Efficient simulation of contacts,friction and constraints using a modified spectral projected gradient method

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ABSTRACT

This work introduces a modified version of the Spectral Projected Gradient method that can be used for simulating dynamical systems with complex joints and frictional contacts. The proposed method is able to solve for unknown reactions in systems with large number of colliding shapes and articulated mechanisms. This method couples the ability of solving complementatity constraints, typical of fixed point iterations used in real-time applications, with the superior convergence of Krylov iterations for linear problems, hence making it attractive as a general purpose solver for both linear and non linear problems.

Keywords

Contacts, simulation, complementarity, spectral gradient.

1 INTRODUCTION

Motion of rigid parts with frictional contacts and impacts can be described by Measure Differential Inclusions (MDI), where the non-smooth equations of motion include set-valued forces [Mor99]. The non-smooth approach is expecially appreciated in applications that call for efficient and real-time performances, such as in video games, virtual reality, etc.

Among the methods to perform the time integration of MDI problems, we refer to the approach of [AT10], that expresses the problem as a sequence of Variational Inequalities (VI) to be solved at each time step.

The problem of Variational Inequalities (VI) is stated as the problem of finding $\mathbf{x}$ subject to:

$$\mathbf{x} \in \mathscr{K} \quad : \quad \langle \mathbf{g}(\mathbf{x}), \mathbf{y}-\mathbf{x} \rangle \geq 0 \quad \forall \mathbf{y} \in \mathscr{K} \tag{1}$$

where $\mathscr{K} \subset \mathbb{R}^n$ is a closed and convex set, and $\mathbf{g}(\mathbf{x}) : \mathscr{K} \rightarrow \mathbb{R}^n$ is a continuous function.

Alternatively, Eq.(1) can be formulated using normal cones:

$$\mathbf{x} \in \mathscr{K}\ ,\ \mathbf{g}(\mathbf{x}) \in -\mathsf{N}_{\mathscr{K}}(\mathbf{x}) \tag{2}$$

We recall the definition of a normal cones $\mathsf{N}_{\mathscr{K}}(\mathbf{x})$ to the set $\mathscr{K}$ at the point $\mathbf{x}$ as:

$$\mathsf{N}_{\mathscr{K}}(\mathbf{x}) = \{\mathbf{y} \in \mathbb{R}^n : \langle \mathbf{y}, \mathbf{x}-\mathbf{z} \rangle \geq 0, \forall \mathbf{z} \in \mathscr{K}\} \tag{3}$$

A special VI of particular interest in mechanics happens when $\mathbf{g}(\mathbf{x})$ is an affine linear mapping $\mathbf{g}(\mathbf{x}) = A\mathbf{x} + \mathbf{b}$ and when, introducing primal variables $\mathbf{y} \in \mathbb{R}^m$ and dual variables $\mathbf{x}$, we can write:

$$\left\{ \begin{array}{c} \mathbf{0} \\ \mathbf{g}(\mathbf{x}) \end{array} \right\} = \left[\begin{array}{cc} W & J \\ J^T & E \end{array} \right] \cdot \left\{ \begin{array}{c} \mathbf{y} \\ -\mathbf{x} \end{array} \right\} - \left\{ \begin{array}{c} \mathbf{k} \\ \mathbf{c} \end{array} \right\} \tag{4}$$

Furthermore we introduce the Schur complement

$$A = \left[J^T W^{-1} J - E \right] \tag{5}$$

$$\mathbf{b} = -\mathbf{c} + J^T W^{-1} \mathbf{k} \tag{6}$$

for the term $\mathbf{g}(\mathbf{x}) = A\mathbf{x} + \mathbf{b}$ in the VI of Eq.(1).

Following the approach of [AT10], in Eq.(4) the W matrix is the mass matrix, $\mathbf{y}$ represents changes in generalized velocities during Δt, $\mathbf{x}$ are impulses in contacts, $\mathbf{k}$ contains applied impulses, $\mathbf{c}$ is a stabilization term, and E represents compliance, if any. In this class of problems, one solves a VI at each step, where $\mathscr{K}$ in Eq.(4) is the Cartesian product of n_f second-order Lorentz cones $\mathscr{K} = (\bigtimes_{k=1}^{n_f} \mathscr{F}_k)$; each being a Coulomb-Amontons friction cone $\mathscr{F}_k \subset \mathbb{R}^3$. If n_j bilateral constraints are added too (as in mechanical joints), one still has a VI, and the cones associated to the single scalar constraints are just $\mathbb{R}$, thus $\mathscr{K} = (\bigtimes_{k=1}^{n_f} \mathscr{F}_k) \bigtimes \mathbb{R}^{n_j}$. Details can be found in [TA11].

