EFFICIENT WATER USE IN CALIFORNIA: ECONOMIC MODELING OF GROUNDWATER DEVELOPMENT WITH APPLICATIONS TO GROUNDWATER MANAGEMENT

PREPARED IN PART FOR THE CALIFORNIA STATE ASSEMBLY RULES COMMITTEE
AND IN PART UNDER A GRANT FROM THE ROCKEFELLER FOUNDATION

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PREFACE

This report is one of seven documenting the findings of Rand's study of water use efficiency in California. The study was commissioned by the California State Assembly in the autumn of 1976 and was supported in part by a grant from the Rockefeller Foundation. Its purpose was to examine current water use efficiency and to suggest ways to improve it. The focus of the study was deliberately set statewide, although particular areas of the state receive attention in some reports. It was not designed to solve problems that have drawn recent attention, such as the 1976-77 drought and its exacerbation of the groundwater overdraft problem in the San Joaquin Valley. Rather, the focus has been widened across a broad expanse of time and across all sources and uses of water. It is a study of long-range water use problems and solutions, rather than of immediately perceived short-term problems.

This report in the series develops simple economic models for groundwater development and discusses some of their implications for policymakers considering ways to improve the efficiency of water use. The report analyzes the long-run qualitative implications of both unmanaged and socially optimal groundwater development. The companion reports are:

Efficient Water Use in California: Executive Summary, by Charles E. Phelps, Morlie H. Graubard, David L. Jaquette, Albert J. Lipson, Nancy Y. Moore, Robert Shishko, and Bruce Wetzel, R-2385-CSA/RF, November 1978.

Efficient Water Use in California: Water Rights, Water Districts, and Water Transfers, by Charles E. Phelps, Nancy Y. Moore, and Morlie H. Graubard, R-2386-CSA/RF, November 1978.

Efficient Water Use in California: Groundwater Use and Management, by David L. Jaquette and Nancy Y. Moore, with Albert J. Lipson, R-2387/1-CSA/RF, November 1978.

Efficient Water Use in California: The Evolution of Groundwater Management in Southern California, by Albert J. Lipson, R-2387/2-CSA/RF, November 1978.

Efficient Water Use in California: Conjunctive Management of Ground and Surface Reservoirs, by David L. Jaquette, R-2389-CSA/RF, November 1978.

Efficient Water Use in California: Water Supply Planning, by Nancy Y. Moore, Morlie H. Graubard, and Robert Shishko, R-2390-CSA/RF, November 1978.

These reports should be of interest to those concerned with California water policy and its implications for other states and the nation.

SUMMARY

This report provides a theoretical economic analysis of the "common-pool" problems associated with groundwater development. The problem arises when several producers who make independent economic decisions develop groundwater from the same aquifer or basin.

We approach the problem through an analysis of the individual net-benefit-maximizing behavior of producers over time when they make pumping-rate decisions. Our analysis shows that individual producers will only partially internalize the costs of using groundwater stocks. When a large number of producers each have a small relative share of the total pumping, they will tend to ignore the effect of stored groundwater on their pumping costs. The incentive to correctly value groundwater resources exists only in the case where a single producer makes all pumping-rate decisions.

From this initial analysis, we consider how a social manager (i.e., a central manager for groundwater pumping) for the basin's producers could provide economic incentives to achieve optimal groundwater development. We consider primarily the use of optimal pump taxes, with some discussion of optimal quota assignments. The optimal pump tax for a producer depends on

- An appropriate rate of discount,
- Aggregate pumping costs per foot of lift,
- The response of the water table level to pumping, and
- The fraction of the social marginal value of groundwater stocks he *fails* to internalize.

When some of the basin's producers already partially internalize the value of groundwater stocks, a set of individualized tax rates should be used to reflect this behavior. Otherwise, a single tax rate would be optimal.

A workable management program based on pump taxation must also include a mechanism for tax revenue redistribution. This is a necessary ingredient, since all producers would not necessarily benefit from groundwater management with some forms of tax distribution. We discuss the following two types of redistribution mechanisms for pump tax revenues under simplified economic and hydrological conditions to illustrate the basic effects of the programs:

- A per-unit subsidy for surface water use, and
- A per-unit subsidy for land use.

The first leads to a net increase in total water use through increased surface water imports; the second increases land use by producers. The effect of the second mechanism on water use depends on the degree to which water and land are complements or substitutes. If they are complements, water use will increase, while if they are substitutes, water use will decrease.

If tradable pumping quotas are assigned to each producer in lieu of a taxation policy, producers will not necessarily reach the socially efficient allocation of quotas through trade. Even if socially optimal quotas are assigned, there may be economic

incentives for producers to trade away from this allocation when they differ in their private marginal costs for pumping an additional quota unit.

The key element in any successful groundwater management program is cooperation by the basin's producers. This report shows that groundwater management tools based on properly designed economic incentives will help ensure this cooperation.

ACKNOWLEDGMENTS

I would like to thank Glenn Gotz, of The Rand Corporation, Bart McGuire, and Rodney Smith for their helpful reviews of my original manuscript. Based on their comments, I performed a major revision which constitutes the present report. I am solely responsible for any remaining errors. Finally, I want to thank Charles Phelps for providing me the opportunity to work on the "groundwater management problem" as part of the Rand study of improved efficiency of water use in California.

CONTENTS

I. INTRODUCTION

The production of pumped groundwater, like any other technical production process, results from the combination of several significant inputs or factors. To develop groundwater from an aquifer, a producer typically relies on a well to lift the water from soil or rock below the land surface. The well, together with its pump and motor, represent capital inputs. The energy used to operate the well's pump and motor represents another, though more variable, input. Less obviously, the water table level also represents an input, since it too affects the costs of groundwater production. However, the water table level, unlike the other factors mentioned, cannot generally be controlled by any one producer. When more than one producer develops groundwater from the same aquifer, the water table level (and, equivalently, the water in storage) represents a shared factor of production. In the absence of appropriate legal and economic incentives, individual producers will tend to overdevelop their commonly held groundwater resources.[1]

This report presents an analysis of the economic aspects of groundwater resource development, with attention to the economic incentives implied under differing concepts of property rights to groundwater.[2] The results we obtain provide conceptual answers to the following:

- What is the socially optimal way for a basin's producers to value their groundwater resources?
- What economic management tools could a social manager employ to encourage optimal development of the basin's groundwater?

By answering the first question we obtain a way to assign an economic value to the water table level as a factor in groundwater production. Answers to the second question provide the basis for designing a groundwater management program. Whether or not the program is workable will, of course, depend on its attractiveness to the basin's producers. Each producer will view the program as an investment opportunity and will evaluate it in much the same way that he would any other potential investment. We show that by participating in a properly managed groundwater program, a producer essentially makes an investment in a future higher water table level. Thus, the economic incentives of the program must be designed

[1] This shared nature of a basin's groundwater resources is an example of the general "common-pool" problem exhibited in the economic development of many natural resources. For example, oil extraction represents development of a nonreplenishable natural resource, while fishing represents development of a replenishable natural resource. For this study, we consider groundwater resources as a replenishable natural resource. Whether or not this is actually the case depends on the particular climatic and hydrological conditions of the groundwater basin under study. A fairly extensive literature on the economics of natural-resource production exists. Notable general treatments are those by Smith (1968), Herfindahl and Knesse (1974), and Clark (1976).

[2] In this report, we use the phrase "property right to groundwater" to mean the right to derive income from groundwater as a natural resource. We assume that a producer acquires this right by virtue of his pumping activity. We distinguish between two cases: one where he must first extract the water to derive income from it, and one where he is issued a marketable pumping right of a fixed amount which is valid for a fixed time period. In the first case, the right is uncertain in both quantity and time, while in the second case, the right is certain in both dimensions.

so that they will command enough political support to gain the cooperation of the basin's producers.[3]

The approach we take to the groundwater management problem relies principally on work done by Brown and McGuire (1967) and by Brown and Deacon (1972). Both papers derived an optimal taxation policy for groundwater use based on aggregate benefits and costs, and on a simple specification of the hydrology. We depart from these two treatments by considering individual producers and their wells and a somewhat more general specification for the groundwater hydrology.

The basic problem we consider in Sec. II deals with how a basin's producers value the water table level as a factor in groundwater production. To approach this problem, we formulate and solve a deterministic model of groundwater development for an aquifer where an arbitrary number of producers extract water for their own use. We treat time as a continuous variable. Our solution methods derive from optimal control theory.

Our model supposes that a producer accounts for the effects of his own wells' pumping on the water table level for the basin. He maximizes his total discounted net benefit subject to this constraint. We term a producer who behaves according to this model a "foresighted" producer.

The solution of the optimization problem of this model reveals that producers value the water table level in terms of the costs in perpetuity of pumping water from a lower level. For a producer, this imputed value consists of pumping costs for his own wells. He gives no consideration to the effects of his own pumping on extraction costs of other producers. Only in the case of sole ownership of the basin's groundwater does the producer's private net-benefit-maximizing behavior attribute the optimal value to the water table level or groundwater stocks.

In Sec. III, the sole-ownership case serves as our basis for setting optimal pump tax rates or optimal pumping-rate quota assignments. Which of these economic management tools might work better depends upon the prevailing legal system for groundwater. Under a system where a producer can derive income from groundwater only by first extracting it from the aquifer, a pump tax policy would be appropriate. Alternatively, with a legal system that allows producers to derive income from groundwater whether or not it is pumped, assignment of tradable pumping-rate quotas would be an alternative policy. We discuss the incentives for trading under both types of legal systems.

Section IV deals with mechanisms for pump tax redistribution. We consider two alternatives:

- Pump taxation with a per-unit subsidy for imported surface water, and
- Pump taxation with a per-unit subsidy for land.

The first alternative represents taxation of one factor of production and redistribution of these taxes through another closely substitutable factor.[4] In this case, total water use and, more specifically, imported surface water use expands. The second

[3] With or without complete agreement for the program, there are incentives for individual producers to cheat, so measures for enforcement will be needed in addition to the usual administrative and monitoring procedures (e.g., water table measurement and pump metering).

[4] For simplicity in our analysis, we assume that groundwater and surface water have the same productivity. In principle, we could handle differences in water quality by letting each have a separate marginal productivity, but for our analysis we assume that water quality is not an issue.

alternative illustrates an example of pump tax redistribution through a factor which may be complementary or substitutable (though not to the extent of surface water) with groundwater. In this case, a per-unit subsidy for land leads to increased land use, while the effect on water use depends on the relationship between land and water.

In Sec. V, we present the conclusions from our analyses that would be important considerations for implementing a groundwater management program.

II. BASIC MATHEMATICAL MODELS FOR GROUNDWATER EXTRACTION

In this section, we formulate a simple model for groundwater extraction. We assume that the groundwater basin consists of a single isolated aquifer and that the costs of operating each well depend on the rate of extraction and the depth to water from the land surface where the well is located. Pumping costs increase as each well's rate of pumping increases and as the pumping lift (i.e., depth to water) increases. However, the costs of operation of the different wells are not independent, since the depth to water at a well is influenced by the total rate of extraction from the aquifer.

