Elastic Kirchhoff Migration for Vertical Seismic Profiles

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Summary

Elastic Kirchhoff migration is implemented for the VSP recording geometry. The resulting migration formula requires measurement of the stress as well as the displacement. Since stress is not measured in a VSP, and in many cases the horizontal component of displacement is not measured, approximate migration formulas are given for these cases. The elastic migration formula for the case where only the vertical components are available, is the same as the acoustic migration formula, where the pressure data are replaced by the magnitudes of the elastic data as reconstructed from the vertical components, and the acoustic Green's functions are replaced with either the P or S wave elastic Green's functions. Two expressions for migration of two component displacement data are presented. In the first, the terms involving traction data are simply ignored. In the second, an improved backpropagation operator for the displacement field is obtained by replacing the traction data in the Kirchhoff integral by displacement data using Hooke's law. The migration expressions for the cases where two component data are available produce images which are less contaminated by artifacts than the migration images of one component data.

The Elastic Kirchhoff Integral

The elastic Kirchhoff integral describes how to forward propagate the displacement field when the displacement field and the stress field have been recorded on some surface. Kuo and Dai (1984) obtained a formula for elastic Kirchhoff migration for the case of constant background parameters with sources and receivers on a free surface. Elastic Kirchhoff migration for the VSP geometry is described here.

The elastic Kirchhoff integral in the frequency domain is

$$\vec{u}(\vec{x},\omega) = \int_S \Big\{ \vec{t}(\vec{x}',\omega) \cdot \overleftrightarrow{G}(\vec{x}|\vec{x}',\omega) - \vec{u}(\vec{x}',\omega) \cdot \left(\hat{n}' \cdot \overleftrightarrow{\Sigma}(\vec{x}|\vec{x}',\omega) \right) \Big\}\, dS' , \tag{1}$$

where $\hat{n}$ is the unit vector normal to the surface S, ω is angular frequency, $\vec{u}$ is the displacement, the traction vector, $\vec{t}$, is

$$\vec{t} = \hat{n}' \cdot \overleftrightarrow{T} = \overleftrightarrow{T} \cdot \hat{n}' , \tag{2}$$

the stress tensor, $\overleftrightarrow{T}$, is

$$\overleftrightarrow{T} = \lambda \overleftrightarrow{I} (\nabla \cdot \vec{u}) + \mu (\nabla \vec{u} + \vec{u} \nabla) , \tag{3}$$

where λ is Lame's parameter, μ is the shear modulus, $\overleftrightarrow{I}$ is the unit dyadic, and the Green's displacement tensor, $\overleftrightarrow{G}$, and the Green's stress tensor, $\overleftrightarrow{\Sigma}$ can be written in terms of the Green's functions

$$g_p(\vec{x}|\vec{x}',\omega) = A_p(\vec{x}'|\vec{x}) e^{i\omega\tau_p(\vec{x}'|\vec{x})} , \tag{4a}$$

$$g_s(\vec{x}|\vec{x}',\omega) = A_s(\vec{x}'|\vec{x}) e^{i\omega\tau_s(\vec{x}'|\vec{x})} . \tag{4b}$$

where A is the amplitude, and τ is the travel time.

For 2-D wavefield extrapolation, the Kirchhoff integral can be written more simply as

$$\vec{u}(\vec{x},\omega) = \frac{1}{4\pi} \int_s \Big\{ A\, \vec{t}(\vec{x}',\omega) \frac{1}{\rho v_p^2} g_p(\vec{x}|\vec{x}',\omega) + B\, \vec{t}(\vec{x}',\omega) \frac{1}{\rho v_s^2} g_s(\vec{x}|\vec{x}',\omega) - C\, \vec{u}(\vec{x}',\omega) \frac{i\omega}{v_p} g_p(\vec{x}|\vec{x}',\omega) - D\, \vec{u}(\vec{x}',\omega) \frac{i\omega}{v_s} g_s(\vec{x}|\vec{x}',\omega) \Big\}\, ds' , \tag{5}$$

where ρ is density, v_p and v_s are the P and S wave velocities, and the integration over s, is now a line instead of the surface S. The 2×2 matrices A, B, C, and D are functions of $\frac{v_s}{v_p}$, and the incident angle at the receiver.

