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EIGENVALUES AND STABILITY PROBLEMS OF ROTORS

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The essential theoretical results of the application of a developed transfer matrix method to the free transverse vibration of a rotor are shown. Gyrosccopic and shear effects, rotary inertia, and external and internal damping as well as the influence of sleeve bearings and rotor supports are taken into consideration. The eigenvalues of the motion equations of the rotor are searched by using a modified determinant method.

INTRODUCTION

An essential feature of the presented method is the application of transfer matrices to uniform and flexible continuous elements of a rotor. These elements are assumed to be the basic elements of the rotor. The motion equations of the remaining rotor elements (rigid discs or complex continuous elements) can be obtained by a suitable modification of the motion equations of the basic rotor element. In this case, quantitative properties of the basic rotor element ought to be the same as in the case when these different rotor elements coexist altogether in a real rotor. In particular, the quantitative properties of the spectrum (set of eigenvalues) of the motion equations of the rotor are preserved.

The first part of the paper is devoted to solving the motion equation of the basic elements and rotor support construction in transfer matrix form. Subsequently, a numerical method of eigenvalues of the motion equations of the whole rotor and its support construction is explained. The final part of the paper contains the analysis of the spectrum of the basic element, which is treated as a simple rotor.

A very similar transfer matrix method was successfully applied to solve the forced vibration of the real rotors of an energy plant turboset (ref. 1). The computer program designed for computation of rotor forced vibration makes possible the evaluation of the critical speeds of the rotor (i.e., speed of resonance peak) with high numerical accuracy and stability. The recently created free rotor vibration computer program, which is based on the method presented in the paper, has the same properties.

SYMBOLS

Oxyz, $O\xi\eta z$	inertial and rotating coordinate systems (fig. 1)
ρ, E, G, b_i	density, Young's and shear moduli, coefficient of internal damping, respectively
R, A, I, β	radius, area, moment of inertia, and shear coefficient of rotor cross section, respectively
b_e	coefficient of external damping

L	length of rotor element
Ω	rotational speed of rotor
Ω_F	angular frequency of exciting force
λ	eigenvalue, $\gamma + j\omega$
$g(z,t)$; $f(z)$	complex rotor dynamic deflection and static deflection, respectively
$G(z)$	free vibration mode of rotor
ϕ	complex rotor cross-section rotary angle
M,T	complex internal bending moment and shearing force
m,q	complex intensity of bending moment and complex continuous load of rotor
μ,ν,κ,Λ,B	parameters of motion equation (3)
u,ζ	complex displacement of bearing oil film and support
$C^{xx},C^{xy},C^{yx},C^{yy}$ $\lambda^{xx},\lambda^{xy},\lambda^{yx},\lambda^{yy}$	elastic and damping hydrodynamic characteristics of bearing oil film
$\Delta^{xx},\Delta^{xy},\Delta^{yx},\Delta^{yy}$	dynamic influence coefficients of rotor supports
R	reaction of support
$\{w\}$	complex state vector of rotor cross section, col$\{G,\phi^*,M,T\}$
$\{G(z)\}$	auxiliary vector of rotor cross section, col$\{G(z),G'(z),G''(z),G'''(z)\}$
$\{\widetilde{w}\}$	real state vector of rotor cross section, col{Ke{w},|m{w}|}
D_k; $\widetilde{D}_k$	complex and real transfer matrix of rotor element
H_k; $\widetilde{H}_k$	complex and real transfer matrix of rotor support and bearing (support construction)
B; $\widetilde{B}$	complex and real transfer matrix throughout rotor with fulfilled boundary conditions at both ends of rotor; dim B = 2x2; dim $\widetilde{B}$ = 4x4
δ	complex characteristic determinant, det B
$\widetilde{\delta}$	real characteristic determinant, det $\widetilde{B}$
$\widetilde{B}_G$; $\widetilde{\delta}_G = \det \widetilde{B}_G$	general real transfer matrix and its determinant throughout rotor with fulfilled boundary conditions at both ends of rotor; dim $\widetilde{B}_G$ = =8x8

β_n	shape number of vibration mode of simple rotor, $n\pi/L$
g_e	acceleration of gravity
t	time
$j^2 = -1$	imaginary unit
e	base of natural logarithm
n	natural number

The physical vector quantities are described by complex variables. Their components on axes Ox and Oy or ($O\xi$ and $O\eta$) establish the real and imaginary parts of these variables, respectively.

