Crit. Poles	Crit. Zeros
Nº	Description		
1	Base Case	+.646+j5.391	+.049+j5.908
2	Lower Transfer	+.428+j5.610	−.023+j5.958
3	No AVR in G_1,G_2,G_3	+.667+j5.315	−.242+j5.660
4	PSS's in G_1,G_2,G_3	+.656+j5.380	−.562+j5.044
5	PSS in G_1	+.652+j5.386	−.427+j5.835

Table 2. Critical Poles and Zeros for $\Delta w^4(s)/\Delta V_r^4(s)$

Results on the Brazilian Interconnected System

The power system analysed is a 616 bus-50 generator model of the South-Southeast Brazilian Interconnected System [8]. The least damped eigenvalues of this system, in the absence of power system stabilizers, are presented in Table 5 of the next section. The reader should refer to [8] for additional information on this system model.

Reference [8] described results showing that the inter-area mode ($\lambda_1 = -0.0017 + j\,3.511$) could be stabilized through a properly tuned SVC located at the terminals of the Jacui generating plant. The effectiveness of a SVC at this bus in damping this inter-area mode actually depends on whether its bus voltage signal is local or remote.

There exists a zero $z_1 = -0.03537 + j\,3.535$ in the transfer function $\Delta V^1(s)/\Delta B_v^1(s)$, where V^1 denotes the voltage magnitude of the Jacui generator bus. The proximity of zero z_1 caused pole λ_1 to be invariably attracted to it as the SVC gain was raised.

Therefore, a local voltage signal is ineffective in damping λ_1.

The zero finding algorithms were used to determine which voltage magnitude signal in the interconnected system could be effective in damping λ_1. After the calculation of zeros for many different transfer functions, the function $\Delta V^{549}(s)/\Delta B_V^{1}(s)$ was seen to have no zeros in the vicinity of λ_1. The bus numbered 549 is actually in the Southeast area while the Jacui generator is in the Southern area. Eigenvalue results confirmed that λ_1 could be effectively damped through a feedback loop on the function $\Delta V^{549}(s)/\Delta B_V^{1}(s)$.

The group of converged zeros obtained by the Simultaneous Iteration algorithm with 8 independent trial vectors and complex shift $q = 0 + j\,4.0$ are shown in Table 3 for the transfer functions $\Delta V^{1}(s)/\Delta B_V^{1}(s)$ and $\Delta V^{549}(s)/\Delta B_V^{1}(s)$. The three troublesome zeros of these two functions are bold-faced in Table 3 and required between 4 to 8 iterations to converge to within a tolerance 10^{-5}. Both functions have badly located zeros of frequency around 5 rad/s, indicating that this SVC, controlling either a local or remote bus voltage, is ineffective in damping another critical system mode: $\lambda_2 = -0.022 + j\,5.374$.

$\Delta V^{1}(s)/\Delta B_V^{1}(s)$	$\Delta V^{549}(s)/\Delta B_V^{1}(s)$
+0.0111 +j 5.355	**+0.0325 +j 5.038**
−0.0354 +j 3.535	−0.2160 +j 6.474
−1.1970 +j 2.222	−1.1889 +j 2.231
−2.2556 +j 3.251	−2.1770 +j 3.196
−0.1503 +j 6.099	−0.1380 +j 6.055

Table 3. Transfer Function Zeros Obtained by Simultaneous Iteration

The order of the *augmented system equations* for the Brazilian system is 2207. The augmented system matrix is factorized in 5.5 seconds on a VAX 11/780. Every iteration of the Simultaneous Iteration algorithm takes 25 seconds of C.P.U. and involves 16 repeat solutions, large vector multiplications and the eigensolution of a (8x8) complex matrix.

IX. COMPARATIVE RESULTS ON THE ORIGINAL AND IMPROVED AESOPS ALGORITHMS

Results on the New England System

The original AESOPS algorithm [16] was enhanced in [3] by working with the full nodal admittance matrix of the network rather than with this matrix reduced to the generator terminal buses. The results obtained in [3] for the New England Test System using the original AESOPS algorithm are here displayed together with those of the improved AESOPS algorithm (Table 4), so that their performances can be comparatively evaluated. The eigenvalues shown in Table 4 differ slightly from those of [16], since speed-governor and exciter saturation effects are neglected here.

Table 4 compiles the results of the original and improved AESOPS obtained from 30 different program runs, and shows that the two algorithms converged to the same system eigenvalues except for 2 cases. Therefore, the improved AESOPS algorithm, with the initialization proposed in section VI, has the same desirable characteristic as the original one, i.e., it converges to an electromechanical eigenvalue which is dominant at the disturbed generator. The difference, however, is that the improved AESOPS has a much faster convergence rate.

