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ELASTIC BUCKLING ANALYSIS OF UNIAXIALLY COMPRESSED CCCC STIFFENED ISOTROPIC PLATES

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This paper reports a research study that investigated buckling of stiffened rectangular isotropic plates elastically restrained along all the edges (CCCC) under uniaxial in-plane load, using the work principle approach. The stiffeners were assumed to be rigidly connected to the plate. Analyses for critical buckling of stiffened plates were carried out by varying parameters, such as the number of stiffeners, stiffness properties and aspect ratios. The study involved a theoretical derivation of a peculiar shape function by applying the boundary conditions of the plate on Taylor Maclaurin's displacement function and substituted on buckling equation derived to obtain buckling solutions. The present solutions were validated using a trigonometric function in the energy method from previous works. Coefficients, *K*, were compared for various numbers of stiffeners and the maximum percentage difference obtained within the range of aspect ratios of *1.0* to *2.0* is shown in Figs 2 - 7. A number of numerical examples were presented to demonstrate the accuracy and convergence of the current solutions.

Key words: buckling, governing equation, polynomial function, uniaxially stiffened plate, work principle method.

1. Introduction

Stiffened plates are a critical class of structural elements widely used in aerospace, marine, nuclear, mechanical and structural engineering (Zhang and Lin [1]). Research in stiffened plate construction has gained attention in recent years as a result of its economic and structural benefits. The advantage of stiffening a plate lies in achieving an economical, lightweight design. A number of methods have been suggested from literature for the prediction of the global buckling load of stiffened plates.

A numerical approach such as the conventional finite element method is a versatile method and has been widely used in the study of stiffened plates to obtain approximate solutions as in Guo and Harik [2], Wang and Yuan [3].The FEM is computationally efficient for predicting buckling coefficients irrespective of boundary conditions, stiffeners shapes and orientation. However, it requires great computational efforts and

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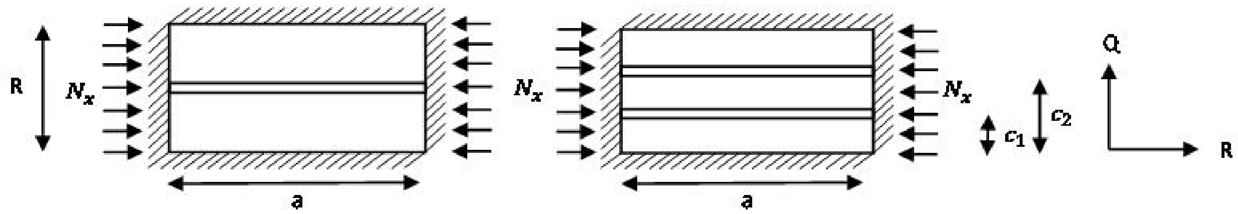
lengthy simulation time, due to the large number of finite units involved. Bisagni and Vescovini [4] presented an analytical formulation for local buckling and post-buckling analysis of stiffened laminated panels and noted that the FEM procedure is somewhat slowed down by the mesh generation time. In recent work, Deng *et al*. [5] noted that in the design of stiffened system that it is not efficient to employ the FEM during the preliminary design stage, since the dimensions of the stiffened panels and stiffeners are not finalized to be optimally designed. Hence, the need for analytical formulations which gives exact solutions.

The analytical solutions for buckling are presented in [4, 6-9]. A number of researchers have used both single and double Fourier series as displacement functions to evaluate the values of buckling coefficients for stiffened systems, but no theoretical solutions exist for more complicated boundary conditions of stiffened plates as in Nildem [10], Bisagni and Vescovini [4]. Analytical methods such as the energy method from literature covered only few cases of edge supports. Most researchers have applied a trigonometric shape function in analyzing stiffened plates with all edges simply supported. However, it is difficult to apply the trigonometric shape function in analyzing stiffened plates with complex boundary conditions.

ANSI/AISC 360-16 [11] recommended elastic and inelastic analyses as two approaches to the direct analysis method in solving stability problems. Hence, the main objective of this work is to present solutions for buckling analysis of stiffened rectangular isotropic plates elastically restrained along all the edges using the work principle and polynomial function intended for design of stiffened systems in accordance with AASHTO [12] specifications.

