Efficient Three-Party Computation from Cut-and-Choose

Seung Geol Choi^1 and Jonathan Katz² and Alex J. Malozemoff² and Vassilis Zikas³ $\ensuremath{\mathsf{Z}}$

¹United States Naval Academy

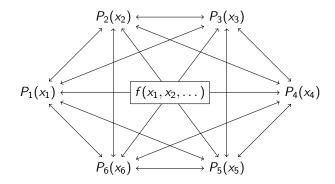
²University of Maryland

³ETH Zurich

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Background

Secure Computation: Parties $P_1, P_2, ..., P_n$ compute some (common) function $f(x_1, x_2, ..., x_n)$ while keeping $x_1, x_2, ..., x_n$ private, even if n - 1 parties are corrupt!



Note: Interested in **malicious** security, where adversaries can deviate *arbitrarily*

Secure Computation: 2PC vs. MPC

Considered separately in the literature:

2PC

- Two parties, 1 corruption
- Many efficient constructions
- Most based on *garbled circuits*
 - Boolean circuits
 - *O*(1) rounds
 - Preprocessing time: none
 - Online time: fast

MPC

- n parties, $\leq n-1$ corrupt
- Fewer efficient constructions
- Most efficient scheme: SPDZ
 - Arithmetic circuits
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Question: Say we want to do secure computation with (fixed) n > 2. Do we need all the MPC machinery?

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Question: Say we want to do secure computation with n = 3. Do we need all the MPC machinery?

Three-Party Computation: Challenges

- 1. Not 2PC, so not clear that two-party protocols/ideas apply
 - e.g., cut-and-choose, oblivious transfer, authenticated bits
- 2. Do not want to resort to complexity/cost of full MPC
 - Only need efficiency for three parties, not arbitrary n

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Efficient $\mathcal{O}(1)$ -round maliciously-secure 3PC protocol for Boolean circuits

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- Requires almost entirely two-party communication
 - Only three broadcasts needed
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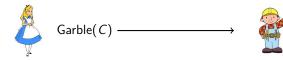
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 - Roughly $8 \times$ more expensive than underlying 2PC scheme
- Requires almost entirely two-party communication
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- Faster start-to-finish running time versus SPDZ
 - SPDZ has faster on-line running time

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Semi-Honest 2PC (High-Level Idea):



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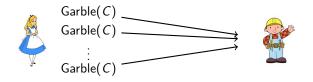






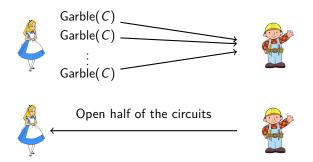
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Cut-and-Choose (High-Level Idea) [LP07]:



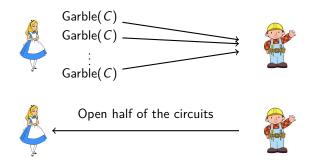
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Cut-and-Choose (High-Level Idea) [LP07]:



If "checked" circuits constructed correctly, w.h.p. majority of unopened garbled circuits constructed correctly

3PC: High-level Idea

How to lift cut-and-choose 2PC protocol to three-party setting:

 $\widehat{\pi}(S,R)$: cut-and-choose 2PC protocol between sender S and receiver R

- S generates many garbled circuits using a *circuit garbling scheme*
- R does cut-and-choose on circuits

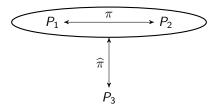


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We emulate $\widehat{\pi}$ using three parties:

- P_1 and P_2 run two-party protocol π emulating S
 - In particular, the *circuit garbling scheme* of S
- P₃ plays role of R

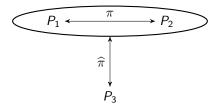


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Note: using generic 2PC schemes for $\hat{\pi}$ and π not efficient!

3PC: Main Steps

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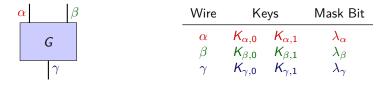
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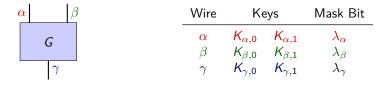
Wire	Keys		Mask Bit
lpha	$K_{lpha,0}$	$K_{\alpha,1}$	λ_{lpha}
β	$K_{eta,0}$	$K_{eta,1}$	λ_eta
γ	$K_{\gamma,0}$	$K_{\gamma,1}$	λ_γ



