Electric Power Network Oligopoly as a Dynamic Stackelberg Game

Pedro A. Neto[a∗] Terry L. Friesz[b†] Ke Han[c‡]

[a]*Strategic Planning and Modeling (SPaM), HP Inc., CA 94304, USA*

[b]*Department of Industrial and Manufacturing Engineering, Pennsylvania State University, PA 16802, USA*

[c]*Department of Civil and Environmental Engineering, Imperial College London, SW7 2BU, United Kingdom*

Abstract

Over the last two decades, the electricity industry has shifted from regulation of monopolistic and centralized utilities towards deregulation and promoted competition. With increased competition in electric power markets, system operators are recognizing their pivotal role in ensuring the efficient operation of the electric grid and the maximization of social welfare. In this article, we propose a hypothetical new market of dynamic spatial network equilibrium among consumers, system operators and electricity generators as the solution of a dynamic Stackelberg game. In that game, generators form an oligopoly and act as Cournot-Nash competitors who non-cooperatively maximize their own profits. The market monitor attempts to increase social welfare by intelligently employing equilibrium congestion pricing anticipating the actions of generators. The market monitor influences the generators by charging network access fees that influence power flows towards a perfectly competitive scenario. Our approach anticipates uncompetitive behavior and minimizes the impacts upon society. The resulting game is modeled as a Mathematical Program with Equilibrium Constraints (MPEC). We present an illustrative example as well as a stylized 15-node network of the Western European electric grid.

Keywords: Energy Economics, Electricity Markets, Game Theoretic Models, Stackelberg Game.

1 Introduction

With increased competition in electric power markets, system operators are recognizing their pivotal role in ensuring the efficient operation of the electric grid and the maximization of net social welfare. For now, we define social welfare as a measure of the aggregate utility of a set of economic decisions upon a society. This need primarily stems from the fact that over the last two decades, the electricity industry has seen shifted from regulation of monopolistic and centralized utilities towards deregulation and promoted competition. These efforts were made by governments hoping a renaissance of competition among firms would lead to increased societal benefits such as lower prices, increased innovation and reduced barriers to entry (Nanduri and

∗email: pneto.a@gmail.com

†email: tlf13@psu.edu

‡Corresponding author, email: k.han@imperial.ac.uk

Das, 2009; Momoh, 2009). Therefore, the need for more advanced decision support models has arisen for both private and public parties. In this article, we provide a framework of modeling a hypothetical new electricity power market as a dynamic Stackelberg game. Dynamic optimization games provides increased model fidelity as well a robust framework to study inter temporal links that may go unseen otherwise.

We model the dynamic spatial network equilibrium among consumers, the market monitor and electricity generators as a solution of a differential Stackelberg game. In the game, generators form an oligopoly and act as Cournot-Nash competitors who non-cooperatively maximize their own profits. The market monitor acts as the Stackelberg leader and maximizes total economic surplus by deciding the access charges generators pay to transmit electricity. The differential game combining the leaders and the followers behaviors is expressed as a mathematical program with equilibrium constraints (MPEC). The market monitor is an independent entity charged with task of minimizing uncompetitive behavior in the market.

The advantage of this proposed mechanism is that the market monitor has new found intelligence instead of simply reacting to what dominant firms have already decided. The market monitor derives their strategic insights by employing a new market mechanism of equilibrium congestion pricing. The market monitor sets the charges of transmitting power not only to efficiently clear the transmission market (i.e. allocate transmission capacity with the least amount of congestion) but also to increase social welfare. Interestingly, we show in our numerical results that it possible to increase social welfare with equilibrium congestion pricing when compared to the Cournot-Nash model. Thus, social welfare can be increased by allowing the market monitor to employ equilibrium congestion pricing assuming our proposed hypothetical market.

1.1 Electricity Market Design

A key feature that distinguishes game-theoretic models of electricity markets is the treatment of bid and transactions of power. Bid-based systems, typically referred to as POOLCO models, represent pool based systems where firms bid a supply curve to the central operator of the grid Ventosa et al. (2005). The supply curve is typically submitted as quantity increments and corresponding prices. The central operator collects all bids from firms and then decides how best to operate the grid. The specific decisions depend on the planning horizon the operator is interested. For example, a day ahead planning model may be referred to as a unit commitment model (UC) where the central operator will inform each firm which of the generation assets will need to be operational the next day. The central planner may not yet know how much power will be required from each firm but allows firms to plan their own operations with greater certainty. An economic dispatch problem (EDP) is typically run either an hour or five minutes ahead of when the power will be consumed. In this scenario, the central operator dispatches the generator to transmit certain amounts of power into the grid Wood and Wollenberg (1996). These models require a fixed demand for the planning-horizon of interest.