2. Governing equation

The stiffened plate with all edges clamped and having a stiffener (s) running in longitudinal direction is shown in Fig.1. In this study, stiffeners are considered as line continuum.



a): Plate with one longitudinal stiffener b): Plate with two longitudinal stiffeners

Fig.1.Stiffened plates with all edges clamped under in - plane load.

The equation presented in Ventsel and Krauthammer [9] that describes the behaviour of a thin elastic plate under in - plane load along the x – coordinate based on Kirchhoff's and Venant hypothesis can be written as

$$D\left[\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right]+N_x\frac{\partial^2 w}{\partial x^2}=0\,. \tag{2.1}$$

From the principles of the theory of elasticity, the governing equation for a linear continuum on a plate element is derived as

$$\sum_{i=0}^{n}\left[EI_i\frac{\partial^4 w}{\partial x^4}+N_x\cdot\frac{A_i}{bh}\frac{\partial^2 w}{\partial x^2}\right]_{y=ci}=0\,, \tag{2.2}$$

ci is the distance of the stiffeners from the edge $y = 0$, b and h are the width and thickness of the plate, respectively.

Applying super position principle, Eqs (2.1) and (2.2) are added to give

$$D\left[\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right] + N_x \frac{\partial^2 w}{\partial x^2} + \sum_{i=0}^{n}\left[EI_i \frac{\partial^4 w}{b\partial x^4} + N_x . \frac{A_i}{bh}\frac{\partial^2 w}{\partial x^2}\right]_{y=ci} = 0 . \tag{2.3}$$

Expressing the independent coordinates whose length in the x and y directions are a and b in the form of non-dimensional coordinates R and Q, yields

$$y = bQ; \qquad 0 \le Q \le 1, \tag{2.4}$$

$$x = aR, \qquad 0 \le R \le 1. \tag{2.5}$$

As in Timoshenko and Gere [6], let the aspect ratio be represented as

$$P = a/b, \qquad \text{that is} \qquad a=Pb. \tag{2.6}$$

Applying Eqs (2.4)- (2.6) in Eq.(2.3) and expanding we obtain

$$\begin{gathered}\frac{1}{P^4}\frac{\partial^4 w}{\partial R^4} + \frac{2}{P^2}.\frac{\partial^4 w}{\partial R^2 \partial Q^2} + \frac{\partial^4 w}{\partial Q^4} + \frac{1}{P^4}.\sum_{i=1}^{n}\gamma_i\left(\frac{\partial^4 w}{\partial R^4}\right)_{Q=ci} + \\ +\frac{b^2}{P^2}.\frac{N_x}{D}.\frac{\partial^2 w}{\partial R^2} + \frac{b^2}{P^2}.\frac{N_x}{D}.\sum_{i=1}^{n}\delta_i\left(\frac{\partial^2 w}{\partial R^2}\right)_{Q=ci} = 0\end{gathered} \tag{2.7}$$

where;

$\gamma_i = \dfrac{EI_i}{Db}$ = ratio of bending stiffness rigidity of stiffeners to the plate,

$\delta_i = \dfrac{A_i}{bh}$ = ratio of cross-sectional area of the stiffeners to the plate.

2.1. Work principle

For the combined action of work done by the compressive and resistive force on the stiffened system through a distance w, applied in Eq.(2.7) as in [13], we get

$$\frac{1}{2}\left[\begin{gathered}\frac{A^2}{P^4}.H\frac{\partial^4 H}{\partial R^4} + \frac{2A^2}{P^2}.H\frac{\partial^4 H}{\partial R^2 \partial Q^2} + A^2\frac{H.\partial^4 H}{\partial Q^4} + \frac{A^2}{P^4}.\sum_{i=1}^{n}\gamma_i\left(\frac{H.\partial^4 H}{\partial R^4}\right)_{Q=ci} + \\ +\frac{b^2}{P^2}.\frac{N_x A^2}{D}.H\frac{\partial^2 H}{\partial R^2} + \frac{b^2}{P^2}.\frac{N_x}{D}.A^2\sum_{i=1}^{n}\delta_i\left(\frac{H.\partial^2 H}{\partial R^2}\right)_{Q=ci}\end{gathered}\right] = ei \tag{2.8}$$

