EIGENFILTER DESIGN OF REAL AND COMPLEX COEFFICIENT PROTOTYPES FOR UNIFORM AND NONUNIFORM FILTER BANKS

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ABSTRACT

In this study we propose a new method to design Pseudo-QMF prototype filters to implement Near Perfect Reconstruction (NPR) cosine-modulated filter banks. The method is based on the eigenfilter approach, that is simple to implement, but, nevertheless, is very efficient in designing high attenuation prototype filters. The method also allows to design complex coefficient prototypes that can be used to build nonuniform filter banks. The effectiveness of the method is demonstrated by means of some examples of design of both uniform and nonuniform filter banks.

1. INTRODUCTION

The eigenfilter approach is an efficient method to design a large variety of digital filters having both Finite Impulse Response (FIR) [1]-[3] and Infinite Impulse Response (IIR) [4][5]. The method is flexible and is easily implemented, since the problem is reduced to finding the eigenvector corresponding to the minimum eigenvalue of a positive-definite matrix. Applications of the eigenfilter approach to multirate signal processing are given in [6]. In [7] it has been used to design two-channel QMF banks by finding the minimum of a mixed time/frequency domain based cost function.

The purpose of this paper is extending the application of the eigenfilter approach to design linear phase prototypes with real and complex coefficients to implement uniform and nonuniform NPR cosine-modulated M-channel filter banks. A nonuniform analysis/synthesis filter bank with integer decimation factors is shown in Fig. 1. If all the downsampling/upsampling factors in each branch are equal to M, we have a uniform filter bank.

In uniform M-channel cosine-modulated filter banks, the impulse responses of the analysis and synthesis filters are given by

$$\begin{aligned} f_k(n) &= 2h(n)\cos((2k+1)\tfrac{\pi}{2M}(n-\tfrac{N-1}{2})+\theta_k) \\ g_k(n) &= 2h(n)\cos((2k+1)\tfrac{\pi}{2M}(n-\tfrac{N-1}{2})-\theta_k) \\ &= f_k(N-1-n) \end{aligned} \quad (1)$$

where k=0,1,...,$M-1$, n=0,1,...,$N-1$, and $\theta_k = (-1)^k\pi/4$. The prototype $h(n)$ is assumed to have length N and to be symmetric. The prototype frequency response must satisfy, at least approximately, the Power Complementary (PC) property , i.e.,

$$\begin{cases} |H(\omega)|^2 + |H(\frac{\pi}{M}-\omega)|^2 = M & 0 \le \omega \le \pi/M \\ |H(\omega)|^2 + |H(-\frac{\pi}{M}-\omega)|^2 = M & -\pi/M \le \omega \le 0 \end{cases} \quad (2)$$

For nonuniform filter banks, several prototypes can be used [8]-[10]. In this case, a different sequence $\{h(n)\}$ is used in each branch of the bank, whereas the phase terms θ_k of adjacent branches must be chosen so that they differ of $\pi/2$. In [8], prototypes with different transition bandwidths are modulated to build the final nonuniform bank. To fill the gaps between the frequency responses of filters derived from different prototypes, the modulation of a prototype having a nonsymmetric amplitude frequency response and such to match the transition bands of the neighboring filters is needed. This allows the cancellation, at least approximately, of the *main aliasing* components (Pseudo-QMF banks). Hence, a complex coefficient filter satisfying the PC property must be designed.

The method proposed in this study can be applied to design both real and complex coefficient PC prototypes (see Section 3). In the next section, we will address the problem of eigenfilter design of complex coefficient FIR filters having a linear phase.