There are several pricing mechanisms to determine what price each generator receives. We refer the readers to (Meier, 2006; Momoh, 2009; Wood and Wollenberg, 1996) for a more detailed overview of the pricing mechanisms and the general operation of electricity markets. In contrast to pool based systems, bilateral transaction markets feature generators directly exchanging electricity and money. The central operator is therefore primarily concerned with the security and reliability of the market. Hybrid systems also exist in which pool based systems also allow bilateral transactions between agents.

Both the UC and EDP are tactical planning tools. The central planner in conjunction with the transmission grid operator must also, typically in parallel, solve an optimal power flow (OPF) model. OPF models usually resemble UC and EDP model closely with the addition of transmission, security and reliability constraints Momoh (2009). These constraints ensure the the resultant flows from the UC and EDP model are feasible for the physical transmission network. One common area of infeasibility is the overloading of the thermal capacity of a transmission line. Such an infeasibility, if allowed to transpire, would cause potential loss of power to customers as well as reduced reliability of the transmission network far beyond the local area surrounding the transmission line Meier (2006). In this article, we provide models for both pool based and bilateral transaction markets with emphasis on the latter.

The market monitor, or often referred to as market monitoring and mitigating group, is an independent entity charged with minimizing uncompetitive behavior in electricity markets and ensuring the market power among participants does not endanger other grid participants (Güler and Gross, 2005). Market monitors have a plethora of tools at their disposable but a majority of them are ex-ante tools. A more detailed discussion about the role that market monitors play in electricity market may be found in (Rahimi and Sheffrin, 2003; Güler and Gross, 2005).

We use game theoretic models to facilitate the computation of interactions between market participants resulting in an equilibrium. The models serve as a decision support tool rather than as a replacement, to the economic dispatch, unit commitment or other scheduling models. There exists some overlap in the phenomena both types of models try to represent; however game theoretic models are proxies for real world models so that market designs and other theoretical exercises may be performed with increased fidelity. Game theoretic models typically do not intend to model all portions of the system models. Rather they focus on mimicking certain aspects of the model to explore various interactions and outcomes that result from competition and interaction of generators, consumers, operators and regulators. Game theoretic models do not perfectly represent the actual models that central planners use to operate the grid. However, they do provide insight that allows the exploration of decision support and the potential for new market design and operational policies.

Our proposed model is primarily focused on the equilibrium of market participants. Equilibrium models are especially suited for decision support since the resulting model output is derived from the interaction of market participants rather than specifically assuming behavior of other participants. Ventosa et al. (2005) makes the distinction between equilibrium models and single firm optimization problems as well as simulation models. Single firm optimization models assume a market participant maximizes their own objection function, typically profit or social welfare, within a known competing market. Market quantities, such as prices and quantities, are usually derived from functions given the single firm's decision variables. Simulation models represent market phenomena that may be too complex to model in traditional optimization or equilibrium models. Simulation models are descriptive models and offer insight how phenomena of interest may behave. In contrast, prescriptive models such as optimization and equilibrium models, offer the modeler information on how to make a decision.

We also focus on modeling imperfectly competitive markets as they offer the biggest challenge for market participants and regulators in modeling decision and behavior. Our proposed market assumes oligopolistic competition where several firms have the ability to influence the price of electricity by their actions for a sustained period of time.

1.2 Preliminary Mathematics

1.2.1 Differential Nash Game

The concept of a differential Nash game is at the core of the type of model we set forth in the article. We present the concise and abstract differential Nash game published in Friesz (2010).

Suppose there are N agents, each of which chooses a feasible strategy vector u^i from the strategy set Ω_i which is independent of the other players' strategies. Furthermore, every agent $i \in \{1, N\}$ has a cost (disutility) functional $J_i(u) : \Omega \rightarrow \mathbb{R}^1$ that depends on all agents' strategies where

$$\Omega = \prod_{i=1}^{N} \Omega_i, \qquad u = (u^i \; : \; i = 1, ..., N) \tag{1.1}$$

Every agent $i \in \{1, N\}$ seeks to solve the problem

$$\min J_i(u^i, u^{-i}) = K_i \left[x^i(t_f), t_f \right] + \int_0^{t_f} \Theta_i(x^i, u^i, x^{-i}, u^{-i}, t) dt \tag{1.2}$$

subject to

$$\frac{dx^i}{dt} = f^i(x^i, u^i, t) \tag{1.3}$$

$$x^i(t_0) = x_0^i \tag{1.4}$$

$$\Psi[x^i(t_f), t_f] = 0 \tag{1.5}$$

$$u^i \in \Omega_i, \tag{1.6}$$

for each fixed yet arbitrary non-own control tuple

$$u^{-i} = (u_j \; : \; j \neq i) \tag{1.7}$$

where x_0^i is a vector of initial values of x^i, the state tuple of the i^{th} agent and

$$x^{-i} = (x_j \; : \; j \neq i) \tag{1.8}$$

is the corresponding non-own tuple.