We remark that, as special case, one might have n_j bilateral joints only: this would lead to a simple linear problem because $\mathscr{K} = (\bigtimes_{k=1}^{n_j} \mathbb{R}) = \mathbb{R}^{n_j}$, and $\mathsf{N}_{\mathscr{K}}(\mathbf{x}) = \{\emptyset\}$. For this type of problems, one could use iterative solvers of Krylov type, such as CG or MINRES, that have very fast convergence; however these solvers do not work in case of unilateral and frictional constraints.

On the other side, there are not so many options for the full VI problem. In most cases, projected-Jacobi fixed point iterations are used, as in [TA11, NTM12]. These solvers are very robust, easy to implement, and fit well in real-time scenarios and GPU parallel versions, but their convergence is very poor, especially when one has to simulate articulated shapes and odd mass ratios.

We found that a type of Spectral Projected Gradient (SPG) method can offer the benefits of Krylov iterations and yet retain the broad applicability of the projected-Jacobi fixed point iterations. We will discuss this in the paper.

First, note that the convex set $\mathscr{K}$ is a Cartesian product of low-dimensional convex sets. This means that the projection of a value $\mathbf{x}$ onto the $\mathscr{K}$ set is relatively easy to implement as a sequence of simple projections onto its subsets.

Moreover, with affine $\mathbf{g}(\mathbf{x})$, one can see that Eq.(2) is the first order optimality condition of a Quadratic Program (QP) with convex constraints, where $\mathbf{g}(\mathbf{x}) = \nabla_{\mathbf{x}} f(\mathbf{x})$:

$$\min \quad f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T A\mathbf{x} + \mathbf{x}^T\mathbf{b} \qquad (7)$$

$$\text{s.t.} \quad \mathbf{x} \in \mathscr{K} \qquad (8)$$

This means that, in order to solve VIs with affine $\mathbf{g}(\mathbf{x})$ and convex $\mathscr{K}$, one can leverage on SPG methods; the attractive feature of the recent SPG methods is that they provide fast enough convergence yet relying only on three computational primitives: product of matrix by a vector, inner products, and projection on convex sets.

Spectral-gradient (SG) methods originated from [BB88] as iterative solvers for unconstrained QPs. A globalization strategy was added in [Ray97] for solving generic non-linear optimization problems. A further advancement was the projected version of solver, that is the SPG presented in [BMR99]. The SPG method solves convex-constrained optimization problems by performing a gradient projection at each step of Barzilai-Borwein iterations. A line search with the Grippo-Lampariello-Lucidi (GLL) strategy can handle the non-monotone nature of the method [GLL86].

We report here our implementation of a preconditioned P-SPG method, similar to the scheme introduced in [BR05], with few modifications. The method requires the following parameters: two safeguards $\alpha_{\min}$ and $\alpha_{\max}$ for the spectral step length (respectively 10^{-9} and 10^9 in our tests), two safeguards for the line search $0 < \sigma_{\min} < \sigma_{\max} < 1$, an integer N_{GLL} (a value about 10 works well in most cases), the Armijo decrease parameter $\gamma \in (0,1)$, and a small value τ_g.