2. LINEAR PHASE COMPLEX COEFFICIENT EIGENFILTERS

Let $h(n)$, $n = 0,1,\ldots,N-1$, be the frequency response of a low-pass filter with, in general, complex coefficients. Let $D(\omega)$ be the desired frequency response and, as shown in Fig. 2, let $D_1(\omega)$ and $D_2(\omega)$ denote the desired response at the negative and positive frequencies, respectively. A subscript *s*, *t* or *p* will be added to denote the desired frequency response in the stopband, transition band or passband. In the eigenfilter approach, a weighted cost function E is defined as a function of the error between $H(\omega)$, where $H(\omega) = \sum_{n=0}^{N-1} h(n)e^{-j\omega n}$, and the desired target function. The cost function is expressed as a quadratic form, i.e., $E = y^T\mathbf{P}y$, where y is related to the filter coefficients and $\mathbf{P}$ is a positive-definite matrix [1]. In [3] $\mathbf{P}$ is a *complex coefficient* $N \times N$ Hermitian matrix, whereas in [2] $\mathbf{P}$ is a *real coefficient* $2N \times 2N$ symmetric matrix. In both cases, the eigenvalues of $\mathbf{P}$ are real and the minimum of the quadratic form under the constraint $||y||_2 = constant$ is reached for y equal to the eigenvector corresponding to the minimum eigenvalue of $\mathbf{P}$.

We will formulate the eigenfilter method by exploiting the fact that the filters to be designed are linear phase, as they are those used in [8]. This property simplifies the structure of the matrix $\mathbf{P}$ that becomes a *real coefficient* $N \times N$ positive-definite symmetric matrix. Suppose that $h(n) = h^*(N-1-n)$, i.e., $h_R(n) = h_R(N-1-n)$ and $h_I(n) = -h_I(N-1-n)$, where the sequences $\{h_R(n)\}$ and $\{h_I(n)\}$ are the real and imaginary part of $\{h(n)\}$, respectively. The frequency response $H(\omega)$ satisfies

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$H(\omega) = \mathcal{H}(\omega)e^{-j\omega\frac{N-1}{2}}$, where $\mathcal{H}(\omega)$ is a real valued function. It can be easily verified that, for even N, we have

$$\mathcal{H}(\omega) = \sum_{k=0}^{N/2-1} 2h_R(\tfrac{N}{2}+k)\cos((2k+1)\tfrac{\omega}{2}) - \sum_{k=0}^{N/2-1} 2h_I(\tfrac{N}{2}+k)\sin((2k+1)\tfrac{\omega}{2}) = \mathbf{h_{LP}}^T\mathbf{c} \quad (3)$$

where

$$\mathbf{h_{LP}} = [h_R(\tfrac{N}{2})\ h_R(\tfrac{N}{2}+1)\ \ldots\ h_R(N-1)\ h_I(\tfrac{N}{2})\ h_I(\tfrac{N}{2}+1)\ \ldots\ h_I(N-1)]^T \quad (4)$$

$$\mathbf{c} = [2\cos\tfrac{\omega}{2}\ \ 2\cos\tfrac{3}{2}\omega\ \ldots\ 2\cos\tfrac{N-1}{2}\omega\ \ -2\sin\tfrac{\omega}{2}\ \ -2\sin\tfrac{3}{2}\omega\ \ldots\ -2\sin\tfrac{N-1}{2}\omega]^T \quad (5)$$

Similar expressions hold for odd N, but they are omitted here for brevity's sake. Also the desired frequency response $D(\omega)$ must have linear phase, i.e., $D(\omega) = \mathcal{D}(\omega)e^{-j\omega\frac{N-1}{2}}$, where $\mathcal{D}(\omega)$ is a real function asymmetrical around $\omega = 0$.

Consider now the positive frequency axis. In the stopband $(\omega_{s,2}, \pi)$, we have $\mathcal{D}_{s,2}(\omega) = 0$. A weighted cost function can be defined as

$$\begin{aligned} E_{s,2} &= \int_{\omega_{s,2}}^{\pi} \mid \mathcal{D}_{s,2}(\omega) - \mathcal{H}(\omega) \mid^2 W_{s,2}(\omega)d\omega = \\ &= \int_{\omega_{s,2}}^{\pi} \mid \mathbf{h_{LP}}^T\mathbf{c} \mid^2 W_{s,2}(\omega)d\omega \\ &= \mathbf{h_{LP}}^T \left[\int_{\omega_{s,2}}^{\pi} \mathbf{c}\mathbf{c}^T W_{s,2}(\omega)d\omega \right] \mathbf{h_{LP}} \\ &= \mathbf{h_{LP}}^T\,\mathbf{P_{s,2}}\,\mathbf{h_{LP}} \end{aligned} \quad (6)$$

where $W_{s,2}(\omega)$ (as well as $W_{p,2}(\omega)$ and $W_{t,2}(\omega)$ defined in (7) and (8)) is a positive-valued weighting function.