1.2.2 Complementarity

The principle of complementarity is a powerful tool in solving differential and dynamics games in electricity markets and several other domains (Gabriel and Leuthold, 2010). Complementarity allows for the efficient computation of certain types of optimization problems that model game-theoretic models. A prerequisite in understanding complementarity is the concept of orthogonality. The symbol $\perp$ signifies orthogonality of two vectors. For example, consider the vectors A and B such that $A = (a_1, a_2, ...a_i)$ and $B = (b_1, b_2, ...b_i)$ with the same cardinality (i.e. $|A| = |B|$). The orthogonality of A and B , $0 \leq A \perp B \geq 0$ suggests:

$$a_i \cdot b_i = 0 \quad \forall i,$$
$$a_i \geq 0$$
$$b_i \geq 0$$

Orthogonality describes a relationship of vectors where their product is zero but both have to be nonnegative. This property is especially convenient because we often see this relationship among constraints in optimization problems. It is seen when relating constraints with their associated dual variables. This occurs since a dual variable can only be greater than zero when a constraint is non-binding. A complementarity problem is a specific type of optimization that uses the principle of orthogonality. The properties of the underlying vectors describe the type of complementarity problem is constitutes. In this article, we are concerned with mixed complementarity problems defined below

$$y \quad \text{solves} \quad MCP\big(h(x,\cdot), \mathbf{B}\big) \tag{1.9}$$

The variables x refer to the upper level variables while the variables y are associated with the lower level as described by the MCP above consisting of both the function $h(x,\cdot)$ and bounds $\mathbf{B}$. The constraints g can be a function of both types of variables. The variables y are a solution to the MCP. A point y with $a_l \leq y_l \leq b_l$ solves (1.9) if, for each l, at least one of the following holds

$$h_l(x,y) = 0 \tag{1.10}$$

$$h_l(x,y) \geq 0, \;\; y_l = a_l \tag{1.11}$$

$$h_l(x,y) \leq 0, \;\; y_l = b_l \tag{1.12}$$

1.3 Contributions

A key contribution of this article is the proposed market design where the market monitor serves as an intelligent agent representing both the operation of the power grid and societal contributor. The traditional role of the market monitor is that of penalizing generator's behavior that it has deemed *uncompetitive*. This has been replaced with a proposal of anticipating both generators' and consumers' behaviors and reactions to intelligently use equilibrium congestion pricing to increase social welfare. The dynamic Stackelberg model considers a new market design mechanism that also includes the following numerous realistic and computable features: oligopolistic competition, inter-temporal constraints, dynamic production constraints, time-varying demand, transmission constrained network and multi generator assets.

1.4 Organization

We organize the article as follows: Section 2 provides a brief literature review of welfare economics and industrial organization, dynamic and differential games, electricity markets and different type of dynamics relevant to electricity markets. Section 3 sets forth a new paradigm in modeling how a proposed market monitor or central planners utilizes equilibrium congestion pricing to increase social welfare in oligopolistic competition as a dynamic Stackelberg game. Section 4 provides numerical examples to demonstrate the computational efficiency of our proposed model and formulation. We end the article with some conclusions presented in Section 5.

2 Literature Review

Our work is in the domain of computable game theory and equilibria with the specific application of electricity markets. Game theory allows us to model decision making under competition. The computability of our state-space game allows us to apply our framework to large problems that would not have been able to be accomplished with normal form games. Specifically, our framework lies within the area of differential games where the state of the game evolves with time according to a differential equation. Our application area of electricity markets poses unique challenges, primarily stemming from a physical transmission network connecting agents and properties of electricity generation, consumption and transmission. The economic conditions of electricity markets further serve as a connection between the electric grid and the computable framework we put forth. The subsequent sections divulge further details of the literature that exists in each of the above areas and fields of study.

2.1 Differential and Dynamic Games

Game theory can be originally attributed to the works of VonNeumann and Morgenstern (1944). However, the computable game theory framework that we build upon in this research is largely a result of Isaacs (1999). We have chosen to accept the widely-held definition that a dynamic game requires that the game evolves over time (Dockner et al., 2000). We utilize the normal form of games, or referred to as state-space games, where variables representing states describe the behavior of the game at any point in time (Mehlmann, 1988). Our differential game is a subset of dynamic games where we utilize ordinary differential equations to describe the evolution over time of the state of the game. We limit our analysis to non-cooperative differential games where decision makers, or agents, do not enter in agreements with other agents. Our work also employs deterministic functions where stochasticity is not used but simple scenario-based perturbation can be employed to mimic levels of degrees of uncertainty.

