## Efficient Three-Party Computation from Cut-and-Choose

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## Background

Secure Computation: Parties $P_{1}, P_{2}, \ldots, P_{n}$ compute some (common) function $f\left(x_{1}, x_{2}, \ldots, x_{n}\right)$ while keeping $x_{1}, x_{2}, \ldots, x_{n}$ private, even if $n-1$ parties are corrupt!


Note: Interested in malicious security, where adversaries can deviate arbitrarily

## Secure Computation: 2PC vs. MPC

Considered separately in the literature:

## 2PC

- Two parties, 1 corruption
- Many efficient constructions
- Most based on garbled circuits
- Boolean circuits
- $\mathcal{O}(1)$ rounds
- Preprocessing time: none
- Online time: fast


## MPC

- $n$ parties, $\leq n-1$ corrupt
- Fewer efficient constructions
- Most efficient scheme: SPDZ
- Arithmetic circuits
- $\mathcal{O}$ (depth) rounds
- Preprocessing time: slow
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Question: Say we want to do secure computation with (fixed) $\boldsymbol{n}>2$. Do we need all the MPC machinery?

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Question: Say we want to do secure computation with $\boldsymbol{n}=3$. Do we need all the MPC machinery?

## Three-Party Computation: Challenges

1. Not 2PC, so not clear that two-party protocols/ideas apply

- e.g., cut-and-choose, oblivious transfer, authenticated bits

2. Do not want to resort to complexity/cost of full MPC

- Only need efficiency for three parties, not arbitrary $n$


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- Requires almost entirely two-party communication
- Only three broadcasts needed
- Existing schemes require broadcast in every round


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- Roughly $8 \times$ more expensive than underlying 2 PC scheme
- Requires almost entirely two-party communication
- Only three broadcasts needed
- Existing schemes require broadcast in every round
- Faster start-to-finish running time versus SPDZ
- SPDZ has faster on-line running time


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Cut-and-Choose: Lifts semi-honest 2PC schemes to malicious security

Cut-and-Choose (High-Level Idea) [LP07]:


## Open half of the circuits

If "checked" circuits constructed correctly, w.h.p. majority of unopened garbled circuits constructed correctly

## 3PC: High-level Idea

How to lift cut-and-choose 2PC protocol to three-party setting:
$\widehat{\pi}(S, R)$ : cut-and-choose 2PC protocol between sender $S$ and receiver $R$

- $S$ generates many garbled circuits using a circuit garbling scheme
- $R$ does cut-and-choose on circuits



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How to lift cut-and-choose 2PC protocol to three-party setting:
We emulate $\widehat{\pi}$ using three parties:

- $P_{1}$ and $P_{2}$ run two-party protocol $\pi$ emulating $S$
- In particular, the circuit garbling scheme of $S$
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- $P_{1}$ and $P_{2}$ run two-party protocol $\pi$ emulating $S$
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Note: using generic 2PC schemes for $\widehat{\pi}$ and $\pi$ not efficient!

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## Recall: (Single-Party) Circuit Garbling Scheme



| Wire | Keys |  | Mask Bit |
| :---: | :---: | :---: | :---: |
| $\alpha$ | $K_{\alpha, 0}$ | $K_{\alpha, 1}$ | $\lambda_{\alpha}$ |
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Garbled Gate:

| 0 | 0 | $\operatorname{Enc}_{K_{\alpha, 0}, K_{\beta, 0}}\left(K_{\gamma, G\left(\lambda_{\alpha}, \lambda_{\beta}\right) \oplus \lambda_{\gamma}} \\| G\left(\lambda_{\alpha}, \lambda_{\beta}\right) \oplus \lambda_{\gamma}\right)$ |
| :--- | :--- | :--- |
| 0 | 1 | $\operatorname{Enc}_{K_{\alpha, 0}, K_{\beta, 1}}\left(K_{\gamma, G\left(\lambda_{\alpha}, \lambda_{\beta} \oplus 1\right) \oplus \lambda_{\gamma}} \\| G\left(\lambda_{\alpha}, \lambda_{\beta} \oplus 1\right) \oplus \lambda_{\gamma}\right)$ |
| 1 | 0 | $\operatorname{Enc}_{K_{\alpha, 1}, K_{\beta}, 0}\left(K_{\gamma, G\left(\lambda_{\alpha} \oplus 1, \lambda_{\beta}\right) \oplus \lambda_{\gamma}} \\| G\left(\lambda_{\alpha} \oplus 1, \lambda_{\beta}\right) \oplus \lambda_{\gamma}\right)$ |
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Note: This is standard Yao using point-and-permute

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\left.\left.\begin{array}{lll}
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Note: Garbling party knows keys/tags being encrypted