Superscription

$(\ )^*$	quantity connected only with rotor bending
$(\ )^{**}$	quantity connected only with rotor shearing
$(\dot{\ })$	derivative with respect to time t
$(\ )'$	derivative with respect to z coordinate
$(\bar{\ })$	complex conjugate quantity
$(\ )^+$; $(\ )^-$	quantities connected with right or left side of rotor cross section

Subscription

k, k+1	indices of both element ends
cr	critical values of parameters
$(\ )_+$; $(\ )_-$	quantities connected with vibrations when their angular speeds are $+\omega$ and $-\omega$, respectively

FREE MOTION OF UNIFORM CONTINUOUS ROTOR ELEMENT

Using the technical theory of bending of beams and taking into consideration the sign convention of components of vector quantities shown in figure 1, the following basic equations are obtained:

$$\left.\begin{aligned} &T' = -q; \qquad M' + m = -jT \\ &M = jEIg^{*''}; \qquad \phi^* = jg^{*'}; \qquad \phi^{**} = jg^{**'} \\ &T = \frac{AG}{\beta} g^{**'}; \qquad g = g^* + g^{**}; \qquad \phi = \phi^* + \phi^{**} \end{aligned}\right\} \tag{1}$$

and

$$\left.\begin{aligned} q &= -\rho A\ddot{g} = b_e\dot{g} \\ m &= Ib_i(\Omega g^{*\prime\prime\prime} + jg^{*\prime\prime\prime}) - I\rho(2\Omega\dot{g}^{*\prime} + j\ddot{g}^{*\prime}) \end{aligned}\right\} \tag{2}$$

In these expressions, the shear effect is taken according to Timoshenko's theory and the internal damping according to the theory of Voight-Kelvin. After elimination of quantities connected only with pure bending and shearing, the final form of the motion equation can be obtained:

$$\kappa B\ddddot{g} + \kappa B(\mu - 2j\Omega)\dddot{g} + (1 - 2j\Omega\kappa\mu B)\ddot{g} + \mu g - \upsilon B\dddot{g}'' + [(j\Omega\upsilon - \Lambda - \nu\mu)B - \kappa]\ddot{g}''$$
$$+ [(j\Omega\upsilon - \Lambda)B\mu + 2j\Omega\kappa]\dot{g}'' + \upsilon g'''' + (\Lambda - j\Omega\upsilon)g'''' = 0 \tag{3}$$

where

$$\mu = \frac{b_e}{A\rho}\ ; \quad \upsilon = \frac{Ib_i}{A\rho}\ ; \quad \kappa = \frac{I}{A}\ ; \quad \Lambda = \frac{EI}{A\rho}\ ; \quad B = \beta\frac{\rho}{G} \tag{4}$$

Solution of equation (3) can be predicted as

$$g(z,t) = e^{\lambda t}G(z) \tag{5}$$

After applying solution (5) to equation (3) the equation of vibration modes is obtained:

$$WG''''(z) + VG''(z) + UG(z) = 0 \tag{6}$$

where

$$W = \upsilon\lambda + \Lambda - j\Omega\upsilon$$

$$\left.\begin{aligned} V &= -\upsilon B\lambda^3 + [(j\Omega\upsilon - \Lambda - \mu\upsilon)B - \kappa]\lambda^2 + [(j\Omega\upsilon - \Lambda)B\mu + 2j\Omega\kappa]\lambda \\ U &= \kappa B\lambda^4 + \kappa B(\mu - 2j\Omega)\lambda^3 + (1 - 2j\Omega\kappa\mu B)\lambda^2 + \mu\lambda \end{aligned}\right\} \tag{7}$$