2.2 Electricity Markets

Electricity market models typically describe the generation, sales and flows of electricity on a transmission network. The literature varies with the information structure of the games, agents and markets as well as time horizon, degree of uncertainty and the objective or goal of the analysis. Ventosa et al. (2005) classified electricity market models into three distinct categories. First, the optimization problem for one firm, in which the firm assumes either an exogenous price or has a known demand function. Second, simulation models are descriptive models that attempt to describe market interaction typically via discrete-event simulation or agent based simulation. Lastly, market equilibrium firms consider all firms and are prescriptive in nature, and typically utilizing a Cournot or supply function equilibrium viewpoint. Cournot competition is a structure in which firms compete by deciding their quantities of production. Supply function equilibrium models require generation firms to submit offer curves to the system operator. In this article, we focus on the market equilibrium considering all firms, assuming Cournot competition. A more detailed survey of Cournot and supply function equilibrium models can be found in Hobbs (2001) and Day et al. (2002), respectively. Wu et al. (1996) provides a readable summary and interpretation of folk theorems that have been developed in the electricity market literature with respect to transmission access. Specifically, the authors provide details of "Nodal prices, congestion revenues, transmission capacity rights and compensation of transmission access." Key formulations of the economic dispatch and optimal power flow models are provided for spot and bilateral transaction markets.

A common goal of system planners is to mitigate market power, typically done by incentivizing or penalizing firms that earn excess profit above a certain threshold, or are excessively depended upon for the successful operation of the grid. This concern is valid when the market exudes imperfect competition. Our proposed model assumes imperfect competition. Specifically, we focus on oligopolies where a few firms dominate the market and can directly influence the price of electricity. Nanduri and Das (2009) and Blumsack et al. (2002) provides a brief survey of market mitigation and imperfect competition literature.

Rivier et al. (2001) presents a hydrothermal coordination assuming Cournot equilibrium and utilizes a Mixed Complementarity Problem (MCP) framework. Wei and Smeers (1999) used Variational Inequalities (VI) to analyze congestion in a spatial network assuming Cournot equilibrium. Mookherjee et al. (2010) utilized a MCP to analyze a Cournot-Nash equilibrium between generation firms competing as oligopolies, simultaneously with a transmission clearing system operator.

Our proposed model goes one step beyond imperfect competition and utilizes a Stackelberg framework. A Stackelberg game is a bilevel game, often represented as a Mathematical Program with Equilibrium Constraints (MPEC), where the leader has the advantage of deciding their variables before the followers decide their own variables; Ventosa et al. (2002); Murphy and Smeers (2005), Gabriel and Leuthold (2010) and Hobbs (2001) all provide unique contributions to the MPEC problem. A bilevel game in which there are multiple leaders is defined as an Equilibrium Problem with Equilibrium Constraints (EPEC). For a more comprehensive review of literature of equilibrium in electricity markets see Ventosa et al. (2005); Nanduri and Das (2009); Daxhalet and Smeers (2001); Yao et al. (2008) and Hobbs and Helman (2004).

Hobbs et al. (2000) set forth a framework to model imperfect competition (electricity prices rise above marginal cost) of electricity markets via an MPEC approach. The model and procedure calculates oligopolistic price equilibria assuming "supply function equilibrium" in which generators decide on their bid curves (acceptable price vs quantity offer curved are generated for each firm) on the belief that rival generators will not change their own bid curves. Specifically, the model is relevant when the economy consists of several dominant firms. The dominant firm acts as the leader (upper level) of the MPEC. They choose their bid curves first and in anticipation of rival generators' bids and the Independent System Operator's (ISO) actions. The ISO, modeled as the follower, solves a single commodity spatial price equilibrium problem. The ISO decides quantity of power as to maximize social welfare given the upper level bid curves. Quantity is an input to bid curves and price equilibrium emerges. Thus, the dominant firms can strategically set their bid curves to maximize profit while anticipating the actions of the ISO and non dominant firms.

2.3 Differential and Dynamic Games

Game theory can be originally attributed to the works of VonNeumann and Morgenstern (1944). However, the computable game theory framework that we build upon in this research is largely a result of Isaacs (1999). Nash (1951) brought forth the foundational concept of a Nash equilibrium strategy where N players cannot increase their utility by deviating from their own strategy. Stackelberg (1952) introduced a different behavioral assumption where a leader can anticipate other player's equilibrium strategies in a bilevel game. We have chosen to accept the widely-held definition that a dynamic game requires that the game evolves over time (Dockner et al., 2000). We utilize the normal form of games, or referred to as state-space games, where variables representing states describe the behavior of the game at any point in time. Our differential game is a subset of dynamic games where we utilize ordinary differential equations to describe the evolution over time of the state of the game. We limit our analysis to non-cooperative differential games where decision makers or agents do not enter in agreements with other agents. Our work also employs deterministic functions where stochasticity is not used but simple scenario-based perturbation can be used to mimic levels degrees of uncertainty.