where: w is the deflection function, and equals AH; "ei" is the introduced error, "i" is the number of points on the continuum. Integrating Eq.(2.8) twice with respect to R and Q and minimizing, we obtain

$$N_{x(cri)} = \frac{D\int_0^1\int_0^1\left[\frac{H}{P^2}.\frac{\partial^4 H}{\partial R^4} + 2\mathrm{H}.\frac{\partial^4 H}{\partial R^2 \partial Q^2} + \mathrm{P}^2\frac{H.\partial^4 H}{\partial Q^4} + \frac{1}{P^2}.\sum_{i=1}^{n}\gamma_i\left(\frac{H.\partial^4 H}{\partial R^4}\right)_{Q=ci}\right]\partial R\partial Q}{-b^2\int_0^1\int_0^1\left[\frac{H.\partial^2 H}{\partial R^2} + \sum_{i=1}^{n}\delta_i\left(\frac{H.\partial^2 H}{\partial R^2}\right)_{Q=ci}\right]\partial R\partial Q}. \tag{2.9}$$

Equation (2.9) is the buckling equation for a rectangular plate stiffened longitudinally.

2.2. Displacement function for CCCC stiffened plate

A formulated polynomial shape function for rectangular plates from Taylor-McLaurin's series was introduced in the work of Ibearugbulem *et al*. [13, 14] for solution of rectangular thin isotropic plates subjected to in-plane loading. The displacement function which satisfies Eq.(2.9) and approximately describes the deflection of the stiffened plate under in-plane loading is given as

$$w = \sum_{m=0}^{4}\sum_{n=0}^{4} a_m b_n R^m Q^n . \tag{2.10}$$

By expanding Eq.(2.10) we get25-term finite series, for $m = n=4$

$$\begin{aligned} w &= a_0b_0 + a_0b_1Q + a_0b_2Q^2 + a_0b_3Q^3 + a_0b_4Q^4 + a_1b_0R + \\ &+a_1b_1RQ + a_1b_2RQ^2 + a_1b_3RQ^3 + a_1b_4RQ^4 + a_2b_0R^2 + \\ &+a_2b_1R^2Q + a_2b_2R^2Q^2 + a_2b_3R^2Q^3 + a_2b_4R^2Q^4 + a_3b_0R^3 + a_3b_1R^3Q + \\ &+a_3b_2R^3Q^2 + a_3b_3R^3Q^3 + a_3b_4R^3Q^4 + a_4b_0R^4 + a_4b_1R^4Q + \\ &+a_4b_2R^4Q^2 + a_4b_3R^4Q^3 + a_4b_4R^4Q^4. \end{aligned} \tag{2.11}$$

Boundary conditions along the R – direction

$$\left(\frac{\partial w}{\partial R}\right)_{R=0 \text{ or } 1} = 0, \tag{2.12}$$

$$(w)_{R=0 \text{ or } 1} = 0. \tag{2.13}$$

Boundary conditions along the Q – direction

$$\left(\frac{\partial w}{\partial Q}\right)_{Q=0 \text{ or } 1} = 0. \tag{2.14}$$

By applying Eqs (2.12) - (2.15) for all edged clamped system in Eq.(2.11) we get

$$w = A\left(R^2 - 2R^3 + R^4\right)\left(Q^2 - 2Q^3 + Q^4\right). \tag{2.15}$$

3. Formation of stability equation for CCCC stiffened plate

The numerical formulations for the stiffened systems arrangements shown in Fig.1 were carried out for the case of one stiffener, two stiffeners and three stiffeners.

3.1. Case of one stiffener

Consider Fig.1a, the stiffener divides the plate into two equal parts.

For The Stiffener /Rib: when there is only one stiffener for $1 \le Q \le 1$; $1 \le R \le 1$, we have;

$$(H)_{y=b/2} = (H)_{Q=1/2} = \left(R^2 - 2R^3 + R^4\right)\left(Q^2 - 2Q^3 + Q^4\right) = 0.0625\left(R^2 - 2R^3 + R^4\right), \quad (3.1)$$

$$\int_0^1\int_0^1 \left(H.\frac{\partial^2 H}{\partial R^2}\right)_{Q=1/2} = -0.7440*10^{-4}, \quad (3.2)$$

$$\int_0^1\int_0^1 \left(H.\frac{\partial^4 H}{\partial R^4}\right)_{Q=1/2} = 0.003125, \quad (3.3)$$

