# Efficient Mini-batch Training for Stochastic Optimization


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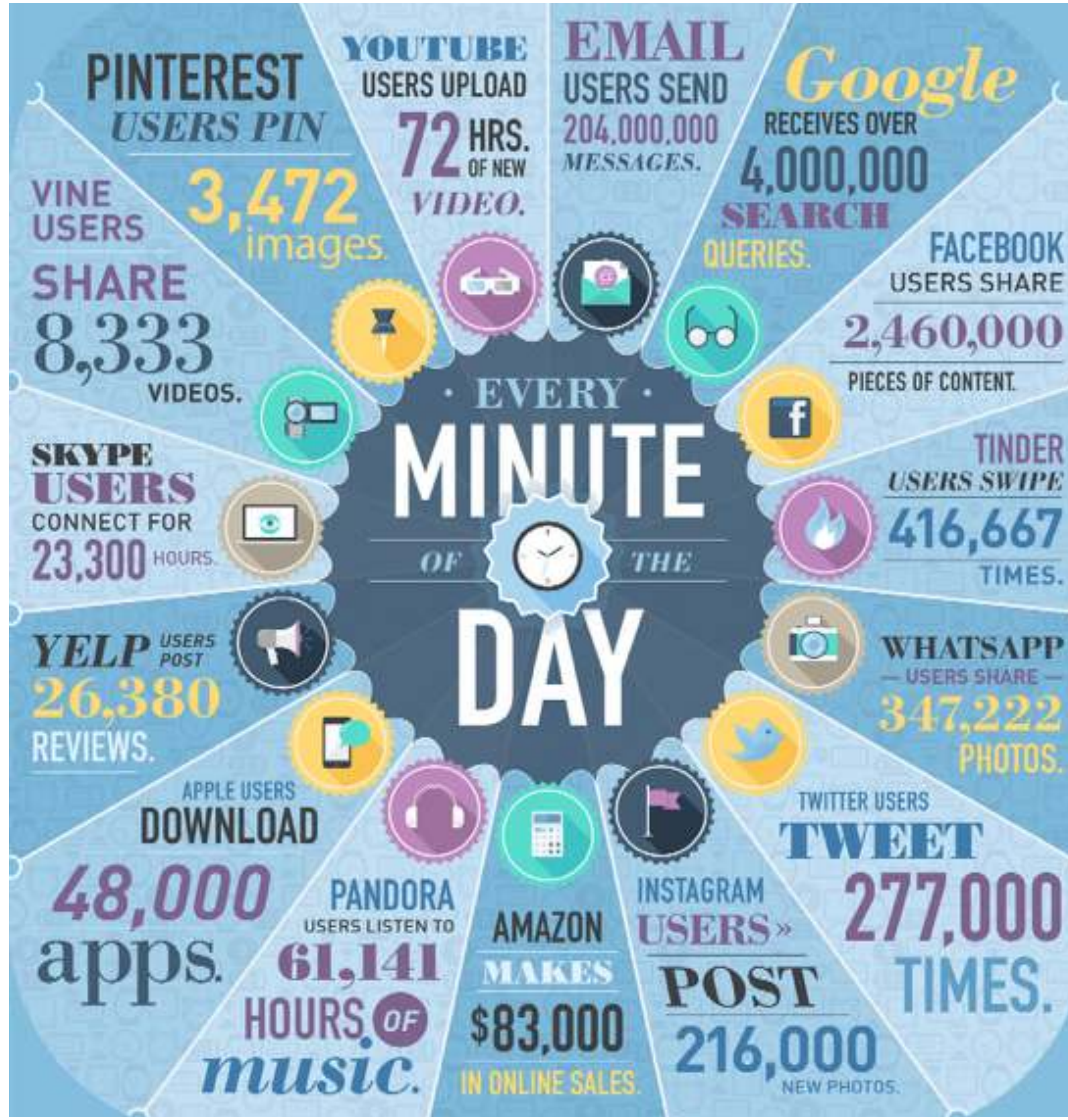
# Big data

- A lot of "big" data
  - ✓ texts
  - ✓ images
  - ✓ voices
  - ✓ videos
- Most of them are user activities
  - ✓ can be modeled as supervised learning



# Objective

- Convex optimization:

$$\min_{w \in \Omega} \frac{1}{n} \sum_{i=1}^{n} \phi_i(w)$$

- For example: Risk minimization

$$\phi_i(w) = \ell(x_i, y_i, w) + \lambda c(w)$$

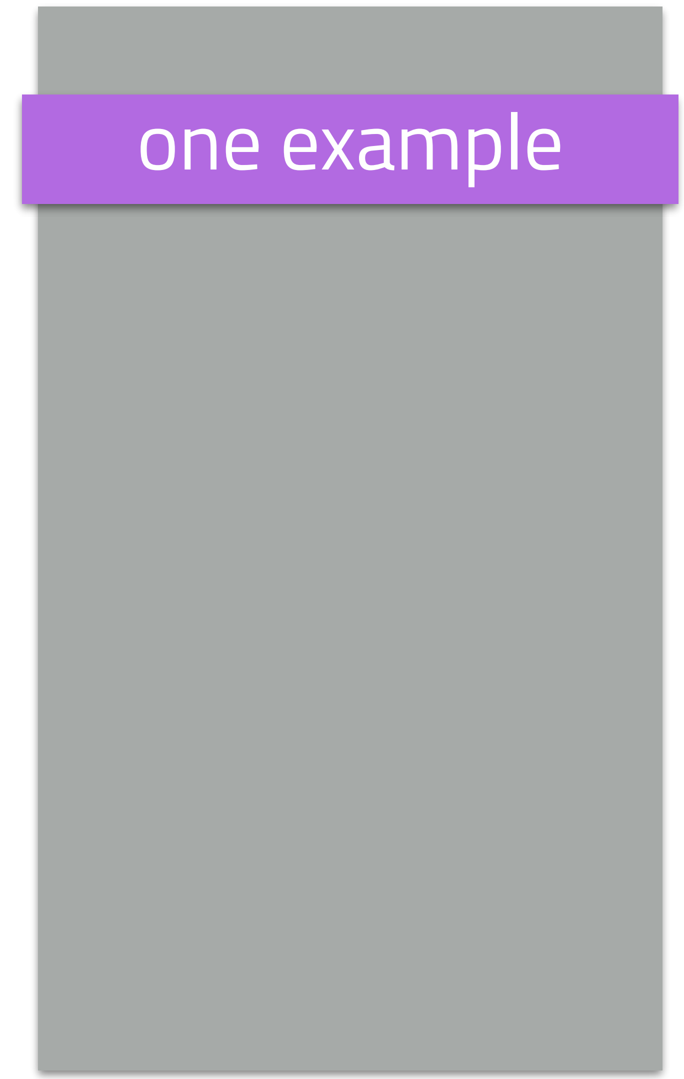
  - ✓ *($x_i$, $y_i$)* are example pairs