Elastic Kirchhoff Migration

Equation (5) above describes how to forward extrapolate the displacement field when the displacement and traction are measured on the surface s. The process of migration requires that the field be propagated backwards into the medium where it will focus at the scatterers and hence form an image or pseudo-reflection coefficient. In order to change Equation (5) from forward propagation to backward propagation, the complex conjugates $\overleftrightarrow{G}^*$ and $\overleftrightarrow{\Sigma}^*$ will be used in place of the Green's displacement and stress tensors, $\overleftrightarrow{G}$ and $\overleftrightarrow{\Sigma}$. The resulting equation is evaluated at $t = 0$ (imaging condition) and one can obtain, for example, a P to S pseudo-reflection coefficient in the form

$$R_{PS}(\vec{x}) \equiv \frac{1}{4\pi A_p^{src}(\vec{x}|\vec{x}_o)} \left| \int_s \frac{1}{\rho v_s^2} A_s^{rcv}(\vec{x}|\vec{x}') \Big\{ B\, \vec{t}(\vec{x}', \tau_s^{rcv} + \tau_p^{src}) \; D\, \dot{\vec{u}}(\vec{x}', \tau_s^{rcv} + \tau_p^{src}) \Big\}\, ds' \right| . \tag{6}$$

Pseudo-reflection coefficients for R_{PP}, R_{SP}, and R_{SS} can also be obtained.

Migration Without Stress Data

Stress data are not measured in a VSP. Since the stress data cannot be obtained from a single measurement of displacement at a given receiver location, the migration cannot be implemented as expressed by Equation (6).

However, if we utilize the assumptions of the migration process, stress terms can be replaced with displacement terms in the Kirchhoff integral. These assumptions are: 1) the data are in the far field; 2) the medium parameters are known (i.e., a background medium is assumed, through which the field is backpropagated); and, 3) the scatterers are point scatterers, and their locations are known (i.e., the process of focusing at an image point assumes the image point is a scatterer).

These assumptions allow us to use Equation (3) to determine the stress from the measured displacement along the diffraction curve as due to a point source located at the image point. Assumption (1) allows the spatial derivatives in Equation (3) to be expressed as temporal derivatives, and assumptions (2) and (3) allow the directions of the gradients in Equation (3) to be determined by ray tracing. Note that for a given receiver location, the stress determined by this method will be different for each image point.

If the image point is not actually a scatterer, the reconstructed stress terms will be incorrect. Hopefully, this will not be a very serious problem. If the image point is not a scatterer, the data along the diffraction curve will interfere destructively instead of constructively (assuming sufficient aperture), and, since no image will be produced, it should not matter whether the stress terms were reconstructed correctly or not.

It should be emphasized that the stress data *cannot* be obtained from a single measurement of displacement at a receiver location. The process described above does not create a stress data set from the observed displacement data set. It utilizes the assumptions of Kirchhoff migration as implemented by a ray method to replace the stress terms with displacement terms *in the Kirchhoff migration integral* (Equation (6)). For a given receiver, this "reconstructed" stress will be different for each image point. The true stress is never known. What this procedure has allowed us to do is to obtain an improved backpropagation operator (i.e., an improved weighting function) which operates only on displacement data. Using the above procedure, Equation(6) can be rewritten as

$$R_{PS}(\vec{x}) \equiv \frac{1}{4\pi A_p^{src}(\vec{x}|\vec{x}_o)} \left| \int_s \frac{A_s^{rcv}(\vec{x}|\vec{x}')}{v_s} \left\{ K\, \dot{\vec{u}}(\vec{x}', \tau_s^{rcv} + \tau_p^{src}) + D\, \dot{\vec{u}}(\vec{x}', \tau_s^{rcv} + \tau_p^{src}) \right\} ds' \right| , \quad (7)$$

where $K\,\vec{u} = B\,F\,\vec{u}$, and the 2×2 matrix F is a function of $\frac{v_s}{v_p}$, and the incident angle at the receiver.

Migration with only the Vertical Components of Displacement

In many VSP's only the vertical component of displacement is measured. If Equation (6) is implemented without using the horizontal components (i.e., setting them equal to zero), the locations of the scatterers can still be imaged. As in the case where the stress data are not available, when the horizontal components of displacement are unavailable, one can make use of the assumptions of Kirchhoff migration as implemented with a ray method to replace the horizontal component data with vertical component data. The resulting weighting function is simply $\cos\alpha$, which is the same weighting as for the acoustic migration. Thus, elastic migration for the case where only the vertical components are available, can be implemented using the acoustic migration formula where the pressure data are replaced by the magnitudes of the elastic data as reconstructed from the vertical components (i.e., replace the pressure P with $\frac{1}{\sin\alpha_p} u_V$ when computing R_{PP} or R_{SP} and with $\frac{1}{\cos\alpha_s} u_V$ when computing R_{PS} or R_{SS}), and the acoustic Green's functions are replaced by the elastic Green's functions.

Example

In this example, a synthetic VSP is generated using paraxial ray tracing (see Beydoun and Keho, 1987) from a model consisting of two flat interfaces. The recording geometry is shown in Figure 1. The ray codes consisted of a direct P wave, and a primary reflected P wave and a primary reflected S wave at each of the two interfaces. The vertical and horizontal components of displacement are shown in Figures 2 and 3, respectively.

