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Einstein-Podolsky-Rosen Paradox and Bell's Inequalities — Source link

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Einstein-Podolsky-Rosen paradox and Bell's inequalities

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Abstract

Considering the *Gedankenexperiment* of Einstein, Podolsky, and Rosen as example the nonlocal character of quantum mechanics is discussed. The attempt to complement quantum mechanics by a supplementary, local causal theory of 'hidden-variables' and the fundamental incompatibility of both theories due to Bell's inequalities is shown. Some experiments clearly favouring quantum mechanics are outlined.

1 Introduction

In 1935, Einstein, Podolsky, and Rosen (EPR) introduced a *Gedankenexperiment* [1] which showed that quantum mechanics has a nonlocal character, more precisely a measurement on one system can influence another spatial separated system. This violated their assumed non-existence of action-at-a-distance. They concluded that the quantum-mechanical 'wave function does not provide a complete description of physical reality' and postulated a complementary, local causal theory to be discovered.
This caused a controversial discussion among the physical society and prominent physicists like Bohr [2] disagreed with their argumentation.
The discussion changed dramatically when Bell [4] showed in 1964 (unfortunately nine years after Einstein's death, who spent the last years of his life searching for such a theory) that quantum-mechanical predictions disagree with *all* types of local 'hidden-variable' theories, and thus, if quantum mechanics was incomplete, it could not simply be complemented but had to be replaced by a totally new theory. Further improvements of his idea by Bell [5] and Clauser *et al* (CHSH) [8] made it possible to experimentally test quantum mechanics vs. local hidden-variable theories and all performed experiments clearly disagree with this class of local theories and agree with quantum mechanics.

2 The EPR paradox

In their paper [1], Einstein, Podolsky, and Rosen define elements of physical reality as physical quantities, the values of which can be predicted with certainty 'without in any way disturbing the system'. They assume that every element of physical reality needs to have a counterpart in a complete physical theory.
They then consider a *Gedankenexperiment* where two systems interact at some time t after which there should be no further interaction. Depending on a measurement of position or momentum on system *one*, due to the reduction of the wave packet both momentum and position of system *two* could become an element of reality by their definition. But since the operators for momentum and position do not commute, they do not both have a simultaneous counterpart in quantum mechanics. Thus, they concluded quantum mechanics to be incomplete.
We will discuss here in detail a simplified form of their hypothetical experiment, introduced by Bohm [3], in order to illustrate 'certain apparently paradoxical predictions of current quantum theory'.
Consider two spin-1/2 particles prepared in such a way that their total spin is zero, i.e., if the spin of particle one in some direction is 1 (in suitable units), then it is -1 for particle two in that direction. Those particles may be emitted by an appropriate source and propagate in different directions. After both particles propagated some distance their spins may be measured (for an outline see Fig.1). It will not be discussed here how to prepare such a state, however, we will assume to have two spin-1/2 atoms, the spins of which could be measured e.g. by *Stern-Gerlach* magnets.
The wave function of the two particles will then be

$$\Psi = \frac{1}{\sqrt{2}} \left(|+\rangle_1 |-\rangle_2 - |-\rangle_1 |+\rangle_2 \right) , \tag{1}$$

where $|+\rangle_1$ refers to the wave function of the state in which particle 1 has a spin +1, etc. This kind of wave function is often called an *entangled state*, i.e., the total wave function cannot be factorized in a

