EFFICIENT OPTIMIZATION OF TRANSIENT DYNAMIC PROBLEMS FOR A MICRO ACCELEROMETER USING MODEL ORDER REDUCTION

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Summary One of the main obstacles to including transient dynamic effects into the performance functions of a structural optimization is the high computational cost of each time-dependent simulation. The focus of this paper is on the application of model order reduction techniques to reduce the transient analysis time for the attainable optimization process. The software mor4ansys developed at IMTEK performs model reduction via the Arnoldi algorithm directly to ANSYS finite element models. We adopt a micro accelerometer as an example to demonstrate the advantages of this approach. The harmonic and transient results of a reduced model of the accelerometer yield very good agreement with those from the original high dimensional ANSYS model. The use of model reduction within the optimization iterations produces almost the same results as without order reduction and speeds up the total computation by about an order of magnitude.

INTRODUCTION

Transient dynamic problems with very large numbers of degrees of freedom are solvable in feasible computational time nowadays due to the tremendous improvement of computers. When a design optimization, however, includes time-dependent performance functions, the optimization often does not fit within a reasonable design process time because of the repeated simulations. In order to alleviate this difficulty, we utilize techniques of model order reduction for efficient time-dependent simulations during the design optimization. In this paper, we introduce fully automatic software for model order reduction to generate a reduced model of a second-order linear system and to achieve its integration into structural optimizations, and provide an example of successful application to optimization of a micro accelerometer.

METHODS

Model order reduction for second order systems

A structural mechanics problem, after the discretization in space, is described by a system of ordinary differential equations (ODEs) of the second order in time

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{B}\mathbf{u}(t) \tag{1}$$

where **M**, **C**, and **K** are the structural mass, damping, and stiffness matrices, **B** is the scattering matrix to distribute the inputs **u**(t) on the domain and **x** is a state vector. It happens that in most cases the trajectory of the state vector can be well approximated by a low-dimensional subspace **V**, and projecting of ODEs on that subspace yields quite and accurate reduced models [1]. The goal of model order reduction is to find a new system of equations

$$\mathbf{M}_r\ddot{\mathbf{z}}(t) + \mathbf{C}_r\dot{\mathbf{z}}(t) + \mathbf{K}_r\mathbf{z}(t) = \mathbf{B}_r\mathbf{u}(t), \quad \mathbf{x} = \mathbf{V}\mathbf{z} + \varepsilon \tag{2}$$

with a lower number of equations and a lower dimensional state vector **z** such that its transfer function matches that of the original system. Our method to find an appropriate low-dimensional subspace is based on moment-matching via the Arnoldi process. The software mor4anys applies this procedure directly to ANSYS finite element models [2].

Integration of optimization and model order reduction

The optimization process integrates several heterogeneous programs, and their communication is implemented via file transfers. First, a parametric finite element model is developed as an ANSYS script. The DOT optimizer [4] changes the design parameters to reach the optimal value of the objective function by satisfying all the constraints. The latter is summarized by three steps: 1) ANSYS is called to mesh the model for given design variables and to produce an element matrix (EMAT file), a FULL file, lists for the displacement boundary conditions and output degrees of freedom; 2) The software mor4ansys uses these files as input and generates a reduced model by means of the Arnoldi algorithm; 3) The postprocessing, that is, the solution of a reduced ODE system, as well as computing its transfer function, is performed in Mathematica. Finally, the objective and constraint functions for DOT are calculated from the postprocessed data. The optimization stops or continues according to convergence criteria. During the optimization, the gradient is estimated by central differencing. A modified feasible direction algorithm, available in DOT, is used to obtain solutions.

A PIEZORESISTIVE MICRO ACCELEROMETER

A piezoresistive cantilever beam micro accelerometer for automotive airbag applications [3] is studied to demonstrate this approach. The micro accelerometer has a symmetrically bonded silicon proof-mass, a self-diagnostic resistor and a sensing part (see Fig. 1). The design objective is to make the output sensitivity (S) as close as possible to a target value (M_S) subject to a set of constraints on the resonant frequency, beam deflection, and impact endurance of the structure.