In the passband $(0, \omega_{p,2})$, the error is measured between $\mathcal{H}(\omega)$ and its value in $\omega = 0$ [1], i.e., $\mathcal{H}(0) = \mathbf{h_{LP}}^T\mathbf{c_0}$, where, for even N, $\mathbf{c_0} = [1\ 1\ \ldots\ 1\ 0\ 0\ldots\ 0]$ is a vector with $N/2$ entries equal to 1 and $N/2$ entries equal to 0. The cost function in this region can be expressed as

$$\begin{aligned} E_{p,2} &= \int_0^{\omega_{p,2}} \mid \mathcal{D}_{p,2}(\omega) - \mathcal{H}(\omega) \mid^2 W_{p,2}(\omega)d\omega \\ &= \mathbf{h_{LP}}^T \left[\int_0^{\omega_{p,2}} (\mathbf{c_0}-\mathbf{c})(\mathbf{c_0}-\mathbf{c})^T W_{p,2}(\omega)d\omega \right] \mathbf{h_{LP}} \\ &= \mathbf{h_{LP}}^T\,\mathbf{P_{p,2}}\,\mathbf{h_{LP}} \end{aligned} \quad (7)$$

In the transition band $(\omega_{p,2}, \omega_{s,2})$, we will assume that $\mathcal{D}_2(\omega)$ is given. We consider the error between $\mathcal{H}(\omega)$ and a scaled version of $\mathcal{D}_2(\omega)$ [2]. Let $\mathcal{D}_{t,2}(\omega) = [\mathcal{D}_2(\omega)/\mathcal{D}_2(\omega_{t,2})]\mathcal{H}(\omega_{t,2})$, where $\omega_{t,2}$ is a frequency belonging to the transition band. By using $\mathcal{H}(\omega_{t,2}) = \mathbf{h_{LP}}^T\mathbf{c_{t,2}}$, where $\mathbf{c_{t,2}}$ is $\mathbf{c}$ computed for $\omega = \omega_{t,2}$, we have $\mathcal{D}_{t,2}(\omega) = \mathbf{h_{LP}}^T\,[\mathcal{D}_2(\omega)/\mathcal{D}_2(\omega_{t,2})]\,\mathbf{c_{t,2}}$. Therefore, the cost function in the transition region is given by:

$$\begin{aligned} E_{t,2} &= \int_{\omega_{p,2}}^{\omega_{s,2}} \mid \mathcal{D}_{t,2}(\omega) - \mathcal{H}(\omega) \mid^2 W_{t,2}(\omega)d\omega \\ &= \mathbf{h_{LP}}^T \left[\int_{\omega_{p,2}}^{\omega_{s,2}} ([\mathcal{D}_2(\omega)/\mathcal{D}_2(\omega_{t,2})]\mathbf{c_{t,2}} - \mathbf{c})\cdot \right. \\ &\left. \cdot([\mathcal{D}_2(\omega)/\mathcal{D}_2(\omega_{t,2})]\mathbf{c_{t,2}} - \mathbf{c})^T W_{t,2}(\omega)d\omega \right] \mathbf{h_{LP}} = \\ &= \mathbf{h_{LP}}^T\,\mathbf{P_{t,2}}\,\mathbf{h_{LP}} \end{aligned} \quad (8)$$

