EFFICIENT SUBSPACE DETECTION FOR HIGH-ORDER MIMO SYSTEMS

Hadi Sarieddeen, Mohammad M. Mansour, and Ali Chehab

Department of Electrical and Computer Engineering
American University of Beirut, Beirut, Lebanon
{has63, mmansour, chehab}@aub.edu.lb

Introduction

We propose two computationally efficient subspace detection algorithms, based on a preprocessing stage that consists of special layer ordering, followed by permutation-robust QR decomposition (QRD) and elementary matrix operations.

Reference MIMO Detection Schemes:

- ZF and MMSE
- ML detection
- Sphere detection
- Subspace detection (SD)
  Decomposes an effective channel matrix into lower order subchannels
  Reduces the number of jointly detected streams
- Layered orthogonal lattice detector (LORD)
  A special class of SD

System Model

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

H: $N \times N$ channel matrix

x: transmitted QAM symbols

n: complex additive white Gaussian noise - zero mean and variance σ^2

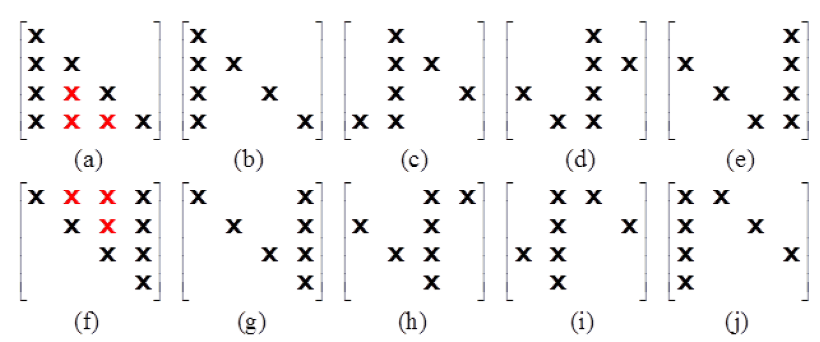
$$\sigma^2 = \frac{N_t}{\text{SNR}}$$

$$d^{\text{ML}} = \min_{\mathbf{x}\in\mathcal{X}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2$$

$$\lambda_{n,k}^{\text{ML}} = \min_{\mathbf{x}\in\mathcal{X}_{n,k}^{(0)}} d(\mathbf{x}) - \min_{\mathbf{x}\in\mathcal{X}_{n,k}^{(1)}} d(\mathbf{x})$$

WR Decomposition

QRD/QLD followed by matrix puncturing



4x4 channel matrix structures: (a,f) LORD; (b-e,g-j) SD

Detection Algorithm

CYSD: Streams are decoupled, one at a time, by cyclically shifting the columns of H and generating the punctured R

$$\tilde{\mathbf{y}} = \mathbf{W}^*\mathbf{y} = \mathbf{R}\mathbf{x} + \mathbf{W}^*\mathbf{n} = \mathbf{R}\mathbf{x} + \tilde{\mathbf{n}}$$

$$\tilde{\mathbf{y}} = \begin{bmatrix}\tilde{\mathbf{y}}_1 \\ \tilde{y}_2\end{bmatrix}, \qquad \mathbf{R} = \begin{bmatrix}\mathbf{A} & \mathbf{b} \\ 0 & c\end{bmatrix}, \qquad \mathbf{x} = \begin{bmatrix}\mathbf{x}_1 \\ x_2\end{bmatrix}$$

$$\mathbf{x}^{\text{WR}} = \arg\min_{\mathbf{x}\in\mathcal{X}} \|\tilde{\mathbf{y}} - \mathbf{R}\mathbf{x}\|^2$$

$$\mathbf{x}^{\text{WR}} = \arg\min_{x_2\in\mathcal{X}_N} \left(|\tilde{y}_2 - c x_2|^2 + \|\tilde{\mathbf{y}}_1 - \mathbf{A}\hat{\mathbf{x}}_1 - \mathbf{b}x_2\|^2\right)$$