Differential equation (6) can be solved by using a known method, and its solution has the form

$$G(z) = \sum_{i=1}^{9} c_i e^{r_i z} \tag{8}$$

where c_i are complex constants and r_i are complex roots of the following biquadratic equation:

$$Wr^4 + Vr^2 + U = 0 \tag{9}$$

If a dependence between auxiliary vectors $\{G(z)\}$ at both element ends ($z = 0$; $z = L$) is searched, the constants c_i can be eliminated:

$$\{G\}_{k+1} = RER^{-1}\{G\}_k \tag{10}$$

where

$$\left.\begin{aligned} &\{G\}_{k+1} = \{G(L)\} \\ &\{G\}_k = \{G(0)\} \\ &\text{and} \\ &E = \mathrm{diag}\left[e^{r_1 L};\ e^{r_2 L};\ e^{r_3 L};\ e^{r_4 L}\right];\quad R = \begin{bmatrix} 1 & 1 & 1 & 1 \\ r_1 & r_2 & r_3 & r_4 \\ r_1^2 & r_2^2 & r_3^2 & r_4^2 \\ r_1^3 & r_2^3 & r_3^3 & r_4^3 \end{bmatrix} \end{aligned}\right\} \tag{11}$$

In order to obtain the transfer matrix, a dependence between the auxiliary vector $\{G(z)\}$ and the state vector $\{w\}$ has to be estimated. This dependence can be expressed by a matrix Γ, namely

$$\{w\} = \Gamma\ \{G(z)\} \tag{12}$$

Elements of the matrix Γ can be obtained by using equations (1) and (2):

$$\Gamma = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & X_1 & 0 & X_2 \\ X_3 & 0 & X_4 & 0 \\ 0 & X_5 & 0 & X_6 \end{bmatrix} \tag{13}$$

and

$$X_1 = j\left(1 - \frac{B}{A\rho}\right)X_5$$

$$X_2 = -j\,\frac{B}{A\rho}\,X_6$$

$$X_3 = -jEIB(\lambda^2 + \mu\lambda)$$

$$X_4 = jEI$$

$$X_5 = \alpha\{\kappa(\lambda^2 - 2j\Omega\lambda) + B(\lambda^2 + \mu\lambda)[\Lambda + \upsilon(\lambda - j\Omega)]\}$$

$$X_6 = -\alpha[\Lambda + \upsilon(\lambda - j\Omega)]$$

$$\alpha = A\rho[1 + B\kappa(\lambda^2 - 2j\Omega\lambda)]^{-1}$$

Applying equation (12) to equation (10), we can obtain the form of the complex transfer matrix of the considered rotor element:

$$D_k = \Gamma R E R^{-1} \Gamma^{-1} \tag{14}$$

Although in many papers the motion equation has less complicated form, our equation (3) follows from accepted assumptions and mathematical transformation. If damping forces are neglected ($\upsilon = \mu = 0$), equation (3) has the well-known form demonstrated by Timoshenko (ref. 2), Dimentberg (ref. 3), and Tondl (ref. 4). Additionally, the state vector of rotor cross section has to contain the rotary angle ϕ^*, which is connected only with pure bending. This dependence on ϕ^* complicates the form of equation (13). Such an approach allows us to satisfy the continuity conditions at common boundary cross sections of two adjacent rotor elements and to obtain the transfer matrix throughout the rotor by multiplying transfer matrices of successive rotor elements and supports. Note that only if the shear and gyroscopic effects as well as internal damping are omitted ($B = \kappa = \mu = 0$) does the matrix (13) reduces to the widely known form:

$$\Gamma = \mathrm{diag}[1;\ j;\ jEI;\ -EI] \tag{15}$$

TRANSFER MATRIX OF ROTOR BEARING AND SUPPORT

The basic system of the motion equations of rotor support construction becomes

For bearing oil film reaction:

$$R = Cu + \widetilde{C}\bar{u} + \Lambda u + \widetilde{\Lambda}\bar{u} \tag{16}$$

$$\left.\begin{aligned} C &= \frac{1}{2}\left[\left(c^{xx} + c^{yy}\right) + j\left(c^{yx} - c^{xy}\right)\right]; \quad \Lambda = \frac{1}{2}\left[\left(\lambda^{xx} + \lambda^{yy}\right) + j\left(\lambda^{yx} - \lambda^{xy}\right)\right] \\ \widetilde{C} &= \frac{1}{2}\left[\left(c^{xx} - c^{yy}\right) + j\left(c^{yx} + c^{xy}\right)\right]; \quad \widetilde{\Lambda} = \frac{1}{2}\left[\left(\lambda^{xx} - \lambda^{yy}\right) + j\left(\lambda^{yx} + \lambda^{xy}\right)\right] \end{aligned}\right\} \tag{17}$$