Assume that there is an initial stock of water in the aquifer and that the initial depth of the water table level is specified for each well. Groundwater stocks change continuously with time, and the instantaneous time rate of change of stocks depends on the difference between the total rate of extraction and the natural rate of replenishment. Symbolically, we have

$X(0)$ = the initial stock of water in the aquifer, (2.1)

$z_i(0)$ = the initial water table level for well i, $i = 1, 2, \ldots, n$,

$R(t)$ = the instantaneous rate of natural replenishment to the aquifer at time $t \geq 0$,

$Q_i(t)$ = the instantaneous rate of extraction of water from the i^{th} well at time $t \geq 0$,

$Q(t)$ = the total rate of extraction at time $t \geq 0$, equal to $\sum_j Q_j(t)$,

$X(t) = X(0) + \int_0^t [R(u) - Q(u)]\, du$, the stock of water in the aquifer at time $t \geq 0$,

$\dot{X}(t) = (dX(t))/dt = R(t) - Q(t)$, the instantaneous rate of change of groundwater stocks with time.

Finally, we define the depth to water at time t for the i^{th} well:

$$d_i(t) = z_i(0) + f_i(Q_i(t)) + \beta[X(0) - X(t)]^{\gamma}, \text{ for } X(0) > X(t) \tag{2.2}$$

and

$$d_i(t) = z_i(0) + f_i(Q_i(t)) - \beta[X(t) - X(0)]^{\gamma}, \text{ for } X(t) > X(0).$$

The first term on the right-hand side of Eq. (2.2) was defined previously as the initial water table level and represents the depth from the land surface to the initial level of water-bearing soil. The second term, $f_i(Q_i(t))$, represents the "drawdown" at the well from pumping at a rate $Q_i(t)$. The function $f_i(\cdot)$ is an increasing function of its argument. For our later work, we assume that $f_i(\cdot)$ varies directly with $Q_i(t)$:[1]

$$f_i(Q_i(t)) = \nu_i Q_i(t), \text{ where } \nu_i > 0.$$

[1] A more general treatment of the drawdown component of the depth to water of a well would consider the influence of neighboring wells' rates of pumping on drawdown. Such considerations give

The sum of the first and third right-hand-side terms we define as the *water table level* at time t; we denote it by $z_i(t)$.

The third term on the right-hand side of Eq. (2.2) represents the effect of changes in groundwater stocks on the well's depth to water. Pumping by any one well affects groundwater stocks and *changes* in water table levels uniformly throughout the aquifer. Decreased stocks of groundwater from their initial level lower the water table level (i.e., $z_i(t)$ increases). If $X(t) < X(0)$ for $t > 0$, then we say that the pumpers of the basin are "exploiting" the aquifer's groundwater. Similarly, increased stocks of groundwater over their initial level raise water table levels (i.e., $z_i(t)$ decreases). In this case, we say that the pumpers of the basin are "conserving" groundwater. The parameters $\gamma > 0$ and $\beta > 0$ generalize this relationship. The parameter γ specifies the geometry of the aquifer. For example, if initial stocks exceed stocks at time t (i.e., $X(0) > X(t)$), then $z_i(t)$ exceeds $z_i(0)$. And as a function of $X(0) - X(t)$, then:

$$\frac{dz_i(t)}{d[X(0)-X(t)]} = \gamma \beta \, [X(0)-X(t)]^{\gamma-1} , \tag{2.3}$$

which is positive, and

$$\frac{d^2 z_i(t)}{d[X(0)-X(t)]^2} = \gamma (\gamma-1) \beta \, [X(0)-X(t)]^{\gamma-2}, \tag{2.4}$$

which is positive, negative, or zero as $\gamma > 1$, $1 > \gamma > 0$, or $\gamma = 1$, respectively. Figure 2.1 shows how $z_i(t)$ depends on γ when $X(0) - X(t)$ is positive. In this case, when $\gamma > 1$ the water table level falls at an increasing rate; when $1 > \gamma > 0$ the water table level falls at a decreasing rate; and when $\gamma = 1$ the water table level falls at a constant rate. Alternatively, with $\gamma > 1$ the cross-sectional area of the aquifer decreases with a falling water table; with $1 > \gamma > 0$ the cross-sectional area increases with a falling water table; and with $\gamma = 1$ the aquifer has constant cross-sectional area. Figure 2.2 shows a constant cross-sectional-area aquifer with straight sides.

The parameter β accounts for soil water-holding characteristics and converts the factor $[X(0) - X(t)]^{\gamma}$ to units of length. For example, in a rectangular aquifer as shown in Fig. 2.3, $\beta = 1/\ell w \rho$. In this case, ρ is the percentage of the physical volume of the aquifer that contains water.

Figure 2.4 shows the various components of the depth to water for a well.

Next, we define some notation for benefit and cost functions for use later. Each function depends on time explicitly in addition to its other arguments.

$H_i(Q_i(t), z_i(t), t)$ = the instantaneous total cost of pumping at time t for the i^{th} well; this function depends on the water table level $z_i(t)$ and the rate of withdrawal $Q_i(t)$.

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rise to problems of well spacing and represent a considerably more complex modeling effort than we aim for in this report. We are primarily interested in modeling the effects of changes in the level of the water table on pumping lifts. We include the simple drawdown relationship for completeness.

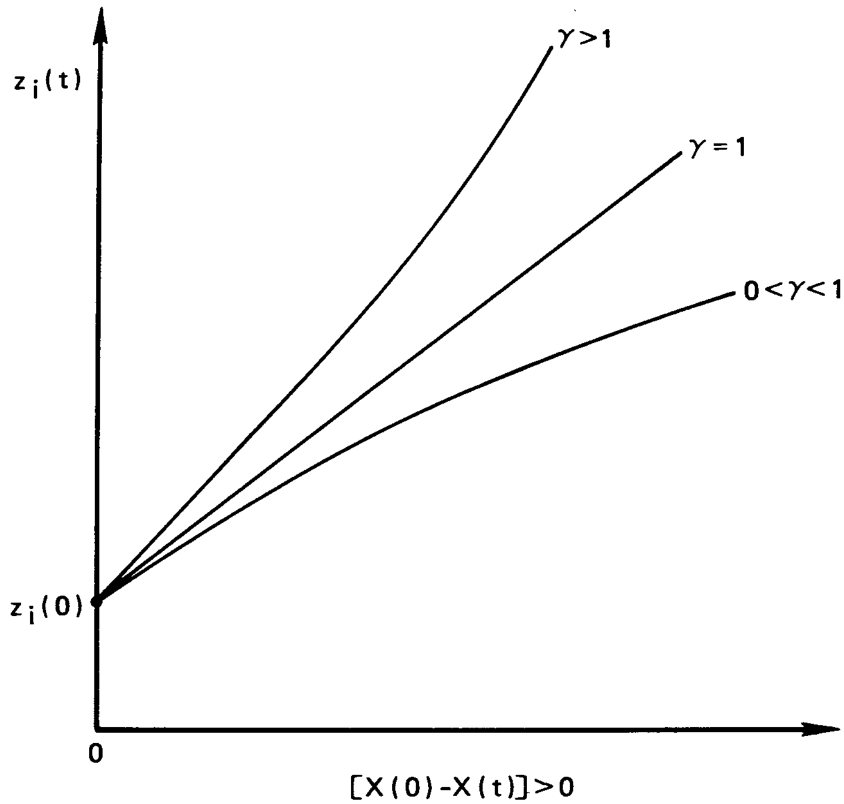
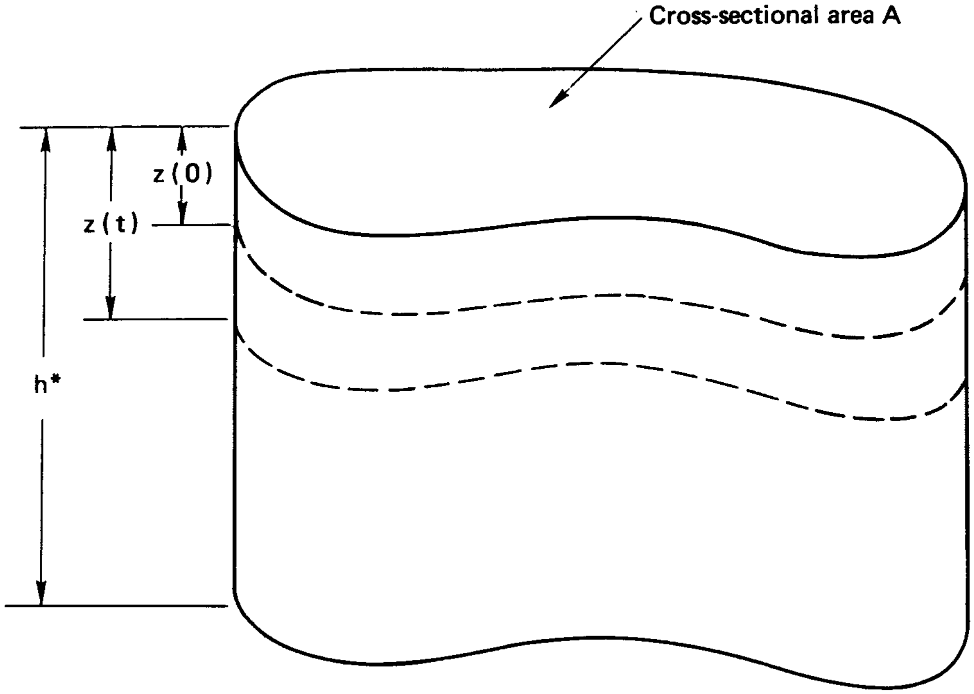


Fig. 2.1—Example of how the function $z_i(t)=z_i(0)+\beta[X(0)-X(t)]^{\gamma}$ depends on the parameter γ



$$V_{max} = Ah^*$$

$$V(t) = Qt = \text{volume of water withdrawn in } (0, t) \text{ at constant rate } Q$$

$$\therefore z(t) = \frac{Qt}{A\rho} + z(0)$$

Fig. 2.2—Constant cross-sectional aquifer with straight sides

$MB_i(Q_i(t),t)$ = the instantaneous marginal benefit derived from operating the i^{th} well at a withdrawal rate $Q_i(t)$; this function defines a price-quantity relationship that represents the demand for water produced by the i^{th} well. It can be thought of as a derived demand for water as a factor of production in some profitable economic activity like farming.

$NB_i(Q_i(t), z_i(t),t) = \int_0^{Q_i(t)} MB_i(q,t)dq - H_i(Q_i(t),z_i(t),t)$ = the net benefit attributed to the i^{th} well at time t; basically, this definition assumes that the owner of the i^{th} well as the producer of water from the well is also the consumer of the water.