$\hat{\mathbf{x}}_1$ is the sliced output of the projection of x_2 on upper layers ($\mathbf{A}$ is real diagonal)

$$\boldsymbol{u}_{n,k}^{\text{WR}} = \arg\min_{\mathbf{x}\in\mathcal{X}_{n,k}^{(0)}} \|\tilde{\mathbf{y}} - \mathbf{R}\mathbf{x}\|^2, \qquad \boldsymbol{v}_{n,k}^{\text{WR}} = \arg\min_{\mathbf{x}\in\mathcal{X}_{n,k}^{(1)}} \|\tilde{\mathbf{y}} - \mathbf{R}\mathbf{x}\|^2$$

$$\lambda_{n,k}^{\text{WR}} = \left\|\tilde{\mathbf{y}} - \mathbf{R}\boldsymbol{u}_{n,k}^{\text{WR}}\right\|^2 - \left\|\tilde{\mathbf{y}} - \mathbf{R}\boldsymbol{v}_{n,k}^{\text{WR}}\right\|^2$$

Proposed Low-Complexity SD

Two proposed layer ordering schemes resulted in two SD schemes:

- **Single-Permutation Subspace Detection (SPSD)**
  Swapping each layer of interest with the last layer
- **Pairwise Subspace Detection (PWSD)**
  Lumping the channel columns in pairs and handling each at a time

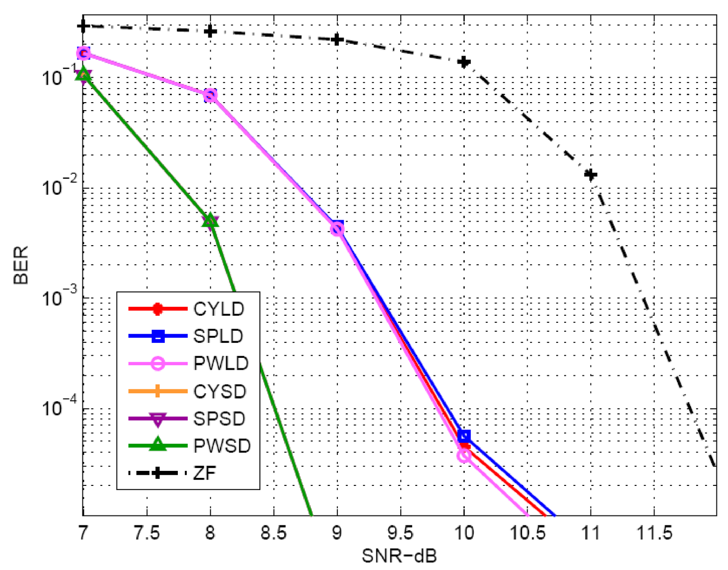
WR decomposition is then carried in two stages:

- **Permutation-Robust QR Decomposition (PR-QRD)**
  With proposed layer ordering, successive decompositions are one swap apart
  Part of the decomposition result remains unaltered

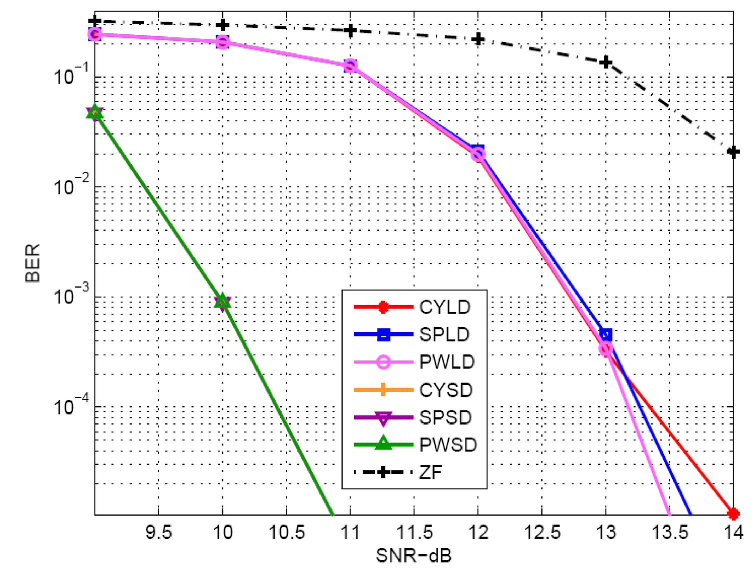
$$\begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44} \end{bmatrix} = \begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{21} & q_{22} & q_{23} & q_{24} \\ q_{31} & q_{32} & q_{33} & q_{34} \\ q_{41} & q_{42} & q_{43} & q_{44} \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ 0 & r_{22} & r_{23} & r_{24} \\ 0 & 0 & r_{33} & r_{34} \\ 0 & 0 & 0 & r_{44} \end{bmatrix}$$