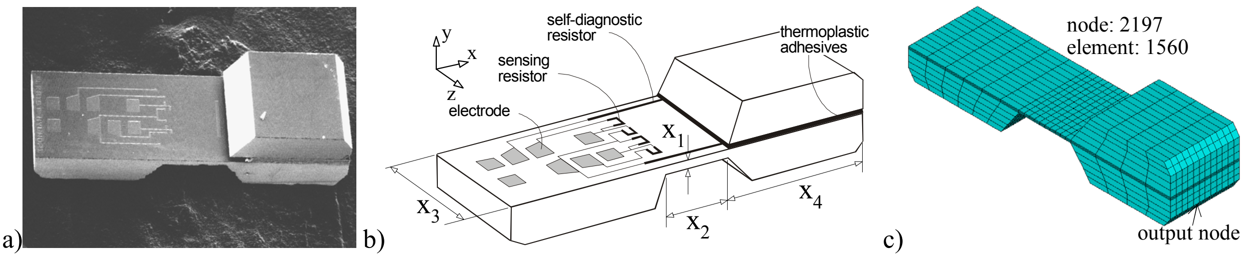
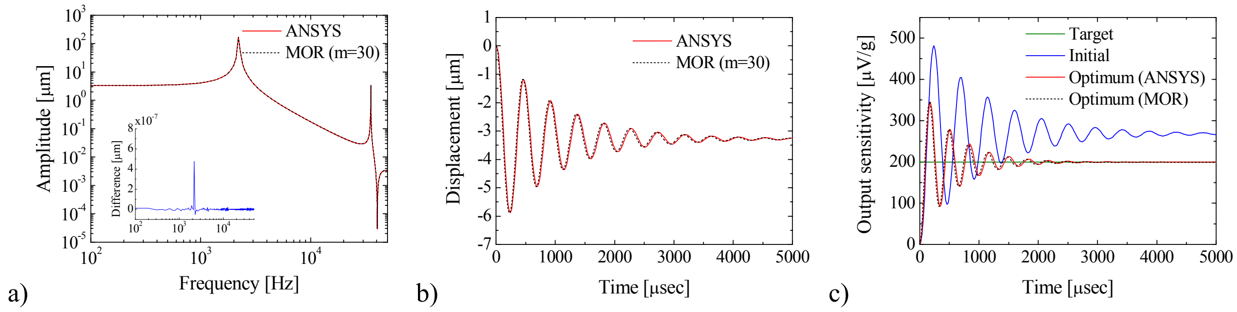


Figure 1: Piezoresistive micro accelerometer. a) SEM; b) definition of design variables; c) FE mesh and an output node



a) b) c)

Figure 2: Comparison for reduced and full ANSYS models. a) harmonic; b) transient; c) output sensitivity

$$\begin{aligned} &\text{Minimize} \quad \int_0^{t_a} \left| S - M_s \right| dt \\ &\text{subject to} \quad f_1 \ge f_{req}, \quad \delta^t_{50g} \le 0.3 d_{gap}, \quad \sigma_{2000g} \le \sigma_{yield} \end{aligned} \tag{3}$$

where f_1, δ^t_{50g}, d_{gap}, and δ_{2000g} denote the first natural frequency, the beam deflection under an acceleration of 50 g (gravity), a gap of 10 μm, and the von Mises stress under an impact loading of 2000 g, respectively. The output sensitivity is calculated from $S=0.5K\varepsilon V_a/a_y$, where K and ε mean a gage factor of the piezoresistive material and strain on the sensing resistor. An applied voltage of 5 volts and y-directional acceleration of 50 g are taken as an actual case. A target output sensitivity of 200 μV/g is assumed. The four design variables for the optimization are illustrated in Fig. 1b).

Harmonic and transient simulation

Harmonic responses at the output node to a y-directional harmonic excitation for an original ANSYS and a reduced model of order 30 perfectly match to a high frequency (Fig. 2a). It is remarkable that even a reduced model of order 5 shows almost perfect agreement. Figure 3b) illustrates transient responses with a Rayleigh damping of α=0 and β=10 μsec at the output node to a step load for the ANSYS and reduced model. Again, the reduced model catches the transient behaviour almost perfectly.

Structural optimization

Design optimization of (3) has been performed using the time-dependent information from ANSYS and the model order reduction. The two optimizations take almost the same number of design iterations and produces nearly identical optimal shapes. The time-varying output sensitivity of the micro accelerometer before and after optimization is shown in Fig. 2c). The output sensitivity of the optimums oscillates less and converges faster into the target value than the initial design. The total computational cost for the optimization using the reduced model decreases to approximately 1/5 of that by using the original ANSYS model alone.

CONCLUSIONS

We have applied model order reduction techniques to reduce the time for transient simulations in the design optimization process. The use of model reduction within the optimization iterations produces almost the same results and reduces the total computational cost by about an order of magnitude. This approach, although only illustrated for a structural problem, can be easily applied to other linear physical problems as well, where transient information is important but the computational cost needs to be reduced for reasonable design times.

References

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