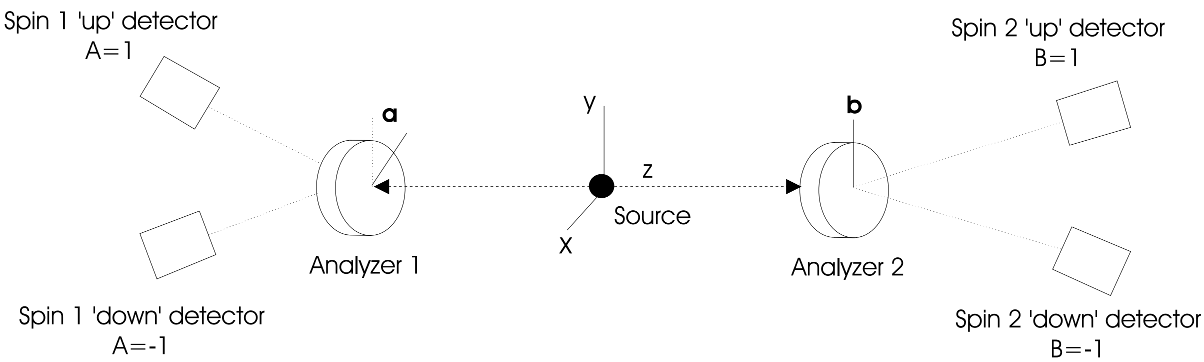

Figure 1: Outline of a principle EPR experiment, introduced by Bohm [3]. A source emits particles with correlated spins in different directions, after some distance the spins are measured in directions $\boldsymbol{a}$ and $\boldsymbol{b}$.

product of the single particle wave functions. The interesting thing about this state is, as we will see, that there is a very strong correlation between the two particles, although they might be far away from each other.
If now a measurement is performed at particle one, according to the outcome, the wave function of particle one will collapse to one of the *eigenstates* $|+\rangle_1$ or $|-\rangle_1$. But, and this is the essential point, *immediately* also the wave function of particle two will collapse to $|-\rangle_2$ or $|+\rangle_2$, respectively. This means that a measurement on one particle can immediately influence another particle which is spatially separated. Thus, after a measurement on particle one, we can instantaneously predict the result of a future measurement performed on particle two.
This 'spooky action in a distance' did not seem reasonable to EPR.

3 Why paradox?

But is this really a paradox situation? Is it not an experience of everyday life? Imagine e.g. you know that you have your keys in the pockets of your jacket, but you do not know if they are in the left or in the right one. If you then do not find them in your left pocket, you immediately know that they will be found in the right one. No one would call this a spooky action in a distance. Of course, everyone will be sure that the keys have always been in the right pocket and one just did not know in which, before performing a 'measurement'. No one will believe that the keys have somehow been in both pockets and just decided to be in one particular when one tried to find them.
We will see that things are different in the 'microscopic' world, i.e., the two atoms do *not* decide which of them has spin 1 and which -1 until a measurement is performed. It is not clear where exactly the border between this microscopic world and the macroscopic world is and the question becomes rather philosophical, similar to the well-known *Schrödinger's cat*.
However, coming back to the original problem, it seems to be natural to assume that when the two entangled particles are emitted also their spins are determined, but we just do not know how. Thus, the quantum-mechanical wave function does not contain all possible information but is a statistical average over yet unknown 'hidden variables'. It will now be shown that this is impossible.

4 Bell's inequalities

In 1964, Bell [4] showed that quantum-mechanical predictions disagree with a very broad class of 'locally causal' theories. The problem of his original proof is that it only holds for ideal systems, and thus, is never testable in an experiment. Therefore, a further modulation of his proof [5], including experimental errors, will be discussed here which is similar to a proof by Clauser *et al* (CHSH) [8].
Consider again a singlet state of two spin-1/2 particles propagating in different directions towards two measuring devices, e.g. Stern-Gerlach magnets, which measure the spin along directions $\boldsymbol{a}$ and $\boldsymbol{b}$, respectively. The results of the measurements may be denoted by A and B. We are now interested in the expectation value $E(\boldsymbol{a},\boldsymbol{b})$ of the product AB.
Quantum-mechanically, the expectation value is given by

$$E(\boldsymbol{a},\boldsymbol{b})_{QM} = \langle \sigma_1 \boldsymbol{a}, \sigma_2 \boldsymbol{b} \rangle = -\boldsymbol{a}\cdot\boldsymbol{b} = -\cos\alpha\,, \tag{2}$$

with the spin matrices σ_i and α being the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$. If we also take imperfections of the analyser adjustment into account, we obtain

$$E(\boldsymbol{a},\boldsymbol{b})_{QM} = -C\boldsymbol{a}\cdot\boldsymbol{b} = -C\cos\alpha\,, \tag{3}$$