For empirical work, Brown and McGuire (1967) successfully used a relatively simple specification for the cost function H_i. If we allow the parameters of their cost function to depend on time, then in our notation the form of their cost function is

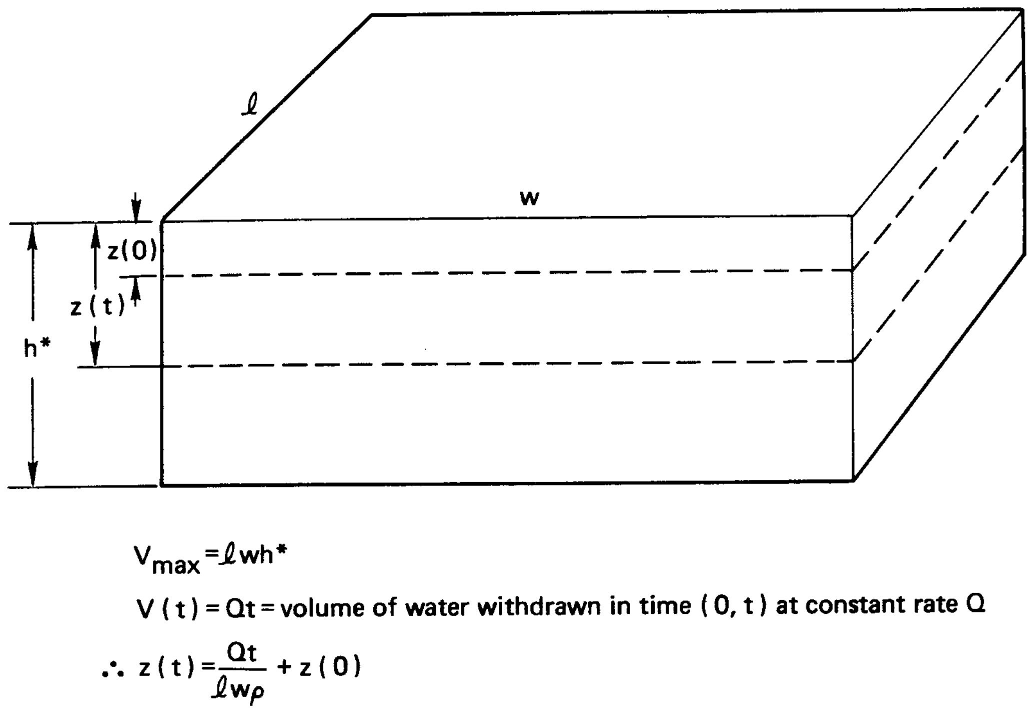


Fig. 2.3—Rectangular aquifer

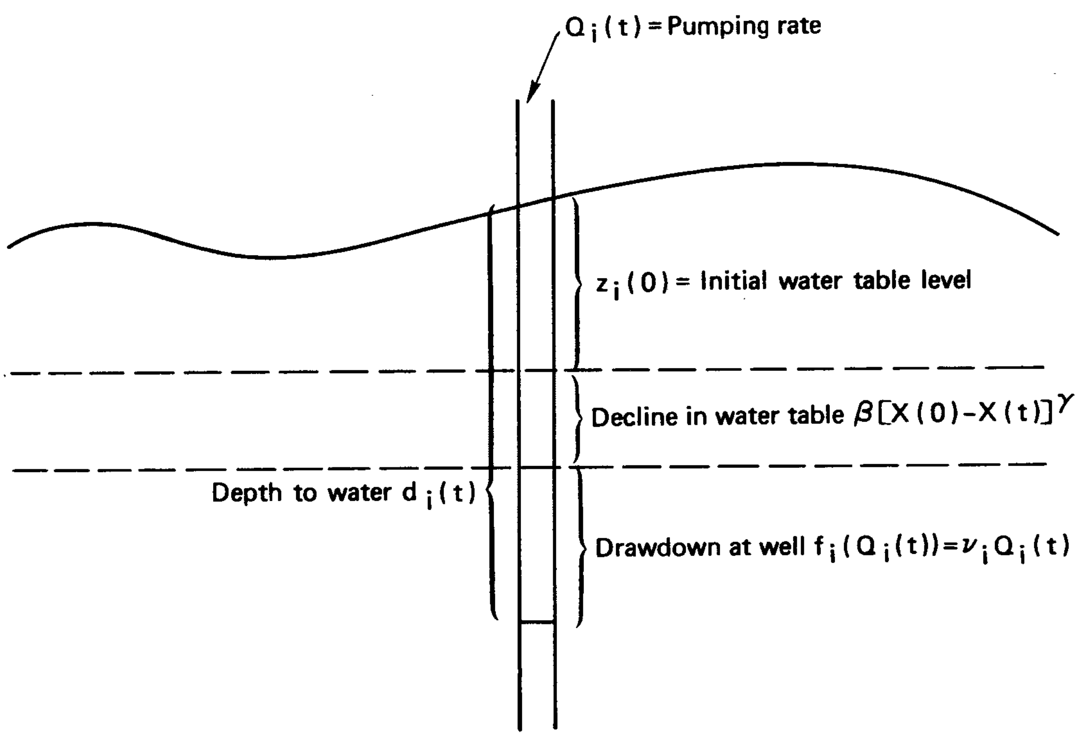


Fig. 2.4—Components of depth to water $z_i(t)$

$$H_i(Q_i(t),z_i(t),t) = a_i(t) + (b_i(t) + c_i(t)Q_i(t))(d_i(t)) \text{ , or}$$

$$H_i(Q_i(t),z_i(t),t) = a_i(t) + (b_i(t) + c_i(t)Q_i(t))\,(z_i(t) + f_i(Q_i(t))) \,. \tag{2.5}$$

The parameter $a_i(t)$ represents a time-varying fixed cost of pumping. The marginal pumping costs per foot of lift are[2]

$$\frac{\partial H_i}{\partial z_i(t)} = \frac{\partial H_i}{\partial d_i(t)} = b_i(t) + c_i(t)Q_i(t) \tag{2.6}$$

The marginal costs of production for groundwater are

$$\frac{\partial H_i}{\partial Q_i(t)} = c_i(t)(z_i(t) + f_i(Q_i(t)) + (b_i(t)+c_i(t)Q_i(t))\frac{\partial f_i}{\partial Q_i(t)} \,. \tag{2.7}$$

To emphasize the general dependence of $\partial H_i/\partial Q_i(t)$ on $Q_i(t)$ and $z_i(t)$, we write

$$\frac{\partial H_i}{\partial Q_i(t)} = mcp_i(Q_i(t)\,,\, z_i(t)) \tag{2.8}$$

The expression $\partial H_i/\partial Q_i(t)$ represents the current marginal cost of producing or extracting an additional unit of groundwater.

We state relationships (2.5)–(2.7) now so that we can use them to state more specifically some of the results we derive from our models.

In addition to assuming as given the initial stock of water in the aquifer, the aquifer parameters, the initial pumping lifts, and the cost and benefit functions, we assume that the number of wells is also given. Moreover, we assume that it does not change with time. This last point contrasts with the approach of other authors who have analyzed production from natural resources with the assumption that the number of producing units (i.e., the amount of capital) varies with time. Smith (1968) and Brown (1974) developed general models of production from natural resources based on an aggregate production function with the amount of capital as a variable. Brown states that we can think of the variable factor capital as being defined in terms of the "number and location of wells in the instance of groundwater."[3] The problem of describing analytically the degree of interference among wells as a function of their rates of pumping is nontrivial. Also, the data requirements for implementing any relevant policy recommendations from such a model could very easily preclude implementing the policy at all. Describing the time-varying nature of the number of wells in operation is analogous to the complex problem of describing the economics of entry and exit of firms in an industry. Such a model is beyond the level of analysis intended in this report; thus, for our models we assume that the number of wells is given.

[2] We use the notation $\partial F/\partial \mathbf{x}(t)$ to mean we partially differentiate the functions F with respect to one of its arguments x(t).

[3] Brown (1974), p. 165.

INDIVIDUAL EXPLOITATION WITH FORESIGHT

For this model of groundwater development, we assume that a producer maximizes *his own* discounted net benefit over an infinite time horizon. We use the term *foresight* to mean that he considers the effects of his own pumping on the level of groundwater stocks.

Suppose that the ℓ^{th} groundwater producer owns n_ℓ of the n wells in the basin.[4] We assume that his aggregate net benefit from groundwater production is just the sum of the total net benefits for each of his wells. His optimization problem is

Maximize:

$$\int_0^\infty \sum_{i \in n_\ell} NB_i(Q_i(t), z_i(t), t)\, e^{-rt}\, dt\,, \qquad (2.9)$$

Subject to:

$$\dot{X}(t) = R(t) - \sum_j Q_j(t)\,,$$

where the $Q_j(t)$ are given for $j \notin n_\ell$, together with $z_i(0)$ and $X(0)$. Also, $Q_i(t) \geq 0$ and $z_i(t) \geq 0$, for $i \in n$.

In solving this problem we include the relationship between $z_i(t)$ and $X(t)$ implied by Eq. (2.2):

$$z_i(t) = z_i(0) - \beta(X(t) - X(0))^\gamma\,, \text{ or}$$

$$z_i(t) = z_i(0) + \beta(X(0) - X(t))^\gamma\,.$$

This relationship, however, is not a constraint for the optimization problem.

Problem (2.9) is a problem in optimal control theory, where the control variables $Q_i(t)$ and the state variable $X(t)$ are conditional on the $Q_j(t)$ for $j \notin n_\ell$. The current-value Hamiltonian[5] is

$$\Theta_\ell = \sum_{i \in n_\ell} NB_i(Q_i(t), z_i(t), t) + \lambda_\ell(t)[R(t) - \sum_j Q_j(t)]. \qquad (2.10)$$

The subscripted auxiliary variable $\lambda_\ell(t)$ represents the imputed marginal value of groundwater stocks at time t for the producer. The necessary conditions for an interior maximum to problem (2.9) are

$$0 = \frac{\partial \Theta_\ell}{\partial Q_i(t)}\,,$$

and (2.11)

$$\dot{\lambda}_\ell(t) - r\lambda_\ell(t) = -\frac{\partial \Theta_\ell}{\partial X(t)}, \text{ for } i \in n_\ell\,.$$

The transversality conditions that $\lambda_\ell(t)$ must satisfy are

$$\lim_{t \to \infty} X(t)\lambda_\ell(t)e^{-rt} = 0, \text{ and } \lim_{t \to \infty} \lambda_\ell(t)e^{-rt} = 0. \qquad (2.12)$$

[4] We use the notation n_ℓ for two purposes: In the first usage, n_ℓ represents a numerical quantity, the number of wells that the ℓ^{th} producer owns. In the second usage, n_ℓ stands for a *subset* of the positive integers 1, 2, . . ., n, which we can think of as the labels for the wells in the basin.

[5] Arrow (1967), p. 13.

The factor e^{-rt} is a continuous time discount factor, where r is the constant instantaneous discount rate. Conditions (2.12) imply that the total value $X(t)\lambda_\ell(t)$ and the marginal value $\lambda_\ell(t)$ of groundwater stocks very far into the future should be negligible when discounted to time t=0.

After we evaluate the partial derivatives of (2.11) and include the equality constraint of (2.9), we obtain the following:

$$MB_i(Q_i(t),t) = \frac{\partial H_i}{\partial Q_i(t)} + \lambda_\ell(t), \text{ for } i \epsilon n_\ell \tag{2.13}$$

$$\dot{\lambda}_\ell(t) - r\lambda_\ell(t) = \sum_{i \epsilon n_\ell} \frac{\partial H_i}{\partial z_i(t)} \frac{\partial z_i}{\partial X(t)}, \tag{2.14}$$

and

$$\dot{X}(t) = W(t) - \sum_j Q_j(t), \tag{2.15}$$

with the $Q_j(t)$ given for $j \notin n_\ell$. To interpret (2.13), we must understand more clearly the significance of $\lambda_\ell(t)$. A solution to (2.14) that satisfies the transversality conditions (2.12) is

$$\lambda_\ell(t) = -e^{rt} \int_t^\infty \sum_{i \epsilon n_\ell} \frac{\partial H_i}{\partial z_i(u)} \frac{\partial z_i}{\partial X(u)} e^{-ru} du. \tag{2.16}$$

This expression for $\lambda_\ell(t)$ is positive, since the partial derivative $\partial z_i / \partial X(u)$ is found to be negative, from Eq. (2.2). It represents the change in the pumping lift for the i^{th} well from a change in the stocks of groundwater. The partial derivative $\partial H_i / \partial z_i(u)$ is just the marginal cost of a unit change in pump lift for the i^{th} well. The product of these two partial derivatives represents the instantaneous marginal value (in terms of marginal pump lift costs) for the i^{th} well of an additional unit of groundwater stocks. Thus, we interpret $\lambda_\ell(t)$ as the total discounted value to time t of these future marginal effects for all wells owned by the ℓ^{th} producer.