The original AESOPS algorithm is known to ocasionally present problems of slow convergence [17] due to its heuristic nature. These problems do not occur with the improved AESOPS algorithm since the Newton-Raphson method possess quadratic convergence in the neighborhood of the solution. The convergence criteria used to obtain the results of Table 4 is based on the residue vector $\underline{t} = \mathbf{A}\underline{x} - \lambda\underline{x}$. When all elements in $\underline{t}$ have magnitudes below 10^{-6} the case is considered to be converged.

Disturb Gener. at Bus	Initial Eigenval. Estimate	Converged Eigenvalue for I.A.	Nº of Iter.	
			I.A.	O.A.
30	.0 +j 4.0	−.1117 +j 7.094	8	24
30	.0 +j 7.5	−.1117 +j 7.094	6	19
30	.0 +j 9.0	−.1117 +j 7.094	7	22
31	.0 +j 4.0	−.2968 +j 6.956*	14	17
31	.0 +j 7.5	−.2817 +j 7.537	7	9
31	.0 +j 9.0	−.2817 +j 7.537	7	9
32	.0 +j 4.0	−.2817 +j 7.537*	17	15
32	.0 +j 7.5	−.2817 +j 7.537	6	8
32	.0 +j 9.0	−.2817 +j 7.537	6	10
33	.0 +j 4.0	−.2489 +j 3.687	9	12
33	.0 +j 7.5	−.3707 +j 8.613	7	17
33	.0 +j 9.0	−.3707 +j 8.613	5	17
34	.0 +j 4.0	−.2489 +j 3.687	8	28
34	.0 +j 7.0	−.2834 +j 6.282	8	19
34	.0 +j 9.0	−.2968 +j 6.956	10	14
35	.0 +j 4.0	−.2489 +j 3.687	9	14
35	.0 +j 7.0	−.2968 +j 6.956	8	17
35	.0 +j 9.0	−.4670 +j 8.963	7	7
36	.0 +j 4.0	−.2489 +j 3.687	10	13
36	.0 +j 7.0	−.2968 +j 6.956	8	16
36	.0 +j 9.0	−.4670 +j 8.963	6	11
37	.0 +j 4.0	−.4117 +j 8.778	8	28
37	.0 +j 7.0	−.1117 +j 7.094	12	45
37	.0 +j 9.0	−.4117 +j 8.778	6	18
38	.0 +j 4.0	−.2489 +j 3.687	9	28
38	.0 +j 5.0	−.3008 +j 5.793	8	17
38	.0 +j 7.5	−.2834 +j 6.282	10	15
39	.0 +j 4.0	−.2489 +j 3.687	6	12
39	.0 +j 7.0	−.2489 +j 3.687	6	13
39	.0 +j 9.0	−.2489 +j 3.687	7	13

Table 4. Eigenvalues for the New England Test System

Note: Both original and improved AESOPS algorithms converged to the same eigenvalues, except for the two cases marked by the asterisk (*), where the original AESOPS algorithm converged to $\lambda = -.2834 + j\,6.282$. I.A. and O.A. are abbreviations for improved AESOPS and original AESOPS.

Figure 4 shows the Bode plot for $\Delta T_m(s)/\Delta\delta(s)$ of equation (8) for generators at buses 32 and 39. These plots provide a pictorial explanation as to why the improved AESOPS algorithm applied to:

a. generator at bus 39 will almost always converge to $z = -0.2489 + j\,3.687$;

b. generator at bus 32 will converge to $z_1 = -0.2834 + j\,6.282$, $z_2 = -0.2817 + j\,7.537$ and $z_3 = -0.2489 + j\,3.687$, depending on the chosen initial value s_0.

Note that $z = -0.1117 + j\,7.094$, the lesser damped system mode, only caused a very slight magnitude dip in these two Bode plots because it is not dominant at generators 32 and 39.

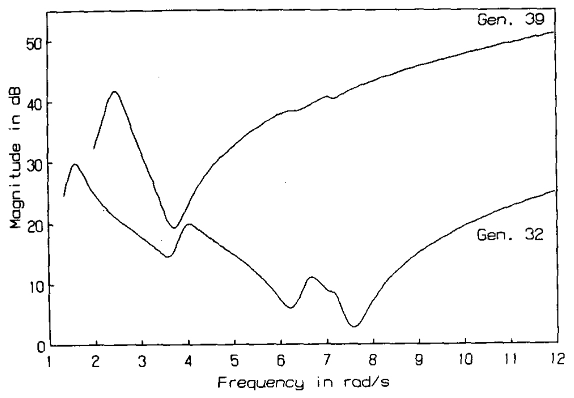

Figure 4. Bode plot of $\Delta T_m(s)/\Delta\delta(s)$ for generators 32 and 39

Results on the Brazilian Interconnected System

Table 5 shows some brief results on the original and the improved AESOPS algorithms for the Brazilian Interconnected System. The superiority of the latter algorithm over the former is very clear.

