SERBIAN JOURNAL OF ELECTRICAL ENGINEERING
Vol. 1, No. 2, June 2004, 249 - 254

Efficient Modeling of Chiral Media Using SCN-TLM Method

M. I. Yaich, Mohsine Khalladi[1], M. Essaaidi

Abstract: An efficient approach allowing to include linear bi-isotropic chiral materials in time-domain transmission line matrix (TLM) calculations by employing recursive evaluation of the convolution of the electric and magnetic fields and susceptibility functions is presented. The new technique consists to add both voltage and current sources in supplementary stubs of the symmetrical condensed node (SCN) of the TLM method. In this article, the details and the complete description of this approach are given. A comparison of the obtained numerical results with those of the literature reflects its validity and efficiency.

Keywords: Bi-isotropic-chiral materials; Time-domain SCN-TLM method.

1 Introduction

The study of the interaction between electromagnetic (EM) waves and chiral media attracts the attention of a constantly growing scientific community. Both the bianisotrpic and dispersive characteristics of chiral media makes very delicate the analytical study of the behaviour of EM waves in them. Using three different approaches, the TLM method with the symmetrical condensed node (SCN) has been extended to describe the time-domain propagation of EM waves in the dispersive media. The first approach describes the behaviour of the medium by equivalent node sources [1]. The second one is based on the Z-transform technique [2]-[3] while the third one models the dispersive media by adding voltage or current sources in supplementary stubs of the hybrid SCN [4]-[5] or the standard SCN [6]. In this paper the third approach with the standard SCN is extended to chiral media. The dispersive properties of such media are accounted for using a new technique based on the SCN and both voltage and current sources.

2 Formulation

Generally, bianisotropic media are described by the following constitutive equations:

[1] Electronic & Microwaves Group Faculty of Sciences, Abdelmalek Essaaidi University
P.O. Box: 2121, Tetuan 93000 Morocco
E-mail: khalladi@fst.ac.ma, M_khall@ieee.org

$$\begin{bmatrix} D \\ B \end{bmatrix} = \begin{bmatrix} \varepsilon_0 E \\ \mu_0 H \end{bmatrix} + \begin{bmatrix} \varepsilon_0 \overline{\overline{\chi}}_{el} & \overline{\overline{\xi}}_r / c \\ \overline{\overline{\zeta}}_r / c & \mu_0 \overline{\overline{\chi}}_{ma} \end{bmatrix} * \begin{bmatrix} E \\ H \end{bmatrix}, \tag{1}$$

where $\overline{\overline{\chi}}_{el}$, $\overline{\overline{\chi}}_{ma}$, $\overline{\overline{\xi}}_r$ and $\overline{\overline{\zeta}}_r$ are respectively the electric susceptibility, the magnetic susceptibility and the magneto-electric tensors [7]. In the particular case of a uniaxial medium, the constitutive parameters are given by uniaxial scalar expressions, which can be written in the time domain as follows [3]:

$$\chi_{el}(t) = \mathrm{Re}\left(\frac{-j\chi_{el_0}\omega_0^2}{\beta} e^{-(\delta - j\beta)t} \right); \tag{2}$$

$$\chi_{ma}(t) = \mathrm{Re}\left(\frac{j\chi_{ma\infty}(\delta - j\beta)^2}{\beta} e^{-(\delta - j\beta)t} \right); \tag{3}$$

and

$$\xi_r(t) = -\zeta_r(t) = \mathrm{Re}\left(\frac{j\tau\omega_0^2(\delta - j\beta)}{\beta} e^{-(\delta - j\beta)t} \right). \tag{4}$$

Where Re is the real operation, χ_{el0} is the static electric susceptibility, ω_0 is the resonance frequency, δ is the damping frequency, $\beta = (\omega_0^2 - \delta^2)^{1/2}$, $\chi_{ma\infty}$ is the optical magnetic susceptibility and τ is the chilarity time constant.