Essentially, $\lambda_\ell(t)$ represents the producer's evaluation of future marginal pump lift costs that he will incur by a unit increase in the pumping rate for any one of his wells. In other words, $\lambda_\ell(t)$ represents the marginal savings in pump lift costs he realizes from a unit decrease in the pumping rate for one of his wells.

The right-hand side of Eq. (2.13) represents the producer's private marginal cost function. It consists of two terms: The first represents his marginal cost of production, and the second represents his marginal cost due to increased pumping lifts in the future from an increased current rate of pumping by any one of his wells. Although by including this second term in his marginal cost function he attributes some value to groundwater stocks, he does not include their value to other producers in the basin. Thus, he makes his pumping-rate decisions based upon a marginal cost function that does not fully value stocks of groundwater.

If each producer behaved according to this model, then Eqs. (2.13) and (2.14) would hold for all producers, together with Eq. (2.15). In long-run equilibrium where all functions and variables do not change with time, we have the following set of equations:

$$MB_i(Q_i) = \frac{\partial H_i}{\partial Q_i} + \lambda_\ell, \text{ for } i \epsilon n_\ell, \text{ for each producer } \ell \quad (2.17)$$

$$\lambda_\ell = \frac{1}{r} \sum_{i \epsilon n_\ell} \left(\frac{\partial H_i}{\partial z_i}\right)\left(-\frac{\partial z_i}{\partial X}\right), \text{ for each producer } \ell \quad (2.18)$$

Economic equilibrium (2.17)–(2.18)

$$R = \sum_j Q_j \quad (2.19)$$

Hydrologic equilibrium (2.19)

Since we can substitute the expression for λ_ℓ of Eq. (2.18) for λ_ℓ in Eq. (2.17), we actually have n + 1 equations in the n + 1 variables $Q_1, Q_2, \ldots, Q_n$, and X. The equilibrium values for these variables determine the equilibrium water table level for each well.

TWO IMPORTANT SPECIAL CASES

Within the framework of this simple model we have two important special cases. First, if each producer ignored the intertemporal relationship between his current groundwater pumping and future water table levels, then he would base his marginal pumping decisions on each well's current marginal benefit and its current marginal costs of production. He would attribute no value to groundwater stocks. This type of decisionmaking by a producer represents myopic or shortsighted behavior. Mathematically, this simply means that $\lambda_\ell(t)$ is set to zero and each well's pumping rate is determined from the condition

$$MB_i(Q_i(t),t) = mcp_i(Q_i(t),z_i(t),t)\ , \quad (2.20)$$

given $z_i(0)$, and X(t) for each $i \epsilon n_\ell$.

Second, if there is only one producer in the basin, then the term $\lambda_\ell(t)$ of his private marginal cost function becomes $\lambda(t)$:

$$\lambda(t) = -\, e^{rt} \int_t^\infty \sum_{i=1}^{n} \frac{\partial H_i}{\partial z_i(u)} \frac{\partial z_i}{\partial X(u)}\, e^{-ru} du\ . \quad (2.21)$$

And in long-run equilibrium we have

$$\lambda = \frac{1}{r} \sum_{i=1}^{n} \left(\frac{\partial H_i}{\partial z_i}\right)\left(-\frac{\partial z_i}{\partial X}\right). \quad (2.22)$$

In both Eqs. (2.21) and (2.22) the summation index ranges over all wells for the basin (i.e., $n_\ell = n$, the total number of wells in the basin). More importantly, this imputed value of groundwater stocks for the sole-ownership case results from the maximization of the *aggregate* discounted net benefit for the *entire* basin over an infinite time horizon. Since this objective function is equivalent to the one that would be assumed for socially optimal groundwater development, the solution to the sole-ownership case also provides the socially optimal solution.

LONG-RUN WATER TABLE LEVELS AND PRODUCTION COSTS

With more than one groundwater producer in the basin and in the absence of an optimal management program, the long-run equilibrium water table levels will be lower than optimal. Consequently, marginal costs of production (mcp_i) for each well will also be greater than optimal. These results can be demonstrated mathematically by a careful analysis of the equations of long-run equilibrium for the basin under the various types of behavior by individual producers.

If we let $z_i^{(s)}$ be the long-run equilibrium water table level under sole ownership, and if $z_i^{(o)}$ is the level with neither sole ownership nor socially optimal management, then we can state the above result as an inequality:

$$z_i^{(s)} < z_i^{(o)}. \tag{2.23}$$

Since for a given pumping rate, Q_i, the marginal costs of production increase with a falling water table level (i.e., $\partial mcp_i/\partial z_i > 0$), we have a second inequality:

$$mcp_i(Q_i,z_i^{(s)}) < mcp_i(Q_i,z_i^{(o)}) \; . \tag{2.24}$$

With the formula for mcp_i of Eq. (2.7), which is linear in z_i, we can write a simple expression for the difference in mcp_i's:

$$\delta_i = c_i(z_i^{(o)} - z_i^{(s)}) \; . \tag{2.25}$$

For a given pumping rate Q_i^*, the area between these two curves represents an instantaneous rate of cost savings (e.g., units of dollars per unit of time) for the well from socially optimal management of groundwater development. Figure 2.5 illustrates this result. Mathematically, the formula for the rate of cost savings is

$$CS_i = \delta_i Q_i^* \; ,$$

which is positive, since $\delta_i > 0$.

Unless the producers of the basin agree to some form of collective management, the potential production cost savings that are possible can never be achieved. The next major topic of this report deals with how optimal collective groundwater management can be achieved through taxation or quota assignments.

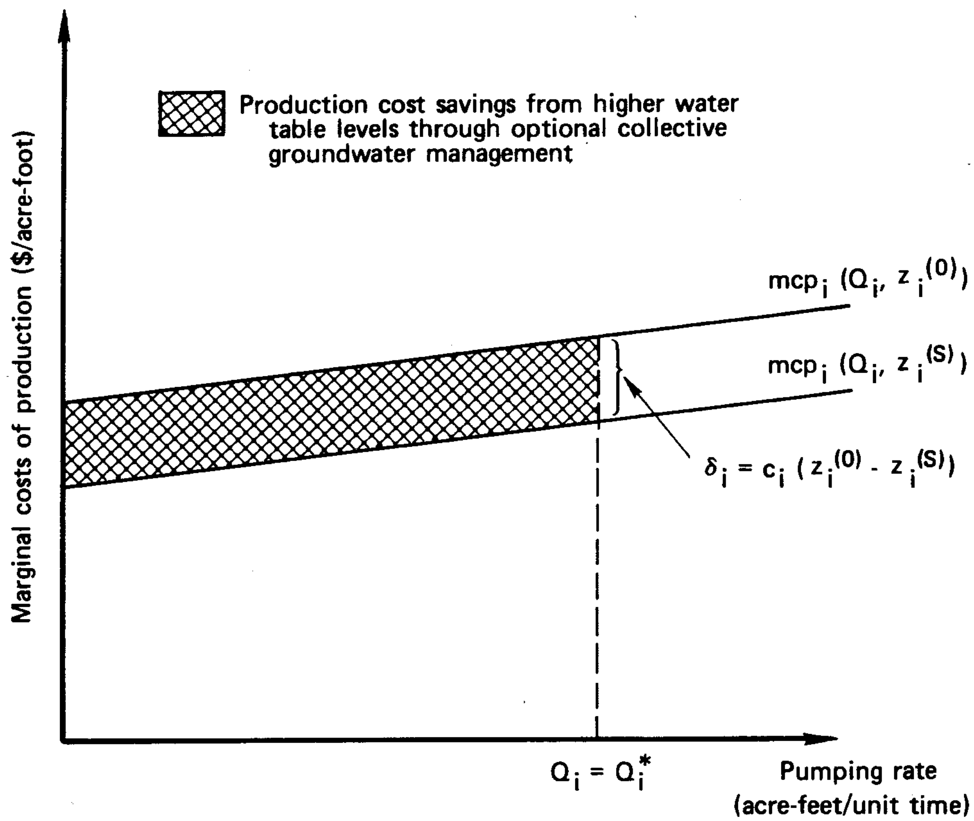


Fig. 2.5—Differences in marginal costs of production in long-run equilibrium

III. SOCIALLY OPTIMAL GROUNDWATER MANAGEMENT

A manager for the basin's groundwater resources could use the marginal cost function for each well based on the sole-ownership model to assign either optimal tax rates or pumping-rate quotas. In general, the socially optimal policy depends on the extent to which producers already account for the value of groundwater stocks in making their individual (private) pumping decisions.

To realize the maximum benefits from optimal social management, all producers of the basin would have to participate. Enforcement of pumping-rate quotas and the collection of pump taxes would be part of the social manager's job.

If a policy of pump taxation is chosen, then redistribution of the pump tax revenues back to the groundwater producers would be crucial. Pump tax revenues (if they are not returned) are subtractions from each producer's welfare. Even if the rate of cost savings a producer realizes from the management program approximately equals his rate of pump tax payments, he will have no incentive to participate. Also, a successful mechanism must avoid providing any incentives for producers to increase their groundwater production, since such incentives would tend to defeat the purposes of having a groundwater management program. Thus, a politically acceptable, nondistortive redistribution mechanism for pump tax revenues plays a crucial role in successful groundwater management.

If the social manager chooses to assign optimal quotas to each producer, he avoids the problem of choosing a workable redistribution mechanism for pump tax revenues. However, in the absence of a water market, there is likely to be a conflict over how the quotas are assigned, since each producer experiences welfare losses from decreased pumping rates. Thus, by assigning optimal quotas that are tradable through a water market, a social manager could mitigate conflicts over the means for determining pumping-rate quotas.

Since collective groundwater management is not a "free good," its operation could be financed out of the pump tax revenues. With quota assignments, a small pump tax could be levied, while the water market operations could be financed through a small sales tax per unit of water traded. Also, lump-sum taxes could be levied per producer or per well to help cover management costs.

When there are major variations among producers in their internal valuations of the last unit of water pumped, there are economic incentives for producers to trade. If tradable pumping quotas have been assigned (i.e., property rights to a given amount of groundwater have been quantified), then producers can either exercise their right to pump groundwater or sell that right to some other producer. In this case, no physical quantities of water need be exchanged.

Under a program that employs pump taxation without allocating groundwater pumping rights to producers, trading in groundwater would necessarily involve actual physical transfers. This difference results from the way a producer acquires the right to derive income from groundwater. Basically, he acquires this right only by "capturing" the water (i.e., by pumping the water from beneath the ground). Thus, a producer would have to extract the water and transport it to his trading partner. The costs of transportation may effectively preclude the existence of a water market under this "law of capture."[1]

In the remainder of this section we consider how optimal pump taxes or quota assignments could be made based on the sole-ownership case. We also consider the long-run effects of these tools in an environment where producers can import surface water for their use and where they have opportunities to participate in a water market.

Before proceeding, we shall define what we mean by the concept of groundwater management as used in this report. We use this term in an economic sense whereby groundwater producers make their marginal pumping-rate decisions not only on the basis of the marginal costs of producing groundwater, but also with a correct assessment of the value of groundwater stocks for the entire basin. A groundwater management entity provides producers with this valuation through some mechanism like a pump tax or a quota assignment. Although a social manager for a basin may levy a tax or set quotas that imply a total pumping rate that just happens to equal the basin's natural replenishment rate, such hydrological balance does not represent the management objective. His primary objective is to provide incentives based on economic criteria so that groundwater producers correctly account for the economic value of their commonly held groundwater resources.[2]

Groundwater management does not mean reducing total pumping to eliminate groundwater overdraft in the short run. However, it does mean that total pumping for the basin should be managed so that the interdependent extraction costs are considered by producers who develop a common aquifer. Where a basin is being overdrafted, application of management tools based on economic criteria would certainly affect the rate of overdraft and could even eliminate overdrafting, depending upon the hydrology and the economic behavioral responses of producers. The water table level might continue to fall, it might even rise, or it might stabilize; but if overdraft continued in the short run, it would be a *managed* overdraft with consideration for its long-run consequences.