Disturb Gener. at Bus	Initial Eigenval. Estimate	Converged Eigenvalue	Nº of Iter.	
			I.A.	O.A.
Jacui	.0 +j 3.0	−.0017 +j 3.511	9	27
Itauba	.0 +j 3.0	−.0017 +j 3.511	9	13
Itaipu	.0 +j 6.0	−.0810 +j 6.988	7	38
I.Solt.	.0 +j 5.0	−.0220 +j 5.374	9	9
Itumb.	.0 +j 5.0	−.0220 +j 5.374	10	11

Table 5. Eigenvalues for the Brazilian System

The convergence tolerance adopted is the same as in the New England test case (10^{-6}). The original AESOPS showed a very slow convergence for the Itaipu major mode of oscillation ($\lambda = -.0810 + j\ 6.988$). Even when adopting a less stringent tolerance (10^{-3}) it took 18 iterations to converge.

X. CONCLUDING COMMENTS

The work reported in this paper has led to the implementation of the following algorithms to a comprehensive package for the analysis of small-signal stability of power systems:

1. A generalized QZ routine for finding all the zeros of a specified transfer function in the power system dynamic model. This algorithm has its use limited to moderate size systems, having a few hundred state variables.

2. An inverse iteration algorithm for the calculation of a zero which is closest to a given point in the complex plane.

3. A simultaneous iteration algorithm for the calculation of a group of zeros which are near to a given point in the complex plane.

4. A Newton-Raphson algorithm to solve for the zeros of non-proper transfer functions [15], i.e, those functions which need be expressed in terms of not only the state variables but also of the system input and its derivatives. This is actually the case of the improved AESOPS algorithm which is listed in the item 5 below.

5. An improved AESOPS algorithm, neatly formulated as a transfer function zero finding problem and efficiently solved by an exact Newton-Raphson method.

6. An algorithm based on the same concept as AESOPS to find the modes which are dominant at the bus terminals of static VAr compensators or HVDC links.

All these algorithms, except the QZ routine, are applied to the *augmented system equations* to solve large scale system problems.

The original AESOPS algorithm is normally presented together with a long technical justification, based on engineering considerations and intuition [16]. The formal mathematical description provided here for the improved AESOPS algorithm obviates the need for such considerations.

The algorithm described in item 6 produced correct results but at this preliminary stage was not found to be advantageous in the practical analysis of low frequency oscillation problems. It is foreseen that such algorithm may find use in studies of higher frequency oscillatory instability problems in systems with HVDC links [18] or multiple static compensator applications [19].

Power system control specialists have recently applied Prony analysis [20] to calculate the major poles and zeros of specified transfer functions directly from field tests on large power systems. The algorithms described in this paper produce results which can be cross–checked with those from Prony analysis, allowing more effective validation of the power system data and computer models utilized.

The algorithms listed in items 1 to 4 of this section were extended to the multi–input–multi–output case to find the invariant zeros of the system matrix. A brief result is presented in section VIII.

The algorithms described in items 1 to 4 of this section generate a number of extraneous zeros which created no practical difficulties to date but are a cause of concern. There is therefore a need for more refined algorithms which should operate on matrices of the same order as the number of finite zeros of the specified transfer function [23].

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APPENDIX

Augmented System Equations

The power system electromechanical stability problem can be modelled by a set of non-linear differential and algebraic equations to be solved simultaneously:

$$\begin{aligned} \dot{\underline{x}} &= f(\underline{x},\underline{r}) \\ \underline{0} &= g(\underline{x},\underline{r}) \end{aligned} \qquad (A.1)$$

where $\underline{x}$ is the state vector and $\underline{r}$ is a vector of algebraic variables.

Small-signal stability analysis involves the linearization of (A.1) around a system operating point $(\underline{x}_0,\underline{r}_0)$:

$$\begin{pmatrix} \Delta\dot{\underline{x}} \\ \hline \underline{0} \end{pmatrix} = \left(\begin{array}{c|c} J_1 & J_2 \\ \hline J_3 & J_4 \end{array}\right) \begin{pmatrix} \Delta\underline{x} \\ \hline \Delta\underline{r} \end{pmatrix} \qquad (A.2)$$

The power system state matrix A can be obtained by eliminating the vector of algebraic variables $\Delta\underline{r}$ in equation (A.2):

$$\Delta\dot{\underline{x}} = (J_1 - J_2 J_4^{-1} J_3)\,\Delta\underline{x} = A\,\Delta\underline{x} \qquad (A.3)$$

The eigenvalues of A provide information on the singular point stability of the system. The symbol Δ signifies an incremental change from a steady-state value and is often omitted along the text of this paper.

Matrix A is non sparse in this application and therefore all needed computer calculations become prohibitively expensive for large order systems. References [2,3,4,5,6,7,8,9,10] presented algorithms for the calculation of eigenvalues, right and left eigenvectors, frequency response plots, transfer function residues and step response plots, which do not require the explicit formation of the power system state matrix. These algorithms are directly applied to the large and highly sparse Jacobian matrix of equation (A.2). This Jacobian matrix equation will be here referred to as the *augmented system equations*.