For The Plate Element: We have;

$$\int_0^1\int_0^1 H.\frac{\partial^4 H}{\partial R^4}\partial R\partial Q = 0.0012698, \quad (3.4)$$

$$\int_0^1\int_0^1 H.\frac{\partial^4 H}{\partial Q^4}\partial R\partial Q = 0.0012698, \quad (3.5)$$

$$\int_0^1\int_0^1 H.\frac{\partial^4 H}{\partial R^2 \partial Q^2}\partial R\partial Q = 0.00036281, \quad (3.6)$$

$$\int_0^1\int_0^1 H.\frac{\partial^2 H}{\partial R^2}\partial R\partial Q = -3.0234*10^{-5}. \quad (3.7)$$

Substituting Eqs (3.2) - (3.7) into Eq.(2.9), yielded

$$\therefore N_x = \frac{D\left[\frac{0.0012698}{P^2} + 2\left(0.00036281\right) + P^2\left(0.0012698\right) + \frac{\left(0.003125\right)}{P^2}\gamma\right]}{-b^2\left[-3.0234*10^{-5} - 0.744*10^{-4}\delta\right]}, \quad (3.8)$$

$$=\frac{D\left[\frac{0.0012698}{P^2}+7.2562*10^{-4}+P^2\left(0.0012698\right)+\frac{\left(0.003125\right)}{P^2}\gamma\right]}{-b^2\left[-3.0234*10^{-5}-0.744*10^{-4}\delta\right]}, \tag{3.9}$$

$$N_{x(cri)}=\frac{\pi^2 D}{b^2P^2}\frac{\left[4.2554+2.4317P^2+4.2554P^4+10.4726\ \gamma\right]}{\left[1+2.4608\delta\right]}, \tag{3.10}$$

$$K=\frac{\left[4.2554+2.4317P^2+4.2554P^4+10.4726\ \gamma\right]}{\mathrm{P}^2\left[1+2.4608\delta\right]}. \tag{3.11}$$

3.2. Case of two stiffeners

As shown in Fig.1b, C_1 and C_2 are the distances of the stiffeners from the edge $y=0$. Stiffeners are assumed to be symmetrical, hence

$$\delta_1=\delta_2=\ \delta \quad \text{and} \quad \gamma_1=\gamma_2=\ \gamma:\ C_1=\frac{1}{3}, \qquad C_2=\frac{2}{3}.$$

Following the same procedure in section (3.1), we obtain the equations as follows

$$\left(H\right)_{Q=C1}=0.04938\left(R^2-2R^3+R^4\right), \tag{3.12}$$

$$\left(H\right)_{Q=C2}=0.04938\left(R^2-2R^3+R^4\right), \tag{3.13}$$

$$\int_0^1\int_0^1\left(H.\frac{\partial^2 H}{\partial R^2}\right)_{Q=C1}\partial R\partial Q=\int_0^1\int_0^1\left(H.\frac{\partial^2 H}{\partial R^2}\right)_{Q=C2}\partial R\partial Q=\ -0.46445*10^{-4}, \tag{3.14}$$

$$\int_0^1\int_0^1\left(H.\frac{\partial^4 H}{\partial R^4}\right)_{Q=C1}\partial R\partial Q=\int_0^1\int_0^1\left(H.\frac{\partial^4 H}{\partial R^4}\right)_{Q=C2}\partial R\partial Q=0.0019507\,/\,. \tag{3.15}$$

Substituting Eqs (3.14), (3.15) and Eqs (3.4) - (3.7) into Eq.(2.9), gave

$$N_{x(cri)}=\frac{D\left[\frac{0.0012698}{P^2}+2\left(0.00036281\right)+P^2\left(0.0012698\right)+\left(0.0019507*2\right)\gamma\right]}{-b^2\left[-3.0234*10^{-5}-0.46445*10^{-4}*2\delta\right]}, \tag{3.16}$$

$$\therefore N_{x(cri)}=\frac{\pi^2 D}{b^2P^2}\frac{\left[4.2554+2.4317P^2+4.2554P^4+13.0745\ \gamma\right]}{\left[1+3.0724\delta\right]}. \tag{3.17}$$