# Stochastic gradient descent (SGD)

- Process an example each time

  for $t$ = 1, 2, …, $T$
  
  draw a random example $i_t$
  
  update $w_t = w_{t-1} - \eta_t \nabla \phi_{i_t}(w_{t-1})$

- 👍 Convergence rate $O(1/\sqrt{T})$
- 👎 Sequential, hard to parallelize



# Minibatch SGD

- One example ➠ several examples

  for $t = 1, 2, \ldots, T$

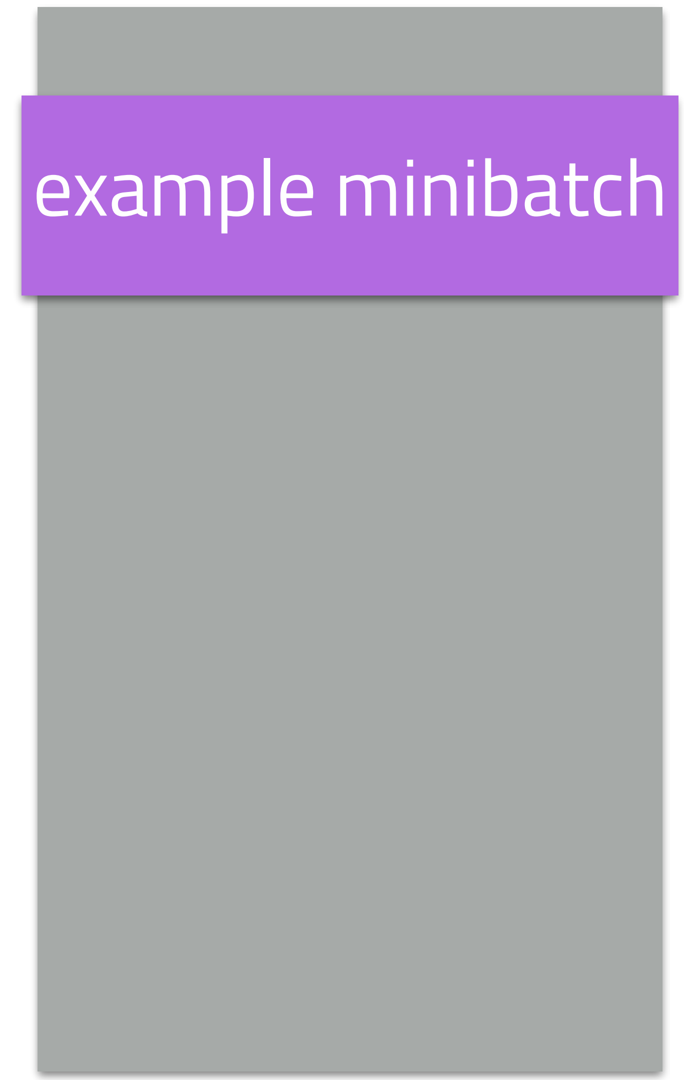
  draw a random minibatch

  $$I_t \subset \{1, \ldots, n\}$$

  update

  $$w_t = w_{t-1} - \eta_t \nabla \phi_{I_t}(w_{t-1})$$

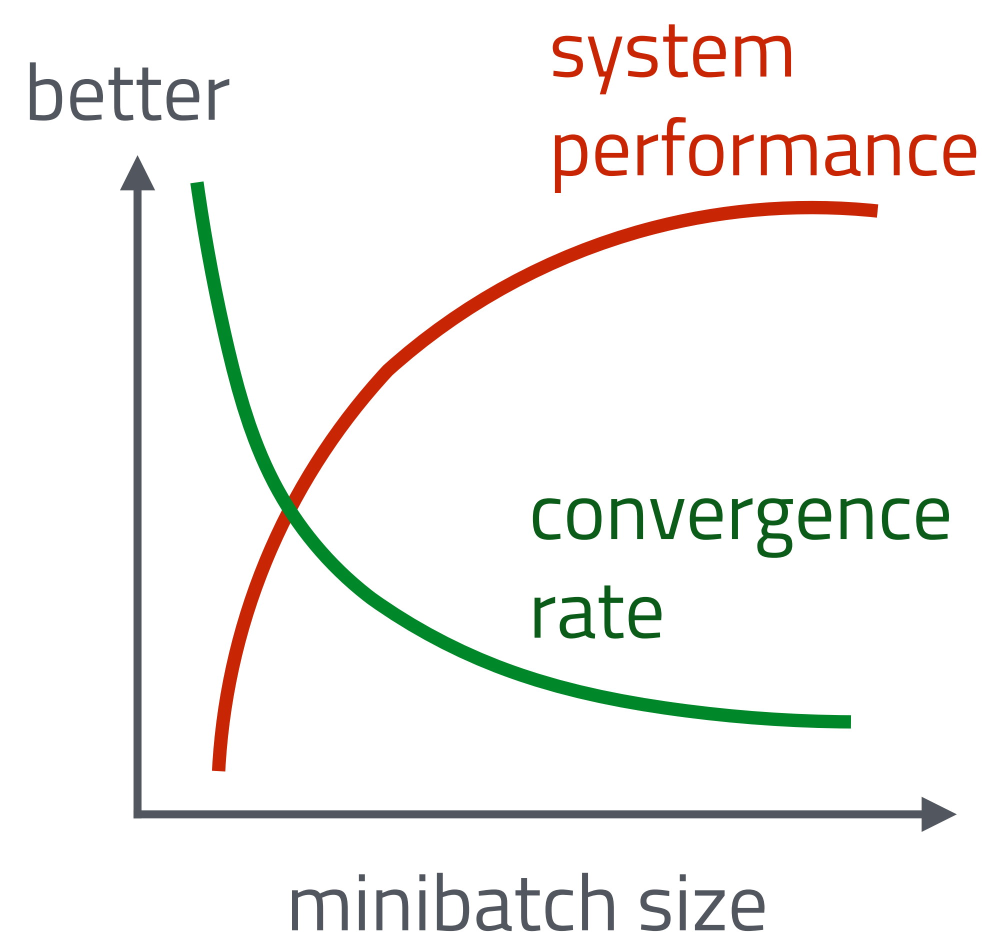
  $$\phi_{I_t}(w) = \frac{1}{|I_t|} \sum_{i \in I_t} \phi_i(w)$$