New matrices $\mathbf{P_{s,1}}$, $\mathbf{P_{p,1}}$ and $\mathbf{P_{t,1}}$ similar to those defined in (6), (7) and (8) can be easily derived relatively to the negative frequency axis. The global cost function can be written as

$$\begin{aligned} E = \ &\mathbf{h_{LP}}^T(k_{s,1}\mathbf{P_{s,1}} + k_{p,1}\mathbf{P_{p,1}} + k_{t,1}\mathbf{P_{t,1}} + k_{s,2}\mathbf{P_{s,2}} + \\ &+ k_{p,2}\mathbf{P_{p,2}} + k_{t,2}\mathbf{P_{t,2}})\,\mathbf{h_{LP}} = \mathbf{h_{LP}}^T\,\mathbf{P}\,\mathbf{h_{LP}} \end{aligned} \quad (9)$$

The constants multiplying the matrices are chosen to weigh the different contributions. The eigenvector of $\mathbf{P}$ relative to the minimum eigenvalue yields the optimum vector $\mathbf{h_{LP}}$ and, by using the symmetries of $h_R(n)$ and $h_I(n)$, the filter coefficients $h(n)$.

3. DESIGN OF PSEUDO-QMF EIGENFILTERS

In this section, we will apply the method previously described to design real and complex coefficient prototypes that are approximately power complementary. Note that a characteristic of the method described in Section 2 is that the desired frequency response is assumed known on the whole interval $(-\pi, \pi)$, i.e., the transition bands are not assumed as "don't care bands". Note also that, by using the definitions given in the previous section, the PC property (2) can be translated into a quadratic constraint that must be satisfied by the prototype filter coefficients. Therefore, the problem could be solved by using a quadratic-constrained least-squares search, that is the formulation given in [11]. To maintain the simplicity of the classical eigenfilter approach, we will assume that $D_1(\omega)$ and $D_2(\omega)$ are defined analytically, so that the procedure described in Section 2 can be used (the real and complex coefficient prototype cases are considered separately).

1) *Real coefficient prototype for uniform M-channel filter banks*

Let $\omega_{s,2} = -\omega_{s,1} = \frac{\pi}{M}$ and $\omega_{p,2} = \omega_{p,1} = 0$. Let $D_1(\omega)$ and $D_2(\omega)$ be defined as follows

$$D_1(\omega) = \begin{cases} 0 & -\pi \le \omega < \omega_{s,1} \\ \sqrt{M}\sin(\frac{\pi}{2}\theta(\frac{\omega-\omega_{s,1}}{\omega_{p,1}-\omega_{s,1}}))e^{-j\omega\frac{N-1}{2}} & \omega_{s,1} \le \omega \le \omega_{p,1} \end{cases}$$

$$D_2(\omega) = \begin{cases} \sqrt{M}\cos(\frac{\pi}{2}\theta(\frac{\omega-\omega_{p,2}}{\omega_{s,2}-\omega_{p,2}}))e^{-j\omega\frac{N-1}{2}} & \omega_{p,2} \le \omega < \omega_{s,2} \\ 0 & \omega_{s,2} \le \omega < \pi \end{cases} \quad (10)$$

where the function $\theta(x)$ satisfies

$$\theta(x) = \begin{cases} 0 & x \le 0 \\ 1 & x \ge 1 \end{cases} \qquad \theta(x) + \theta(1-x) = 1 \quad 0 < x < 1 \quad (11)$$

Functions of this type have been used by Meyer to define the Fourier Transform of orthonormal continuous wavelet functions (see, for example, [12][13]). It can be easily verified that $D_1(\omega) = D_2^*(-\omega)$. Even if the choice $\omega_{p,2} = \omega_{p,1} = 0$ seems to imply the absence of a passband, the definitions of the transitions bands in (10) include also an interval in which the frequency reponse is flat around $\omega = 0$. The higher the regularity of $\theta(x)$ at $x = 0$, the wider the interval in which the response can be considered a passband.