3.3. Case of three stiffeners

For the case of three stiffeners, stiffeners will divide the plate into four equal parts

$$\delta_1 = \delta_2 = \delta_3 = \delta; \quad \gamma_2 = \gamma_2 = \gamma_3 = \gamma : C_1 = \frac{1}{4}, \quad C_2 = \frac{1}{2}, \quad C_3 = \frac{3}{4}.$$

Following the same procedure as above, we obtain

$$N_{x(cri)} = \frac{\pi^2 D}{b^2 P^2} \frac{\left[4.2554 + 2.4317P^2 + 4.2554P^4 + 17.1000\gamma\right]}{\left[1 + 4.0180\delta\right]}. \tag{3.18}$$

4. Results and discussion

Polynomial functions have been successfully applied in analytical methods for the study of thin plates [14 - 16]. However, this study presented buckling analysis for CCCC stiffened plates using the polynomial function and the general solution can be written as

$$N_{x(cri)} = \frac{\pi^2 D}{b^2} K \tag{4.1}$$

where K is the buckling coefficients from the polynomial function for different numbers of stiffeners.

Ibearugbulem *et al*. [14] presented total energy functional from Ritz Method for buckling analysis of thin rectangular plates. Applying energy functional for stiffeners given in Iyengar [16] and solving gave analytical solution in Eq.(4.2)

$$N_{x(cri)} = \frac{\frac{D}{b^2}\int_0^1\int_0^1 \left[\frac{1}{P^2}\cdot\left(\frac{\partial^2 H}{\partial R^2}\right)^2 + 2\left(\frac{\partial^2 H}{\partial R \partial Q}\right)^2 + P^2\left(\frac{\partial^2 H}{\partial Q^2}\right)^2 + \frac{1}{P^2}\cdot\sum_{i=1}^{n}\gamma_i\left(\frac{\partial^2 H}{\partial R^2}\right)^2_{Q=ci}\right]\partial R \partial Q}{\int_0^1\int_0^1\left[\left(\frac{\partial H}{\partial R}\right)^2 + \sum_{i=1}^{n}\delta_i\left(\frac{\partial H}{\partial R}\right)^2_{Q=ci}\right]\partial R \partial Q}. \tag{4.2}$$

By applying the trigonometric function from Iyengar [16] for CCCC boundary conditions as given in Eq.(4.3) into Eq.(4.2), we obtained buckling solutions for the various numbers of stiffeners in Eqs (4.4) – (4.6)

$$H = \left(1 - \cos 2\pi R\right).\left(1 - \cos 2\pi Q\right). \tag{4.3}$$

One stiffener

$$N_{x(cri)} = \frac{4\pi^2 D}{b^2} \times \frac{\left[1 + 2/3P^2 + P^4 + 8/3\gamma\right]}{P^2\left[1 + 8/3\delta\right]}. \tag{4.4}$$

Two stiffeners

$$N_{x(cri)} = 4\frac{\pi^2 D}{b^2} \times \frac{\left[1 + 2/3P^2 + P^4 + 3\gamma\right]}{P^2\left[1 + 3\ \delta\right]}. \tag{4.5}$$

Three stiffeners

$$N_{x(cri)} = 4\frac{\pi^2 D}{b^2} \times \frac{\left[1+2/3P^2+P^4+4\gamma\right]}{P^2\left[1+4\delta\right]}. \tag{4.6}$$

The general solution of the trigonometric shape function is written as

$$N_{x(cri)} = \frac{\pi^2 D}{b^2} K_T . \tag{4.7}$$

Comparing the buckling coefficients K of the present study which made use of the polynomial function in work principle with K_T from an analytical solution that used a trigonometric function, shows good agreement. The average percentage difference is 0.446% for $0.1 \le P \le 2.0$ for the CCCC boundary conditions. Figure 2 shows good convergence for the case of one longitudinal stiffener dividing the plate into two equal parts having, $\gamma = 5$, $\delta = 0.05$. The average percentage difference for the case of two stiffeners is -0.006 with $\gamma = 10$, $\delta = 0.10$ as shown in Fig.3