The EM wave components propagating in the z-direction in a chiral bi-isotropic medium are obtained directly from Maxwell's equations and the constitutive relations. Using a convolution constant discretization, we obtain at time (n+1)Δt the components located in the xoy plane:

$$_{n+1}\begin{pmatrix} E_q \\ Z_0 H_q \end{pmatrix} = \frac{1}{\Delta_o} \begin{pmatrix} 1+\chi_{ma}^0 & -\xi_r \\ -\zeta_r & 1+\chi_{el}^0 \end{pmatrix} \begin{pmatrix} {}_nE_q + \mathrm{Re}\left(\sum_{m=0}^{n-1}\left({}_{n-m}E_q\Delta\chi_{el}^m + Z_0\,{}_{n-m}H_q\Delta\xi_r^m\right)\right) + \frac{\Delta t}{\varepsilon_0}\,{}_{n+1/2}(\nabla\times H)_q \\ Z_0\,{}_nH_q + \mathrm{Re}\left(\sum_{m=0}^{n-1}\left({}_{n-m}E_q\Delta\zeta_r^m + Z_0\,{}_{n-m}H_q\Delta\chi_{ma}^m\right)\right) - \frac{Z_0\Delta t}{\mu_0}\,{}_{n+1/2}(\nabla\times E)_q \end{pmatrix}. \tag{5}$$

Where $q \in \{x, y\}$,

$$\chi_{el}^m(t) = \int_{m\Delta t}^{(m+1)\Delta t} \chi_{el}(t')dt', \quad \chi_{ma}^m(t) = \int_{m\Delta t}^{(m+1)\Delta t} \chi_{ma}(t')dt', \quad \xi_r^m(t) = \zeta_r^m(t) = \int_{m\Delta t}^{(m+1)\Delta t} \xi_r(t')dt',$$

$$\Delta\chi_{el}^m = \chi_{el}^m - \chi_{el}^{m+1},\ \Delta\chi_{ma}^m = \chi_{ma}^m - \chi_{ma}^{m+1},$$

$$\Delta\xi_r^m = \xi_r^m - \xi_r^{m+1},\ \Delta\zeta_r^m = \zeta_r^m - \zeta_r^{m+1} \text{ and } \Delta_o = (1 + \chi_{el}^0)(1 + \chi_{ma}^0) + (\xi r^0)^2.$$

Phenomenologically, similar components can be obtained from the TLM formalism. To this end, we apply to the SCN, to which we add voltage sources (V_{svq}) and current sources (V_{siq}), charge and magnetic flux conservation principles, we use the equations relating incident and reflected pulses for the EM field components [8], then we couple equivalent currents and voltages having the same direction. This leads to the following expression:

$$ {}_{n+1}\begin{pmatrix} V_q \\ I_q \end{pmatrix} = \begin{pmatrix} 1 & C_{el} \\ C_{ma} & 1 \end{pmatrix} \begin{pmatrix} {}_nV_q + \dfrac{1}{4+Y_{oq}}\left(\left({}_{n+1}V_{svq} + {}_nV_{svq}\right) + \dfrac{4\Delta t}{\varepsilon_0}\, {}_{n+1/2}\left(\nabla \times H\right)_q\right) \\ {}_nI_q + \dfrac{1}{4+Z_{sq}}\left(\left({}_{n+1}V_{siq} + {}_nV_{siq}\right) - \dfrac{4Z_0\Delta t}{\mu_0}\, {}_{n+1/2}\left(\nabla \times E\right)_q\right) \end{pmatrix} .(6) $$

The formal equivalence between equations (5) and (6) provides the necessary parameters and relations for the novel proposed TLM approach, namely, the coupling coefficient

$$ \begin{pmatrix} C_{el} \\ C_{ma} \end{pmatrix} = \begin{pmatrix} -\xi_r^0 / (1+\chi_{el}^0) \\ -\zeta_r^0 / (1+\chi_{ma}^0) \end{pmatrix}, \tag{7} $$

the admittance and the impedance of chiral media

$$ \begin{pmatrix} Y_{oq} \\ Z_{sq} \end{pmatrix} = \begin{pmatrix} 4\Delta_0 / (1+\chi_{el}^0) - 4 \\ 4\Delta_0 / (1+\chi_{ma}^0) - 4 \end{pmatrix}, \tag{8} $$