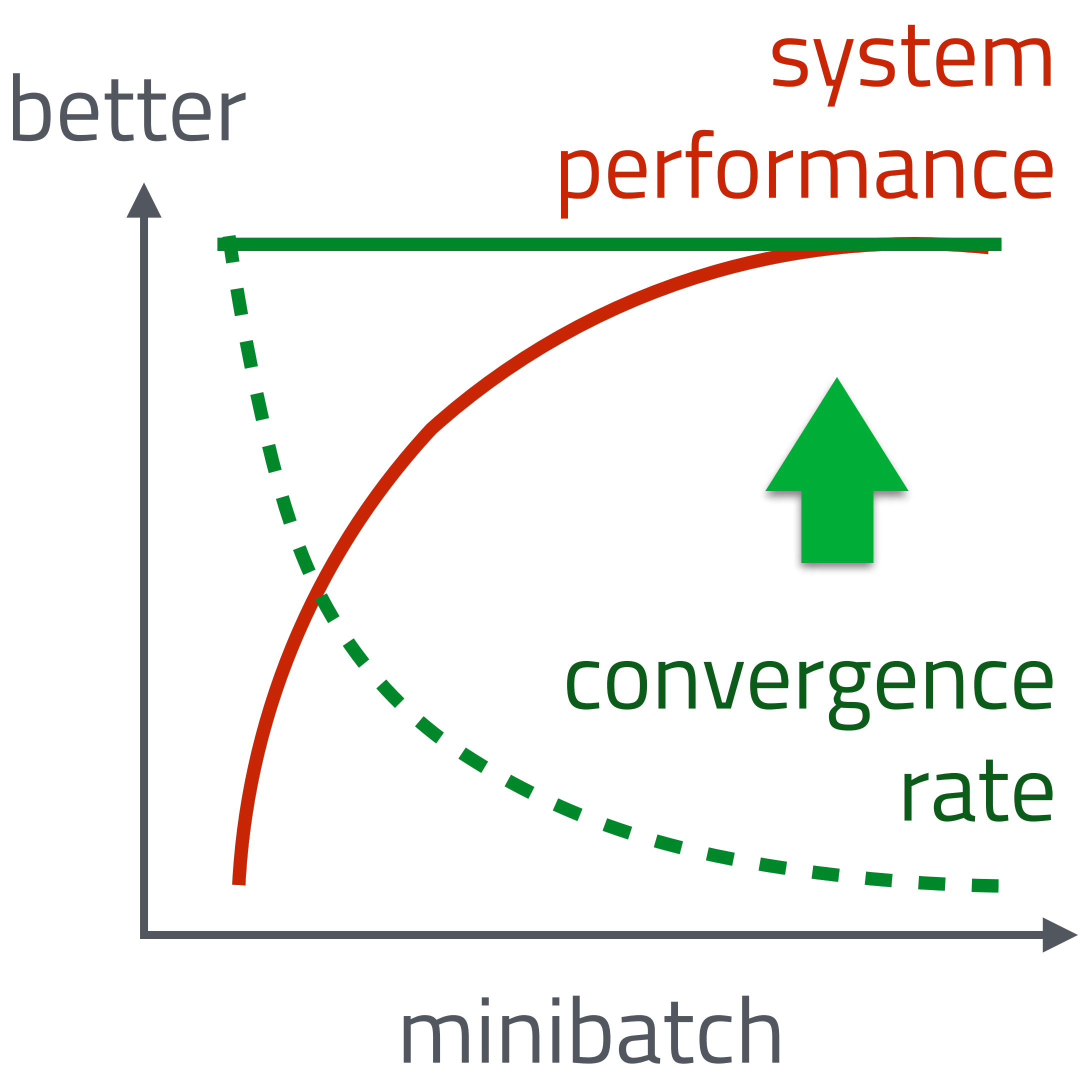

- 👍Efficient parallel/distributed implementation within a minibatch

# Effects of the minibatch size

- Fix #examples $bT$:
- minibatch size $b$↑, then #iteration $T$↓
- 👍System performance↑
  - ✓ #synchronization↓
  - ✓ network communication↓
- 👎Convergence rate↓
  - ✓ it is $\mathcal{O}(1/\sqrt{bT} + 1/T)$
  - ✓ O(1/T)↑



# Our goal




**Key idea:**

When minibatch size↑,
sample variance↓.
Solve a more “accurate” optimization problem over each minibatch.

# Observation

- Rewrite the update rule of minibatch SGD:

$$w_t = \operatorname*{argmin}_{w \in \Omega} \left[ \underbrace{\phi_{I_t}(w_{t-1}) + \langle \nabla \phi_{I_t}(w_{t-1}), w - w_{t-1} \rangle}_{\text{A coarse first-order approximation}} + \underbrace{\frac{1}{2\eta_t} \|w - w_{t-1}\|_2^2}_{\text{a conservative penalty}} \right]$$

- 👍Fast to solve
- 👎Data utilization is low
  - ✓ large switching cost to the next minibatch

# The proposed solution: EMSO

- Solve a conservative subproblem:

$$w_t = \underset{w \in \Omega}{\operatorname{argmin}} \left[ \underbrace{\phi_{I_t}(w)}_{\text{exact objective over minibatch}} + \frac{\gamma_t}{2} \underbrace{\|w - w_{t-1}\|_2^2}_{\text{a conservative penalty}} \right].$$

- 👍achieve a full utilization of the minibatch
- 👍avoid over utilization

# Convergence rate

- Minibatch SGD: $\mathcal{O}(1/\sqrt{bT} + 1/T)$
- EMSO: $\mathcal{O}(1/\sqrt{bT})$
  - ✓ only depends on the #examples
- Can be further improved when the objective is λ-strongly convex