For support displacement:

$$\zeta = \Delta R + \widetilde{\Delta}\bar{R} \tag{18}$$

$$\Delta = \frac{1}{2}\left[\left(\Delta^{xx} + \Delta^{yy}\right) + j\left(\Delta^{yx} - \Delta^{xy}\right)\right]; \quad \Delta = \frac{1}{2}\left[\left(\Delta^{xx} - \Delta^{yy}\right) + j\left(\Delta^{yx} + \Delta^{xy}\right)\right] \tag{19}$$

For forced equilibrium and displacement continuity:

$$G = \zeta + u \quad \text{and} \quad R = T^{+} - T^{-} \tag{20}$$

The hydrodynamic characteristics of a bearing oil film can be obtained as the gradients of oil forces at the static balance point and are calculated by using a linear perturbation method to the Reynolds equation. Only in one case can the elements of the transfer matrix of supports be expressed by complex variables without using any complex conjugate quantities. It takes place when support construction has no couplings ($\widetilde{C} = \widetilde{\Lambda} = \widetilde{\Delta} = 0$); then the transfer matrix of support has the form:

$$H_k = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ h & & & 1 \end{bmatrix} \tag{21}$$

where $h = [C + \Lambda\lambda][1 + (C + \Lambda\lambda)\Delta]^{-1}$. In the general case a real form of this transfer matrix has to be applied. Using equations (16) to (20) and predicting the trajectory of rotor motion in elliptical form (instead of circular form (5)) gives

$$g(z,t) = G_+(z)e^{\lambda t} + G_-(z)e^{\lambda t} \tag{22}$$

The transfer matrix of support construction is obtained as

$$H_k = \left[\begin{array}{cccc|cccc|cccc|cccc}
1 & & & & & & & & & & & & & & & \\
 & 1 & & & & & & & & & & & & & & \\
 & & 1 & & & & & & & & & & & & & \\
h_1 & & & 1 & -h_2 & & & & h_3 & & & & h_4 & & & \\
\hline
 & & & & 1 & & & & & & & & & & & \\
 & & & & & 1 & & & & & & & & & & \\
 & & & & & & 1 & & & & & & & & & \\
h_2 & & & & h_1 & & & 1 & h_4 & & & & -h_3 & & & \\
\hline
 & & & & & & & & 1 & & & & & & & \\
 & & & & & & & & & 1 & & & & & & \\
 & & & & & & & & & & 1 & & & & & \\
h_5 & & & & -h_6 & & & & h_7 & & & 1 & h_8 & & & \\
\hline
 & & & & & & & & & & & & 1 & & & \\
 & & & & & & & & & & & & & 1 & & \\
 & & & & & & & & & & & & & & 1 & \\
-h_6 & & & & -h_5 & & & & -h_8 & & & & h_7 & & & 1
\end{array}\right] \tag{23}$$

The elements of matrix (23) can be calculated from the following matrix equation:

$$\begin{bmatrix} h_1 + jh_2; & h_3 + jh_4 \\ h_5 + jh_6; & h_7 + jh_8 \end{bmatrix} = K^{-1}L \tag{24}$$

where

$$\left.\begin{aligned} L &= \begin{bmatrix} C + \Lambda\lambda; & \tilde{C} + \tilde{\Lambda}\lambda \\ \tilde{C} + \tilde{\Lambda}\bar{\lambda}; & C + \Lambda\bar{\lambda} \end{bmatrix} \\ K &= \begin{bmatrix} 1 + (C + \Lambda\lambda)\Delta + (\tilde{C} + \tilde{\Lambda}\lambda)\bar{\tilde{\Delta}}; & (C + \Lambda\lambda)\tilde{\Delta} + (\tilde{C} + \tilde{\Lambda}\lambda)\bar{\Delta} \\ (\tilde{C} + \tilde{\Lambda}\bar{\lambda})\bar{\Delta} + (C + \Lambda\bar{\lambda})\tilde{\Delta}; & 1 + (\tilde{C} + \bar{\tilde{\Lambda}}\bar{\lambda})\bar{\tilde{\Delta}} + (C + \Lambda\bar{\lambda})\Delta \end{bmatrix} \end{aligned}\right\} \tag{25}$$