2) *Complex coefficient prototypes*

For nonuniform filter banks, as designed in [8], in the simplest case, two prototypes are designed for an M_1 and M_2-channel uniform filter bank. Filters are selected from these banks to build the final nonuniform bank. To join filters coming from different banks, the modulation of a complex coefficient prototype having a nonsymmetric amplitude frequency response, like that shown in Fig. 2, is needed. In fact, its frequency response in the two transition bands must match those of the M_1 and M_2-channel prototypes. Suppose (10) is used to design the prototypes for the uniform $M = M_1$ and $M = M_2$ channel banks. Without loss of generality, assume $M_2 < M_1$, so that a wider transition bandwidth is allowed for the M_2-channel prototype. The downsampling factor associated to the complex coefficient prototype is M_2. Its frequency response in the transition band at positive and negative frequencies ideally should match those defined in (10). Let $\omega_{s,1} = -\frac{\pi}{2M_2} - \frac{\pi}{2M_1}$, $\omega_{p,1} = -\frac{\pi}{2M_2} + \frac{\pi}{2M_1}$, $\omega_{p,2} = 0$ and

$\omega_{s,2} = \frac{\pi}{M_2}$. We define $D_1(\omega)$ and $D_2(\omega)$ as follows

$$D_1(\omega) = \begin{cases} 0 & -\pi \le \omega < \omega_{s,1} \\ \sqrt{M_2}\sin(\frac{\pi}{2}\theta(\frac{\omega-\omega_{s,1}}{\omega_{p,1}-\omega_{s,1}}))e^{-j\omega\frac{N-1}{2}} & \omega_{s,1} \le \omega < \omega_{p,1} \\ \sqrt{M_2}e^{-j\omega\frac{N-1}{2}} & \omega_{p,1} \le \omega \le 0 \end{cases}$$

$$D_2(\omega) = \begin{cases} \sqrt{M_2}\cos(\frac{\pi}{2}\theta(\frac{\omega-\omega_{p,2}}{\omega_{s,2}-\omega_{p,2}}))e^{-j\omega\frac{N-1}{2}} & \omega_{p,2} \le \omega < \omega_{s,2} \\ 0 & \omega_{s,2} \le \omega \le \pi \end{cases} \quad (12)$$

where N is the number of coefficients of the prototype.

The ideal PC prototypes defined by (10) and (12) would allow to implement nonuniform filter banks such that the aliasing cancellation requirement is perfectly satisfied. The eigenfilter approach described in Section 2 allows to compute the filter coefficients that approximate the ideal prototypes in a weighted least-squares sense.

The degree of regularity of $\theta(x)$ at the points $x = 0$ and $x = 1$ induces the degree of regularity of $D(\omega)$. A compromise between a smoothly behaved $D(\omega)$ (high regularity of $\theta(x)$) and not eccessively steep descent transition bands (low regularity of $\theta(x)$) must be achieved. If in the interval $(0, 1)$ $\theta(x)$ is a polynomial function, antisymmetrical around the point $(1/2, 1/2)$, and having a certain number of null derivatives at $x = 0$ and $x = 1$, then its coefficients can be easily computed by solving a linear system. We have found that, for the filter lengths used in our examples, an order equal to nine, i.e., $\theta(x) = 126x^5 - 420x^6 + 540x^7 - 315x^8 + 70x^9$, is often a good choice to limit the PC property approximation error and to obtain high stopband attenuations.

The analysis filter frequency responses of a five-channel nonuniform filter bank are plotted (with a normalized gain equal to unity in the passband) in Fig. 3. The filters have been selected from 8-channel and 4-channel uniform banks, having length $N_1 = 128$ and $N_2 = 64$, respectively. They are joined by a 128 coefficient intermediate filter. The prototypes have been designed by using either (10) or (12) (the order of $\theta(x)$ is nine). All the constants $k_{\cdot,\cdot}$ in (9) have been taken equal to unity. The overall distortion in the absence of aliasing is ± 0.004 dB, whereas the maximum of the uncanceled aliasing components is -71.5 dB.