$$\mathcal{O}(\log T/(\lambda bT) + \lambda/(\sqrt{b}T))$$

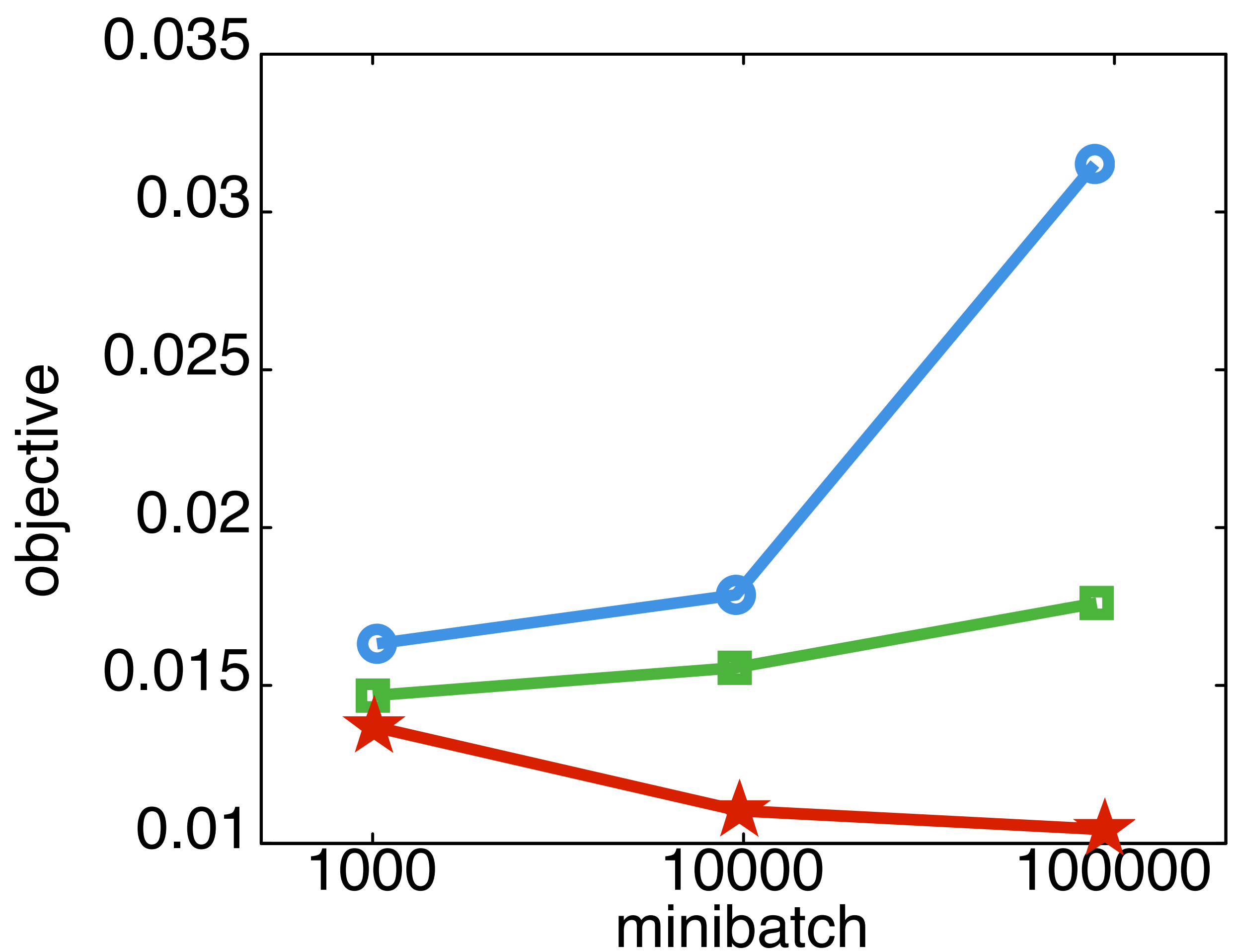
# How to solve the subproblem

$$w_t = \operatorname*{argmin}_{w \in \Omega} \left[ \phi_{I_t}(w) + \frac{\gamma_t}{2} \|w - w_{t-1}\|_2^2 \right].$$

- The conservative subproblem can be solve by standard technologies:
  - ✓ EMSO-GD: by gradient descent
  - ✓ EMSO-CD: by coordinate descent
- No need to solve it exactly
- Early stopping:
  - ✓ fix the #iterations be a small constant

# Convergence does not slow down with minibatch size

- Fix #iterations
- Logistic regression
- Dataset KDD04: 146K examples, 76 features