The state-space description of the system shown in equation (1) of section II can be expressed in terms of the *augmented system equations*:

$$\begin{aligned} \begin{pmatrix} \dot{\underline{x}} \\ \hline \underline{0} \end{pmatrix} &= \left(\begin{array}{c|c} J_1 & J_2 \\ \hline J_3 & J_4 \end{array}\right) \begin{pmatrix} \underline{x} \\ \hline \underline{r} \end{pmatrix} + \begin{pmatrix} \underline{b}_x \\ \hline \underline{b}_r \end{pmatrix} u \\ &= J\,\underline{x}^a + \underline{b}^a\,u \end{aligned} \qquad (A.4)$$

$$y = \left(\begin{array}{c|c} \underline{c}_x^t & \underline{c}_r^t \end{array}\right) \begin{pmatrix} \underline{x} \\ \hline \underline{r} \end{pmatrix} = (\underline{c}^a)^t\,\underline{x}^a$$

where:

J = Jacobian matrix of the system
$\underline{x}^a$ = augmented state vector
$\underline{b}^a$ = augmented input vector
$\underline{c}^a$ = augmented output vector

The basic equation relating state matrix, eigenvalues and eigenvectors is:

$$A\,\underline{x}_i = \lambda_i\,\underline{x}_i \qquad (A.5)$$

where

λ_i = i-th eigenvalue of A
$\underline{x}_i$ = right eigenvector associated with λ_i

The equivalent equation in terms of the *augmented system equations* is:

$$\left[\begin{array}{c|c} J_1 & J_2 \\ \hline J_3 & J_4 \end{array}\right]\left[\begin{array}{c} \underline{x}_i \\ \hline \underline{r}_i \end{array}\right] = \lambda_i \left[\begin{array}{c} \underline{x}_i \\ \hline \underline{0} \end{array}\right] \qquad (A.6)$$

where $(\underline{x}_i^t, \underline{r}_i^t)^t$ is the *augmented eigenvector* associated with λ_i and is denoted $\underline{x}_i^a$.

For a neater computer implementation, the Jacobian equations in (A.2), (A.4) and (A.6) should be reordered as discussed in [3]. However, for the sake of clarity and brevity, the algorithms proposed in this paper will be described using the ordering shown in this Appendix.

Nelson Martins (M'1981) received his B.Sc. degree from the University of Brasilia, Brazil, in 1972. He received the M.Sc. and Ph.D. degrees, both in electrical engineering, from the University of Manchester, UK, in 1974 and 1978 respectively.

Dr. Martins is presently a senior research engineer at CEPEL, the Brazilian electrical energy research center located in Rio de Janeiro. He developed new digital-computer tools and authored many papers in the area of power systems dynamics and control.

Herminio J.C.P. Pinto received his B.Sc. and M.Sc. degrees, both in electrical engineering, from the Federal University of Rio de Janeiro, Brazil, in 1986 and 1990 respectively. Since 1986 he is with CEPEL, in the Systems Department and his current work and interests include power system operation and control and parallel processing.

Leonardo T.G. Lima (M'1990) received his B.Sc. degree in electrical engineering from Federal University of Rio de Janeiro, Brazil, in 1986.

Mr. Lima worked in CEPEL until 1988 and presently works at MARTE Engenharia S.A., an electrical consultant firm in Rio de Janeiro. His current work and interests include power system dynamics and development of large scale power system analysis tools.

DISCUSSION

J.C.Castro (Universidade Federal da Paraíba, João Pessoa, Paraíba, Brasil): The authors are commended for their valuable contribution in developing algorithms for determination of zeros in large power systems.

The authors' concern on the zeros of a system is timely, since the effect of those zeros are usually disregarded on the analysis of power systems and design of their controllers. For instance, the PSS designs are usually concerned with the eigenvalues located on the verge of the instability region, which are the eigenvalues associated with the critical electromechanical modes, without taking into account the effect of the system zeros and the zeros introduced by the PSS. However, the zeros are closely related to controlability and observability of the modes. They also have a great effect on peaks of the response.

The authors say that the open loop transfer function zeros of a plant are not changed by the addition of a feedback controller. This is always true for a SISO system but not rigorously true for a multivariable system as those studied in the paper. The authors themselves show that the troublesome zero of the 5-machine system is changed by applying PSS in G_1, G_2 and G_3. That zero could be shifted by controlling these generators because it is not a zero of their transfer functions.

The discusser would like to raise the following questions that arose from the analysis of the paper:

1. Does the critical zero pattern change with different models representing the generators and the equivalent motor?
2. Is the effect of the troublesome zero observed on the PSS tuning in the field?
3. Why the South Brazilian Grid was represented by only three generators, disregarding, for instance, Salto Ozorio plant?

Again, I commend the authors and encourage them to continue their interest in this field.