where $|C| \leq 1$ and $C = 1$ only in the ideal case.
We now suppose a hypothetical complete description of the initial state in terms of local 'hidden variables' λ, where we will treat λ as if it was a single continuous parameter but it could in general denote a set of variables, set of functions, or whatever. *Hidden* shall denote that the physical behavior of λ is (yet) unknown. Let $\rho(\lambda)$ be the normalized probability distribution of λ for a given quantum-mechanical state. The result $A(=\pm 1)$ of a spin measurement will then surely depend on $\boldsymbol{a}$ and λ, just as $B(=\pm 1)$ will depend on $\boldsymbol{b}$ and λ, but, if we assume locality, A does *not* depend on $\boldsymbol{b}$ and neither does B on $\boldsymbol{a}$. Since the particles are emitted physically independent of $\boldsymbol{a}$ and $\boldsymbol{b}$, we also expect $\rho(\lambda)$ to be independent of them.
Thus, we obtain an expectation value

$$E(\boldsymbol{a}, \boldsymbol{b}) = \int_\Lambda \mathrm{d}\lambda\, \rho(\lambda)\, A(\boldsymbol{a}, \lambda)\, B(\boldsymbol{b}, \lambda)\,, \tag{4}$$

where Λ denotes the whole space of λ.
In a realistic experiment, the apparatus might of course sometimes fail to measure any spin, i.e., $A = 0$ or $B = 0$. We will therefore only assume

$$|A(\boldsymbol{a}, \lambda)| \leq 1 \quad , \quad |B(\boldsymbol{b}, \lambda)| \leq 1 \quad . \tag{5}$$

Let $\boldsymbol{a}'$ and $\boldsymbol{b}'$ be alternative settings of the measuring devices. Then, we have

$$\begin{aligned} E(\boldsymbol{a}, \boldsymbol{b}) - E(\boldsymbol{a}, \boldsymbol{b}') &= \int_\Lambda \mathrm{d}\lambda\, \rho(\lambda) \Big[A(\boldsymbol{a}, \lambda)\, B(\boldsymbol{b}, \lambda) - A(\boldsymbol{a}, \lambda)\, B(\boldsymbol{b}', \lambda) \Big] \\ &= \int_\Lambda \mathrm{d}\lambda\, \rho(\lambda) \Big[A(\boldsymbol{a}, \lambda)\, B(\boldsymbol{b}, \lambda) \big(1 \pm A(\boldsymbol{a}', \lambda)\, B(\boldsymbol{b}', \lambda) \big) \Big] \\ &\quad - \int_\Lambda \mathrm{d}\lambda\, \rho(\lambda) \Big[A(\boldsymbol{a}, \lambda)\, B(\boldsymbol{b}', \lambda) \big(1 \pm A(\boldsymbol{a}', \lambda)\, B(\boldsymbol{b}, \lambda) \big) \Big] \end{aligned} \tag{6}$$

using Eq.(5), we find that

$$\begin{aligned} |E(\boldsymbol{a}, \boldsymbol{b}') - E(\boldsymbol{a}, \boldsymbol{b}')| &\leq \int_\Lambda \mathrm{d}\lambda\, \rho(\lambda) \big(1 \pm A(\boldsymbol{a}', \lambda)\, B(\boldsymbol{b}', \lambda) \big) + \int_\Lambda \mathrm{d}\lambda\, \rho(\lambda) \big(1 \pm A(\boldsymbol{a}', \lambda)\, B(\boldsymbol{b}, \lambda) \big) \\ &= 2 \pm \big(E(\boldsymbol{a}', \boldsymbol{b}') + E(\boldsymbol{a}', \boldsymbol{b}) \big) \end{aligned} \tag{7}$$

or

$$S \equiv |E(\boldsymbol{a}, \boldsymbol{b}) - E(\boldsymbol{a}, \boldsymbol{b}')| + |E(\boldsymbol{a}', \boldsymbol{b}') + E(\boldsymbol{a}', \boldsymbol{b})| \leq 2 \quad . \tag{8}$$