3 Electric Power Network Oligopoly as a Dynamic Stackelberg Game

In this section, we set fourth a hypothetical new market that allows market monitors to ensure uncompetitive behavior is minimized. The proposed framework may serve as a starting point for a new class of models that aids market monitors to anticipate and prevent uncompetitive behavior ex-ante any electricity is transmitted. This is in contrast to most of the tools available to market monitors that look ex-post to the alleged uncompetitive behavior. Specifically, we present a model in the which the market monitor employs equilibrium congestion pricing of a bilateral transaction market in which generators and consumers are in equilibrium.

We consider an electric power grid operating with consumers, a market monitor and generators forming an oligopoly. Specifically, each generator possesses the power of influencing the price of electricity with their own production and sales plan. Our consumers are generalized to include retail consumers, utilities and Load Serving Entities (LSE). We assume the market monitor utilizes equilibrium congestion pricing for the purposes of clearing the transmission market to prevent generators from exceeding the physical limitations of the network. Furthermore, we assume the market monitor has the power to use equilibrium congestion pricing to represent society's interests as a whole; with the objective of maximizing economic surplus or commonly referred to as social welfare. Each generator maximizes individual profit in a Cournot-Nash game with other generators given the access charges the market monitor has set forth. Our unique Stackelberg approach models the market monitor as a single leader, with the generators acting as followers. The leader has complete anticipatory knowledge of the generators' equilibrium problem and decides access charges such that the generators produce a production and sales schedule that is optimal from a societal perspective. Furthermore, we model the interaction of the game's agents with the use of dynamics. The dynamic approach allows us to represent a higher fidelity model, which advances the level of market design tools available for the system operators to analyze the competitive implications of oligopolies in electric power markets.

We start with presenting a general dynamic Stackelberg game of a electric power oligopoly. The market monitor is represented as a leader maximizing social welfare. The lower level consists of a Cournot-Nash equilibrium among generation firms. The section begins with preliminary notation and assumptions while Section 3.2 presents a continuous time formulation of dynamic Stackelberg game. Section 3.3 contains our discrete formulations of the Stackelberg game as well as the Mathematical Program with Equilibrium Constraints (MPCC) into which we reformulate our original game.

3.1 Notation and Assumptions

The price of electricity ($\$/MWH$) of each node where electricity is consumed is a known function of sales and continuous time since we assume an oligopoly market structure. $\pi(t)$ satisfies

$$\pi(t) \in L^2[t_0, t_f]$$

where $L^2[t_0, t_f]$ is the space of square-integrable functions. Moreover, we stipulate that the price is a square-integrable function of time. We further assume that every firm is an oligopoly and that no firms are price-takers. An oligopoly is an economic market structure in which firms can influence the market price through their own sales and generations while price-takers do not have any influence on price and sell at a price determined by the market. Our model is general enough to include price-takers but are omitted at this point to convey a homogenous market structure. Furthermore, we assume our decision structure is deterministic and open loop. "Open loop" signifies that firms simultaneously determine the decision variable for all time periods within the planning horizon. We further assume perfect initial information and a finite time interval $[t_0, t_f] \subseteq \Re^1_+$, where $t_0 \in \Re^1_+$ is the fixed initial time, $t_f \in \Re^f_{++}$ is the fixed terminal time and $t_f > t_0$. The ramping rate $r(t)$ describes each generators' instantaneous rate of change of of output $q(t)$ with respect to time. Each generator's output rate is $q(t)$ with associated generation cost $V(q(t))$ with producing $q(t)$ units of electricity (MW). The rate of power sales is denoted as $c(t)$ while $w(t)$ represents the access charges that the market monitor charges to transmit power from the hub node to the node of interest. The controls

$$r(\cdot) \in L^2[t_0, t_f]$$
$$c(\cdot) \in L^2[t_0, t_f]$$
$$w(\cdot) \in L^2[t_0, t_f]$$

determine the generators' dynamics $q(t)$

$$\frac{dq_i^f(t)}{dt} = r_i^f(t) \tag{3.13}$$

We impose the following upper bounds on generation and ramping respectively

$$q_{max} \in \Re^f_{++}$$
$$r_{max} \in \Re^f_{++}$$
$$-r_{min} \in \Re^f_{++}$$

Our model focuses on spatial equilibrium since any electric grid of interest spans a network. We denote each node of the network using the index i that belongs to the set N consisting of every node of the network. We also define the set of nodes M in which there are markets for power since it is possible to sell and generate power at different nodes simultaneously. Furthermore, we assume a linearized DC power flow as an approximation to the real world AC flow electric grid as published in Schweppe et al. (1988). This common model approximation allows us to easily represent Kirchoff's laws with a parameter $PTDF_{i,a}$ representing the proportion of power that flows on each transmission line of the network when power is transmitted to node i. Furthermore, firms behave non-cooperatively (i.e. no collusion). A summary of sets, variables and parameters is shown in Table 1 in continuous time.