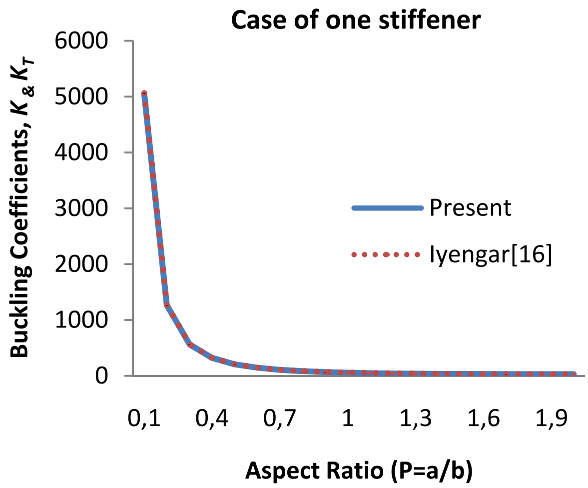


Fig.2. Buckling coefficients vs aspect ratio for CCCC stiffened plate for $\gamma = 5$, $\delta = 0.05$.

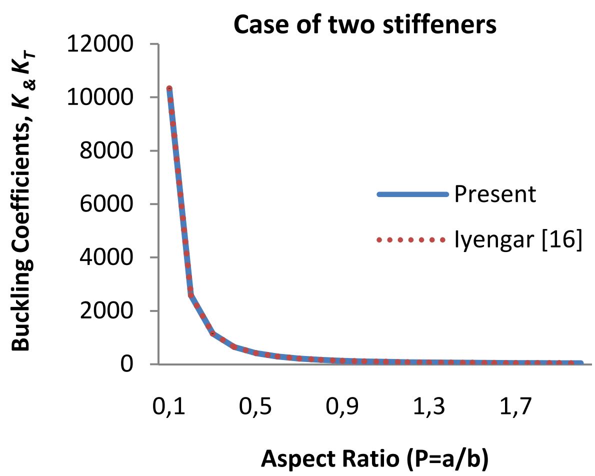


Fig.3. Buckling coefficients vs aspect ratio for CCCC stiffened plate for $\gamma = 10$, $\delta = 0.1$.

The behaviour of buckling coefficients with different aspect ratios, varying stiffness properties and varying number of stiffeners are presented in Figs 4 - 9. Substituting the aspect ratio in the range of $0.1 \leq P \leq 2.0$ into Eqs (3.11), (3.17) and (3.18) gave the figures below. In Figs 4 and 5, the curves show that at a constant value of $\gamma = 10$, the buckling coefficient decreases with an increase in δ from 0.05 to 0.1. The maximum percentage increase in the buckling coefficient is found at $P = 0.1$ which is 20.586 and the minimum at $P = 2$ which is 10.909. In Figs 6 and 7, the curves show that the percentage increase in the buckling coefficient is lager compared to Figs 4 and 5. At a constant value of $\gamma = 5$, observation shows that an increase in the number of stiffeners increases the buckling coefficients rapidly. However, an increase in the aspect ratio cause a decrease in K value.

It is observed from Figs 8 and 9 that at a constant value of $\gamma = 15$ and $\delta = 0.05$, the percentage increase in K value is maximum at $P = 0.1$ which is 21.694 and minimum at $P = 2$ which is 13.815. Similarly, at $\delta = 0.10$, the percentage increase in K value is 15.676 at $P = 0.1$ and 8.186 at $P = 2$. The polynomial series used as the displacement function has been applied in work principle for the buckling of stiffened rectangular isotropic plates. The polynomial series as a shape function show a good convergence from the results above and satisfies all the non-essential (dynamic) boundary conditions of the stiffened plates. Hence, it is a very good approximation of the exact shape function for the stiffened plates. It is therefore claimed that the present theory can accurately predict the critical buckling load of isotropic plates.

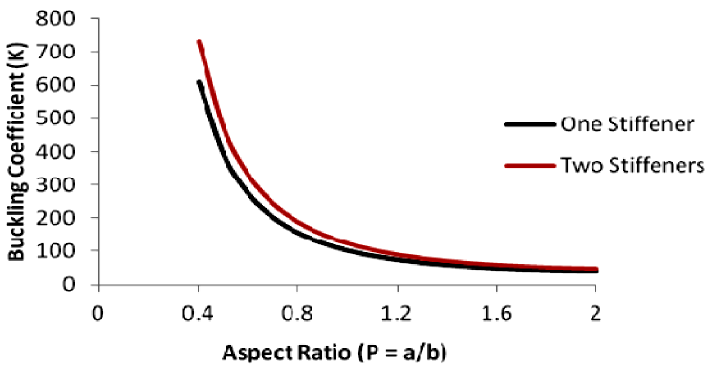


Fig.4. Buckling coefficient vs aspect ratio for CCCC stiffened plate for $\gamma = 10$, $\delta = 0.05$.