Suppose now measurements according to coplanar vectors $\boldsymbol{a}$,$\boldsymbol{a}'$,$\boldsymbol{b}$,$\boldsymbol{b}'$ with relative angles $\angle(\boldsymbol{a}, \boldsymbol{b}) = \angle(\boldsymbol{b}, \boldsymbol{a}') = \angle(\boldsymbol{a}', \boldsymbol{b}') = \pi/4$ then quantum mechanics predicts due to Eq.(3)

$$S_{QM} \equiv \big(|E(\boldsymbol{a}, \boldsymbol{b}) - E(\boldsymbol{a}, \boldsymbol{b}')| + |E(\boldsymbol{a}', \boldsymbol{b}') + E(\boldsymbol{a}', \boldsymbol{b})| \big)_{QM} = 2\sqrt{2}\, |C| \quad . \tag{9}$$

This means that for a very wide range of experimental parameters C quantum mechanics violates equation (8). The violation is maximum in the idealized case $C = 1$ ($\Rightarrow 2\sqrt{2} \leq 2 \lightning$) and we obtain that quantum mechanics is incompatible with all local hidden-variable theories satisfying our assumptions.
Of course, it might turn out that Nature does not violate Eq.(8), and thus, quantum theory was wrong at all, but this can be tested in experiments. One might also assume another hidden mechanism between measuring devices and source, i.e., $\rho = \rho(\lambda, \boldsymbol{a}, \boldsymbol{b})$, but we will see that this is also testable.

5 Experimental violation of Bell's inequalities

Several experiments have been performed to test Bell's inequalities and all clearly favor quantum mechanics. In most experiments, entangled photons are used instead of spin-1/2 particles, but the principle is the same (cf. Fig.2, left).
For example, Aspect, Grangier, and Roger (1982) [10] achieved a violation of an inequality similar to Eq. (8) with an accuracy of more than 45 standard deviations ($S_{exp} = 2.697 \pm 0.015 \leq 2 \lightning$) using the linear-polarization correlation of pairs of photons emitted in a radiative cascade of calcium, whereas their result is in excellent agreement with the quantum-mechanical prediction $S_{QM} = 2.70 \pm 0.05$.
Today's technique makes it even possible to perform such experiments in undergraduate laboratories [12].
It will be outlined here another experiment providing a further feature, namely a changing of the analysers *after* the two entangled photons spatially separated in a time short compared to their transit time.

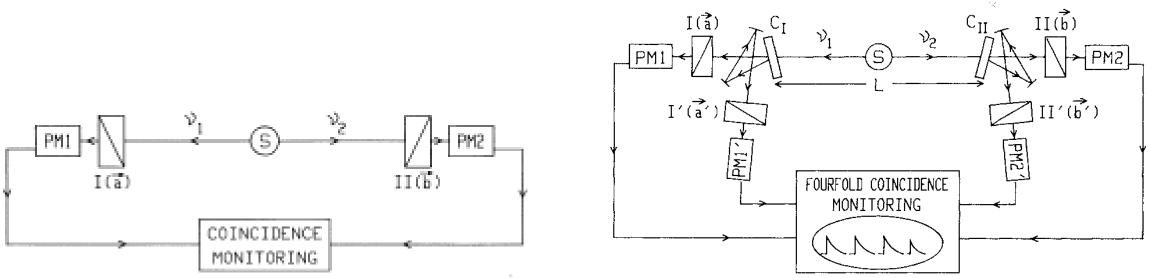

Figure 2: *Left* Optical version of an EPR experiment, the pair of photons ν_1 and ν_2 is analysed by linear polarisers I and II. *Right* Timing experiment with optical switches, each switching device is followed by two polarisers in two different orientations (Figures taken from Ref.[11]).

I.e., it will turn out that $\rho(\lambda)$ does not depend on $\boldsymbol{a}$ or $\boldsymbol{b}$.
Aspect, Dalibard, and Roger [11] realized this situation in an experiment outlined in Fig.2 (right). They used radiative cascades of calcium to emit pairs of photons the linear polarization of which is correlated. According to acousto-optical switches, each photon can be analysed by two polarisers in two different orientations. The main point is that the switching time is small compared to the travel time of the photons, i.e., the decision in which direction the polarisation will be measured is made *after* the two photons were emitted and propagated a certain distance away from each other. The two switches are changed independently and more or less randomly (not randomly enough in the opinion of the authors). Nevertheless, they obtain a violation of a corresponding Bell inequality by five standard deviations ($S_{exp} = 0.101 \pm 0.020 \leq 0\nmid$) and a good agreement with the quantum-mechanical prediction $S_{QM} = 0.112$.
Thus, it is shown that Nature can neither be described by local hidden-variables nor by some kind of unknown distant action between the measuring devices and the emission source. For a fully disagreement with all supplementary-parameter theories (i.e., to eliminate also some possible unknown action between the measurement devices), an experiment with more ideal random switching is still needed.