The following vector concatenations are used, when applicable, to simplify the notation.

$$\begin{array}{l} q^f: \; q_i^f(t) \text{ for all } i \in \mathcal{M}, \; t \in [t_0, t_f] \\ c^f: \; c_i^f(t) \text{ for all } i \in \mathcal{M}, \; t \in [t_0, t_f] \\ r^f: \; r_i^f(t) \text{ for all } i \in \mathcal{M}, \; t \in [t_0, t_f] \\ w: \; w_i(t) \text{ for all } i \in \mathcal{M}, \; t \in [t_0, t_f] \end{array}$$

We distinguish $\pi_i(c, t)$ and $V_i^f(q, t)$ as explicit functions that have both the arguments $c_i^f(t)$ and $q_i^f(t)$ respectively and the time t as a parameter. The variables and parameters presented above will be used in the subsequent section in discrete time by substituting t as a subscript for a continuous function.

Table 1: Notations

	Sets
A:	set of transmission lines (arcs) in the network
F:	set of generating firms
M:	set of nodes at which there are markets for power
N:	set of nodes in the network
T:	Set of time periods in planning horizon
	Variables
$q_i^f(t)$:	generation in MW by firm $f \in F$ at node $i \in N$
$c_i^f(t)$:	sales (consumption) in MW by firm $f \in F$ at market $i \in M$
$w_i(t)$:	access Charge ($/MW) for market $i \in M$
$r_i^f(t)$:	ramping rate for firm $f \in F$ at node $i \in N$
$\pi_i(c,t)$:	inverse demand function($/MW) at market $i \in M$
$V_i^f(q,t)$:	generation cost function for firm $f \in F$ at node $i \in N$
	Parameters
T_a:	transmission capacity of arc a
$q_{i,max}^f$:	upper bound of generation $f \in F$ at node $i \in N$
$\text{PTDF}_{i,a}$:	describes how much MW occurs through transmission line ("arc") a as a result of a unit MW injection at the hub node and a withdrawal at node i.
$r_{i,min}^f$:	minimum ramping for firm $f \in F$ at node $i \in N$
$r_{i,max}^f$:	maximum ramping for firm $f \in F$ at node $i \in N$

3.2 Dynamic Stackelberg Game

We assume the market monitor uses its influence to maximize social welfare or economic surplus. In this article, we assume that the only means of reaching their objective is by way of enforcing access charges paid by the generators per MW of electricity transmitted on the network.

We define access charges as tariffs per units of power transmitted from the hub node to the node of interest. The hub node is single location within the network that all electricity sales are assumed to pass through in order calculate the cost of transmission. For example, if 1 MW is sold from a generator at location A to a consumer at location B, for transmission pricing purposes only, the transaction is divided into two transactions: a 1 MW transfer from location A to the hub node, and a 1 MW transfer from the hub node to location B. The hub node is arbitrarily chosen and the charge can be simply thought as the price to transmit one MW of electricity to the desired node of sales or consumption. We define the access charge to transmit to a node may be either positive or negative.

Every feasible set of unique access charges influences each generator's profit and thus, the Cournot-Nash equilibrium of the generators as a whole. The generators take the access charges as exogenous variables and play a Cournot-Nash game with other generators to maximize their individual profit. The consumers are represented by an inverse demand function at each node, making the game a complete market. Standard congestion pricing formulations set the price of transmitting electricity equal to the difference of marginal pricing between nodes. Locational marginal pricing (LMP) is one of such pricing schemes (Wood and Wollenberg, 1996). Thus, the price of electricity is equal at every node when combined with the transmission cost for each node. We propose a new congestion pricing scheme where the access charge is the market monitor's decision variable. Our formulation gives the responsibility of the market monitor to set the access charges so that: 1) Generators do not transmit electricity beyond the capacity of each transmission line; and 2) The electricity flows between generators and consumers maximize net economic surplus all assuming our proposed hypothetical market.