R.Doraiswami (University of New Brunswick, Fredericton, N.B., Canada): The authors are to be commended for proposing an efficient algorithm for finding the transfer function zeros of large power system dynamic models and for providing an improvement to the well known AESOPS algorithm.

The zeros of the transfer function play an important role in both the design of a controller and in identifying the changes in the system. Simplistic algorithms based directly on the definition of zeros are generally non-robust even for lower order systems. The authors' algorithm is based on the fact that the eigenvalues asymptotically approach the zeros as the feedback gain approaches infinity. It was shown by Davison [12] that this root locus approach is robust.

The authors use *augmented system equations* so as to exploit sparsity and "one root or a set of roots at a time" approach and hence their algorithm can handle large systems.

The authors have touched upon an important issue, namely the role of zeros. The controller design is constrained by the location of zeros especially when they are located in the right-half plane. Further the zero movements reflect the system changes.

It would be instructive if the authors could elaborate on this issue in the context of the examples considered in the paper. What system parameters influence the zero locations?

Could the authors elaborate on the rationale behind their approach of converting the problem of determining the poles of the system by finding the zeros of an improper transfer function (sections V and VI)? It is an interesting idea!

To conclude, it is an excellent paper.

Anan M.A.Hamdan (University of Science & Technology, Irbid, Jordan): The authors are to be commended on their efforts to work out methods for calculating transfer functions zeros for large scale multimachine power systems. I would like to make the following comments:

1. In section 8 of the paper, the opening statement has to be qualified. System zeros are invariant under a range of state and output feedback [A], but they are not invariant under dynamic feedback.
2. The zeros that are important are those of the return ratio of the feedback loop for a SISO system. The excitation loop is shown in Figure 1 for a single machine system. With no stabilization the position of the zeros of $g_1(s)$ in the complex plane are important. We have shown that $g_1(s)$ for a single machine connected to an infinite busbar exhibits RHP zeros at some loading conditions [B]. The same transfer function for a machine connected to a multimachine power system can be calculated by the methods developed by the authors. Such transfer functions are likely to display RHP zeros for some loading conditions as well, or some zeros will be near to the imaginary axis. A speed stabilizer with a double phase advance unit is a dynamic feedback and it changes the zeros of the return ratio of the stabilized system. Assuming $g_1(s) = n_1(s)/D(s)$, $g_2(s) = n_2(s)/D(s)$, $L(s) = a(s)/b(s)$, and ignoring $K(s)$ since for some exciter types it does not contribute any zeros, then we can show that the numerator poly-

nomial of the return ratio is $\phi(s) = n_2(s)\,b(s) + n_1(s)\,a(s)$. The roots of $\phi(s)$ are different than the zeros of the uncompensated system. The parameters of the stabilizer numerator $a(s)$ can be worked out if the desired positions of the roots of $\phi(s)$ are given [B].

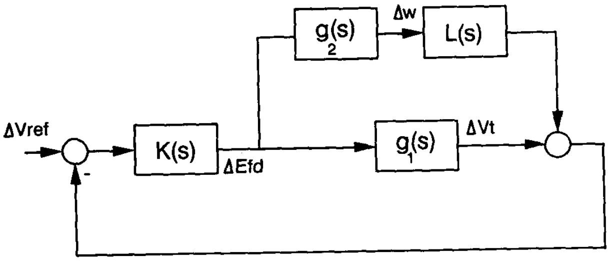

Figure 1 – Excitation loop with power system stabilizer

3. The system zeros of a multiloop feedback system are defined in different ways. However, they are not the zeros of the individual transfer functions. Only square systems with the same number of inputs and outputs have zeros. A multi-machine power system with n machines can be formulated as a n-loop system where each machine has an excitation loop as in Figure 2.

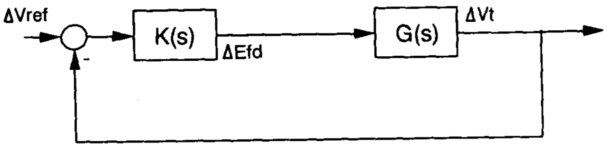

Figure 2 – Excitation loop of a MMPS

The system has a transfer function matrix $V_t = G(s)\,E_{fd}$, where V_t and E_{fd} are n-vectors. $G(s)$ can be obtained from a state space representation with n-inputs and n-outputs as follows: $\mathbf{x} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u}$, $\mathbf{y} = \mathbf{C}\,\mathbf{x}$. The zeros of this MIMO system can be calculated in many ways. In reference [C] we considered a 3-machine system studied in other papers as well [D]. The system zeros were calculated as the eigenvalues of the matrix **N.A.M**, where **M** is a right annihilator of **C** and **N** is a left annihilator of **B**. For the operating condition which was unstable, we calculated three open loop zeros in the RHP at $0.1 \pm j4.59$, $0.009 \pm j7.32$ and $0.11 \pm j8.72$. The presence of these RHP zeros of the open loop system makes it impossible to stabilize the system using constant state or output feedback. The reason is that these zeros are invariant under such feedback and their presence keeps a lot of phase lag in the system. The stabilization scheme using speed signals from each of the three machines with double phase advance units adds more state variables to the system and takes a fresh set of outputs, thus reallocating the open loop zeros of the stabilized system [C]. Actually, with such a stabilization scheme we calculated the open loop zeros of the stabilized system and all of them were shifted to the LHP. If a classical representation of the machines is used with no field windings the resulting MIMO representation has no zeros at all. It is stabilizable by constant state or output feedback in a scheme that is called ideal PSS's [E].