and the voltage and current sources

$$ \begin{pmatrix} {}_{n+1}V_{svq} + {}_nV_{svq} \\ {}_{n+1}V_{siq} + {}_nV_{siq} \end{pmatrix} = \begin{pmatrix} (4+Y_{oq})/\Delta_o \\ (4+Z_{sq})/\Delta_o \end{pmatrix} \begin{pmatrix} (1+\chi_{ma}^0)\,\mathrm{Re}\left(\sum_{m=0}^{n-1}\left({}_{n-m}V_q\Delta\chi_{el}^m + {}_{n-m}I_q\Delta\xi_r^m\right)\right) + (1+\chi_{ma}^0 - \Delta_o)\,{}_nV_q \\ (1+\chi_{el}^0)\,\mathrm{Re}\left(\sum_{m=0}^{n-1}\left({}_{n-m}V_q\Delta\zeta_r^m + {}_{n-m}I_q\Delta\chi_{ma}^m\right)\right) + (1+\chi_{el}^0 - \Delta_o)\,{}_nI_q \end{pmatrix} .(9) $$

The incorporation of these three equations in the TLM method allows to model directly the interaction between EM waves and uniaxial chiral media.

3 Numerical results

To test the proposed model, we have studied the polarization rotation of the incident field in reflection and transmission from a bi-isotropic chiral layer free space and from a bi-isotropic layer with a perfect electric conductor back sheet. For the two examples, the incident field is a normally (Gaussian shaped pulse) plane wave, travelling toward the chiral layer with an electric field component:

$$ \vec{E}^i(z,t) = \hat{x}\, A_0\, e^{-g_0^2\left(\frac{z}{c} - (t - t_{max})\right)^2} . \tag{10} $$

Where g_0 is the spread parameter for a Gaussian pulse, $t_{max} = (\ln 100)^{½}/g_0$ and A_0=1 (V/m). The TLM problem space considered had 220 Δl in z direction, where Δl is the mesh width taken to be Δl = 1 mm. The bi-isotropic chiral layer's thickness is 200 mm occupying cells from 10 to 210. Figure 1 and figure 2 depict respectively the reflection and transmission coefficients of the right and the left hand circularly polarisations (RCP and LCP). Figure 3 shows the co-polarized reflection coefficient of the same layer bounded by a metallic sheet [3]. In both cases, the medium's properties are $\chi_{el0} = \chi_{ma0} = 0.5$, the resonance frequency $\omega_0 = 2\pi E9$ rad/s, the chilarity time constant

$\tau = 0.5\omega_0^{-1}$ and the damping frequency $\delta = 0.1\omega_0$. The background relative permittivity and permeability were selected $\varepsilon_r = \mu_r = 2$. In the three figures, good agreement can be seen between the TLM results and those of the analytical solution.

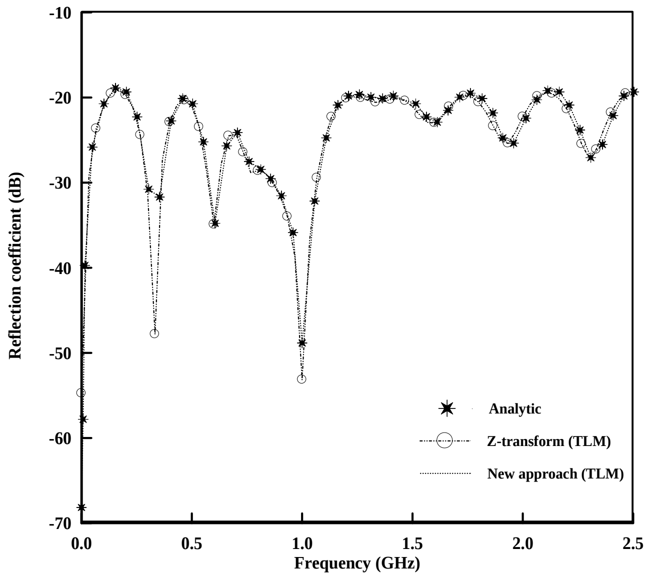

Fig. 1 - *Reflection coefficient magnitudes versus frequency for a plane wave incident on a chiral layer for the right and the left hand circularly polarisations (RCP and LCP).*