3.2.1 Market Monitor's Upper Level Problem

Social welfare, or interchangeably economic surplus, represents the benefits that all agents in a market receive from the economic participation of purchasing and selling goods. Gabriel et al. (2013) states that "It is the standard measure of market efficiency." It is the sum of consumer, producer and market monitor surplus. Each surplus is defined as the monetary gain experienced from purchasing/selling a good for less/more than what they are willing to pay/sell for the good. The consumer surplus can be thought of the gain each consumer receives from willing to pay for a quantity of electricity less what they actually paid for it. It can also be thought as the psychological or perceived benefit derived from consuming a good beyond the opportunity cost to purchase it. Producer surplus is completely synonymous with its profit in producing and selling electricity. Note that in addition to the generation cost of producing electricity, each generator must also pay an access charge to transmit the electricity it produces. The market monitor surplus must also be considered since it is an active player in the proposed market. The market monitor surplus is derived from the network assess charge revenue it collects from the generators. Thus the gain of the market monitor surplus is 100% at the loss of the generator surplus. This access charge is specifically included in the calculation of producer's surplus as seen later within this section. We directly stipulate for a given time period that the total market monitor surplus must be non negative. A negative market monitor surplus would indicate that market monitor would subsidize the users of the network and thus require outside funding. In practice, it may be desired for the market monitor to act as a non-profit entity serving the needs of the proposed market as a whole and thus the revenue would be returned to the users of the network via a financial mechanism.

We present a general form of social welfare that can be evaluated at equilibrium values of power consumption, generation and access charges. The market monitor's objective function is to maximize social welfare $SW(c^*, q^*)$ for every node where a total of c^* units of power were sold, q^* units of power were generated and w^* dollars of access charges summed for every firm, node and time period in the planning horizon. We, for the time being, drop the subscripts for nodes, firms and time for the sake of clear exposition. $SW(c^*, q^*)$ can be generally defined as the summation of all surpluses associated with each agent in the market economy. Our problem of interest consists of consumers, generators (also referred to as producers) and the market monitor. We define consumer surplus $CS(c^*)$ as

$$CS(c^*) = \int_0^{c^*} \pi(x)dx - [\pi(c^*) \cdot c^*] \tag{3.14}$$

where c^* is the equilibrium value of sales, π is the inverse demand function and $\pi(c^*)$ is the equilibrium price that the consumers and producers pay and receive respectively. The first term denotes integrating every consumer's benefit derived from consuming electricity from the the first unit of electricity up to c^* units. Note that consumers do not receive any benefits beyond c^* units simply because we define c^* as the equilibrium units that are sold. The second term refers to the cost, price multiplied by quantity of sales, that the consumers paid to the generators for their consumption.

Producer surplus $PS(c^*, q^*, w^*)$ is defined as the profit of the electricity generation industry given by revenue less costs. We define producer surplus as

$$PS(c^*, q^*, w^*) = \pi(c^*) \cdot c^* - V(q^*) - w^* \cdot q^* \tag{3.15}$$

where $V(q^*)$ is the total cost to generate $q*$. In our current case of dropped node subscripts, $c^* \doteq q^*$ since we have not explicitly defined social welfare for the network. The first term of (3.16) represents the revenue received. The second and third terms denote the generation cost and network access charges respectively associated with q^* and w^*.

The market monitor surplus $MMS(c^*, q^*, w^*)$ is the revenue the market monitor receives from placing access charges on the network. We define market monitor surplus as

$$MMS(c^*, q^*, w^*) = w^* \cdot q^* \tag{3.16}$$

We can now state that social welfare is a summation of all surplus as seen in equations (3.17)-(3.19).

$$SW(c^*, q^*, w^*) = CS(c^*) + PS(c^*, q^*, w^*) + MMS(c^*, q^*, w^*) \tag{3.17}$$

$$= \left\{ \int_0^{c^*} \pi(x)dx - [\pi(c^*) \cdot c^*] + \pi(c^*) \cdot c^* - V(q^*) - w^* \cdot q^* + w^* \cdot q^* \right\} \tag{3.18}$$

$$SW(c^*, q^*) = \int_0^{c^*} \pi(x)dx - V(q^*) \tag{3.19}$$

Simplification of equation (3.18) results in the dropping of the revenue terms of electricity and access charges as they are net neutral in the calculation of social welfare. Equation (3.19) shows that social welfare is not directly influenced by the access charges. Instead, the access charges w^* influence c^* and q^* which turn are used to calculate social welfare.

