

**Thermo Elastic-Plastic Transition of Transversely
Isotropic Thick-Walled Rotating Cylinder
under Internal Pressure**

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Abstract

Thermo elastic-plastic stresses for a transversely isotropic thick-walled rotating cylinder under internal pressure by using transition theory have been discussed numerically and depicted graphically. It has been observed that at room temperature, thick-walled circular cylinder made of isotropic material having thickness ratio $(b/a)=4$ yields at the internal surface at high pressure as compared to cylinder made of transversely isotropic material. For a smaller radii ratio, thick-walled circular cylinder yields at lower pressure. With the introduction of thermal effects and angular speed, cylinder made of isotropic/transversely isotropic material yields at lower pressure. With the increase in angular speed and thermal effects much less pressure is required for initial yielding. It has been observed that for fully plastic state circumferential stress is maximum at external surface. Therefore, circular cylinder under internal pressure made of transversely isotropic material is on the safer side of the design.

Mathematics Subject Classification: 74C20, 74C99

Keywords: Elastic-plastic, Transition, Cylinder, Isotropic

1. Introduction

The constantly increasing industrial demand for axisymmetrical cylindrical components has concentrated the attention of designers and scientists on this particular area of activity. The progressive world-wide scarcity of materials combined with their consequently higher cost, makes it increasingly less attractive to confine design to the customary elastic regime only. Thick-walled cylinders of circular cross-section are used commonly either as pressure vessels intended for storage in industrial gases or as media for transportation of high pressurized fluids. A thick-walled cylinder is also widely used as a structural component in oil refineries, power industries etc. Problem of thick-walled cylinder under internal pressure have been analyzed by many authors [1-4] for isotropic homogeneous elastic-plastic states. In their treatment, the following assumptions were made: (i) the incompressibility conditions, (ii) the deformation is small, (iii) the yield criterion.

Incompressibility of the material in plasticity is one of the most important assumption that simplifies the problem. Infact, in most of the cases, it is not possible to find a solution in closed form without this assumption. Transition theory [2] does not require any of the above assumptions and thus solves a more general problem. This theory utilizes the concept of generalized principal strain measure, which not only gives the well-known strain measures but can also be used to find the stresses in plasticity and creep problems.

In this paper, thermo elastic-plastic stresses for a transversely isotropic thick-walled rotating cylinder under internal pressure have been obtained by using transition theory. Results obtained have been discussed numerically and depicted graphically.

2. Governing Equations

Consider a thick-walled circular cylinder of internal and external radii 'a' and 'b' respectively, subjected to internal pressure (p) and rotating with an angular velocity (ω) and temperature (T_0) applied at the internal surface. The generalized principal components of strain is defined as,

$$e_{ii} = \int_0^{e_{ii}^A} [1 - 2e_{ii}^A]^{n-1} de_{ij}^A = \frac{1}{n} [1 - (1 - 2e_{ii}^A)^n], \quad (i, j = 1, 2, 3) \quad (1)$$

where n is the measure and e_{ij}^A is the principal Almansi finite strain components [4].

The displacement components in cylindrical polar co-ordinates are given by,

$$u = r(1 - \beta), \quad v = 0, \quad w = dz \quad (2)$$

where β is a function of $r = \sqrt{x^2 + y^2}$ only and d is a constant.

The finite components of strain are given as

$$e_{rr}^A = \frac{1}{2}[1 - (r\beta' + \beta)^2], \quad e_{\theta\theta}^A = \frac{1}{2}[1 - \beta^2], \quad e_{zz}^A = \frac{1}{2}[1 - (1-d)^2], \quad e_{r\theta}^A = e_{\theta z}^A = e_{rz}^A = 0 \quad (3)$$

where $\beta' = d\beta/dr$

Substituting equation (3) in equation (1) we get the generalized components of strain as,

$$e_{rr} = \frac{1}{n}[1 - (r\beta' + \beta)^n], \quad e_{\theta\theta} = \frac{1}{n}[1 - \beta^n], \quad e_{zz} = \frac{1}{n}[1 - (1-d)^n], \quad e_{r\theta} = e_{\theta z} = e_{rz} = 0 \quad (4)$$