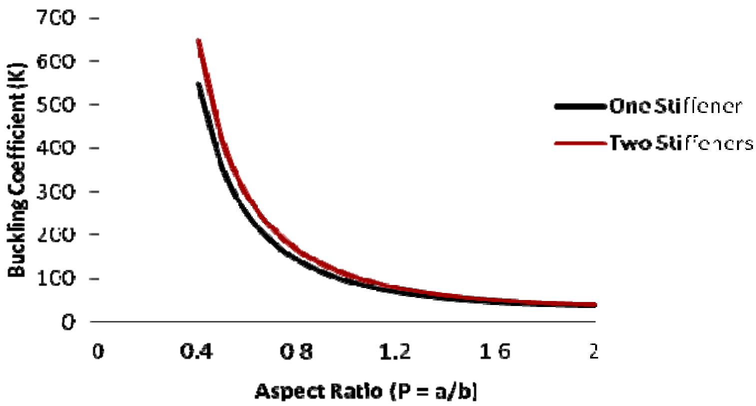


Fig.5. Buckling coefficient vs aspect ratio for CCCC stiffened plate for $\gamma = 10$, $\delta = 0.10$.

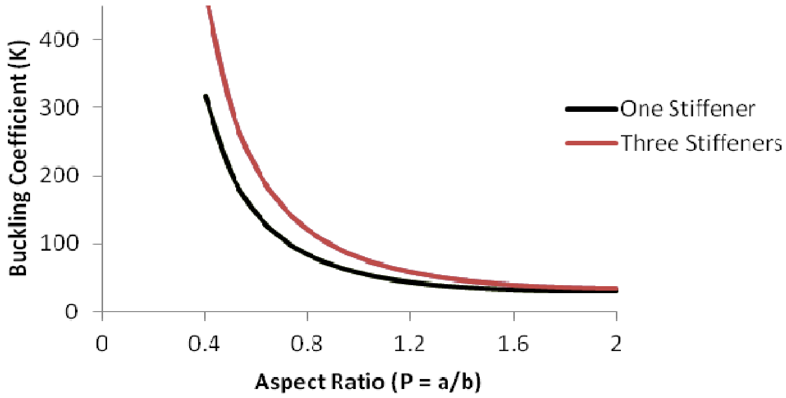


Fig.6. Buckling coefficient vs aspect ratio for CCCC stiffened plate for $\gamma = 5$, $\delta = 0.05$.

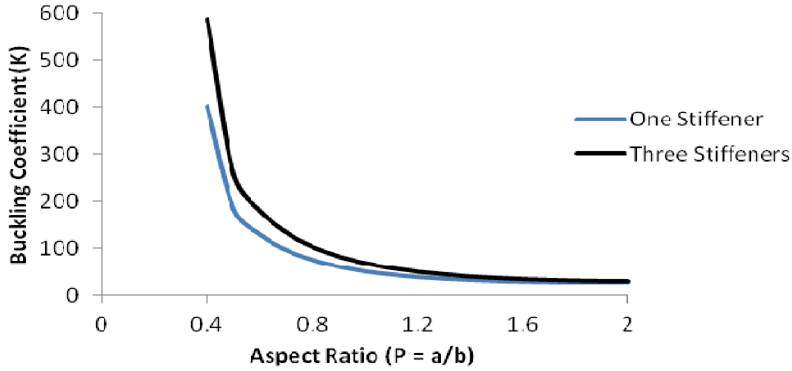


Fig.7. Buckling coefficient vs aspect ratio for CCCC stiffened plate for $\gamma = 5, \delta = 0.10$.