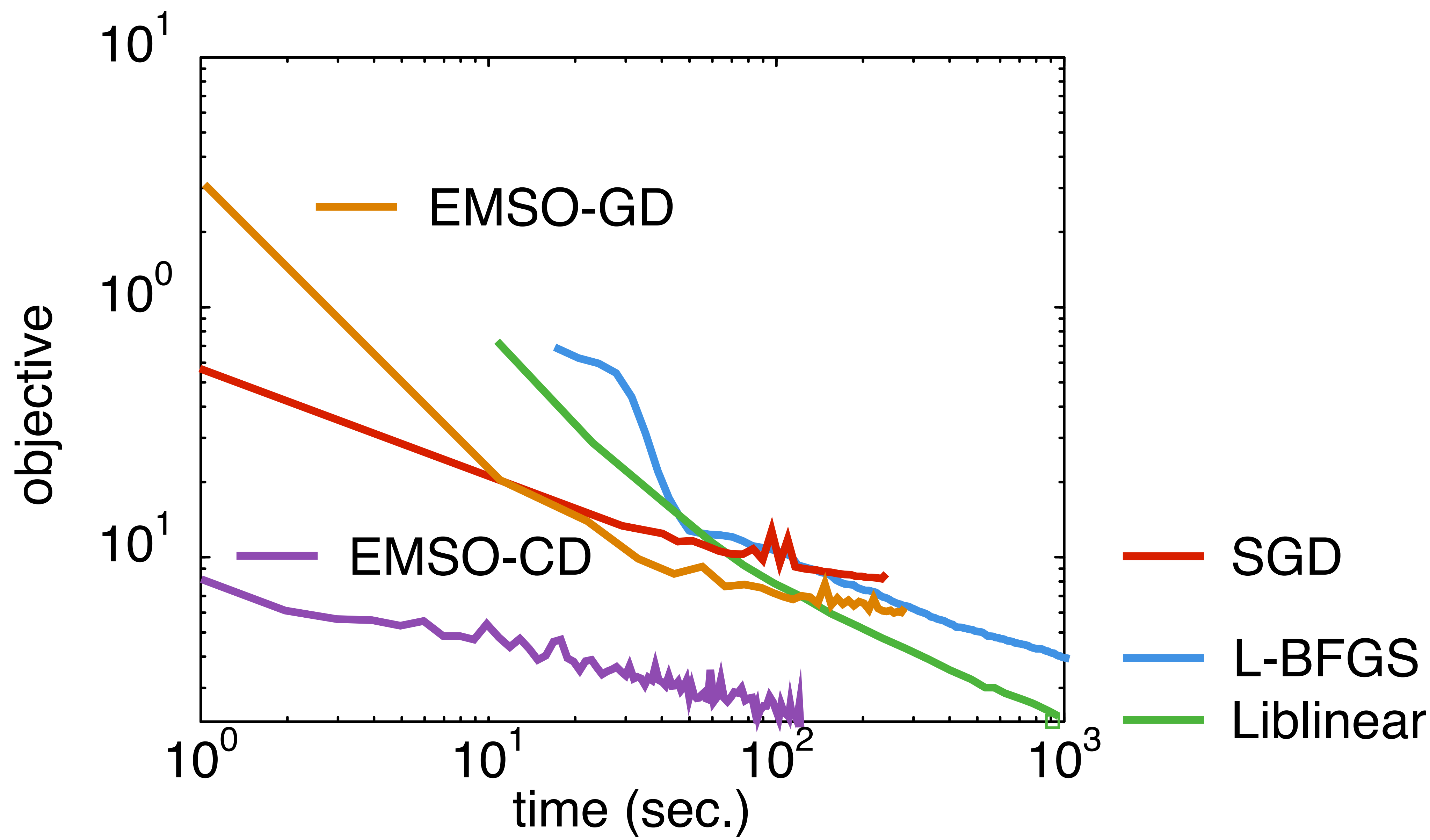

minibatch SGD

EMSO-GD:
pass the minibatch 5 times

EMSO-CD:
pass the minibatch 2 times

# EMSO-CD outperforms other algorithms

- Dataset URL: 2.4M #examples, 3.2M #features



# Distributed model averaging

assume $d$ machines

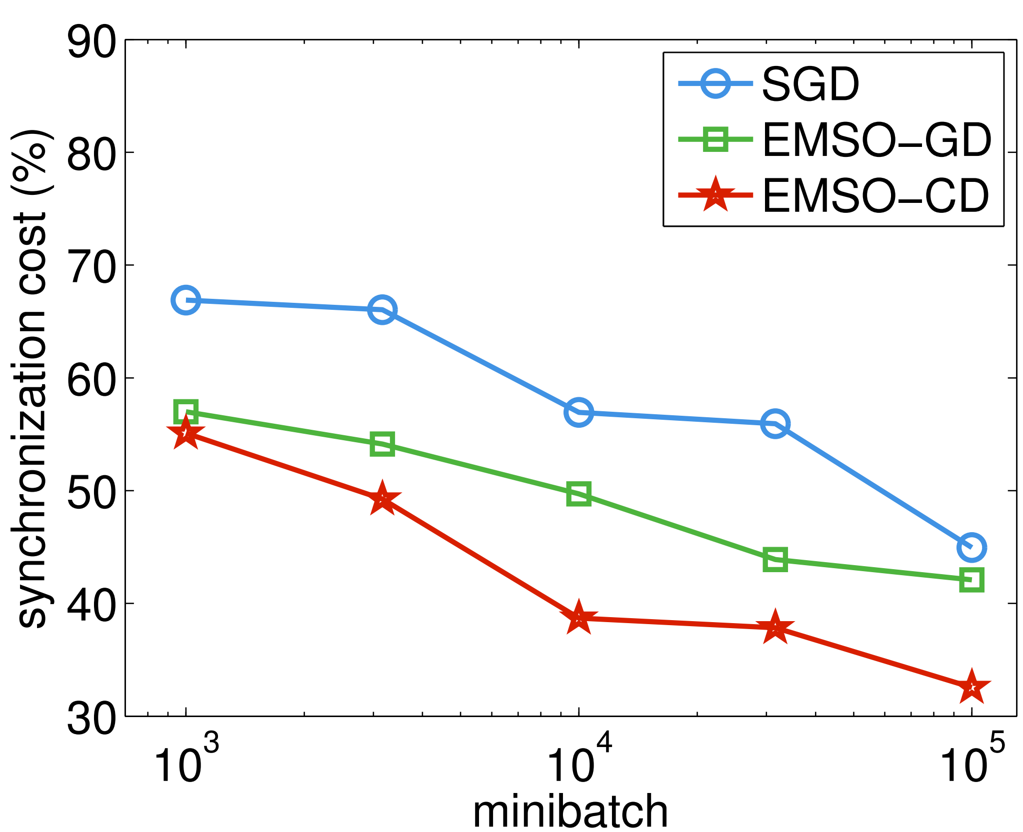
for each minibatch $I_t$:

1. partition $I_t$ into $\{I_{t,1}, \ldots, I_{t,d}\}$
2. machine $i$ solve the conservative subproblem on its own data partition $I_{t,i}$
3. average model via communication

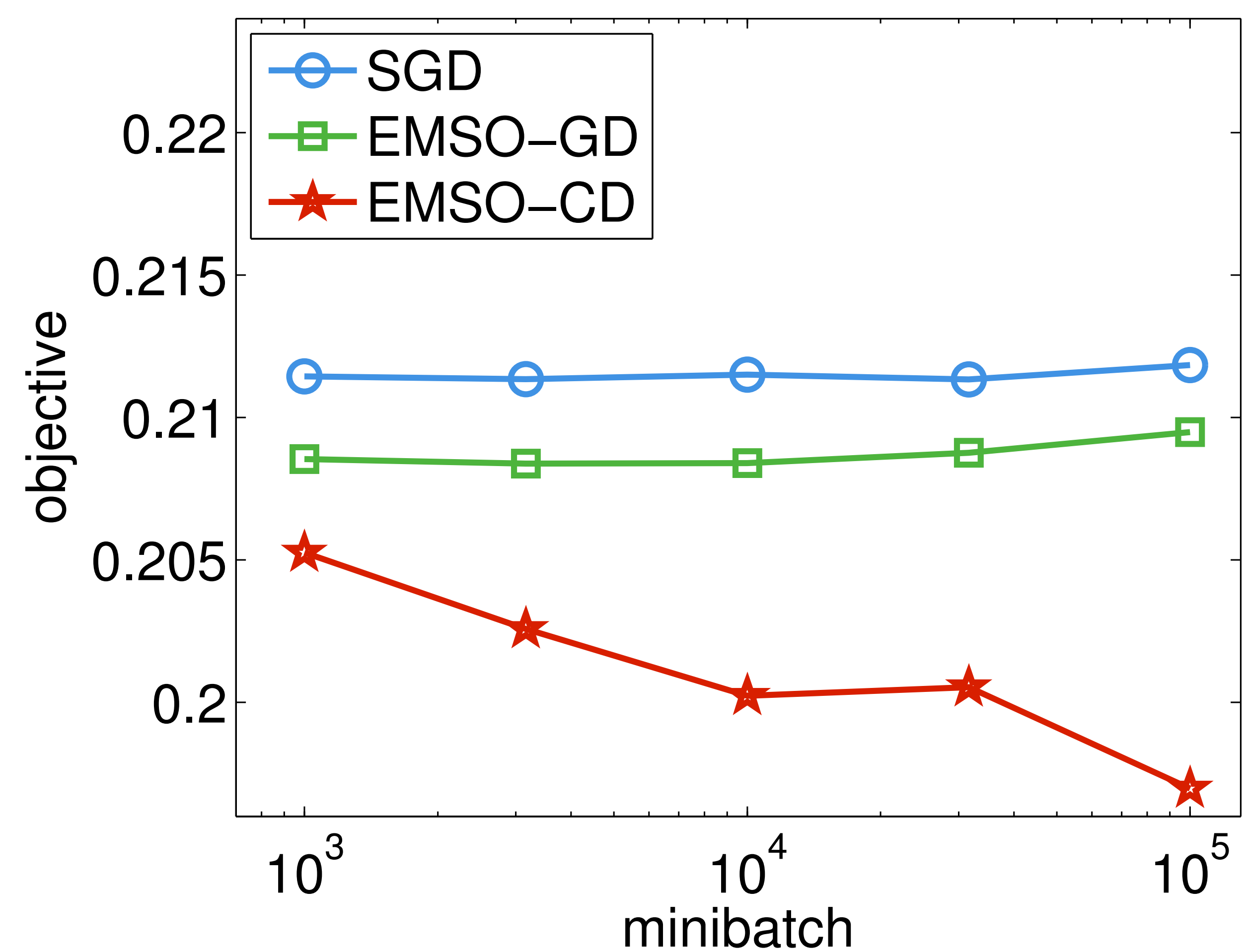
$$w_t = \frac{1}{d}\sum_{i=1}^{d} w_t^{(i)}$$