As the general transfer matrix of the support construction is real, a suitable real transfer matrix of rotor elements has to be created. Using earlier demonstrated results the real transfer matrix of rotor element can be described in block form:

$$\tilde{D}_k = \begin{bmatrix} \tilde{D}_{k+} & 0 \\ 0 & \tilde{D}_{k-} \end{bmatrix} \tag{26}$$

where

$$\tilde{D}_{k\alpha} = \begin{bmatrix} ReD_{k\alpha} & -ImD_{k\alpha} \\ ImD_{k\alpha} & ReD_{k\alpha} \end{bmatrix}$$

and

$$\alpha = + \text{ or } -$$

The matrix D_{k+} is analogous with matrix (14); however, matrix D_{k-} is obtained from matrix (14) by replacing λ by λ. Transfer through rotor elements or rotor supports can be described by using real matrix (26) or (23) and the following real state vector of rotor cross sections:

$$\{W\} = \text{col}\ \{\{\tilde{w}\}_+;\ \{\tilde{w}\}_-\} \tag{27}$$

EIGENVALUES AND MOTION STABILITY

The transfer matrix through the rotor is obtained as the consequent product of transfer matrices of rotor elements and supports. The characteristic matrix is obtained from the transfer matrix through the rotor by inserting an appropriate set of boundary conditions. Let us consider three cases:

(1) The rotor support construction has no couplings: The characteristic matrix (denoted as B) is obtained in complex form, and its characteristic determinant is complex:

$$\delta = \det B \tag{28}$$

(2) The rotor support construction has no couplings: The characteristic matrix (denoted as $\tilde{B}$) is obtained in real form, and it is related to matrix B in the following manner:

$$\tilde{B} = \begin{bmatrix} ReB & -ImB \\ ImB & ReB \end{bmatrix} \tag{29}$$

The determinant of matrix $\tilde{B}$ is real:

$$\tilde{\delta} = \det \tilde{B} \tag{30}$$

(3) The rotor support construction has couplings: The characteristic matrix (denoted as $\tilde{B}_G$) is obtained in real form by using real matrices (23) and (26), and its determinant is real:

$$\delta_G = \det B_G \tag{31}$$

The following relations are valid for these three determinants:

$$\tilde{\delta} = \delta\bar{\delta} \tag{32}$$

$$\tilde{\delta}_G = \tilde{\delta}^2 + \psi \tag{33}$$

Expression (32) results in widely known properties of determinants. Equation (33) is obtained in the same way as (32). The quantity ψ is connected with couplings of the rotor support construction. As the degree of support construction anisotropy is increased, the value of ψ decreases and the following quantities are the measure of the support anisotropy:

$$c^{xx} - c^{yy},\ c^{yx} + c^{xy};\ \lambda^{xx} - \lambda^{yy},\ \lambda^{yx} + \lambda^{xy};\ \Delta^{xx} - \Delta^{yy},\ \Delta^{yx} + \Delta^{xy}$$

In real turbine construction these quantities usually have values such that the determinant $\tilde{\delta}_G$ is positive (known by performing several numerical calculations). It is evident that the determinant $\tilde{\delta}$ is positive.

These conclusions are useful in the numerical computations of eigenvalues (roots of characteristic determinants). As the real determinants are always positive, they can vanish only in places where their local minima in the complex $\lambda = \gamma + j\omega$ plane are observed because the determinants can be considered as the continuous functions of two real variables γ and ω. Computing results given in technical papers (e.g., refs. 5 and 6) point out that eigenvalues are located relatively far from one another. Besides the subspaces of spectrum plane (complex plane of eigenvalues) in which real determinants can decrease monotonically to their minimal values are relatively large. This remark probably explains why Lund's gradient method (ref. 6) as well as Muller's quadratic approximation technique can be successfully applied without any hard restrictions concerning starting points for the iteration procedure. These properties also permit us to use with success the bisection method alternately applied to both real arguments (γ,ω) of the real determinants $\tilde{\delta}_G$ or $\tilde{\delta}$. A very important problem, whether each determinant minimum is its zero, has not been theoretically solved, and one can answer this question by the numerical verification of an appropriate eigenvector mode of free vibration.