## Distributing the Garbling Scheme

Goal: $P_{1}$ and $P_{2}$ together compute garbled circuit such that no party "knows" garbled values

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1. Obliviousness

- Neither party should know key/tag being encrypted


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- If one party malicious, garbled circuit evaluation must either:
- Compute correct answer
- Abort, independent of honest party's input


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Solution: Combine distributed encryption [DIO5] with authenticated bit shares [NNOB12]

## Building Blocks (1): Distributed Encryption Scheme [DI05]

Goal: $P_{1}$ and $P_{2}$ want to encrypt secret shared message $[\mathrm{m}]=m_{1} \oplus m_{2}$ using keys $K_{1}, K_{2}$

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$$
\operatorname{Enc}_{K_{1}, K_{2}}([m])=\left(m_{1} \oplus F_{s_{1}^{1}}^{1}(0) \oplus F_{s_{2}^{1}}^{2}(0), m_{2} \oplus F_{s_{1}^{2}}^{1}(0) \oplus F_{s_{2}^{2}}^{2}(0)\right)
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Note: Encryption is local!
Note: Cost per party (in PRF calls) to encrypt message of length $\ell$ is $2 \ell$

## Building Blocks (2): Functionalities Needed

$\langle\cdot\rangle$ denotes (form of authenticated and linear) bit secret sharing
Note: [•] denotes (standard) secret sharing
$\langle\cdot\rangle^{(i)},[\cdot]^{(i)}$ denotes $P_{i}$ s share

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## Building Blocks (2): Functionalities Needed

Note: efficient maliciously secure constructions exist

- Uses ideas from [NNOB12]
- See paper for details


## Example: Garbling an AND Gate

$$
\lambda_{\alpha}=1, \lambda_{\beta}=0, \lambda_{\gamma}=1
$$

Standard (single-party) garbling:
Step 1: $S$ computes tags:

$$
\begin{array}{ccc}
i & j & A N D\left(\lambda_{\alpha} \oplus i, \lambda_{\beta} \oplus j\right) \oplus \lambda_{\gamma} \\
\hline 0 & 0 & A N D(1 \oplus 0,0 \oplus 0) \oplus 1=1 \\
0 & 1 & A N D(1 \oplus 0,0 \oplus 1) \oplus 1=0 \\
1 & 0 & A N D(1 \oplus 1,0 \oplus 0) \oplus 1=1 \\
1 & 1 & A N D(1 \oplus 1,0 \oplus 1) \oplus 1=1
\end{array}
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## Distributed garbling:

Step 1: $P_{1}$ and $P_{2}$ compute oblivious sharings of tags:

| $i$ | $j$ | $\left\langle A N D\left(\lambda_{\alpha} \oplus i, \lambda_{\beta} \oplus j\right) \oplus \lambda_{\gamma}\right\rangle$ |
| :---: | :---: | :---: |
| 0 | 0 | $\mathcal{F}_{\text {gate }}^{\text {AND }}(\langle 1\rangle \oplus\langle 0\rangle,\langle 0\rangle \oplus\langle 0\rangle) \oplus\langle 1\rangle=\langle 1\rangle$ |
| 0 | 1 | $\mathcal{F}_{\text {gate }}^{\text {AND }}(\langle 1\rangle \oplus\langle 0\rangle,\langle 1\rangle \oplus\langle 1\rangle) \oplus\langle 1\rangle=\langle 0\rangle$ |
| 1 | 0 | $\mathcal{F}_{\text {gate }}^{\text {gate }}(\langle 1\rangle \oplus\langle 1\rangle,\langle 0\rangle \oplus\langle 0\rangle) \oplus\langle 1\rangle=\langle 1\rangle$ |
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## Example: Garbling an AND Gate



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$$

## Standard (single-party) garbling:

Step 2: $S$ encrypts key + tag:

$$
\begin{array}{ccl}
i & j & \\
\hline 0 & 0 & \operatorname{Enc}_{K_{\alpha, 0}, K_{\beta, 0}}\left(K_{\gamma, 1} \| 1\right) \\
0 & 1 & \operatorname{Enc}_{K_{\alpha, 0}, K_{\beta, 1}}\left(K_{\gamma, 0} \| 0\right) \\
1 & 0 & \text { Enc }_{K_{\alpha, 1}, K_{\beta, 0}}\left(K_{\gamma, 1} \| 1\right) \\
1 & 1 & \text { Enc }_{K_{\alpha, 1}, K_{\beta, 1}}\left(K_{\gamma, 1} \| 1\right)
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$$