A SOCIALLY OPTIMAL PUMP TAX POLICY

The marginal cost function for a well implied by the sole-ownership case represents the social marginal cost (SMC_i) function that should be used by individual producers. It consists of two terms, the first being the current marginal costs of production (mcp_i), and the second being the imputed marginal social value (λ) of groundwater stocks for the entire basin:

$$SMC_i(Q_i,z_i) = mcp_i(Q_i,z_i) + \lambda \; . \tag{3.1}$$

[1] In the state of California, a type of "law of capture" governs the right to use groundwater in nonadjudicated groundwater basins. (See Phelps et al., R-2386-CSA/RF.)

[2] Groundwater management based on economic criteria applies not only to managing water table levels, but also to other types of groundwater development problems. Some of these problems, like land subsidence and groundwater pollution, may be physically more "dramatic." However, economic criteria could also be used to assess their relative importance and to evaluate the costs of dealing with them as part of a more inclusive groundwater management program.

However, individual producers in making their private pumping decisions do not account for the full (social) value of groundwater stocks. Their private marginal cost function (PMC_i) is just

$$PMC_i(Q_i,z_i) = mcp_i(Q_i,z_i) + \lambda_\ell , \tag{3.2}$$

for the ℓ^{th} producer.

We have demonstrated that with two or more producers in the basin, the expression for λ_ℓ consists only of terms which reflect future costs of this producer's pumping to himself. It does not include the future costs of his pumping to any of his neighbors who share the same aquifer. Therefore, to encourage an individual producer to make private pumping decisions in a socially optimal manner, a tax could be levied on him. This tax should just include costs which his pumping imposes on his neighbors but which he otherwise ignores. The optimal tax rate is just the difference between the two marginal cost functions SMC_i and PMC_i:

$$t_\ell = SMC_i(Q_i,z_i) - PMC_i(Q_i,z_i) ,$$

$$t_\ell = \lambda - \lambda_\ell . \tag{3.3}$$

We can rearrange this expression as follows:

$$t_\ell = \lambda - \lambda_\ell ,$$

$$t_\ell = \lambda(1 - \lambda_\ell/\lambda) . \tag{3.4}$$

In terms of the previous formulas for λ and λ_ℓ, we can express the ratio λ_ℓ/λ as a ratio of marginal costs:

$$\frac{\lambda_\ell}{\lambda} = \frac{\left(\frac{1}{r}\right) \sum_{j \in n_\ell} \left(\frac{\partial H_j}{\partial z_j}\right)\left(-\frac{\partial z_j}{\partial X}\right)}{\left(\frac{1}{r}\right) \sum_{j=1}^{n} \left(\frac{\partial H_j}{\partial z_j}\right)\left(-\frac{\partial z_j}{\partial X}\right)} , \tag{3.5}$$

$$\frac{\lambda_\ell}{\lambda} = \frac{\sum_{j \in n_\ell} \frac{\partial H_j}{\partial z_j}}{\sum_{j=1}^{n} \frac{\partial H_j}{\partial z_j}} ,$$

since $\partial z_i/\partial X$ does not depend on the summation index "j." The ratio λ_ℓ/λ represents the fraction of total marginal pumping costs per unit of pumping lift for the entire basin and internalized by the ℓ^{th} producer. Hence, his optimal tax rate is just that fraction $(1 - \lambda_\ell/\lambda)$ of the social marginal value of groundwater stocks which he *does not* internalize.

If we assume that $\partial H_j/\partial z_j = cQ_j$ (the same linear function of pumping rate for each well), then the ratio λ_ℓ/λ reduces to the ℓ^{th} producer's share, w_ℓ, of the total pumping rate for the basin. Furthermore, his tax rate becomes

$$t_\ell = \lambda(1 - w_\ell) . \tag{3.6}$$

Thus, with this assumption about pumping costs, the larger a producer's share of the basin's total pumping, the smaller his *tax rate* should be. This is exactly what we would expect, since he already accounts for a larger share of the imputed social marginal value of groundwater stocks.

To summarize our discussion, the optimal taxation policy consists of levying tax rates on producers. These tax rates consist of two factors: (1) λ, the social marginal value of groundwater stocks, and (2) $(1 - \lambda_\ell/\lambda)$, the fraction of λ not internalized by the ℓ^{th} producer.

One could estimate λ by using the long-run equilibrium formula as an approximation:

$$\lambda = \frac{1}{r} \sum_{j=1}^{n} \left(\frac{\partial H_j}{\partial z_j}\right) \left(-\frac{\partial z_j}{\partial X}\right) . \tag{3.7}$$

Finally, the above expression for λ consists of three important quantities. First, λ varies inversely with the discount rate r. Second, λ is directly proportional to the aggregate marginal pumping costs per foot of lift. Third, λ is proportional to $(-\partial z_j/\partial X)$, which we have assumed is independent of the index "j." This factor describes the response of the water table level to a change in the stock of groundwater, and it reflects the geometry of the aquifer.

MEASUREMENT OF WATER TABLE LEVEL RESPONSES

From Eq. (2.2), we find that

$$-\frac{\partial z_j}{\partial X} = \gamma \beta (\Delta X)^{\gamma - 1} \tag{3.8}$$

where we write ΔX for the absolute value of $X(0) - X(t)$. Now, ΔX represents the accumulated change in groundwater stocks from their initial level. We define this level as the level of stocks at the start of the groundwater management program or at some prior time for which data are available. If we let Δz be the absolute accumulated change in the water table level, then we can write

$$\Delta z = \beta (\Delta X)^\gamma . \tag{3.9}$$

Now, since

$$\Delta X = \left| \int_0^t (R(u) - Q(u))du \right| = \left| \int_0^t (Q(u)) - R(u))du \right| ,$$

then ΔX also represents the absolute value of the accumulated total amount pumped less the accumulated total replenishment. Therefore, to estimate values for

β and γ in Eq. (3.9), we could perform a log-linear regression of the time series of accumulated changes in the water table level, Δz, against the time series of accumulated changes in groundwater storage, ΔX. Previous authors (Brown and McGuire (1967), Brown and Deacon (1972)) have assumed that the relationship in Eq. (3.9) is strictly linear, so that $\gamma = 1$. Admittedly, if there is a lack of adequate data on water table levels and changes to groundwater stocks, the linear relationship is the simplest to start with. As more data become available through more accurate monitoring of water table levels, pumping rates, and replenishment rates, data analysis using Eq. (3.9) may show that a value of γ significantly different from 1 more accurately describes the aquifer's response to changes in groundwater stocks.

If we solve Eq. (3.9) for ΔX and substitute this expression into Eq. (3.8), we have

$$-\frac{\partial z_j}{\partial X} = \gamma \, \beta^{1/\gamma} \, (\Delta z)^{1-1/\gamma} . \tag{3.10}$$

If γ is assumed or is shown from data analysis to be essentially 1, then Eq. (3.10) becomes

$$-\frac{\partial z_j}{\partial X} = \beta . \tag{3.11}$$

This very simple expression has the nice property of being independent of the current state of the water table level. When γ turns out to be significantly different from 1, then the current absolute value of the accumulated change in the water table level must be included in setting the tax rate of Eq. (3.7).

ESTIMATION OF PUMPING COSTS

To estimate the marginal pumping cost of increased pumping lifts, we can start with the simple specification in Eq. (2.6):

$$\frac{\partial H_j}{\partial z_j} = b_j + c_j Q_j , \tag{3.12}$$

where we have suppressed the time dependence of each variable. Basically, b_j represents a fixed component (e.g., \$/ft) which might include well maintenance charges that vary with pumping lift. The factor c_j primarily represents a power cost (\$/A$-$F/ft). In many cases, it would be possible to estimate values of c_j from electric utility power rate schedules that apply to groundwater producers.[3] Also, pumping rates Q_j (e.g., A$-$F pumped in the prior time period) would have to be estimated either by metering wells or from data on power consumption by motors used to operate pumps.

Combining Eqs. (3.10) and (3.12) in Eq. (3.7), we have

[3] Bowers (1977) and Moore and Hedges (1962).

$$\lambda = \frac{1}{r} \sum_{j=1}^{n} (b_j + c_j Q_j)\, \gamma_\beta^{\,1/\gamma} (\Delta z)^{1-1/\gamma} \qquad (3.13)$$

We can see from Eq. (3.13) that for given narrow ranges for b_j and c_j, as n (the number of active wells) increases or as the total pumping rate $\Sigma_j Q_j$ increases, then the social marginal vaue λ also increases. For a given value of Δz, a γ less than 1 would tend to decrease the tax rate and a γ greater than 1 would magnify it.

TRADING IN GROUNDWATER UNDER PUMP TAXATION AND THE "LAW OF CAPTURE"

Since each producer determines pumping rates based upon each well's marginal benefit, private marginal costs, and his tax rate, producers will be likely to have different internal prices[4] of groundwater. If these differences across producers are significant enough, then at least two producers may find it mutually beneficial to trade groundwater at an agreed-upon price which falls between their respective internal prices. Under a legal system for groundwater where a producer has the right to derive income from groundwater only if he pumps it out of the ground, trading would involve actual physical transfers of water. Thus, the mutually agreed-upon price for the trade would include costs of transportation (probably shared in some way).

Trading permits both the buyer and the seller to increase their net benefit. The effect of their trade on the other producers in the basin depends on whether or not the parties to the trade increase, decrease, or leave unchanged their total pumping rate. Clearly, if the trading partners leave their combined pumping rate unchanged, no producer has been harmed, while the trading parties have each increased their net benefit.

If the combined pumping rate decreases, then *all* producers are better off: The trading partners each increase their net benefit; and since the total pumping rate for the basin is less as a result of the trade, the tax rate in the next time period will be decreased and groundwater stocks in the current period will be conserved. Clearly, no producer would oppose trading groundwater under these circumstances. But when the trading partners pump at a greater combined rate to meet their trade agreement, other producers will be adversely affected. A total increased pumping rate for the basin means a higher tax rate for *all* producers, while the trading partners obtain the benefits from the trade. In the absence of any means for compensating those parties who are adversely affected, trading would probably be opposed.

Apart from the effects of increased pumping by trading partners on other producers, opposition to groundwater trading would come primarily from those

[4] By "internal price" we mean the common value of marginal benefit and marginal cost (e.g., $MB_i = PMC_i + t_\ell$) when evaluated at the net-benefit-maximizing pumping rate for a given well.

producers who do not participate in trading. As more trading partnerships develop, there would be less opposition to trading. Thus, the likelihood that producers could participate in trading would determine whether or not they would oppose it. Under this system of pump taxation and the right to derive income from pumped water,[5] we would expect that transportation costs could affect the price of traded water to the extent that only a few trading agreements would be made. In those cases where existing surface water conveyance facilities connect enough producers to make transfers possible and where the rental charge for use of the facilities is not too high, trading may involve enough of the producers to be acceptable.

Later in this section we consider the incentives for trading under a groundwater management system that assigns each producer tradable pumping-rate quotas.

SURFACE WATER IMPORTS WITH GROUNDWATER PUMP TAXATION

Suppose that in addition to pumping groundwater to meet their water demands, producers can fully supplement their groundwater supplies by importing water via surface conveyance facilities. Assume that a wholesaler supplies surface water at a uniform market clearing unit price to each producer and that supply is unlimited in quantity to each buyer at this price. Furthermore, assume that each producer purchases some quantity of surface water.