The procedure for computing the minimal values of a real determinant can be simultaneously used to evaluate so-called critical speeds of resonance peak. The real determinant $\tilde{\delta}_G$ is a function of three arguments: $\tilde{\delta}_G = \tilde{\delta}_G\ (\gamma,\ \omega,\ \Omega)$. The critical speeds of resonance peak are calculated under assumptions $\gamma = 0$ and $\omega = \Omega_F$ and are determined by the following condition:

$$\tilde{\delta}_G(0,\ \Omega_F,\ \Omega) = \min \tag{34}$$

where $\Omega_F = \pm\Omega,\ \pm2\Omega,\ \pm3\Omega$, etc.; usually $\Omega_F = \Omega$. From the theory of the linear dynamical systems it is known that any minimum of expression (34) is connected with a certain critical speed of resonance peak.

We can obviously compute only a finite number of eigenvalues. Therefore the stability problem of rotor motion, based on investigation of the real parts of eigenvalues, can be solved partially because the full spectrum of the rotor motion equation always contains an infinite number of elements. In further consideration an example is presented when stable and unstable modes of rotor free vibration exist simultaneously (see case 3.1 next section).

BASIC ROTOR ELEMENT AS A SIMPLE ROTOR

In this section, in order to examine some interesting properties of the spectrum of the rotor motion equations, the single rotor element is treated as a simple rotor. Consider a rotor simply supported on its ends by a rigid support structure. This is equivalent to the following boundary conditions:

$$g(0,t) = g(L,t) = 0 \quad \text{and} \quad M(0,t) = M(L,t) = 0 \tag{35}$$

From equation (12) and matrix (13) these conditions take the form:

$$G(0) = G(L) = 0; \; \chi_3 G(0) + \chi_4 G''(0) = 0; \; \chi_3 G(L) + \chi_4 G''(L) = 0 \tag{36}$$

The equations (36) point out that

$$G(z) = G_0 \sin \beta_n z \tag{37}$$

can be chosen as a mode function. Substituting expression (37) into the motion equation (3) gives the complex determinant δ in the expanded form:

$$\delta = \kappa B\lambda^4 + \left[(\mu - 2j\Omega)\kappa + \upsilon\beta_n^2\right]B\lambda^3 + \Big\{[(\Lambda - j\Omega\upsilon + \mu\upsilon)B + \kappa]\beta_n^2 + (1 - 2j\Omega\kappa\mu B)\Big\}\lambda^2 + \mu + \Big\{[(\Lambda - j\Omega\upsilon)\mu B - 2j\Omega\kappa]\beta_n^2 + \upsilon\beta_n^4\Big\}\lambda + (\Lambda - j\Omega\upsilon)\beta_n^4 \tag{38}$$

The roots of complex determinants δ and $\bar{\delta}$ establish the eigenvalue spectrum. Additionally, the roots of determinant δ are located symmetrically to the roots of determinant $\bar{\delta}$, and the real axis is the axis of symmetry.

In general, the determinant (38) has four complex roots for each parameter β_n. Further some particular cases of the physical model of the simple rotor are considered.

Case 1. $\Omega \equiv 0$ (no rotating rotor): The determinant (38) has only real coefficients. Therefore each spectrum element is a root of determinant δ and determinant $\bar{\delta}$ as well. The rotor motion is not coupled. The motion in planes Oxz and Oyz can be executed independently.

Case 2. $B \equiv 0$ (no shear effect): The number of roots of determinant (38) decreases twice.

Case 3. $\omega = 0$; $\lambda = \gamma$ (critical damping): Substituting the above assumptions into expression (38), we obtain two real equations, for which the dependence among three parameters γ, υ, μ can be investigated.

Case 3.1. $\omega = 0$ and $B \equiv 0$: In this case the equations take the form:

$$\left.\begin{aligned} \left(1 + \kappa\beta_n^2\right)\gamma^2 + \mu\left(+ \upsilon\beta_n^4\right)\gamma + \Lambda\beta_n^4 = 0 \\ 2\kappa\gamma + \upsilon\beta_n^2 = 0 \end{aligned}\right\} \qquad (39)$$

A dependence among three critical parameters is given by

$$\left.\begin{aligned} \gamma_{cr} &= -\frac{\upsilon_{cr}\beta_n^2}{2\kappa} \\ \upsilon_{cr} &= \kappa\left\{\mu_{cr} \pm \left[\mu_{cr}^2 - 4\Lambda\left(1 - \kappa\beta_n^2\right)\beta_n^4\right]^{1/2}\right\}\left[\left(1 - \kappa\beta_n^2\right)\beta_n^2\right]^{-1} \end{aligned}\right\} \qquad (40)$$