## Distributed garbling:

Step 2a: $P_{1}$ and $P_{2}$ compute oblivious sharings of each party's sub-keys:

| $i$ | $j$ |  |  |
| :---: | :---: | :--- | :--- |
| 0 | 0 | $\mathcal{F}_{\text {oshare }}^{1}\left(\langle 1\rangle, s_{\gamma, 0}^{1}, s_{\gamma, 1}^{1}\right)=\left[s_{\gamma, 1}^{1}\right]$ | $\mathcal{F}_{\text {oshare }}^{2}\left(\langle 1\rangle, s_{\gamma, 0}^{2}, s_{\gamma, 1}^{2}\right)=\left[s_{\gamma, 1}^{2}\right]$ |
| 0 | 1 | $\mathcal{F}_{\text {oshare }}^{1}\left(\langle 0\rangle, s_{\gamma, 0}^{1}, s_{\gamma, 1}^{1}\right)=\left[s_{\gamma, 0}^{1}\right]$ | $\mathcal{F}_{\text {osshare }}^{2}\left(\langle 0\rangle, s_{\gamma, 0}^{2}, s_{\gamma, 1}^{2}\right)=\left[s_{\gamma, 0}^{2}\right]$ |
| 1 | 0 | $\mathcal{F}_{\text {oshare }}^{1}\left(\langle 1\rangle, s_{\gamma, 0}^{1}, s_{\gamma, 1}^{2}\right)=\left[s_{\gamma, 1}^{1}\right]$ | $\mathcal{F}_{\text {oshare }}^{2}\left(\langle 1\rangle, s_{\gamma, 0}^{2}, s_{\gamma, 1}^{2}\right)=\left[s_{\gamma, 1}^{2}\right]$ |
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## Distributed garbling:

Step 2b: $P_{1}$ and $P_{2}$ use distributed encryption to encrypt:

| $i$ | $j$ |  |
| :---: | :---: | :---: |
| 0 | 0 | $\operatorname{Enc}_{K_{\alpha, 0}, K_{\beta, 0}}\left(\left[s_{\gamma, 1}^{1}\right]\left\\|\left[s_{, 1}^{2}\right]\right\\|\langle 1\rangle\right)$ |
| 0 | 1 | $\operatorname{Enc}_{K_{\alpha, 0}, K_{\beta, 1}}\left(\left[s_{\gamma, 0}^{1}\right]\left\\|\left[s_{, 0}^{2}\right]\right\\|\langle 0\rangle\right)$ |
| 1 | 0 | Enc $_{K_{\alpha, 1}, K_{\beta, 0}}\left(\left[s_{\gamma, 1}^{1}\right]\left\\|\left[s_{\gamma, 1}^{2}\right]\right\\|\langle 1\rangle\right)$ |
| 1 | 1 | $\operatorname{Enc}_{K_{\alpha, 1}, K_{\beta, 1}}\left(\left[s_{\gamma, 1}^{1}\right]\left\\|\left[s_{\gamma, 1}^{2}\right]\right\\|\langle 1\rangle\right)$ |

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## Distributed garbling:

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| 1 | 0 | Enc $_{K_{\alpha, 1}, K_{\beta, 0}}\left(\left[s_{\gamma, 1}^{1}\right]\left\\|\left[s_{\gamma, 1}^{2}\right]\right\\|\langle 1\rangle\right)$ |
| 1 | 1 | Enc $_{K_{\alpha, 1}, K_{\beta, 1}}\left(\left[s_{\gamma, 1}^{1}\right]\left\\|\left[s_{\gamma, 1}^{2}\right]\right\\|\langle 1\rangle\right)$ |

Note: Cost of encryption $8 \times$ cost in 2 PC setting

## 3PC: Main Steps

## Two main steps:

1. Distribute S's circuit garbling scheme between two parties
2. Modify existing 2PC protocol to use distributed circuit garbling scheme

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- Take existing cut-and-choose protocol (e.g., [LP07, LP11, Lin13])
- Replace sender's circuit generation by distributed circuit generation


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Note: Not exactly this straightforward; see paper for details

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## Security Intuition

- Exactly one of $P_{1}$ or $P_{2}$ malicious: garbled circuits either correct or abort independent of input
- Both $P_{1}$ and $P_{2}$ malicious: cut-and-choose by $P_{3}$ detects cheating
- $P_{3}$ malicious: covered by security of garbling protocol


## Summary

Can "lift" cut-and-choose 2PC protocols to 3PC setting

- Provides efficient constant round 3PC protocol
- Only $\approx 8 \times$ slower than underlying 2PC protocol
- Approach works for combination of [LP07, LP11] and [Lin13]
- Only three broadcast calls needed
- Important in WAN settings where broadcast is expensive
- Faster start-to-finish time than existing 3PC solutions


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## Future Work:

- Support free-XOR
- Optimize distributed encryption scheme (à la JustGarble [BHKR13])


## Thank You

## Any questions?

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