The stress-strain relations for transversely isotropic material are

$$\begin{aligned} T_{rr} &= C_{11}e_{rr} + (C_{11} - 2C_{66})e_{\theta\theta} + C_{13}e_{zz} - \beta_1 T, \\ T_{\theta\theta} &= (C_{11} - 2C_{66})e_{rr} + C_{11}e_{\theta\theta} + C_{13}e_{zz} - \beta_2 T \\ T_{zz} &= C_{13}e_{rr} + C_{13}e_{\theta\theta} + C_{33}e_{zz} - \beta_2 T, \quad T_{rz} = T_{\theta z} = T_{r\theta} = 0 \end{aligned} \quad (5)$$

where $\beta_1 = C_{11}\alpha_1 + 2C_{12}\alpha_2$; $\beta_2 = C_{12}\alpha_1 + (C_{22} + C_{33})\alpha_2$;

C_{ij} 's = Material constants; T = Temperature change;

α_1 = Coefficient of linear thermal expansion along the axis of symmetry,

α_2 = Corresponding quantities orthogonal to axis of symmetry.

Using equations (4) in equations (5), we have

$$\begin{aligned} T_{rr} &= (C_{11}/n)[1 - (\beta + r\beta')^n] + [(C_{11} - 2C_{66})/n][1 - \beta^n] + C_{13}e_{zz} - \beta_1 T \\ T_{\theta\theta} &= [(C_{11} - 2C_{66})/n][1 - (\beta + r\beta')^n] + (C_{11}/n)[1 - \beta^n] + C_{13}e_{zz} - \beta_2 T \\ T_{zz} &= (C_{13}/n)[1 - (\beta + r\beta')^n] + (C_{13}/n)[1 - \beta^n] + C_{33}e_{zz} - \beta_2 T \\ T_{r\theta} &= T_{\theta z} = T_{rz} = 0 \end{aligned} \quad (6)$$

Equations of equilibrium are all satisfied except,

$$\frac{d}{dr}(T_{rr}) + \left(\frac{T_{rr} - T_{\theta\theta}}{r} \right) + \rho r \omega^2 = 0 \quad (7)$$

where ρ is the density of the material. The temperature field satisfying Laplace equation ($\nabla^2 T = 0$) and $T = T_0$ at $r = a$, $T = 0$ at $r = b$

where T_0 is a constant given by

$$T = \frac{T_0}{\log(a/b)} \log(r/b). \quad (8)$$

Substituting equation (6) and (8) in equation (7), we get a non-linear differential equation in β as,

$$\begin{aligned} nPC_{11}\beta^{n+1}(1+P)^{n-1} \frac{dP}{d\beta} &= -nPC_{11}\beta^n(1+P)^n - (C_{11} - 2C_{66})nP\beta^n + 2C_{66}[1 - \beta^n(1+P)^n] \\ &\quad - 2C_{66}(1 - \beta^n) - n\beta_1\bar{T}_0 + (\beta_2 - \beta_1)n\bar{T}_0 \log(r/b) + \rho nr^2\omega^2 \end{aligned} \quad (9)$$

where $r\beta' = \beta P$ and $\bar{T}_0 = T_0/\log(a/b)$.

The transitional points of P in equation (9) are $P \rightarrow -1$ and $P \rightarrow \pm\infty$.