The choice of defining analytically the frequency response in the transition bands may seem a too severe constraint and such to prevent from good prototype design. However, the technique described in this section, associated with a proper choice of the weighting functions, allows to obtain high stopband attenuation filters (greater than 110 dB) even when the number of coefficients of the filter and the number of channels of the bank are relatively high, e.g., $N = 512, 1024$ and $M = 32, 64$. Recursively updated weighting functions have often been associated to the eigenfilter approach [1][2][3][5]. Let $H^{(n)}(\omega)$ be the frequency response at the *n-th* step of an iterative procedure. $H^{(n)}(\omega)$ is obtained by applying the eigenfilter approach with $W_{\cdot,\cdot}^{(n)}(\omega)$ as weighting functions. $W_{\cdot,\cdot}^{(n)}(\omega)$ are updated as follows until a stop criterion is met.

1) *Real coefficient prototypes for uniform M-channel banks*

Consider the following definitions:

$$e_{t,2}^{(n)}(\omega) = \left| \; | H^{(n)}(\omega) |^2 + | H^{(n)}(\omega - \frac{\pi}{M}) |^2 - M \right| \quad (13)$$

for $0 \le \omega \le \pi/M$ and

$$e_{s,2}^{(n)}(\omega) = | H^{(n)}(\omega) | \quad (14)$$

for $\pi/M \le \omega \le \pi$. The first expression measures the deviation from the PC property of $H^{(n)}(\omega)$ in the transition band. The second measures the deviation from zero in the stopband. The weighting functions are updated according to

$$W_{t,2}^{(n+1)}(\omega) = W_{t,2}^{(n)}(\omega)\,\mathrm{env}(e_{t,2}^{(n)}(\omega)) \quad (15)$$

$$W_{s,2}^{(n+1)}(\omega) = W_{s,2}^{(n)}(\omega)\,\mathrm{env}(e_{s,2}^{(n)}(\omega)) \quad (16)$$

where $\mathrm{env}(\cdot)$ is the envelope function, computed as a linear interpolation of the maxima of its argument.

2) *Complex coefficient prototypes*

Assume that two prototypes $h_1(n)$ and $h_2(n)$ for uniform M_1 and M_2-channel banks have been designed by using the weighting functions updating described in the previous item. The introduction of $W_{t,2}(\omega)$, based on (15) and (13), let $H_1(\omega)$ and $H_2(\omega)$ move from the nominal functions defined by (10). Therefore, the frequency response of the complex coefficient prototype to be used along with $h_1(n)$ and $h_2(n)$ should have, as target functions, the transition bands of $H_1(\omega)$ and $H_2(\omega)$ instead of those defined in (12). The weighting functions, in the different bands, must be based on the deviation from PC property, i.e.,

$$e_{t,1}^{(n)}(\omega) = \left| \; | H^{(n)}(\omega) |^2 + | \sqrt{\frac{M_2}{M_1}} H_1(-\frac{\pi}{2M_2} - \frac{\pi}{2M_1} - \omega) |^2 - M_2 \right| \quad (17)$$

for $-\frac{\pi}{2M_2} - \frac{\pi}{2M_1} \le \omega < -\frac{\pi}{2M_2} + \frac{\pi}{2M_1}$,

$$e_{t,2}^{(n)}(\omega) = \left| \; | H^{(n)}(\omega) |^2 + | H_2(\frac{\pi}{M_2} - \omega) |^2 - M_2 \right| \quad (18)$$

for $0 \le \omega \le \frac{\pi}{M_2}$, and

$$e_{p,1}^{(n)}(\omega) = \left| \; | H^{(n)}(\omega) |^2 - M_2 \right| \quad (19)$$

for $-\frac{\pi}{2M_2} + \frac{\pi}{2M_1} \le \omega < 0$. The error in the stopbands at positive and negative frequencies is measured as in (14). The weighting function update is performed analogously as done in (15) and (16).