ALGORITHM P-SPG-FB(A, $\mathbf{b}$, $\mathbf{x}_0$, $\mathscr{K}$, $P \mapsto \mathbf{x}$)
- $\mathbf{x}_0 := \Pi_{\mathscr{K}}(\mathbf{x}_0)$, $\mathbf{x}_{FB} = \mathbf{x}_0$, $\hat{\alpha}_0 \in [\alpha_{min}, \alpha_{max}]$
- $\mathbf{g}_0 := A\mathbf{x}_0 + \mathbf{b}$, $f(\mathbf{x}_0) = \frac{1}{2}\mathbf{x}_0^T A\mathbf{x}_0 + \mathbf{x}_0^T\mathbf{b}$, $w_0 = 10^{29}$
- **for** $j := 0$ **to** N_{max}
  - $\mathbf{p}_j = P^{-1}\mathbf{g}_j$
  - $\mathbf{d}_j = \Pi_{\mathscr{K}}(\mathbf{x}_j - \hat{\alpha}_j \mathbf{p}_j) - \mathbf{x}_j$
  - **if** $\langle \mathbf{d}_j, \mathbf{g}_j \rangle \geq 0$
    - $\mathbf{d}_j = \Pi_{\mathscr{K}}(\mathbf{x}_j - \hat{\alpha}_j \mathbf{g}_j) - \mathbf{x}_j$
  - $\lambda := 1$
  - **while** line search
    - $\mathbf{x}_{j+1} := \mathbf{x}_j + \lambda \mathbf{d}_j$
    - $\mathbf{g}_{j+1} := A\mathbf{x}_{j+1} + \mathbf{b}$
    - $f(\mathbf{x}_{j+1}) = \frac{1}{2}\mathbf{x}_{j+1}^T A\mathbf{x}_{j+1} + \mathbf{x}_{j+1}^T\mathbf{b}$
    - **if** $f(\mathbf{x}_{j+1}) > \max_{i=0,..,\min(j,N_{GLL})} f(\mathbf{x}_{j-i}) + \gamma\lambda \langle \mathbf{d}_j, \mathbf{g}_j \rangle$
      - define $\lambda_{\text{new}} \in [\sigma_{\min}\lambda, \sigma_{\max}\lambda]$ and repeat line search
    - **else**
      - terminate line search
  - $\mathbf{s}_j = \mathbf{x}_{j+1} - \mathbf{x}_j$
  - $\mathbf{y}_j = \mathbf{g}_{j+1} - \mathbf{g}_j$
  - **if** j is odd
    - $\hat{\alpha}_{j+1} = \frac{\langle \mathbf{s}_j, P\mathbf{s}_j \rangle}{\langle \mathbf{s}_j, \mathbf{y}_j \rangle}$
  - **else**
    - $\hat{\alpha}_{j+1} = \frac{\langle \mathbf{s}_j, \mathbf{y}_j \rangle}{\langle \mathbf{y}_j, P^{-1}\mathbf{y}_j \rangle}$
  - $\hat{\alpha}_{j+1} = \min(\alpha_{\max}, \max(\alpha_{\min}, \hat{\alpha}_{j+1}))$
  - $w_{j+1} = \left|\left| \left[\mathbf{x}_{j+1} - \Pi_{\mathscr{K}}(\mathbf{x}_{j+1} - \tau_g \mathbf{g}_{j+1})\right] / \tau_g \right|\right|_2 = ||\varepsilon||_2$
  - **if** $w_{j+1} \leq \min_{k=0,..,j} w_k$
    - $\mathbf{x}_{FB} = \mathbf{x}_{j+1}$
- **return** $\mathbf{x}_{FB}$

This algorithm adds preconditioning, alternate spectral step sizes and a safe fallback strategy to the original SPG method presented in [BMR99].

A preconditioned SG is discussed in [LRGH02], and a preconditioned SPG method is presented in [BR05]. By adopting a left-right symmetry-preserving preconditioning with Hermitian $P = LL^T$, exploiting $L^{-T}L^T = I$ and recalling $\mathbf{g} = A\mathbf{x} + \mathbf{b}$, we have $\hat{\mathbf{g}} = \hat{A}\hat{\mathbf{x}} + \hat{\mathbf{b}}$ with

$$\hat{A} = L^{-1}AL^{-T}, \quad \hat{\mathbf{b}} = L^{-1}\mathbf{b}, \quad \hat{\mathbf{x}} = L^T\mathbf{x}, \quad \hat{\mathbf{g}} = L^{-1}\mathbf{g} \tag{9}$$