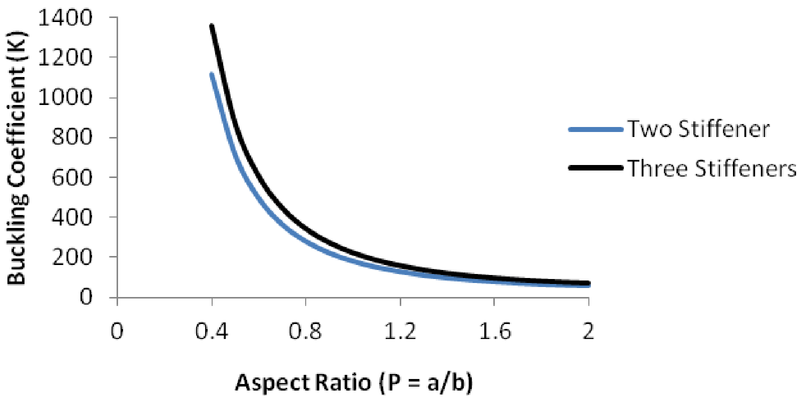


Fig.8. Buckling coefficient vs aspect ratio for CCCC stiffened plate for $\gamma = 15$, $\delta = 0.05$.

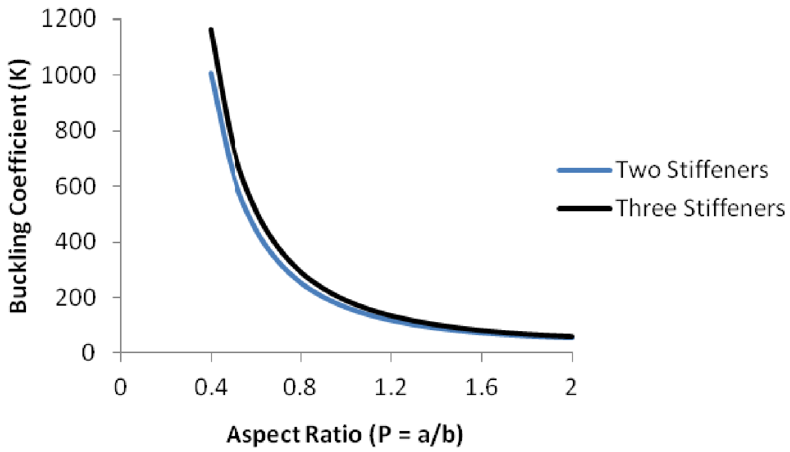


Fig.9. Buckling coefficient vs aspect ratio for CCCC stiffened plate for $\gamma = 15$, $\delta = 0.10$.

4. Conclusion

Buckling of stiffened rectangular isotropic plates elastically restrained along all the edges (CCCC) was investigated. Theoretical analyses were conducted and the work principle approach based on the polynomial function method was proposed. A numerical model was developed to determine the buckling coefficients intended for the design of stiffened systems. The proposed solution was validated and compared with the solution of the trigonometric function from previous works. From the results, polynomial series applied as displacement function shows good convergence from the figures above and satisfies the complex boundary conditions of the stiffened plates. Hence, it is therefore claimed that the present theory can accurately predict the critical buckling load of stiffened isotropic plates regardless of the boundary conditions.

Nomenclature

A	– amplitude of the deflection function
AASHTO	– American Association of State Highway and Transportation Officials
AISC	– American Institute of Steel Construction
ANSI	– American National Standards Institute
$\boldsymbol{A_i}$	– cross-sectional area of the stiffeners
a	– length of the plate
b, h	– width and thickness of the plate
C	– Clamped edge
CCCC	– rectangular stiffened plate clamped along all the four edges
ci	– stiffeners distance from the edge $y = 0$
D	– plate flexural rigidity in the elastic range
E	– Young's modulus
ei	– introduced error
H	– shape function
I_i	– moment of inertia of stiffener
K	– buckling coefficient for polynomial function
K_T	– buckling coefficient for trigonometric shape function
N_x	– in - plane uniform compression compressive load on x-plane
$N_{x\,(cr)}$	– critical buckling load
P	– aspect ratio
R, Q	– non dimensional coordinates along the x and y directions
w	– deflection function

x, y – Cartesian coordinates in the horizontal and vertical direction, respectively
γ_i – ratio of bending stiffness rigidity of stiffeners to the plate
$\boldsymbol{\delta_i}$ – ratio of cross-sectional area of the stiffeners to the plate

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