The boundary conditions are

$$T_{rr} = -p \text{ at } r = a \text{ and } T_{rr} = 0 \text{ at } r = b \quad (10)$$

The resultant force normally applied to the ends of cylinder is

$$2\pi \int_a^b r T_{zz} dr = \pi a^2 p \quad (11)$$

3. Solution through the Principal Stresses

It has been shown [5-9] that the asymptotic solution through the principal stress leads from elastic to plastic state at the transition point $P \rightarrow \pm\infty$. For finding the plastic stresses at the transition point $P \rightarrow \pm\infty$, we define the transition function R as,

$$R = 2(C_{11} - C_{66}) + nC_{13}e_{zz} - nT_{rr} - n\beta_1 T = \beta^n [C_{11} - 2C_{66} + C_{11}(1+P)^n] \quad (12)$$

Taking the logarithmic differentiation of equation (12) with respect to 'r', and taking the asymptotic value as $P \rightarrow \pm\infty$, and integrating, we get

$$R = A_1 r^{-C_1} \quad (13)$$

where A_1 is a constant of integration and $C_1 = 2C_{66}/C_{11}$.

Using equation (13) in equation (12), we get

$$T_{rr} = C_3 - \beta_0 \log(r/b) - (A_1/n)r^{-C_1} \quad (14)$$

where $C_3 = [2(C_{11} - C_{66}) + nC_{13}e_{zz}]/n$ and $\beta_0 = \beta_1 \bar{T}_0$.

Using boundary condition (10) in equation (14), we get

$$A_1 = nb^{C_1} \left[\frac{p - \beta_0 \log(a/b)}{(b/a)^{C_1} - 1} \right], \quad C_3 = \left[\frac{p - \beta_0 \log(a/b)}{(b/a)^{C_1} - 1} \right] \quad (15)$$

Substituting the value of A_1 and C_3 in equation (14), we get

$$T_{rr} = \left[\frac{p - \beta_0 \log(a/b)}{(b/a)^{C_1} - 1} \right] \left[1 - \left(\frac{b}{r} \right)^{C_1} \right] - \beta_0 \log\left(\frac{r}{b} \right) \quad (16)$$

Using equation (16) in equation (7), we have

$$T_{\theta\theta} = \left[\frac{p - \beta_0 \log(a/b)}{(b/a)^{C_1} - 1} \right] \left[1 - (1 - C_1) \left(\frac{b}{r} \right)^{C_1} \right] - \beta_0 \left[1 + \log\left(\frac{r}{b} \right) \right] + \rho r^2 \omega^2 \quad (17)$$

The axial stress is obtained from equation (5) as

$$T_{zz} = \frac{C_{13}}{2(C_{11} - C_{66})} [T_{rr} + T_{\theta\theta}] + \left[\frac{C_{33}(C_{11} - C_{66}) - C_{13}^2}{C_{11} - C_{66}} \right] e_{zz} \quad (18)$$

$$+ \frac{T}{2(C_{11} - C_{66})} [C_{13}(\beta_1 + \beta_2) - 2\beta_2(C_{11} - C_{66})]$$

Applying the end condition (11) in equation (18), the axial strain is given by,

$$e_{zz} = \frac{(C_{11}-C_{66})}{[C_{33}(C_{11}-C_{66})-C_{13}^2]} \left[\begin{array}{l} \frac{a^2 p}{b^2-a^2} - \frac{a^2 C_{13}(p-\beta_0 \log(a/b))}{(C_{11}-C_{66})(b^2-a^2)} + \frac{\beta_0 C_{13} a^2 \log(b/a)}{(C_{11}-C_{66})(b^2-a^2)} \\ - \frac{C_{13}}{4(C_{11}-C_{66})} \rho \omega^2 (b^2+a^2) - \frac{\beta_0}{2(C_{11}-C_{66})} \left[\begin{array}{l} C_{13} \left(1 + \frac{\beta_2}{\beta_1} \right) \\ - 2 \frac{\beta_2}{\beta_1} (C_{11}-C_{66}) \end{array} \right] \\ \left[-\frac{1}{2} + \frac{a^2 \log(b/a)}{(b^2-a^2)} \right] \end{array} \right] \quad (19)$$