From each producer's point of view, the price of surface water determines each well's optimal pumping rate together with his total rate of surface water use. Under pump taxation, we have the following set of equations for the ℓ^{th} producer:

$$MB_i\,(Q_i + y_i) = p_s,\ i \epsilon n_\ell \qquad (3.14)$$

$$p_s = mcp_i + t_\ell,\ i \epsilon n_\ell\ , \qquad (3.15)$$

where Q_i = the i^{th} well's pumping rate,
y_i = the rate of surface water imports attributable to the i^{th} well,
p_s = the unit price of surface water,
PMC_i = the private marginal cost for groundwater at the i^{th} well (recall that $PMC_i = mpc_i + \lambda_\ell$),
MB_i = the marginal benefit function for water at the i^{th} well, and
t_ℓ = the optimal pump tax rate for the ℓ^{th} producer, as discussed previously.

Figure 3.1 illustrates how the pumping rate, Q_i, and surface water import rate, y_i, are determined together with the producer's total water use rate, $Q_i + y_i$. Initially, producers will decrease their pumping rates from Q'_i to Q_i and increase their surface water use from y'_i to y_i. Essentially, producers will substitute surface water for groundwater because the pump tax has made groundwater more expensive relative to surface water.

[5] I.e., where physical quantities of water must be transferred to derive trade income.

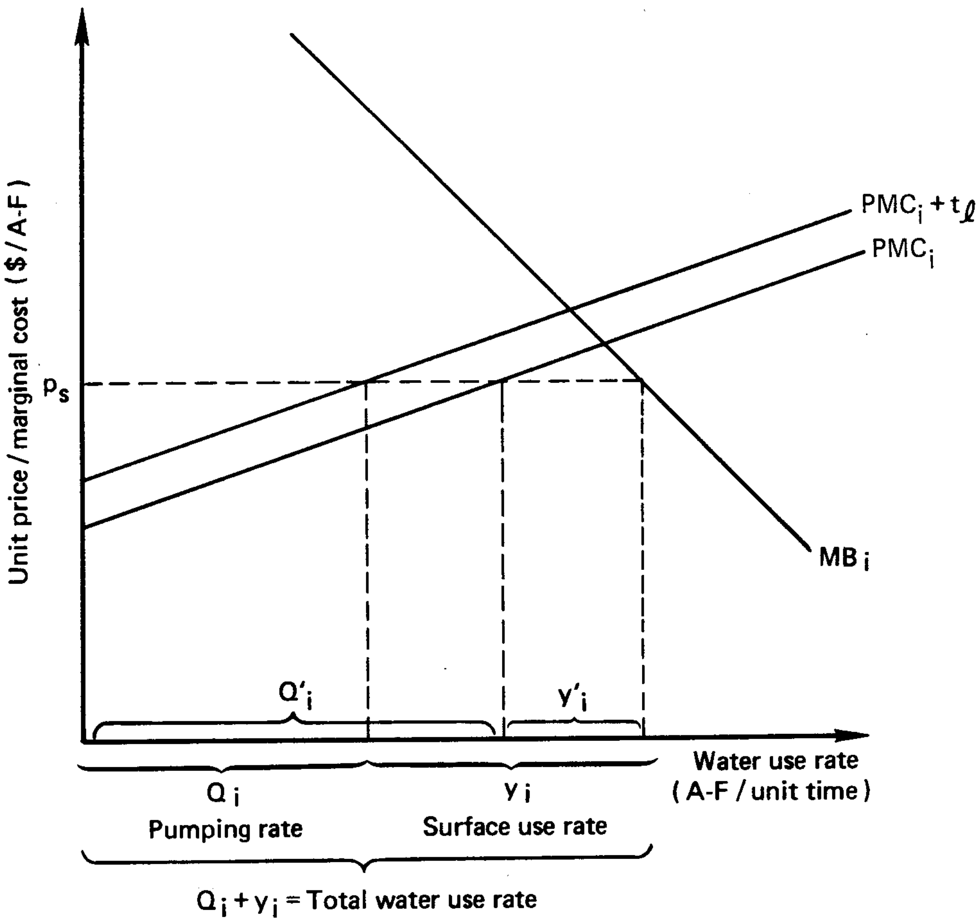


Fig. 3.1—Determination of optimal pumping rate and surface water use rate

If the groundwater basin is not yet in hydrological equilibrium, the water table level will either rise or fall. This in turn depends on whether or not the total pumping rate falls short of or exceeds the natural replenishment rate. These effects take place independently of surface water use. However, surface water imports do influence their magnitude. The total basin pumping rate will be less than it would be without surface imports. This in turn implies that the long-run equilibrium water table level will be higher than it would be without any imported surface water.

Since the main effect of the pump tax is to cause *all* producers to cut back their groundwater pumping activity, the long-run water table level will be higher under the tax policy than it would be without it. As we demonstrated previously, higher water table levels translate into production cost savings (i.e., the mcp_i curve will be lower). In the aggregate, these savings represent the *potential* social benefits from a groundwater management program. The *actual* social gains must, of course, include a subtraction for the costs of administering the program.

If the basin is assumed to be in both economic and hydrological equilibrium, then in addition to Eqs. (3.14) and (3.15) for each producer and his wells, we have the hydrological balance equation:

$$\sum_{j=1}^{n} Q_j = R \,, \tag{3.16}$$

where R is the long-run natural replenishment rate for the basin. These 2n + 1 equations allow us to determine the long-run equilibrium values of pumping rates $Q_1, Q_2, \ldots, Q_n$, surface water import rates $y_1, y_2, \ldots, y_n$, and the level of groundwater stocks X. From the relation $\Delta z = (|X(0) - X|)^{\gamma}$, we can find the absolute change in the water table level Δz.

Suppose that in this equilibrium the price of surface water increases slightly but not enough to make any one producer cease taking surface water. Also, assume that each producer's fraction λ_ℓ / λ remains stable. Initially, groundwater pumping rates for producers would tend to increase. However, since the tax rates depend directly on rates of pumping, each producer's tax rate would also increase.[6] Alternatively, a slight decrease in the surface water price would encourage decreased pumping rates by all producers. Decreased pumping rates would imply lower tax rates, which would then provide incentives for increased pumping. In either case, changes in the price of surface water tend to cause like-signed changes in each producer's tax rate.

SOCIAL MANAGEMENT BY QUOTA ASSIGNMENTS

In principle, the optimal quotas to assign to each well result from equating the well's social marginal cost function to its marginal benefit function. The pumping rate for which this equality holds is the well's optimal quota assignment. Unfortunately, the information requirements (e.g., knowledge of social marginal cost and marginal benefit functions for each well) needed to set optimal quotas are probably unrealistic. However, if one could periodically determine an optimal total pumping rate for the basin and periodically assign tradable quotas in an arbitrary manner, then would the optimal quota assignment discussed above evolve from trade?

Suppose that each producer imports surface water and pumps groundwater so that optimal quotas result from equating the unit price of surface water with each well's social marginal cost function. This allocation of quotas represents economically efficient social allocation, since social marginal cost functions for pumping water are equated through their equality with the price of surface water. If producers can freely trade their quotas, then trading decisions will be made on the basis of each producer's marginal evaluation for changing his quota assignment. Suppose that Q_i is the quota assigned to the i^{th} well and this well is owned by the ℓ^{th} producer. His marginal evaluation of this quota is just the difference between the price of surface water, p_s, and his well's private marginal cost, $PMC_i(Q_i)$. Where producers differ in their private cost conditions, they will also differ in their marginal evaluation of optimal quota assignments. And with differing marginal evalua-

[6] The water table level will also respond to an increased rate of pumping, so that marginal private costs for each producer would evenually rise. However, the tax rates respond with a lag of at most one time period, whereas the water table level may take several time periods to fall.

tions there will be economic incentives for trading some of their quotas until their marginal evaluations are equal throughout the basin. This further implies that trading will result in equal private marginal costs of pumping for the final allocation of quotas. However, the social marginal costs of pumping this reallocation of quotas will no longer be equal across wells.[7] From society's point of view, this resulting allocation would be economically inefficient.

The above discussion argues that by allowing trade of optimal quotas among producers who differ in their marginal evaluation of their quotas, the initial socially efficient allocation would become socially inefficient. Therefore, if quotas were allocated in an arbitrary manner from some optimally determined total, trading among producers would not necessarily evolve to the socially optimal allocation.

These results contrast with the use of optimal pump taxes as a management tool. In principle, with optimal taxation, producers make both their marginal pumping and trading decisions based on social marginal costs rather than on private marginal costs as in the case of quotas.

As a practical matter, although an arbitrary assignment of quotas together with trading would not necessarily lead to the socially optimal allocation, it could improve the allocational efficiency of pumping activity within the basin. This type of quota system might also simplify the problem of administering the groundwater management program, since no pump tax revenues would be collected and thus no tax redistribution mechanism would be needed. Essentially, the costs and benefits of the program are confined to the basin's groundwater producers, and this helps to mitigate the income transfer effects involving other groups.

[7] This is true as long as producers differ in their tax rates. The potential asymmetry between optimal quota assignments and optimal pumping rates arrived at through optimal pump taxation was pointed out by Rodney Smith in his review of the original draft of this report.

IV. MECHANISMS FOR PUMP TAX REVENUE REDISTRIBUTION

Without a mechanism to redistribute pump tax revenues, they are net subtractions from each producer's welfare. The only benefits that a producer "receives" are the cost savings from being able to achieve lower private marginal costs under the management program. To a first (and at best very rough) approximation, total tax payments by producers would equal their total cost savings in the long run, so they would essentially be indifferent to, or more likely against, groundwater management. Thus, an appropriately designed redistribution mechanism is necessary. The legal environment that governs the formation of a groundwater management program would probably constrain it to be a non-profitmaking institution. Also, the redistribution mechanism chosen must be nondistortive of groundwater-use decisions; otherwise, the potential cost savings from the management program cannot be realized.

In this section, we consider two redistribution mechanisms and analyze their effects when combined with a pump tax policy. We compare each program on the basis of the long-run equilibrium results with the case of no management.

The first redistribution mechanism we consider is a uniform per-unit subsidy on the price of imported surface water for groundwater producers. The social manager for the basin chooses the surface water subsidy so that total subsidy payments equal total pump tax revenues.

We show that the subsidy to producers leads to increased total water use though expanded surface water imports. Since producers make their pumping-rate decisions based on the subsidized surface water price, this leads to a higher long-run equilibrium water table level than would result without the subsidy.

In the second redistribution mechanism, the social manager provides a per-unit subsidy for land used by producers. To analyze the effects of this program, we enlarge our model to include both water and land use by producers. We show that when water and land complement each other as factors of production, the land subsidy leads to an increase in both water and land use. Total water use expands though increased surface water imports. With water and land as substitutes, the land subsidy encourages producers to increase their land use, while water use falls due to decreased surface water imports.

The discussion we present does not treat the problem at its most general level. We simply aim to illustrate the basic economic effects. To do this, we make several simplifying assumptions:

1. Our analysis is purely partial-equilibrium in nature, so that subsidies to surface water or land use do not affect the prices of these factors of production.
2. We assume that every producer has the same parametric pumping cost function, similar to Eq. (2.5), and the same number of wells.
3. The price of surface water determines each well's pumping rate.

4. We simplify the hydrology by assuming a straight-sided aquifer so that the response of water table levels to pumping does not depend on current water table levels.
5. We assume that the depth from the land surface to the initial water table level is the same for each well (i.e., $z_i(0) = z(0)$ for each well i).