4. EXPERIMENTAL RESULTS

Several uniform M-channel banks have been designed. The weighting functions updating was based on the discussion made in the previous section. A ninth order polynomial $\theta(x)$ was used in all the examples. Table 1 reports some characteristics of the banks designed with $M = 4, 8, 16, 32, 64$: A_{stop} is the maximum stopband gain, E_r is the maximum deviation from unity of the overall distortion function in the absence of aliasing and E_{al} is the maximum value of the uncanceled aliasing components.

As an example of nonuniform filter bank a twenty subbands filter bank, as shown in Fig. 4, was designed. The involved downsampling factors are $64, 32, 16, 8$. The prototypes described in Table 1 have been used as well as transition filters with length $1024, 512, 256$. For this bank, we obtained $E_r = 6.18E-04$ and $E_{al} = -98.0$ dB. As can be seen, even if long filters and large decimation factors are involved, the procedure yields a high reconstruction performance.

5. CONCLUSIONS

In this study, we have proposed an extension of the eigenfilter approach to design approximately power complementary prototypes having both real and complex coefficients. In spite of its simplicity, when associated to suitable weighting functions, it allows to design high attenuation prototypes that allow to implement uniform and nonuniform banks with good recontruction properties.

Table 1: Characteristics of uniform filter banks designed with the eigenfilter procedure.

M	N	A_{stop} (dB)	E_r	E_{al} (dB)
4	64	−111.4	$5.85E-04$	−106.2
8	128	−111.9	$6.36E-04$	−106.4
16	256	−111.9	$6.78E-04$	−107.6
32	512	−112.7	$6.12E-04$	−107.2
64	1024	−111.0	$5.66E-04$	−102.6

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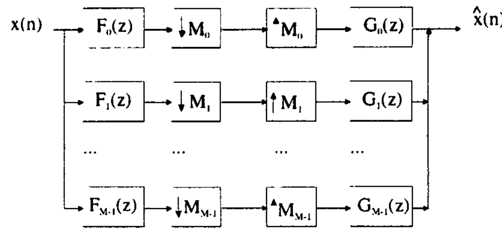

Figure 1: Uniform M-channel analysis/synthesis banks.

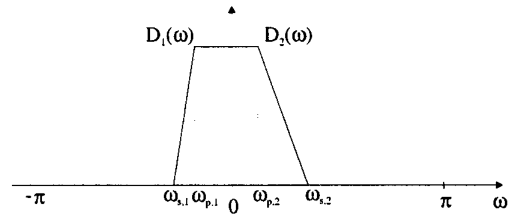

Figure 2: Desired frequency response $D(\omega)$.

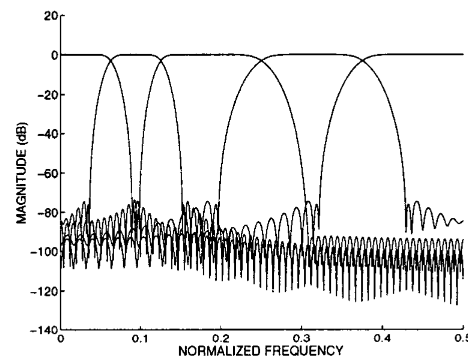

Figure 3: Analysis filter frequency responses relative to a nonuniform 5-channel bank. The prototypes are designed by using the eigenfilter method to approximate the ideal frequency responses defined in either (10) or (12).

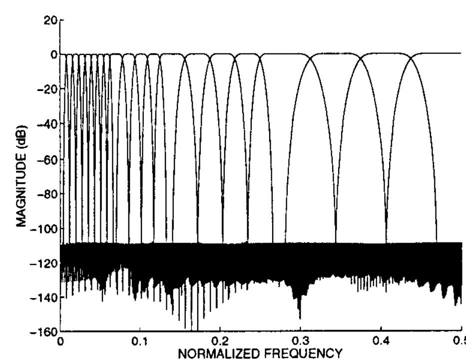

Figure 4: Analysis filters frequency responses relative to a nonuniform 20-channel bank. The prototype are designed by using recursively updated weighting functions.