Substituting equation (19) in equation (18), we get

$$T_{zz} = \frac{C_{13}}{C_{11}(2-C_1)} \left[\left(\frac{p-\beta_0 \log(a/b)}{(b/a)^{C_1}-1} \right) \left(2 - (2-C_1) \left(\frac{b}{r} \right)^{C_1} \right) \right] + \frac{a^2 p}{b^2-a^2} - \frac{C_{13} \beta_0}{C_{11}(2-C_1)} \left[1 + 2 \log \left(\frac{r}{b} \right) \right] \\ - \frac{2a^2 p C_{13}}{C_{11}(2-C_1)(b^2-a^2)} + \frac{C_{13} \rho r^2 \omega^2}{C_{11}(2-C_1)} - \frac{C_{13} \rho \omega^2 (b^2+a^2)}{2C_{11}(2-C_1)} \quad (20) \\ - \frac{\beta_0}{C_{11}(2-C_1)} \left[C_{13} \left(1 + \frac{\beta_2}{\beta_1} \right) - 2 \frac{\beta_2}{\beta_1} (C_{11}-C_{66}) \right] \left[\frac{a^2 \log(b/a)}{(b^2-a^2)} - \frac{1}{2} - \log \left(\frac{r}{b} \right) \right]$$

From equation (16) and (17), we get

$$T_{\theta\theta} - T_{rr} = \left[\frac{p - \beta_0 \log(a/b)}{(b/a)^{C_1} - 1} \right] C_1 \left(\frac{b}{r} \right)^{C_1} - \beta_0 + \rho r^2 \omega^2 \quad (21)$$

It is found that the value of $|T_{\theta\theta} - T_{rr}|$ is maximum at $r = a$, which means yielding of the cylinder will take place at the internal surface. Therefore, we have

$$|T_{\theta\theta} - T_{rr}|_{r=a} = \left[\frac{p - \beta_0 \log(a/b)}{(b/a)^{C_1} - 1} \right] C_1 \left(\frac{b}{a} \right)^{C_1} - \beta_0 + \rho a^2 \omega^2 \equiv Y(\text{say}) \quad (22)$$

The relation between pressure and temperature for initial yielding is given by,

$$P_i = \frac{p}{Y} = \left(\frac{1 + (\beta_0/Y) - (\rho a^2 \omega^2 / Y)}{C_1 (b/a)^{C_1}} \right) \left[\left(\frac{b}{a} \right)^{C_1} - 1 \right] + \frac{\beta_0}{Y} \log(a/b) \quad (23)$$

Using equation (23) in equation (16), (17) and (20), we get transitional stresses as

$$\begin{aligned}
\sigma_r &= \frac{T_{rr}}{Y} = \left[\frac{P_i - (\beta_0/Y) \log(a/b)}{(b/a)^{C_1} - 1} \right] \left[1 - \left(\frac{b}{r} \right)^{C_1} \right] - \frac{\beta_0}{Y} \log\left(\frac{r}{b} \right) \\
\sigma_\theta &= \frac{T_{\theta\theta}}{Y} = \left[\frac{P_i - (\beta_0/Y) \log(a/b)}{(b/a)^{C_1} - 1} \right] \left[1 - (1 - C_1) \left(\frac{b}{r} \right)^{C_1} \right] - \frac{\beta_0}{Y} \left[1 + \log\left(\frac{r}{b} \right) \right] + \frac{\rho r^2 \omega^2}{Y} \\
\sigma_z &= \frac{T_{zz}}{Y} = \frac{C_{13}}{C_{11}(2 - C_1)} \left[\left(\frac{P_i - (\beta_0/Y) \log(a/b)}{(b/a)^{C_1} - 1} \right) \left(2 - (2 - C_1) \left(\frac{b}{r} \right)^{C_1} \right) \right] + \frac{a^2 P_i}{b^2 - a^2} \\
&\quad - \frac{C_{13}(\beta_0/Y)}{C_{11}(2 - C_1)} \left[1 + 2 \log\left(\frac{r}{b} \right) \right] + \frac{C_{13}}{C_{11}(2 - C_1)} \frac{\rho r^2 \omega^2}{Y} - \frac{2a^2 C_{13} P_i}{C_{11}(2 - C_1)(b^2 - a^2)} \\
&\quad - \frac{C_{13}}{2C_{11}(2 - C_1)} \frac{\rho \omega^2 b^2}{Y} - \frac{C_{13}}{2C_{11}(2 - C_1)} \frac{\rho \omega^2 a^2}{Y} \\
&\quad - \frac{(\beta_0/Y)}{C_{11}(2 - C_1)} \left[C_{13} \left(1 + \frac{\beta_2}{\beta_1} \right) - 2 \frac{\beta_2}{\beta_1} (C_{11} - C_{66}) \right] \left[\frac{a^2 \log(b/a)}{b^2 - a^2} - \frac{1}{2} - \log\left(\frac{r}{b} \right) \right]
\end{aligned} \tag{24}$$