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S.G.Jalali and F.L.Alvarado (The University of Wisconsin-Madison): This paper continues on the outstanding task of working with augmented matrices established by the authors in prior papers. The paper develops tools to calculate the zeros of $H(s)$ using feedback theory.

$$H(s) = \underline{c}^t\,(s\mathbf{I} - \mathbf{A})^{-1}\,\underline{b}$$

It is shown that the values of λ obtained from the solution of equation (4) approach the zeros of $H(s)$ as $-1/K$ approaches zero.

$$\left(\begin{array}{c|c} \mathbf{A} & \underline{b} \\ \hline \underline{c}^t & -1/K \end{array}\right)\begin{pmatrix} \underline{x} \\ \hline u \end{pmatrix} = \lambda \left(\begin{array}{c|c} \mathbf{I} & \underline{0} \\ \hline \underline{0}^t & 0 \end{array}\right)\begin{pmatrix} \underline{x} \\ \hline u \end{pmatrix} \qquad (4)$$

The discussers would like offer a more direct way to reach the same result. The numerator of $H(s)$ is a sum of the transposed co-factors of $(s\mathbf{I} - \mathbf{A})$, weighted according to the nonzero values of $\underline{b}$ and $\underline{c}^t$. It can be verified that the same sum of co-factors is obtained from the evaluation of the determinant of:

$$\left|\begin{array}{c|c} s\mathbf{I} - \mathbf{A} & \underline{b} \\ \hline -\underline{c}^t & 0 \end{array}\right|$$

While the co-factors are not the way to perform computations, they do provide a very direct proof of the same results in the paper without the need to resort to feedback theory and limits. This approach can give us additional insight about equation (4). For example, if $\underline{b}$ and $\underline{c}^t$ are singletons, the rows and columns of the A matrix which correspond to the non zero elements of $\underline{b}$ and $\underline{c}^t$ do not contribute to the location of the zeros. Therefore, they may be set to zero.

H.E.Peña and **A.S.Silva** (Universidade Federal de Santa Catarina, Florianópolis, Brasil): The authors are to be complimented for their continuous work on the problem of power system dynamics. The search for zeros of power system transfer functions is an interesting topic to be dealt with and a valuable and timely contribution to the subject is the paper under discussion.

Through the analysis of the paper some questions arose that the discussers would like to put forward:

1. In the five machine example presented, one of the conclusions that the authors bring about is that the undamped or poorly damped poles problem can be solved through an appropriate choice of sites for PSS application, as shown in reference [1], and that installing PSS on sites

other than the one chosen would in fact move critical zeros to more appropriate locations. Would the authors care to comment on how could the concepts of mode controlability and observability factors be used so as to determine the PSS site for zero relocation? Could this be thought of as a "dual" of the poorly damped poles problem?

2. Can it be concluded that the problem of appropriate PSS location is now augmented with another problem, that is of PSS site for critical zero reallocation?

3. The discussers feel that the sequential nature of the approach presented in the paper creates the situation of having to close other loops so as to relocate troublesome zeros as it is suggested in the paper under discussion. Do the authors foresee that the same situation would necessarily appear if a global, coordinated approach for PSS setting in multimachine power system is employed? Our experience with a method that presents such global characteristics and whose preliminary results are reported in [2] does not seem to indicate that the identification and relocation of zeros would be a problem if this approach is used.

Finally the discussers would like to thank the authors for their time in answering the above questions.

[1] **N. Martins & L.T.G. Lima**, "Determination of Suitable Locations for Power System Stabilizers and Static VAr Compensators for Damping Electromechanical Oscillations in Large Scale Power Systems", *IEEE Trans. on Power Systems*, Vol.5, no.4, pp.1455–1469, November 1990.

[2] **H.E.Peña & A.J.A.Simões Costa**, "Controle Ótimo Descentralizado Aplicado ao Projeto de Estabilizadores de Sistemas de Potência", in *Proc. of VIII Congresso Brasileiro de Automatica*, Vol.2, pp.762–768, September 1990 (in portuguese).

CLOSURE

Nelson Martins, Herminio J.C.P. Pinto & Leonardo T.G. Lima: We thank the discussers for their valuable comments and questions. Many of these constitute contributions to the subject of the paper. We failed to make adequate reference to prior research on system zeros and references [C1 to C3] are now included to partially fulfill this gap. Reference [C2] was suggested to us by Dr. G. Verguese, from MIT, as essentially representing the state of the art on computational methods for moderate-size problems (with no sparsity concerns). The text of our paper is sometimes imprecise and we thank the discussers for having pointed some of the necessary corrections.