6 Communication faster than speed of light?

The instantaneous effect on one particle by a measurement on another, spatially separated, particle seems to violate the principle of Einstein's general relativity theory, that no cause and effect can be faster than light speed. Could the set-up described by EPR maybe be used to communicate faster than speed of light?
Unfortunately not. Quantum mechanics solves this situation quite elegant, since the outcomes of measurements on both sides are completely random, the correlation can only be observed when the results of both sides are compared, which is of course limited by the speed of light.

7 Summary and outlook

We have seen that it is impossible to supplement quantum mechanics by a huge class of locally causal, perhaps more intuitive, theories and probably several generations of further physics students will have to learn it. Quite the contrary, nonlocality seems to be an essential feature of quantum mechanics and in special situations even single particles can show nonlocal properties [13, 14].
It is a current field of research how to use this features, where quantum computing, quantum cryptography, and quantum teleportation are some of the major names. For those who are interested, Ref.[15] is recommended, where the application of Bell's inequalities in quantum cryptography is outlined in just three pages.

References

[1] A. Einstein, B. Podolsky, and N. Rosen, *Can quantum-mechanical description of physical reality be considered complete?*, Phys. Rev. **47**, 777 (1935).

[2] N. Bohr, *Can quantum-mechanical description of physical reality be considered complete?*, Phys. Rev. **48**, 696 (1935).

[3] D. Bohm and Y. Aharonov, *Discussion of experimental proof for the paradox of Einstein, Rosen, and Podolsky*, Phys. Rev. **108**, 1070 (1957).

[4] J.S. Bell, *On the EPR paradox*, Phys. **1**, 195 (1964).

[5] J.S. Bell, *Introduction to the Hidden-Variable Question*, Foundations of Quantum Mechanics, ed B. d'Espagnat (Academic, New York, 1971), 171-181.

[6] J.S. Bell, K. Gottfried, and M. Veltmann, *On the foundations of quantum mechanics* (World Scientific, Singapore, 2001).

[7] J.S. Bell, *Speakable and unspeakable in quantum mechanics* (Cambridge University Press, 1987), (including a reprint of [4] and [5]).

[8] J.F. Clauser, M.A. Horne, A. Shimony, and R.A. Holt, *Proposed experiment to test local hidden-variable theories*, Phys. Rev. Lett. **23**, 880 (1969).

[9] J.F. Clauser and A. Shimony, *Bell's theorem: experimental tests and implications*, Rep. Prog. Phys. **41**, 1881 (1978).

[10] A. Aspect, P. Grangier, and G. Roger, *Exp. Real. of EPR-Bohm Gedankenexperiment: A New Violation of Bell's Inequalities*, Phys. Rev. Lett. **49**, 91 (1982).

[11] A. Aspect, J. Dalibard, and G. Roger, *Experimental test of Bell's inequalities using time-varying Analyzers*, Phys. Rew. Lett. **49**, 1804 (1982).

[12] D. Dehlinger and M.W. Mitchell, *Entangled photons, nonlocality, and Bell inequalities in the undergraduate laboratory*, Am. J. Phys. **70**, 903 (2002).

[13] G. Björk and P. Jonsson, *Single-particle nonlocality and entanglement with the vacuum*, Phys. Rev. A **64**, 042106 (2001).

[14] B. Hessmo, P. Usachev, H. Heydari, and G. Björk, *An experimental demonstration of single photon nonlocality*, Phys. Rev. Lett **92**, pp. 180401/1-4 (2004).

[15] A.K. Ekert, *Quantum cryptography based on Bell's theorem*, Phys. Rev. Lett **67**, 661 (1991).