Equations (24) give thermo elastic-plastic transitional stresses in thick-walled rotating cylinder under internal pressure.

For fully plastic state ($C_1 \rightarrow 0$), equation (22) becomes

$$|T_{\theta\theta} - T_{rr}|_{r=b} = \left[\frac{p - \beta_0 \log(a/b)}{\log(b/a)} \right] - \beta_0 + \rho b^2 \omega^2 \equiv Y^* \text{ (say)} \tag{25}$$

From equation (23), we have

$$P_f = \frac{p}{Y^*} = \left(1 - (\rho b^2 \omega^2 / Y^*) \right) \log(b/a) \tag{26}$$

From equation (24), we get stresses for fully plastic state as

$$\begin{aligned}
\sigma_r^* &= \frac{T_{rr}}{Y^*} = \left(1 - \frac{\rho b^2 \omega^2}{Y^*} \right) \log\left(\frac{r}{b} \right) \\
\sigma_\theta^* &= \frac{T_{\theta\theta}}{Y^*} = \left(1 - \frac{\rho b^2 \omega^2}{Y^*} \right) \left(1 + \log\left(\frac{r}{b} \right) \right) + \frac{\rho r^2 \omega^2}{Y^*} \\
\sigma_z^* &= \frac{T_{zz}}{Y^*} = \frac{C_{13}}{2C_{11}} \left[\left(1 - \frac{\rho b^2 \omega^2}{Y^*} \right) \left(1 + 2 \log\left(\frac{r}{b} \right) \right) \right] + \left(\frac{a^2}{b^2 - a^2} \right) \left[\left(1 - \frac{\rho b^2 \omega^2}{Y^*} \right) \log\left(\frac{b}{a} \right) \right] \\
&\quad - \frac{a^2 C_{13}}{C_{11}(b^2 - a^2)} \left[\left(1 - \frac{\rho b^2 \omega^2}{Y^*} \right) \log\left(\frac{b}{a} \right) \right] + \frac{C_{13}}{2C_{11}} \frac{\rho r^2 \omega^2}{Y^*} - \frac{C_{13}}{4C_{11}} \frac{\rho \omega^2}{Y^*} (b^2 + a^2) \\
&\quad - \frac{(\beta_0/Y^*)}{2C_{11}} \left[C_{13} \left(1 + \frac{\beta_2}{\beta_1} \right) - \frac{2\beta_2(C_{11} - C_{66})}{\beta_1} \right] \left[\frac{a^2 \log(b/a)}{b^2 - a^2} - \frac{1}{2} - \log\left(\frac{r}{b} \right) \right]
\end{aligned} \tag{27}$$

4. Isotropic Case

For an isotropic material, the stresses required for fully plastic state ($c \rightarrow 0$) is given as,

$$\begin{aligned}\sigma_r^* &= \left[1 - \rho b^2 \omega^2 / Y^*\right] \log(r/b), \sigma_\theta^* = \left[1 - \rho b^2 \omega^2 / Y^*\right] \left[1 + \log(r/b)\right] + \rho r^2 \omega^2 / Y^* \\ \sigma_z^* &= \frac{1}{2} \left[\left(1 - \frac{\rho b^2 \omega^2}{Y^*}\right) (1 + 2 \log(r/b)) \right] + \frac{1}{2} \frac{\rho r^2 \omega^2}{Y^*} - \frac{\rho b^2 \omega^2}{Y^*} \frac{(1 + (a/b)^2)}{4} \\ &\quad - \frac{\beta_0}{2Y^*} \left[1 - \frac{\beta_2}{\beta_1} \right] \left[\frac{\log(b/a)}{(b/a)^2 - 1} - \frac{1}{2} - \log(r/b) \right]\end{aligned}\quad (28)$$

These equations are same as obtained by Gupta et.al. [5].