One can rewrite the original SG method using $\hat{A}$, $\hat{\mathbf{b}}$ and $\hat{\mathbf{x}}$, substituting terms in (9) and alternating two different formulas for the computation of the spectral step size, as suggested in [Fle05]: $\alpha_{BB1} = \langle \mathbf{s}, \mathbf{s} \rangle / \langle \mathbf{s}, \mathbf{y} \rangle$ and $\alpha_{BB2} = \langle \mathbf{s}, \mathbf{y} \rangle / \langle \mathbf{y}, \mathbf{y} \rangle$. In the preconditioned case:

$$\hat{\alpha}_{BB1} = \frac{\langle \hat{\mathbf{s}}, \hat{\mathbf{s}} \rangle}{\langle \hat{\mathbf{s}}, \hat{\mathbf{y}} \rangle} = \frac{\mathbf{s}^T LL^T \mathbf{s}}{\mathbf{s}^T LL^{-1}\mathbf{y}} = \frac{\mathbf{s}^T P\mathbf{s}}{\mathbf{s}^T \mathbf{y}} \tag{10}$$

$$\hat{\alpha}_{BB2} = \frac{\langle \hat{\mathbf{s}}, \hat{\mathbf{y}} \rangle}{\langle \hat{\mathbf{y}}, \hat{\mathbf{y}} \rangle} = \frac{(L^T\mathbf{s})^T(L^{-1}\mathbf{y})}{(L^{-1}\mathbf{y})^T(L^{-1}\mathbf{y})} = \frac{\mathbf{s}^T\mathbf{y}}{\mathbf{y}^T P^{-1}\mathbf{y}} \tag{11}$$

This has the beneficial effect of partially smoothing the non-monotone descent as shown in Fig.1.

The continuous nonexpansive projection operator $\Pi(\cdot)$ is a mapping that satisfies $\Pi(\mathbf{x})_{\mathscr{K}} = \arg\min_{\mathbf{z}\in\mathscr{K}} ||\mathbf{z} - \mathbf{x}||$. For the preconditioned iteration we use a custom diagonal preconditioner $P = \overline{\mathrm{diag}}(A)$, where the diagonal elements relative to the same sub-set are averaged.

Finally, a fallback strategy is needed because the method, being non monotone, might experience wild oscillations before settling to a stationary point; if one wishes to truncate prematurely the iteration because of real time requirements, as in Fig.2 or in video games, the last computed value $\mathbf{x}_j$ might be really bad. It is wiser to resort to the vector that performed better among those computed; a metric of good performance is $\left|\left|\mathbf{x}_{j+1} - \Pi(\mathbf{x}_{j+1} - \mathbf{g}_{j+1})\right|\right|_2$.

2 RESULTS

We implemented the preconditioned P-SPG-FB method in our C++ library for multibody simulation, in order to solve the dynamic problems with bilateral constraints and frictional contacts. We tested it in various benchmarks and we obtained remarkable convergence properties, as shown in Figs.3,4,5,6.

3 CONCLUSION

In this work we presented the P-SPG-FB method and we discussed its performance in relation to typical scnearios that are found in complex real-time simulations. The fallback strategy cured the problem of the non-monotone nature of the classical SPG method, and preconditioning made it more robust in case of bad conditioning. A remarkable feature is that in case of simple linear problems SPG converges as fast as state-of-the-art CG or MINRES methods. Also, it fits on GPU processor architectures.

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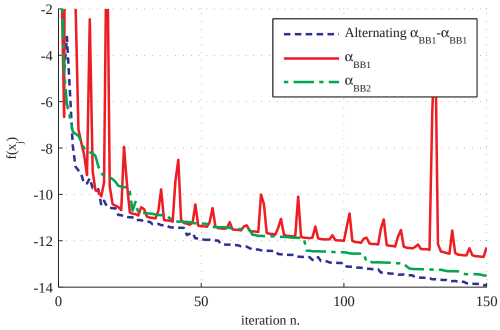

Figure 1: Non-monotone behavior of the SPG method (in benchmark B4).

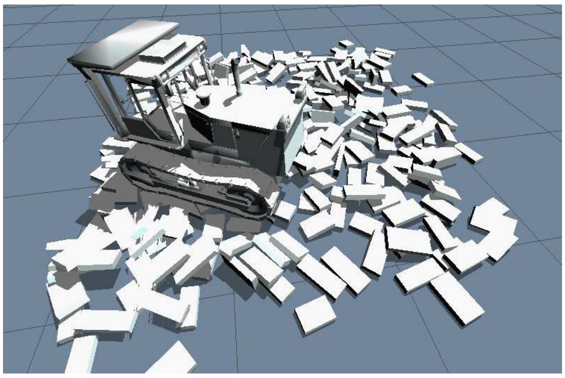

Figure 2: Real-time simulation of a tracked vehicle moving on slabs with rigid frictional contacts.