The quantity υ_{cr} is positive when the expression $(1 - \kappa\beta_n^2)$ is positive (independently of the quantity μ_{cr}). However, υ_{cr} has only real values if

$$\mu_{cr} > \left[2\ \Lambda(1 - \kappa\beta_n^2)\right]^{1/2}\beta_n^2 \qquad (41)$$

For any given rotor size L,R we can determine a value n_0 such that for $n > n_0$ the expression $(1 - \kappa\beta_n^2)$ becomes negative. Thus rotor motion with modes $n > n_0$ is given by

$$n_0 > \frac{2L}{\pi R} \qquad (42)$$

which depends on the geometrical parameters of the rotor.

The next two cases concern the estimations of the spectrum limits. The results are equivalent to the necessary conditions. To find these estimations, one should assume that the variable λ is finite.

Case 4. $n \to \infty$ or $\beta_n \to \infty$ (mode order increases): In this case the spectrum is limited and the boundary values of the parameters satisfy the following equation:

$$\upsilon\lambda^\infty + (\Lambda - j\Omega\upsilon) = 0, \text{ or } \gamma^\infty = -\frac{\Lambda}{\upsilon} \text{ and } \omega^\infty = \Omega \qquad (43)$$

The spectrum is limited only by the internal damping; when $\upsilon \equiv 0$, the spectrum is unlimited. The limit of the damped critical speed is equal to the rotor speed Ω.

Case 5. $\Omega \to \infty$ (rotor speed increases): The characteristic equation takes the form:

$$2\kappa B(\lambda^\infty)^3 + B\left(2\kappa\mu + \upsilon\beta_n^2\right)(\lambda^\infty)^2 + (B\upsilon\mu + 2\kappa)\beta_n^2\lambda^\infty + \upsilon\beta_n^4 = 0 \qquad (44)$$

The coefficients of equation (44) are real, and it has real roots or one real and two complex roots. Therefore the spectrum is limited.

Case 5.1. $\Omega \to \infty$ and $\upsilon \equiv 0$ (without internal damping): From equation (44) follows the next limiting conditions:

$$\lambda^{\infty} = 0 \quad \text{or} \quad \lambda^{\infty} = -\frac{1}{2}\mu \pm \left[\left(\frac{1}{2}\mu\right)^2 - \beta_n^2 B^{-1}\right]^{1/2} \tag{45}$$

The limitation (45) points out that the "bending motion" of the rotor is impossible as ($\lambda^{\infty} = 0$). Only "shearing motion" of the rotor is possible. In this limiting case the rotor shear motion can be either periodic or aperiodic. The critical parameter of external damping is given by

$$\mu_{cr}^{\infty} = 2\beta_n B^{-1/2} \tag{46}$$

Case 5.2. $\Omega \to \infty$ and $B \equiv 0$: The limiting conditions can be expressed as

$$\gamma^{\infty} = -\frac{\upsilon\beta_n^2}{2\kappa} \quad \text{and} \quad \omega^{\infty} = 0 \tag{47}$$

If the internal damping is neglected ($\upsilon = 0$), then $\lambda^{\infty} = 0$ and no rotor motion is possible.

From these results it follows that the internal damping taken according to the hypothesis of Voight-Kelvin has a significant influence on the critical and limiting parameters of motion of the rotor. Additionally, internal damping of this type can lead to the unstable aperiodic motion of the rotor. The spectrum of the rotor motion equations always has an infinite number of elements. Damping of the bearing oil film has the same character as the external damping of the rotor, and it should not affect the quantitative results.

CENTER OF ROTOR MOTION

Usually the line of the rotor motion center is a line of static rotor displacement under its weight. However, in the considered physical model, the motion center has an additional component in horizontal direction caused by the internal damping. The equation of the motion center is given by

$$(\Lambda - j\Omega\upsilon)f'''' = jg_e \tag{48}$$

This additional component increases simultaneously with the speed of the rotor. These conclusions are important because the hydrodynamic characteristics of the bearings of a multisupported rotor (statically indeterminant) should be calculated after solving the static problem for the whole rotor, where a misalignment of the bearings has to be taken into consideration. When the multisupported rotor and its supporting construction are relatively rigid, the static effect of the bearing oil film should be taken into consideration. Although that static problem is nonlinear, it can be solved by the method of successive linear iterations (ref. 7).

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