5. Numerical Illustration and Discussions

As a numerical illustration, the values of pressure (p) required for initial yielding (P_i) and for fully plastic state (P_f) at different temperatures for different angular speeds has been given in table 1.

It has been observed from figure1 that at room temperature, thick-walled circular cylinder made of isotropic material having thickness ratio $(b/a)=4$ yields at the internal surface at high pressure as compared to cylinder made of transversely isotropic material. With the introduction of thermal effects and angular speed, cylinder made of isotropic/ transversely isotropic material yields at lower pressure. With the increase in angular speed and thermal effects much less pressure is required for initial yielding. From table 1, it has been observed that a thick-walled circular cylinder made of transversely isotropic material requires high percentage increase in pressure to become fully plastic from its initial yielding and this percentage increases with the increase in temperature and angular speed. It means that at room temperature, thick-walled cylinder made of isotropic material is to withstand a greater pressure to initiate yielding at the internal surface as compared to cylinder made of transversely isotropic material. With the introduction of thermal effects and angular speed they yield at a lower pressure, where as cylinder made of isotropic material requires less percentage increase in pressure to become fully plastic state from its initial yielding.

In figures 2-3, elastic plastic transitional stresses for fully plastic state have been drawn with radii ratio $R = (r/b)$. For fully plastic state circumferential stress is maximum at external surface. Circumferential stress decreases with the increase in angular speed. It has been observed that for fully plastic state, radial and circumferential stresses are independent of thermal effects.

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Table 1: The pressure required for initial yielding and Fully-plastic state at different Temperatures (β_0/Y) and different angular speeds

$\Omega^2=0$		Transversely Isotropic Material(Magnesium)						Isotropic Material(Brass)					
b/a	P	$\frac{\beta_0}{Y} = 0$	$\frac{\beta_0}{Y} = 0.5$	$\frac{\beta_0}{Y} = 0.75$	$P = \frac{P_f - P_i}{P_i}$ is the percentage increase in pressure required from initial yielding to fully-plastic state at different temperatures.			$\frac{\beta_0}{Y} = 0$	$\frac{\beta_0}{Y} = 0.5$	$\frac{\beta_0}{Y} = 0.75$	$P = \frac{P_f - P_i}{P_i}$ is the percentage increase in pressure required from initial yielding to fully-plastic state at different temperatures.		
					$\frac{\beta_0}{Y} = 0$	$\frac{\beta_0}{Y} = 0.5$	$\frac{\beta_0}{Y} = 0.75$				$\frac{\beta_0}{Y} = 0$	$\frac{\beta_0}{Y} = 0.5$	$\frac{\beta_0}{Y} = 0.75$
2	P_i	0.5742	0.5148	0.4851	20.707	34.6348	42.8778	0.5858	0.5321	0.5053	18.3168	30.2574	37.166
	P_f	0.6931	0.6931	0.6931									
3	P_i	0.82	0.6808	0.6111	34.0195	61.369	79.7742	0.8453	0.7186	0.6553	29.9656	52.8806	67.6484
	P_f	1.0986	1.0986	1.0986									
4	P_i	0.9635	0.752	0.6463	43.8817	84.3484	114.4979	1	0.8069	0.7103	38.63	71.8056	95.1711
	P_f	1.3863	1.3863	1.3863									
$\Omega^2=2$		Transversely Isotropic Material(Magnesium)						Isotropic Material(Brass)					
b/a	P	$\frac{\beta_0}{Y} = 0$	$\frac{\beta_0}{Y} = 0.5$	$\frac{\beta_0}{Y} = 0.75$	$P = \frac{P_f - P_i}{P_i}$ is the percentage increase in pressure required from initial yielding to fully-plastic state at different temperatures.			$\frac{\beta_0}{Y} = 0$	$\frac{\beta_0}{Y} = 0.5$	$\frac{\beta_0}{Y} = 0.75$	$P = \frac{P_f - P_i}{P_i}$ is the percentage increase in pressure required from initial yielding to fully-plastic state at different temperatures.		
					$\frac{\beta_0}{Y} = 0$	$\frac{\beta_0}{Y} = 0.5$	$\frac{\beta_0}{Y} = 0.75$				$\frac{\beta_0}{Y} = 0$	$\frac{\beta_0}{Y} = 0.5$	$\frac{\beta_0}{Y} = 0.75$
2	P_i	0.2871	0.2277	0.198	341.414	404.392	450.051	0.2929	0.2392	0.2124	336.634	389.758	426.318
	P_f	-0.6931	-0.6931	-0.6931									
3	P_i	0.6378	0.4985	0.4289	272.248	320.381	356.144	0.6575	0.5308	0.4675	267.087	306.971	334.995
	P_f	-1.0986	-1.0986	-1.0986									
4	P_i	0.843	0.6316	0.5259	264.448	319.49	363.605	0.875	0.6819	0.5853	258.434	303.3	332.872
	P_f	-1.3863	-1.3863	-1.3863									