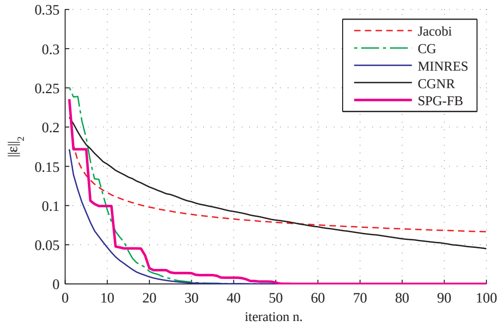

Figure 3: Convergence of benchmark B1: network of 3500 bilateral constraints between 1000 parts.

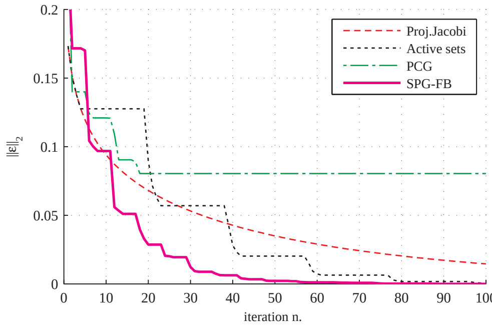

Figure 4: Convergence of benchmark B2: network of 3525 unilateral constraints between 1000 smooth spheres of equal mass.

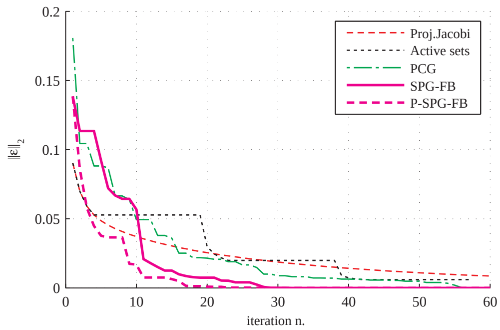

Figure 5: Convergence of benchmark B3: Vertical stack of 20 steel plates, with odd mass ratio.

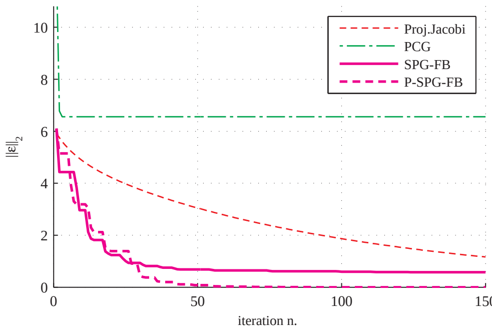

Figure 6: Convergence of benchmark B4: Wrecking ball impacting masonry.

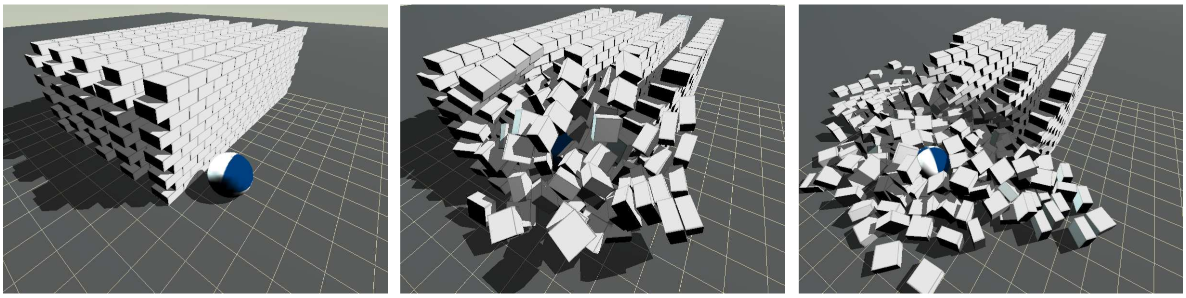

Figure 7: Snapshots from benchmark B4. Simulation of 750 bricks being impacted by a wrecking ball.

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