Under these conditions, each well pumps water at the same equilibrium pumping rate. And each well's pumping rate is the same fraction, 1/n, of the natural replenishment rate, R. Thus, $Q_i = Q = R/n$, where n = the number of wells in the basin. Since each producer has the same number of wells, they each have the same total pumping rate:

$$Q_\ell = mQ \text{ , for each producer } ,$$

where m is the number of wells owned by any one producer. Consequently, each producer will have the same optimal tax rate, so that $t_\ell = t$, for each producer ℓ. Finally, the effect of the pump tax is a higher long-run equilibrium water table level which translates into lower private pumping costs at each well.

The main result we have is that if groundwater management were costless, then it clearly would result in a more economically beneficial steady state than would the case of unmanaged development. To more realistically assess the benefit of groundwater management, we should also weigh (1) the steady-state costs of runnng the program and (2) the administrative plus opportunity costs of moving from some current unmanaged state of the basin to the optimally managed state. These opportunity costs would include the welfare losses suffered by producers from having to "cut back" their pumping activity as would be required with optimal management. Although we do not treat these additional aspects of the problem, we mention them to indicate limitations of the current analysis.

REDISTRIBUTION THROUGH SURFACE WATER SUBSIDIES

Figure 4.1 illustrates the two long-run equilibria for a typical well. The optimal water table level, z_i^*, under just the optimal taxation policy is the value of z for which the curves $SMC(Q_i,z)$ and $PMC(Q_i,z')$ coincide. The value $z = z'$ corresponds to the water table level without management. Because of the subsidized price of surface water, the water table level $z = z''$ is lower than z^*. The curve $SMC(Q_i,z'')$ lies below the $PMC(Q_i,z')$ curve by an amount equal to the per-unit surface water subsidy, S_s. Also, the subsidized price of surface water leads producers to expand their long-run equilibrium surface water use from y_i' to y_i''.

The changes in producers' welfare for the well illustrated in Fig. 4.1 are readily obtained with our simplifying assumptions stated earlier. The downward shift in the PMC curve is $S_s + t$ and represents a per-unit cost savings applied to the pumping rate $Q_i = Q$. The cost savings are

$$CS_i = (t + S_s)Q \text{ .} \tag{4.1}$$

The pump tax payments, TP_i, made by this producer for operating his well are

$$TP_i = tQ \text{ .} \tag{4.2}$$

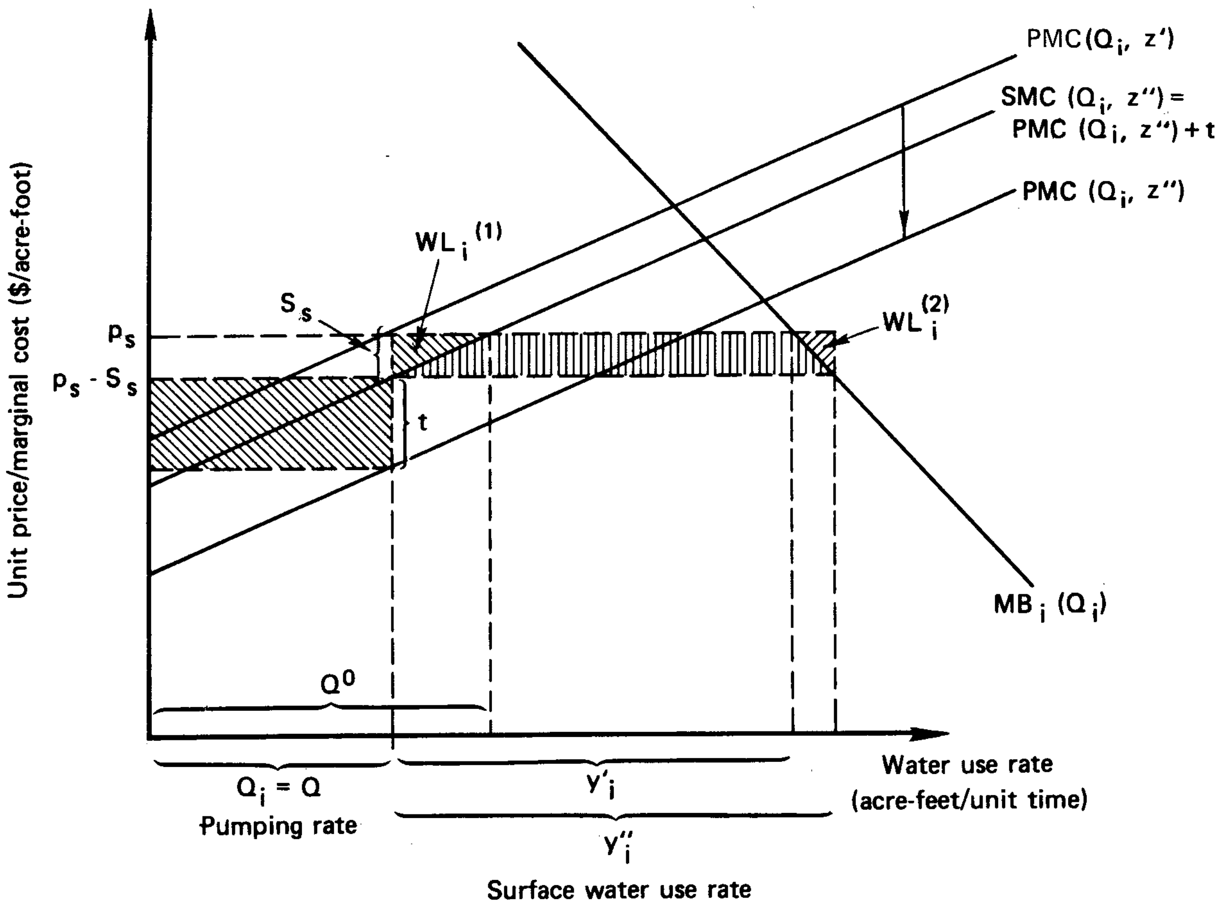


Fig. 4.1—Pump taxation with surface water subsidy

The surface water subsidy payments, SP_i, attributed to this well are

$$SP_i = S_s \cdot y_i'' \ . \tag{4.3}$$

The small triangular area to the left above the $SMC(Q_i, z_i'')$ curve represents a welfare loss due to the use of higher-priced surface water in the amount of $Q^o - Q$. The magnitude of this welfare loss is

$$WL_i^{(1)} = \tfrac{1}{2}(S_s)\,(Q^o - Q) \ . \tag{4.4}$$

A second welfare-loss triangle is shown to the right above the $MB_i(Q_i)$ curve. This loss is due to the subsidized price of surface water which leads to increased total use of water. The magnitude of this welfare loss is

$$WL_i^{(2)} = \tfrac{1}{2}(S_s)\,[(y_i'' + Q) - (y_i' + Q)] \ ,$$

$$WL_i^{(2)} = \tfrac{1}{2}(S_s)\,(y_i'' - y_i') \ . \tag{4.5}$$

The change in welfare is the sum of cost savings and subsidy payments less tax payments and welfare losses:

$$\Delta W_i = CS_i + SP_i - TP_i - WL_i^{(1)} - WL_i^{(2)}. \tag{4.6}$$

Note that ΔW_i has units of dollars per unit time and is a "flow" or rate variable rather than a "stock" variable. The aggregate welfare losses for the basin represent a flow of payments to the water wholesaler. However, since the cost savings exceed the pump tax payments and the two welfare-loss terms are only a fraction of the subsidy payments, the change in welfare is unambiguously positive.

REDISTRIBUTION THROUGH A PER-UNIT LAND SUBSIDY

Since water and land represent two unique factors of production, different producers will generally employ them in varying proportions. We assume that producers employ the profit-maximizing combinations of water and land as dictated by their own economic activities. Since water and land may be substitutes or complements, we consider the effects of both relationships.

Since we include two factors of production, each producer has two corresponding marginal benefit functions: one for water and one for land. Let

$MB_\ell^{(w)}(x_\ell, L_\ell^*)$ = producer ℓ's marginal benefit function from water use, (4.7)

where x_ℓ is the total rate of water use, and L_ℓ^* is some fixed total rate of land use. Also, let

$MB_\ell^{(L)}(x_\ell^*, L_\ell)$ = producer ℓ's marginal benefit function from land use, (4.8)

where L_ℓ is the total rate of land use, and x_ℓ^* is some fixed total rate of water use. Moreover, the variable x_ℓ represents his total water use:

$$x_\ell = Q_\ell + y_\ell \,, \tag{4.9}$$

where Q_ℓ is the total groundwater pumping rate, and y_ℓ is the total rate of surface water use. Thus, the marginal benefit functions from land and water are each functions of the various possible combinations of water, x_ℓ, and land, L_ℓ.

We can now define water and land use as substitutes or complements: Water and land are *substitutes* if

$$\frac{\partial MB_\ell^{(w)}}{\partial L_\ell} > (<)\ 0 \tag{4.10}$$

while

$$\frac{\partial MB_\ell^{(L)}}{\partial x_\ell} < (>)\ 0\,. \tag{4.11}$$

Water and land are *complements* if

$$\frac{\partial MB_\ell^{(w)}}{\partial L_\ell} > (<)\ 0.$$

while

$$\frac{\partial MB_\ell^{(L)}}{\partial x_\ell} > (<)\ 0\ .$$

Basically, if water and land are substitutes, their respective marginal benefit functions shift in opposite directions; if they are complementary, their marginal benefit functions shift in the same direction.

With unmanaged groundwater development, each producer determines his optimal rates of land and water use from the following equations:

$$\begin{cases} MB_\ell^{(w)}\,(mQ + y'_\ell, L'_\ell) = p_s, \\ \qquad p_s = PMC(Q_\ell, z'), \text{ and} \\ MB_\ell^{(L)}\,(mQ + y'_\ell, L'_\ell) = p_L \end{cases} \tag{4.12}$$

When the basin's groundwater pumping is managed optimally through a pump tax, t, accompanied by a per-unit subsidy to the price of land, the following equations determine each producer's optimal rates of land and water use:

$$\begin{cases} MB_\ell^{(w)}\,(mQ + y''_\ell, L''_\ell) = p_s, \\ \qquad p_s = PMC(Q_\ell, z'') + t, \text{ and} \\ MB_\ell^{(L)}\,(mQ + y''_\ell, L''_\ell) = p_L - S_L. \end{cases} \tag{4.13}$$

The total land subsidy payments must just balance pump tax revenues:

$$\sum_\ell t\, Q_\ell = \sum_\ell S_L\, L''_\ell, \text{ for } Q_\ell = mQ \tag{4.14}$$

In addition to the above sets of equations for each producer, the hydrological equilibrium equation,

$$R = \sum_\ell Q_\ell = \sum_\ell mQ\,,$$

must hold to establish the long-run equilibrium water table levels z′ and z″.

Figures 4.2 and 4.3 illustrate the two long-run equilibrium solutions, first for a producer with water and land as complements, and second for a producer with water and land as substitutes. In either case, we write the change in the producer's welfare as follows:

$$\Delta W_{\ell} = CS_{\ell} + SP_{\ell} - TP_{\ell} , \tag{4.15}$$

where CS_{ℓ} = the rate of production cost savings,
SP_{ℓ} = the rate of land subsidy payments, and
TP_{ℓ} = the rate of pump tax revenues.