Fig. 1: Relationship between (β_0/Y) and P_i at different thickness and different angular speed

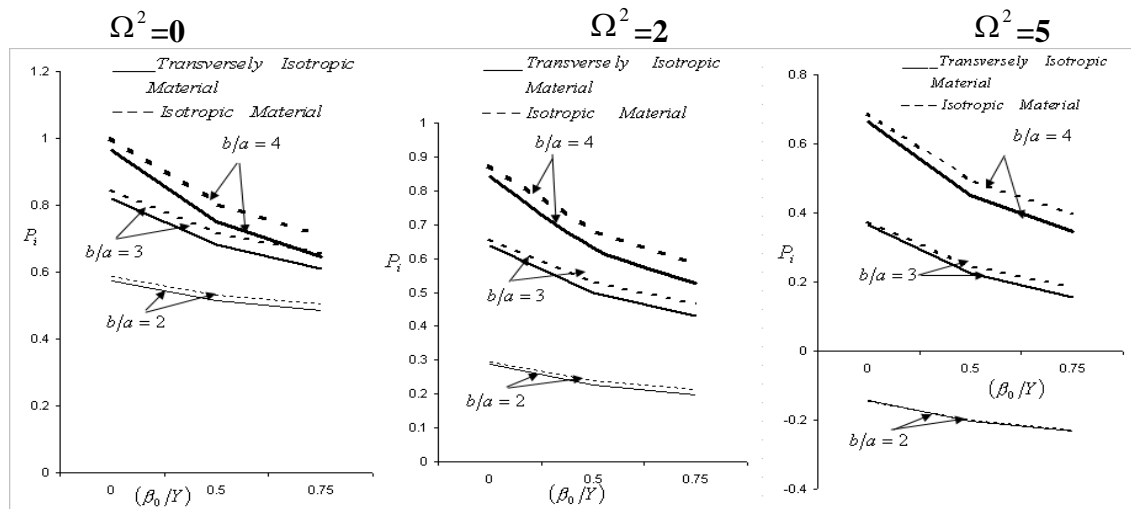


Fig. 2: Fully-plastic Stresses for a Thick-walled Cylinder under Internal Pressure at different temperatures and angular speed ($\Omega^2 = 0$)