- **Efficient matrix puncturing**
  Puncturing is executed via elementary matrix operations

Results



BER performance of 4x4 MIMO detectors with 16-QAM



BER performance of 8x8 MIMO detectors with 16-QAM

RAD: Real Addition Operation
RML: Real Multiplication Operation

	Permutations	Redundant	Saved
SPSD	1: $\mathbf{h}_1\mathbf{h}_2\mathbf{h}_3\mathbf{h}_4$	none	none
	2: $\mathbf{h}_1\mathbf{h}_2\mathbf{h}_4\mathbf{h}_3$	$q_1, q_2, r_{11}, r_{12}, r_{22}$	88 RML+40 RAD
	3: $\mathbf{h}_1\mathbf{h}_4\mathbf{h}_3\mathbf{h}_2$	q_1 r_{11}	28 RML+8 RAD
	4: $\mathbf{h}_4\mathbf{h}_2\mathbf{h}_3\mathbf{h}_1$	none	none
PWSD	1: $\mathbf{h}_3\mathbf{h}_4\mathbf{h}_1\mathbf{h}_2$	none	none
	2: $\mathbf{h}_3\mathbf{h}_4\mathbf{h}_2\mathbf{h}_1$	$q_1, q_2, r_{11}, r_{12}, r_{22}$	88 RML+40 RAD
	3: $\mathbf{h}_1\mathbf{h}_2\mathbf{h}_3\mathbf{h}_4$	none	none
	4: $\mathbf{h}_1\mathbf{h}_2\mathbf{h}_4\mathbf{h}_3$	$q_1, q_2, r_{11}, r_{12}, r_{22}$	88 RML+40 RAD