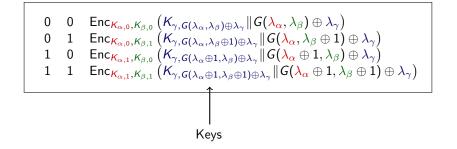
Garbled Gate:

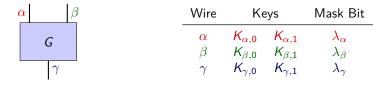
$$\begin{array}{lll} 0 & 0 & \operatorname{Enc}_{\boldsymbol{K}_{\alpha,0},\boldsymbol{K}_{\beta,0}}\left(\boldsymbol{K}_{\gamma,\boldsymbol{G}(\lambda_{\alpha},\lambda_{\beta})\oplus\lambda_{\gamma}}\|\boldsymbol{G}(\boldsymbol{\lambda}_{\alpha},\lambda_{\beta})\oplus\lambda_{\gamma}\right)\\ 0 & 1 & \operatorname{Enc}_{\boldsymbol{K}_{\alpha,0},\boldsymbol{K}_{\beta,1}}\left(\boldsymbol{K}_{\gamma,\boldsymbol{G}(\lambda_{\alpha},\lambda_{\beta}\oplus1)\oplus\lambda_{\gamma}}\|\boldsymbol{G}(\boldsymbol{\lambda}_{\alpha},\lambda_{\beta}\oplus1)\oplus\lambda_{\gamma}\right)\\ 1 & 0 & \operatorname{Enc}_{\boldsymbol{K}_{\alpha,1},\boldsymbol{K}_{\beta,0}}\left(\boldsymbol{K}_{\gamma,\boldsymbol{G}(\lambda_{\alpha}\oplus1,\lambda_{\beta})\oplus\lambda_{\gamma}}\|\boldsymbol{G}(\boldsymbol{\lambda}_{\alpha}\oplus1,\lambda_{\beta})\oplus\lambda_{\gamma}\right)\\ 1 & 1 & \operatorname{Enc}_{\boldsymbol{K}_{\alpha,1},\boldsymbol{K}_{\beta,1}}\left(\boldsymbol{K}_{\gamma,\boldsymbol{G}(\lambda_{\alpha}\oplus1,\lambda_{\beta}\oplus1)\oplus\lambda_{\gamma}}\|\boldsymbol{G}(\boldsymbol{\lambda}_{\alpha}\oplus1,\lambda_{\beta}\oplus1)\oplus\lambda_{\gamma}\right)\end{array}$$

Note: This is standard Yao using point-and-permute

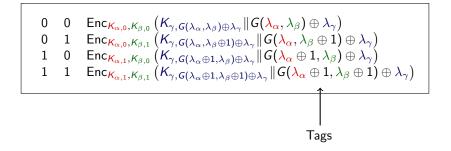


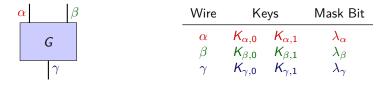
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Note: Garbling party knows keys/tags being encrypted

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Solution: Combine distributed encryption [DI05] with authenticated bit shares [NNOB12]

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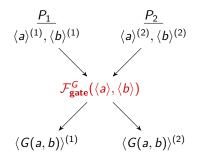
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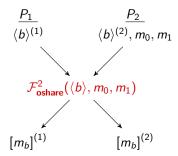
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Note: Cost per party (in PRF calls) to encrypt message of length ℓ is 2ℓ

$\langle \cdot \rangle$ denotes (form of authenticated and linear) bit secret sharing **Note:** [·] denotes (standard) secret sharing $\langle \cdot \rangle^{(i)}$, [·]⁽ⁱ⁾ denotes P_i s share



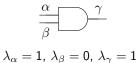
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Note: efficient maliciously secure constructions exist

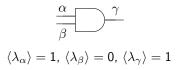
- Uses ideas from [NNOB12]
- See paper for details



Standard (single-party) garbling:

Step 1: S computes tags:

 $\begin{array}{cccc} i & j & AND(\lambda_{\alpha} \oplus i, \lambda_{\beta} \oplus j) \oplus \lambda_{\gamma} \\ \hline 0 & 0 & AND(1 \oplus 0, 0 \oplus 0) \oplus 1 = 1 \\ 0 & 1 & AND(1 \oplus 0, 0 \oplus 1) \oplus 1 = 0 \\ 1 & 0 & AND(1 \oplus 1, 0 \oplus 0) \oplus 1 = 1 \\ 1 & 1 & AND(1 \oplus 1, 0 \oplus 1) \oplus 1 = 1 \end{array}$



Distributed garbling:

Step 1: *P*₁ and *P*₂ compute *oblivious sharings* of tags:

$$\begin{array}{c|cccc} i & j & \langle AND(\lambda_{\alpha} \oplus i, \lambda_{\beta} \oplus j) \oplus \lambda_{\gamma} \rangle \\ \hline 0 & 0 & \mathcal{F}_{gate}^{AND}(\langle 1 \rangle \oplus \langle 0 \rangle, \langle 0 \rangle \oplus \langle 0 \rangle) \oplus \langle 1 \rangle = \langle 1 \rangle \\ 0 & 1 & \mathcal{F}_{gate}^{AND}(\langle 1 \rangle \oplus \langle 0 \rangle, \langle 1 \rangle \oplus \langle 1 \rangle) \oplus \langle 1 \rangle = \langle 0 \rangle \\ 1 & 0 & \mathcal{F}_{gate}^{AND}(\langle 1 \rangle \oplus \langle 1 \rangle, \langle 0 \rangle \oplus \langle 0 \rangle) \oplus \langle 1 \rangle = \langle 1 \rangle \\ 1 & 1 & \mathcal{F}_{gate}^{AND}(\langle 1 \rangle \oplus \langle 1 \rangle, \langle 0 \rangle \oplus \langle 1 \rangle) \oplus \langle 1 \rangle = \langle 1 \rangle \end{array}$$

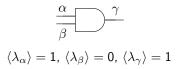


$$\lambda_lpha=$$
 1, $\lambda_eta=$ 0, $\lambda_\gamma=$ 1

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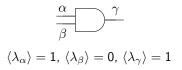
Step 2: *S* encrypts key + tag:

$$\begin{array}{c|cccc} i & j \\ \hline 0 & 0 & \mathsf{Enc}_{K_{\alpha,0},K_{\beta,0}}(K_{\gamma,1}\|1) \\ 0 & 1 & \mathsf{Enc}_{K_{\alpha,0},K_{\beta,1}}(K_{\gamma,0}\|0) \\ 1 & 0 & \mathsf{Enc}_{K_{\alpha,1},K_{\beta,0}}(K_{\gamma,1}\|1) \\ 1 & 1 & \mathsf{Enc}_{K_{\alpha,1},K_{\beta,1}}(K_{\gamma,1}\|1) \end{array}$$



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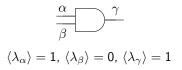
Step 2a: P_1 and P_2 compute *oblivious sharings* of each party's sub-keys:



Distributed garbling:

Step 2b: P_1 and P_2 use *distributed encryption* to encrypt:

$$\begin{array}{c|ccccc} i & j \\ \hline 0 & 0 & \operatorname{Enc}_{K_{\alpha,0},K_{\beta,0}}(\left[s_{\gamma,1}^{1}\right] \parallel \left[s_{\gamma,1}^{2}\right] \parallel \langle 1 \rangle) \\ 0 & 1 & \operatorname{Enc}_{K_{\alpha,0},K_{\beta,1}}(\left[s_{\gamma,0}^{1}\right] \parallel \left[s_{\gamma,0}^{2}\right] \parallel \langle 0 \rangle) \\ 1 & 0 & \operatorname{Enc}_{K_{\alpha,1},K_{\beta,0}}(\left[s_{\gamma,1}^{1}\right] \parallel \left[s_{\gamma,1}^{2}\right] \parallel \langle 1 \rangle) \\ 1 & 1 & \operatorname{Enc}_{K_{\alpha,1},K_{\beta,1}}(\left[s_{\gamma,1}^{1}\right] \parallel \left[s_{\gamma,1}^{2}\right] \parallel \langle 1 \rangle) \end{array}$$



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Note: Cost of encryption $8 \times \text{cost}$ in 2PC setting

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High-Level Idea

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Note: Not exactly this straightforward; see paper for details

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Security Intuition

- Exactly one of P₁ or P₂ malicious: garbled circuits either correct or abort independent of input
- Both P_1 and P_2 malicious: cut-and-choose by P_3 detects cheating
- *P*₃ malicious: covered by security of garbling protocol

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- Approach works for combination of [LP07, LP11] and [Lin13]
- Only three broadcast calls needed
 - Important in WAN settings where broadcast is expensive
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Future Work:

- Support free-XOR
- Optimize distributed encryption scheme (à la JustGarble [BHKR13])



Any questions?

E-mail: amaloz@cs.umd.edu URL: https://www.cs.umd.edu/~amaloz ePrint: https://eprint.iacr.org/2014/128