Figure 4 shows the image produced by the One Component EKM (Elastic Kirchhoff Migration). Strong artifacts can be seen near the borehole (on the left). Figure 5 shows the image produced by the Two Component EKM (Equation 6 without the stress terms). The main artifact has been significantly reduced, but some additional smaller artifacts have been generated. The Two Component Plus Pseudo-stress EKM (Equation 7 — the stress terms are replaced with displacement terms using Hooke's Law), shown in Figure 6, eliminates almost completely all artifacts near the borehole. The image of the two interfaces dies out more rapidly away from the borehole.

Conclusion

The elastic migration formula for the case where only the vertical components are available, is the same as the acoustic migration formula, where the pressure data are replaced by the magnitudes of the elastic data as reconstructed from the vertical components, and the acoustic Green's functions are replaced with either the P or S wave elastic Green's functions. A noticeable difference in migration artifacts is observed when comparing the One Component EKM and both of the two component EKM formulations. The Two Component EKM formulation substantially improves the quality of the image in the vicinity of the borehole. The additional weighting function derived from the stress terms in the elastic Kirchhoff integral produces the Two Component Plus Pseudo-stress EKM which produces an image with almost no migration artifacts in the vicinity of the borehole.

References

Beydoun, W.B., and T.H. Keho, 1987, The paraxial ray method, Submitted to: *Geophysics*.

Kuo, J.T., and T. Dai, 1984, Kirchhoff elastic wave migration for the case of noncoincident source and receiver, *Geophysics*, 49, 1223–1238.

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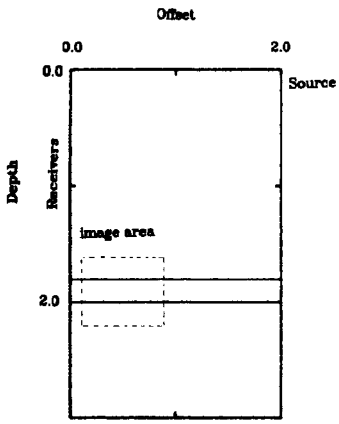

FIG. 1. Recording geometry (constant velocity medium).

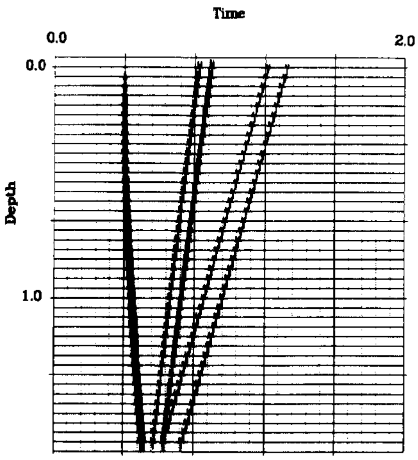

FIG. 2. Synthetic displacement data, vertical components.

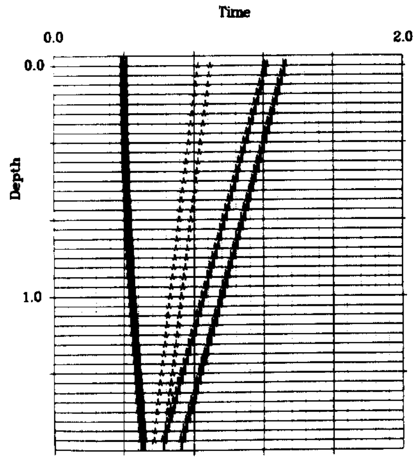

FIG. 3. synthetic displacement data, horizontal components.

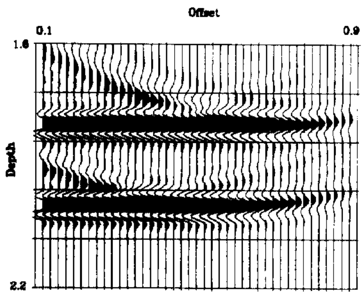

FIG. 4. 1-component EKM.

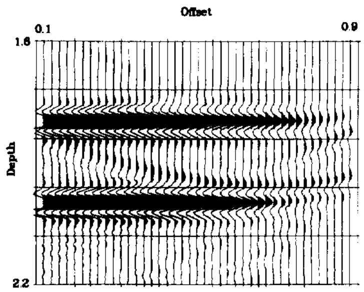

FIG. 5. 2-component EKM.

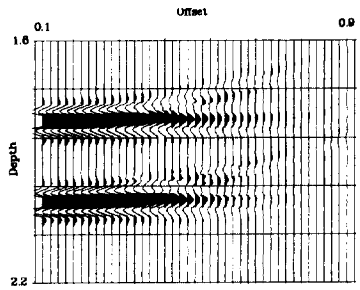

FIG. 6. 2-component plus pseudo-stress EKM.