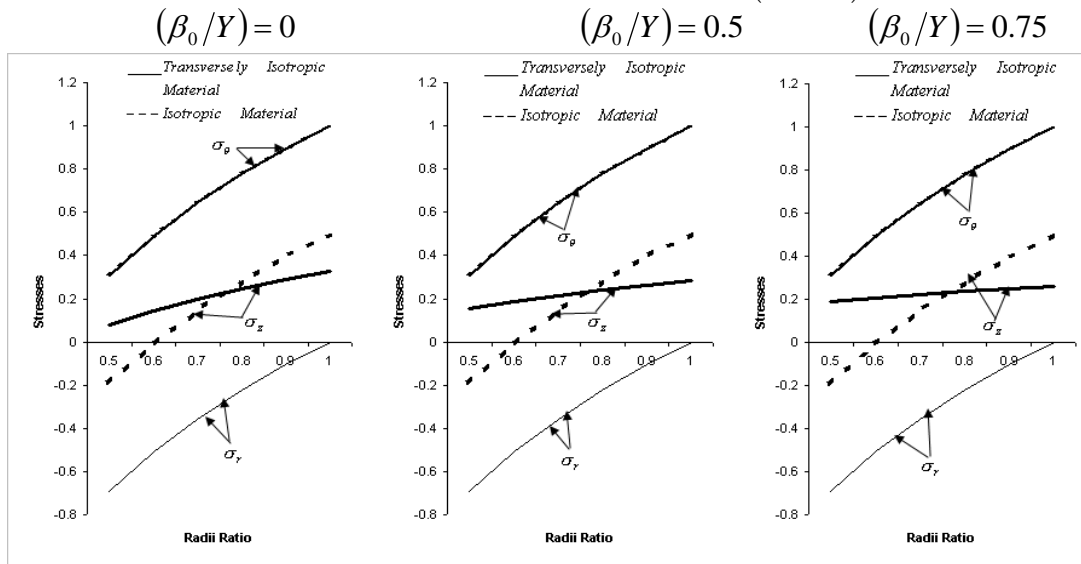
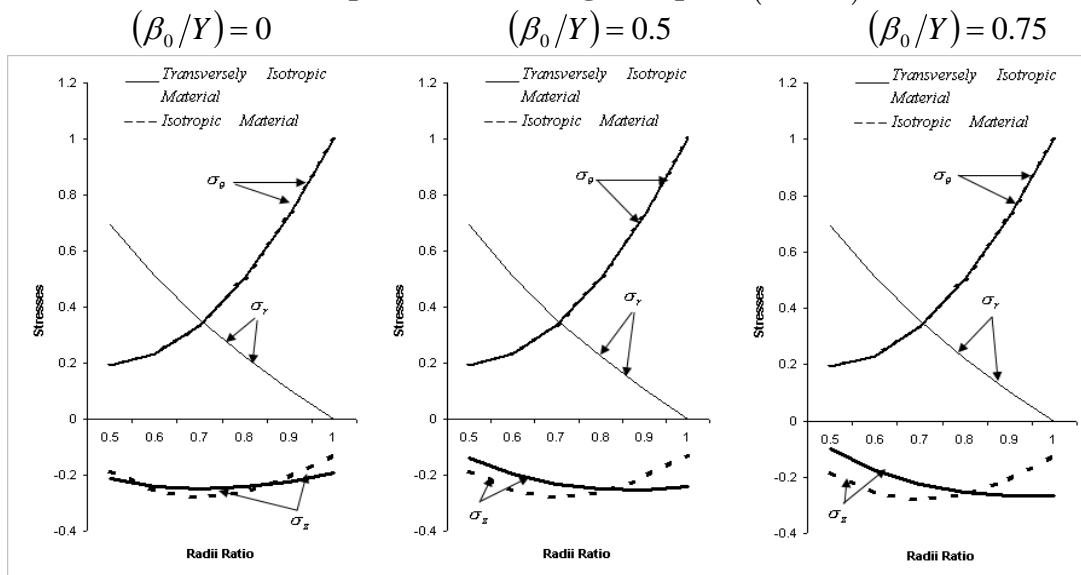


Fig. 3: Fully-plastic Stresses for a Thick-walled Cylinder under Internal Pressure at different temperatures and angular speed ($\Omega^2 = 2$)



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