We will answer to each discusser separately, following the order suggested in letter by IEEE Service Center.

Dr. J.C. Castro:

The authors thank Dr. Castro for his comments on the role of system zeros in controller design.

1) In the five machine system results, the generators were represented by six states: ($\Delta E_d''$, $\Delta E_q''$, $\Delta E_q'$, $\Delta\omega$, $\Delta\delta$, ΔE_{fd}), where ΔE_{fd} is used to model a first-order excitation control system. In order to analyse the effect of system modeling on transfer function zeros, every generator was also represented by only four states: ($\Delta E_q'$, $\Delta\omega$, $\Delta\delta$, ΔE_{fd}). The system, in its base case configuration (case 1 of Table 2), has 28 eigenvalues (or transfer function poles) when considering the 6-state generator model. The function $\Delta\omega^4(s)/\Delta V_r^4(s)$, for the same case and generator model, has 25 zeros. Table 2.b shows the critical zeros for the transfer function $\Delta\omega^4(s)/\Delta V_r^4(s)$ obtained with two degrees of system modeling. Critical zeros are here defined as all those having a damping factor (ξ) smaller than 5%. When all zeros have damping factors larger than this only that one with the smallest damping factor is printed.

The damping of critical system poles is highly dependent on the degree of system modeling. The results of Table 2.b show that the same is true for the critical zeros.

Cases Studied		Generator Model	
N°	Description	6–State	4–State
1	Base Case	+.049+j5.908	–.109+j8.654 –.114+j8.539 +.225+j5.693
2	Lower Transfer	–.023+j5.958	–.111+j8.649 –.118+j8.525 +.188+j5.699
3	No AVR in G_1,G_2,G_3	–.242+j5.660	–.095+j8.655 –.093+j8.543 +.064+j5.552
4	PSS's in G_1,G_2,G_3	–.562+j5.044	–.283+j5.046
5	PSS in G_1	–.427+j5.835	–.107+j8.615 –.156+j5.661

Table 2.b – Critical Zeros for $\Delta w^4(s)/\Delta V_r^4(s)$

Note that a higher number of critical zeros occur when neglecting the damper windings in the generator model. These critical zeros are very close to some of the calculated system poles. They indicate the impossibility to stabilize the low damped electromechanical oscillations between the closely connected generators G_1, G_2 and G_3 from the distant G_4 generator.

In some cases, some transfer functions may show critical zeros whose locations are fundamentally determined by the system operating point and not by the degree of generator modeling. Please find an example in Dr. Hamdan's discussion and our reply to his comments.

2) The RHP complex zero impairs system stabilization through a PSS located at the G_4 generator. The RHP complex zero will cause excessive phase lag in the excitation control loop, which is directly seen in the frequency response measurements in the field. Please refer to the frequency response plots of Figures 19 and 22 of [2] for further information.

3) The 5-machine example studied in this paper and in [2] is an over-simplified model of a part of the South-Southeast Brazilian System. The results obtained with this model cannot be used for practical engineering studies of the real system. Actually, even the results on the 50-generator model of the South-Southeast system, presented in this paper, are not adequate for practical use. This is due to the fact that the Itaipu HVDC link, a major dynamic component in this system, was not adequately represented.

Dr. R. Doraiswami:

The authors thank Dr. Doraiswami for his comments on algorithm robustness and the role of system zeros.

1) Dr. Doraiswami asked us to elaborate on the role of zeros in the context of the examples in this paper. Part of this answer is given in our replies to Dr. Castro, Dr. Hamdan and Messrs. Peña and Silva. Regarding the results of Table 2, one may note that, except for case nº 2, the critical pole remains practically unchanged while the critical zeros experience large changes. The little change observed in the critical pole is a result of the cases chosen for analysis. Consider now another result, case nº 6, which differs from the base case by the absence of the AVR model in G7, the equivalent motor. The critical pole and zero, for case nº 6, are $\lambda = +0.319 + j\,5.279$ and $z = -0.072 + j\,5.884$. In this case it is the critical pole which suffers the largest change.

A badly located zero may represent a physical impossibility to effective control action. Consider the transfer function $\Delta\omega^4(s)/\Delta V_r^4(s)$ and note that $z = +0.049 + j5.908$ (Case 1) changes to $z = -0.242 + j5.66$ (Case 3) when removing the tight AVR control action from buses 1, 2, 3 (please refer to Figure 3). The total impedance load in this area (4631 MW) can only now be effectively modulated by the stabilizer at distant G4 to damp the unstable oscillations.