# EMSO-CD outperforms other algorithms

- Dataset CTR: 142M #examples, 28M #features, raw text data size 300GB
- Distributed over 12 machines
  - Synchronization cost↓ when minibatch size↑




- Fix run time

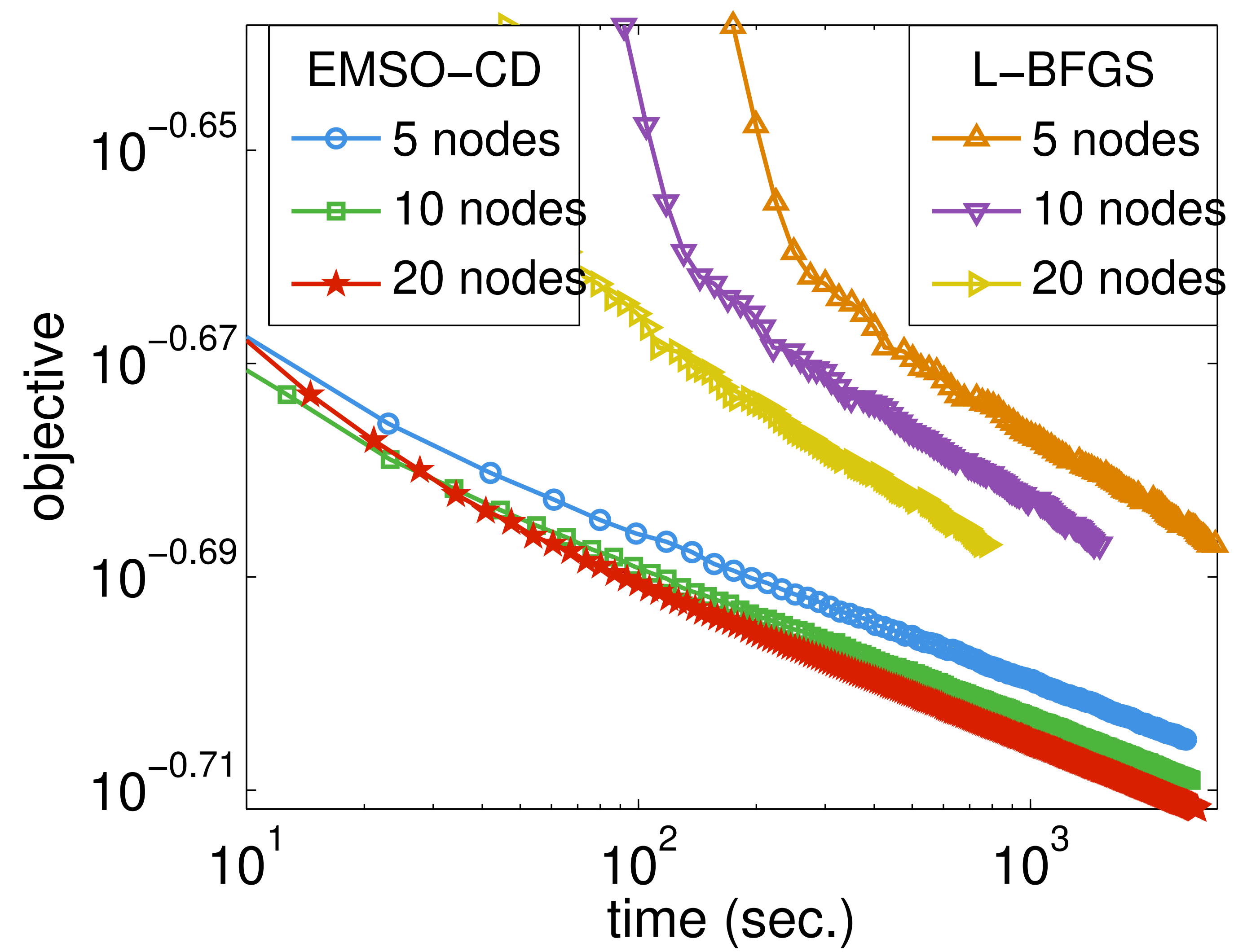


# Scalability of EMSO-CD

- Dataset CTR
- Fix target objective

| #machines | time (sec) | speedup |
|---|---|---|
| 5 | 2439 | 1x |
| 10 | 1367 | 1.78x |
| 20 | 962 | 2.54x |

- Compare to L-BFGS



# Conclusion

- EMSO: solve the conservative subproblem in each minibatch:

$$w_t = \operatorname*{argmin}_{w \in \Omega} \left[ \phi_{I_t}(w) + \frac{\gamma_t}{2} \|w - w_{t-1}\|_2^2 \right].$$

- Faster convergence rate $\mathcal{O}(1/\sqrt{bT})$
  - ✓ Does not slow down when minibatch size↑
- Improve the effective workload
  - ✓ Demonstrated in experiments with real large scale datasets