PWSD/PWLD	Saved Computations
1: $\mathbf{h}_1\mathbf{h}_2\mathbf{h}_3\mathbf{h}_4\mathbf{h}_5\mathbf{h}_6\mathbf{h}_7\underline{\mathbf{h}_8}$	none
2: $\mathbf{h}_1\mathbf{h}_2\mathbf{h}_3\mathbf{h}_4\mathbf{h}_5\mathbf{h}_6\mathbf{h}_8\underline{\mathbf{h}_7}$	1296 RML+816 RAD
3: $\mathbf{h}_1\mathbf{h}_2\mathbf{h}_3\mathbf{h}_4\mathbf{h}_5\mathbf{h}_8\mathbf{h}_7\underline{\mathbf{h}_6}$	920 RML+560 RAD
4: $\mathbf{h}_1\mathbf{h}_2\mathbf{h}_3\mathbf{h}_4\mathbf{h}_8\mathbf{h}_6\mathbf{h}_7\underline{\mathbf{h}_5}$	608 RML+352 RAD
5: $\mathbf{h}_1\mathbf{h}_2\mathbf{h}_3\mathbf{h}_8\mathbf{h}_5\mathbf{h}_6\mathbf{h}_7\underline{\mathbf{h}_4}$	360 RML+192 RAD
6: $\mathbf{h}_1\mathbf{h}_2\mathbf{h}_8\mathbf{h}_4\mathbf{h}_5\mathbf{h}_6\mathbf{h}_7\underline{\mathbf{h}_3}$	176 RML+80 RAD
7: $\mathbf{h}_1\mathbf{h}_8\mathbf{h}_3\mathbf{h}_4\mathbf{h}_5\mathbf{h}_6\mathbf{h}_7\underline{\mathbf{h}_2}$	56 RML+16 RAD
8: $\mathbf{h}_8\mathbf{h}_2\mathbf{h}_3\mathbf{h}_4\mathbf{h}_5\mathbf{h}_6\mathbf{h}_7\underline{\mathbf{h}_1}$	none
8-Stage PWSD/PWLD	**Saved Computations**
1: $\mathbf{h}_1\mathbf{h}_2\mathbf{h}_3\mathbf{h}_4\mathbf{h}_5\mathbf{h}_6\mathbf{h}_8\underline{\mathbf{h}_7}$	none
2: $\mathbf{h}_1\mathbf{h}_2\mathbf{h}_3\mathbf{h}_4\mathbf{h}_5\mathbf{h}_6\mathbf{h}_7\underline{\mathbf{h}_8}$	1296 RML+816 RAD
3: $\mathbf{h}_1\mathbf{h}_2\mathbf{h}_3\mathbf{h}_4\mathbf{h}_7\mathbf{h}_8\mathbf{h}_6\underline{\mathbf{h}_5}$	608 RML+352 RAD
4: $\mathbf{h}_1\mathbf{h}_2\mathbf{h}_3\mathbf{h}_4\mathbf{h}_7\mathbf{h}_8\mathbf{h}_5\underline{\mathbf{h}_6}$	1296 RML+816 RAD
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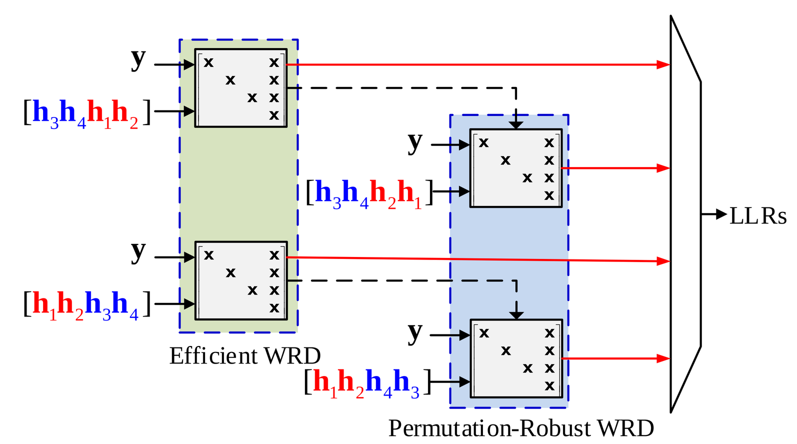
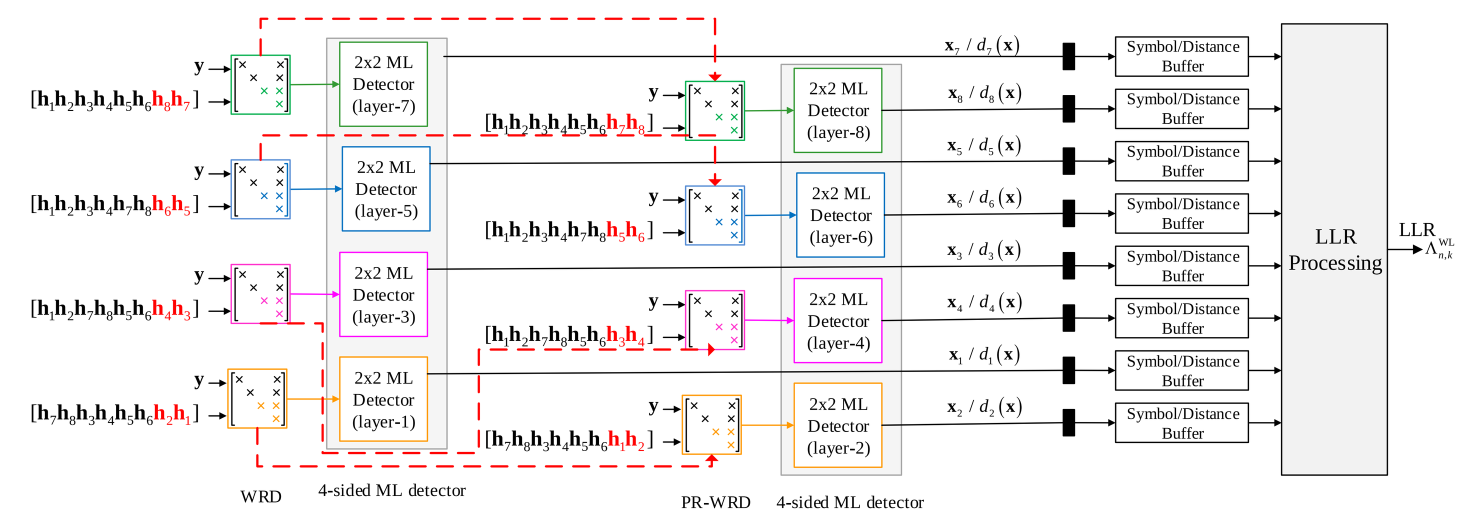