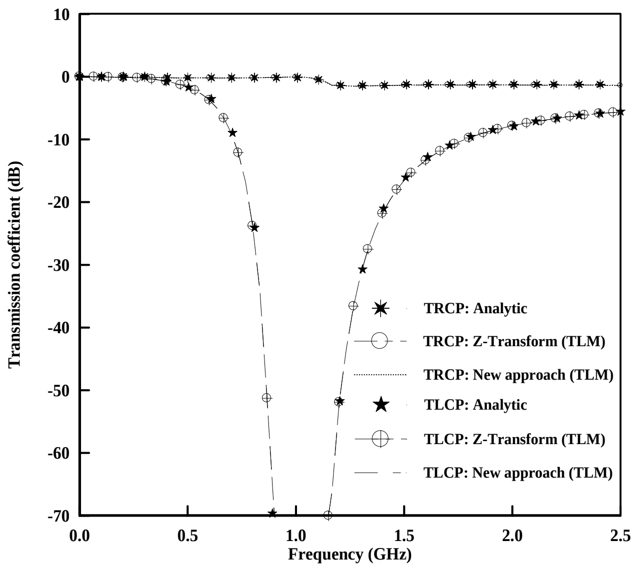

Fig. 2 - *Transmission coefficient magnitudes versus frequency for a plane wave incident on a chiral layer for the right and the left hand circularly polarisations (RCP and LCP).*

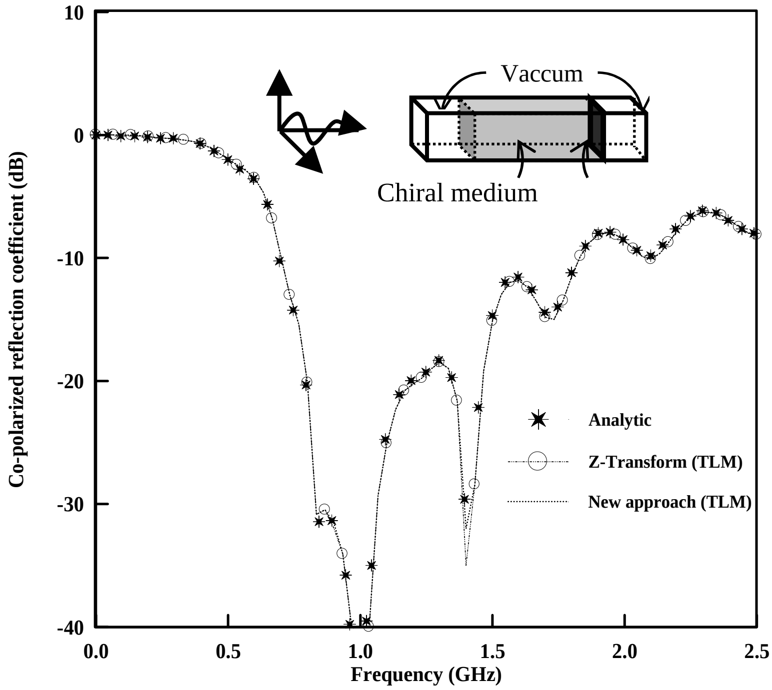

Fig. 3 - *Co-polarized reflection coefficient magnitude versus frequency for a plane wave incident on a metal backed chiral layer.*

4 Conclusion

A novel and robust technique based on the TLM method with both voltage and current sources is proposed for the modeling of linear bi-isotropic chiral media. To illustrate and validate this model, the reflection and transmission of a Gaussian plane wave normally incident on a chiral and metal backed chiral layer have been investigated and excellent agreement between the obtained numerical results and those of the literature is achieved.

5 References

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