2) A desirable characteristic of the AESOPS method is its ability to converge to electromechanical modes which are dominant at the disturbed generator. The desired eigenvalues can be obtained either by calculating the dominant poles of $\Delta\delta^i(s)/\Delta T_m^i(s)$ or the dominant zeros of $\Delta T_m^i(s)/\Delta\delta^i(s)$. We are not aware of an adequate manner to calculate only the dominant poles of $\Delta\delta^i(s)/\Delta T_m^i(s)$, which is a proper transfer function. A good method for obtaining the dominant zeros of the non-proper transfer function $\Delta T_m^i(s)/\Delta\delta^i(s)$ is described in the paper.

The Bode plot of Figure 4 shows strikingly different pole-zero cancellations in the $\Delta T_m^i(s)/\Delta\delta^i(s)$ transfer function for generators 32 and 39.

Every solution to a non-linear problem has its domain of attraction when using iterative methods such as Newton-Raphson. In low-order examples, the domain of attraction may be graphically expressed in terms of the problem variables. Regarding the Bode plot for generator 39 and considering the only problem variable to be the imaginary part of the system zero, one can see the large domain of attraction of the solution $z = -0.2489 + j3.687$. The rationale we are able to offer is however rather simplistic. The desired robustness of the improved AESOPS algorithm was actually only obtained when initializing the iterative process with the augmented initial vector described in Section VI.

Dr. Anan M.A. Hamdan:

We regret having left Dr. Hamdan's work unintentionally out of our list of references. We have followed Dr. Hamdan's work with high interest along the years. His publications [C4,C5] were the basis for some of our controller design work through frequency response techniques.

We were not, at the time, aware of [C], a paper in which Dr. Hamdan shows interesting results on multimachine system zeros. We knew of his results on RHP zeros for single machine infinite bus systems from [C4], a reference which we encourage control oriented engineers to read. In this reference, a complex pair of RHP zeros of $g_1(s) = n_1(s)/D(s)$ are shown to move to the RHP as the torque angle δ exceeds the critical value δ_c. The critical value δ_c corresponds to an operating point beyond which aperiodic stability appears, i.e., the polynomial $D(s)$ shows a real root in the RHP.

Dr. Hamdan favors the use of the return ratio $K(s)\,(g_1(s) + g_2(s)\,L(s))$ which is linear with respect to the AVR gain, but the design of $L(s)$ becomes slightly more laborious. We have favored closing the voltage control loop and using $(K(s)\,/\,(1 + K(s)\,g_1(s)))\,(g_2(s)\,L(s))$ as the return ratio. This is closer to adopted practices in stabilizer tuning and the return ratio becomes linear with respect to the $L(s)$ gain, making simpler the stabilizer design. In his approach the zeros of the return ratio are given by $\phi(s) = n_1(s)\,b(s) + n_2(s)\,a(s)$ while in ours they are given by $\phi(s) = n_2(s)\,a(s)$. Note that in his case the return ratio poles are the roots of $D(s) = 0$ while in ours they are the roots of $d(s)\,D(s) + k\,n_1(s) = 0$; where $K(s) = k/d(s)$.

Messrs. S.G. Jalali and F.L. Alvarado:

We thank the discussers for their encouraging remarks and the alternate proof to reach the results of our paper. We could not yet completely verify their proof but already made valuable use of the additional insight it brings to the problem. By reducing the rank of the determinant one can eliminate the generation of extraneous zeros. We apparently have found a simple way, based on the discussers' comments, to efficiently eliminate the extraneous zeros from our eigensolutions.

Messrs. H.E. Peña and A.S. Silva:

1,2) The discussers are quite right. The same concept of transfer function residues [1] can be used to determine the PSS site for zero reallocation. We have shortly described in [C6] a new methodology for the determination of the most effective set of feedback control loops in large scale dynamic systems. This methodology is based on the efficient calculation of:

a) dominant poles and transmission zeros;
b) transfer function residues [1] associated with the dominant poles and transmission zeros of the already chosen loops in the system.

The methodology, as suggested by the discussers, is sequential: after one loop is chosen, the critical transmission zeros are obtained and their transfer function residues calculated. The transfer function residue with largest modulus identifies the next control loop which, when closed, will shift most the critical transmission zeros. Further details and results on this methodology will be provided in a future publication.

3) The global tuning carried out by the discussers in their reference [2] employs decentralized optimal control techniques. All the generators in their example system have stabilizers. We do not expect troublesome transmission zeros in our return ratio matrix when all excitation control loops in a system are used for stabilization. The troublesome zeros are more prone to appear when choosing only a subset of the total number of generators for damping control. The methodology described in our answer to the previous questions of the discussers will help in identifying the most adequate subset of control loops for system stabilization.

Final Comments:

We apologize for being unable to provide satisfactory answers, within the brief confines of this closure, to some of the questions raised by the discussers. We thank again the discussers for their valuable comments and questions. The discussers' interest in this paper reflects the practical importance of knowing better the role of system zeros in the design of power system controllers. Much work is still needed in this field.

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