6: $\mathbf{h}_1\mathbf{h}_2\mathbf{h}_7\mathbf{h}_8\mathbf{h}_5\mathbf{h}_6\mathbf{h}_3\underline{\mathbf{h}_4}$	1296 RML+816 RAD
7: $\mathbf{h}_7\mathbf{h}_8\mathbf{h}_3\mathbf{h}_4\mathbf{h}_5\mathbf{h}_6\mathbf{h}_2\underline{\mathbf{h}_1}$	none
8: $\mathbf{h}_7\mathbf{h}_8\mathbf{h}_3\mathbf{h}_4\mathbf{h}_5\mathbf{h}_6\mathbf{h}_1\underline{\mathbf{h}_2}$	1296 RML+816 RAD
2-Stage PWSD/PWLD	**Saved Computations**
1: $\mathbf{h}_1\mathbf{h}_2\mathbf{h}_3\mathbf{h}_4\mathbf{h}_5\mathbf{h}_6\mathbf{h}_8\underline{\mathbf{h}_7}$	none
2: $\mathbf{h}_1\mathbf{h}_2\mathbf{h}_3\mathbf{h}_4\mathbf{h}_5\mathbf{h}_6\mathbf{h}_7\underline{\mathbf{h}_8}$	1296 RML+816 RAD
3: $\mathbf{h}_1\mathbf{h}_2\mathbf{h}_3\mathbf{h}_4\mathbf{h}_7\mathbf{h}_8\mathbf{h}_6\underline{\mathbf{h}_5}$	none
4: $\mathbf{h}_1\mathbf{h}_2\mathbf{h}_3\mathbf{h}_4\mathbf{h}_7\mathbf{h}_8\mathbf{h}_5\underline{\mathbf{h}_6}$	1296 RML+816 RAD
5: $\mathbf{h}_1\mathbf{h}_2\mathbf{h}_7\mathbf{h}_8\mathbf{h}_5\mathbf{h}_6\mathbf{h}_4\underline{\mathbf{h}_3}$	none
6: $\mathbf{h}_1\mathbf{h}_2\mathbf{h}_7\mathbf{h}_8\mathbf{h}_5\mathbf{h}_6\mathbf{h}_3\underline{\mathbf{h}_4}$	1296 RML+816 RAD
7: $\mathbf{h}_7\mathbf{h}_8\mathbf{h}_3\mathbf{h}_4\mathbf{h}_5\mathbf{h}_6\mathbf{h}_2\underline{\mathbf{h}_1}$	none
8: $\mathbf{h}_7\mathbf{h}_8\mathbf{h}_3\mathbf{h}_4\mathbf{h}_5\mathbf{h}_6\mathbf{h}_1\underline{\mathbf{h}_2}$	1296 RML+816 RAD



Architecture for a 2-stage 4x4 MIMO PWSD



Architecture for a 2-stage 8x8 MIMO PWSD

CYLD, SPLD, and PWLD are the LORD versions of CYSD, SPSD, and PWSD, with no matrix puncturing.

Saving in QRD overhead are 30% in 8x8 MIMO, but can reach 50% with very high order systems.

The computational saving came at no performance degradation cost.