The rate of production cost savings results from the downward shift in the $PMC(Q_{\ell},z')$ curve under managed development. The social maginal cost curve, $SMC(Q_{\ell},z'')$, coincides with the $PMC(Q_{\ell},z')$ curve so the shift in private cost curves just equals the pump tax rate. Therefore,

$$CS_{\ell} = t\, Q_{\ell} .$$

Each producer's pump tax payments are

$$TP_{\ell} = t(mQ) ,$$

and his land subsidy payments are

$$SP_{\ell} = S_L \cdot L_{\ell}'' .$$

The small triangular area above the marginal benefit functions for land in Figs. 4.2 and 4.3 represents a small loss in economic efficiency from a subsidy to the price of land. Without the subsidized price, producers would use land at a lower rate (e.g., L_{ℓ}^{o}). The subsidized price prompts them to choose a rate $L_{\ell}'' > L_{\ell}^{o}$. The price on these additional units of land exceeds their marginal benefit. The magnitude of this efficiency loss is

$$WL = ½\,(S_{\ell})(L_{\ell}'' - L_{\ell}^{o}) .$$

If a producer owns the land that he uses, he employs these additional units of land by sustaining losses of economic efficiency. However, if he rents land, these efficiency losses convert to decreases in his subsidy payments and thus to decreases in his welfare. In either case, the efficiency loss is just a fraction of the total land subsidy payments.

The change in a producer's welfare after substituting the above expressions for CS_{ℓ}, TP_{ℓ}, and SP_{ℓ}, and after simplifying, becomes

$$\Delta W_{\ell} = S_{\ell}\, L_{\ell}'' . \tag{4.16}$$

Even after we subtract the welfare loss, WL , experienced by producers who rent their land, the change in welfare is still positive.

Producers respond to the per-unit subsidy for land by increasing their equilibrium rates of land use. With a complementary relationship to water, increased land use feeds back to induce increased water use. Such increases in total water use result from expanded rates of surface water use. With a significant degree of substitutability between land and water, expanded land use causes a decrease in total water use, and this decrease comes about through reduced use of surface water.

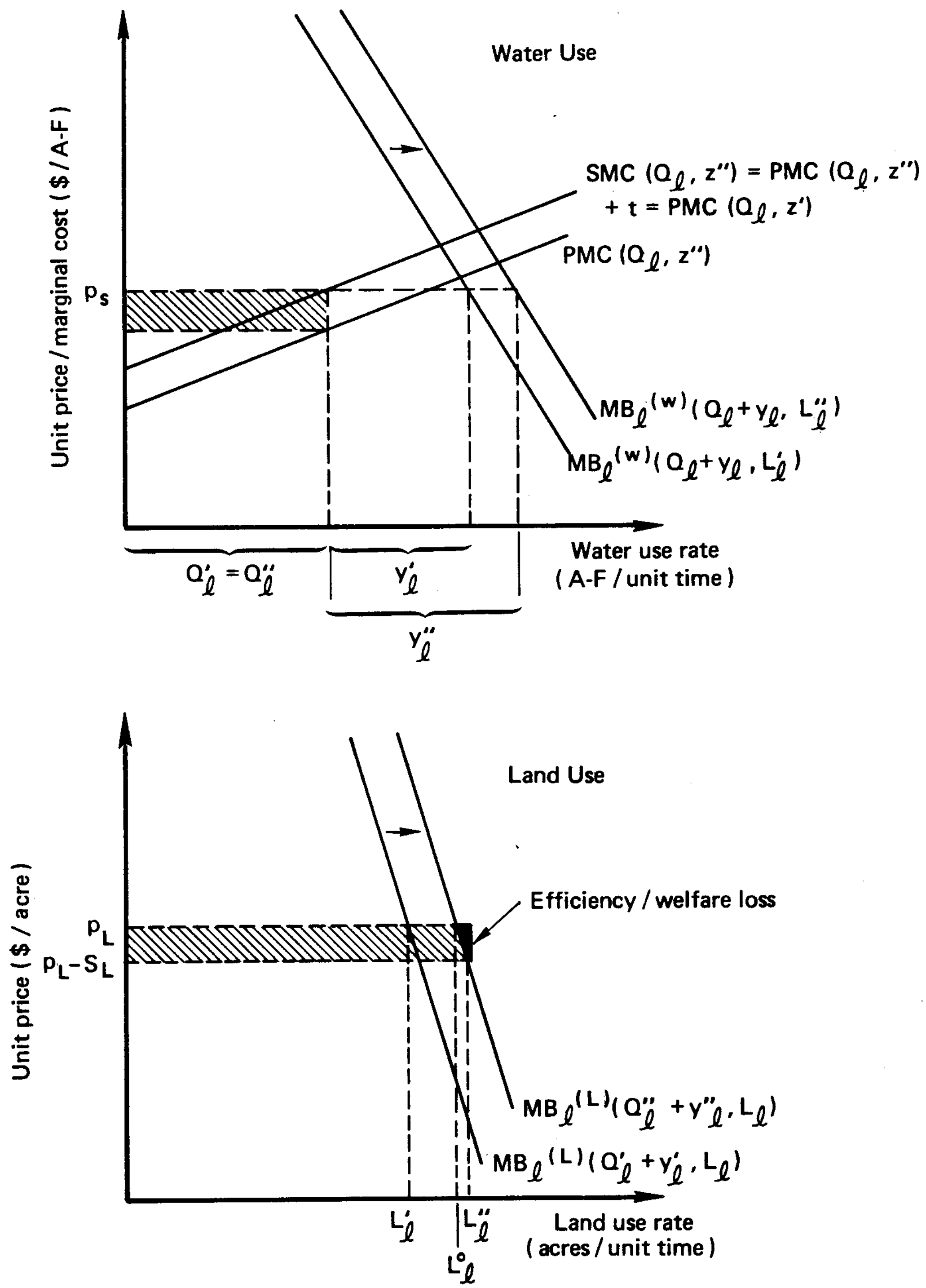


Fig. 4.2—Comparison of long-run equilibrium solutions with water and land as complements (pump tax revenues redistributed through a per-unit subsidy for land)

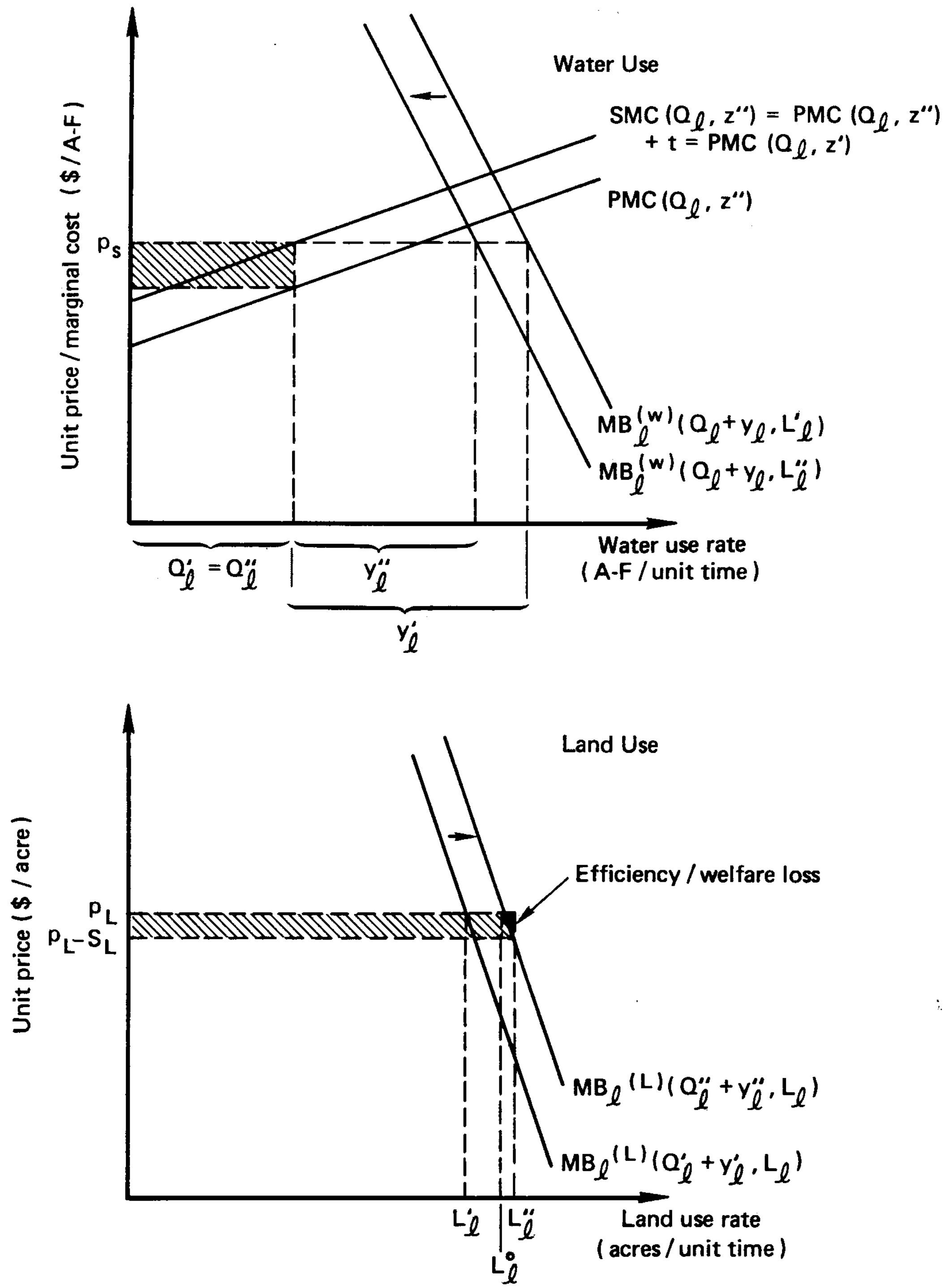


Fig. 4.3—Comparison of long-run equilibrium solutions with water and land as substitutes (pump tax revenues redistributed through a per-unit subsidy for land)

V. CONCLUSIONS

The analysis of the simple mathematical and economic models developed in this report shows that collective management of groundwater resources can potentially result in a more beneficial long-run steady state. (Whether or not actual net benefits result from management depends on the costs of administration and the costs of changing the status quo.) We have shown how optimal pump taxes can provide the incentive for producers to correctly assess the costs of extracting groundwater.

When producers have no legal limits to their pumping activity, a management program should be based on pump taxation. As shown in Sec. IV, the pump tax revenues must be redistributed to the producers to achieve an acceptable program. Otherwise, the producers will be opposed to groundwater management and in fact would probably be better off without it. To illustrate the long-run economic effects of pump taxation with redistribution, we have considered two types of program:

1. Pump taxation with a subsidy for surface water, and
2. Pump taxation with a subsidy for land.

The first program leads to expanded surface water use and a higher water table level than would be obtained with just the pump tax. Producers tend to *overvalue* their groundwater stocks. The second program leads to increased land use, with the effects on water use dependent upon the degree to which water and land substitute for or complement each other in producers' aggregate economic activity.

Under a legal system that permits producers to derive income from groundwater either by pumping it or by selling their rights to pump, assignment of tradable pumping quotas may not be an appropriate management tool. If producers differ in their private cost conditions, then a socially efficient assignment of pumping-rate quotas would tend to evolve to a socially inefficient allocation through trading. This occurs because producers would make their trading decisions based on private marginal costs rather than on social marginal costs. However, this inefficiency should not be a reason for excluding quota assignments as a management tool, since its welfare effects are likely to be small compared to the overall improvements in groundwater use that would result.

The key element in any groundwater management program is cooperation among the basin's producers. In this report, we have emphasized groundwater management tools based on properly designed economic incentives to help ensure this cooperation.

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