We now present a more formal notation and elaboration of social welfare maximization as it relates to our problem, with the inclusion of the subscripts for nodes, firms and time. The market monitor determines access charges $w_{i,t}$ such that social welfare is maximized as seen in equation (3.20). The access charges represent the cost to transmit power from the hub node to node i for time period t. These fees are set to clear the transmission market to ensure that generators do not send power on a transmission line beyond its physical limitations. We sum social welfare across all time periods within the planning horizon, as well as nodes where power is consumed or generated. The summation of V across f represents the total cost of generation for all firms f. With the planning horizon assumed to be approximately one day, net present value (NPV) is not accounted for. Our modeling approach is general enough to include longer time horizons and NPV calculations.

$$\max_{w_i(\cdot)} Z(c^f, q^f) = \int_{t_0}^{t_f} \sum_{i \in N} \left\{ \int_0^{\sum_{g \in F} c_i^g(t)} \pi_i\,(x,t)\,dx - \sum_{f \in F} V_i^f(q_i^f(t)) \right\} dt \tag{3.20}$$

Note that the market monitor's objective function does not specifically contain the decision variable $w_i(t)$. The market monitor uses $w_i(t)$ to influence the equilibrium quantity of c and q determined in the lower level. Specific details of the lower level are presented in section 3.2.2. The clearing of the transmission markets is modeled in Equations (3.21) and (3.22). The quantity $\sum_{f \in F}(c_i^f - q_i^f)$ is the net power flow from the hub node to node i. The parameter $PTDF_{i,a}$ is multiplied with the net power flow to determine what proportion of the power flows on arc a. The summation of all power flows across nodes i results in the total net power flow on arc a. T_a simply bounds the operating capacity of the transmission line. Electricity is modeled as either a positive and negative quantity representing the direction of travel along a transmission line. Therefore, we must account for both directions not exceeding transmission capacity as indicated by Equations (3.21) and (3.22). These two equations represent the upper level constraints of our Stackelberg game.

$$\sum_{i \in N} PTDF_{i,a} \cdot \Big[\sum_{f \in F} (c_i^f(t) - q_i^f(t)) \Big] \leq T_a \quad \forall a \in A \tag{3.21}$$

$$\sum_{i \in N} PTDF_{i,a} \cdot \Big[\sum_{f \in F} (c_i^f(t) - q_i^f(t)) \Big] \geq -T_a \quad \forall a \in A \tag{3.22}$$

The market monitor must also be concerned that the surplus be strictly positive. A negative surplus would indicate a subsidy provided by the market monitor to generators and consumers. Equation (3.23) states the constraint on the market monitor surplus in the upper level problem.

$$\sum_{i \in N} \sum_{F \in F} w_i(t)[c_i^f(t) - q_i^f(t)] \geq 0 \tag{3.23}$$

3.2.2 Generator's Lower Level Problem

Each generating firm acts as a follower to the market monitor leader and plays a Cournot-Nash game with all other firms given access charges $w_{i,t}$ set by the market monitor leader in the upper level. "Cournot" refers to the fact that each firm competes with other firms by determining their "quantity." A "Nash" game indicates that the solution defines an equilibrium such that no firm has an incentive to deviate from their strategy.

Each firm maximizes their individual profit function J_f consisting of production costs and access charges subtracted from revenue to transmit their net balance of sales and production at each node. The firms determine c and production q while in equilibrium with other generation firms. Other firms' sales are denoted by c^{-f} where $c^{-f} : c^g \; \forall g \neq f$. The resulting game is represented in Equations (3.24) through (3.28).

$$\max J_f(c^f, q^f; c^{-f}, w) = \int_{t_0}^{t_f} \Bigg\{ \sum_{i \in M} \pi_i \left(\sum_{g \in F} c_i^g(t) \right) \cdot c_i^f(t) - V_i^f(q_i^f(t)) - w_i(t)[c_i^f(t) - q_i^f(t)] \Bigg\} dt \tag{3.24}$$

subject to

$$\sum_{i \in N} q_i^f(t) = \sum_{i \in M} c_i^f(t) \quad \forall i \in M \tag{3.25}$$

$$q_i^f(t) \leq q_{i,max}^f \quad \forall i \in M \tag{3.26}$$

$$\frac{dq_i^f(t)}{dt} = r_i^f(t) \quad \forall i \in M \tag{3.27}$$

$$r_{i,min}^f \leq r_i^f(t) \leq r_{i,max}^f \quad \forall i \in M \tag{3.28}$$

The oligopoly market structure that we assume has a direct consequence on the equilibrium of the game since every decision to sell power affects the market price of power and thus, all other firm's profit functions. This is contrast to perfectively competitive markets in which firms have no influence on price and thus would be considered price takers. A game theoretic approach is utilized, utilizing of the oligopoly market structure. Each agent in the economy has a direct or indirect influence on the decisions and outcomes made and experienced by other agents. Equation (3.25) ensures that all the power that a firm generates in the boundary of the network is sold due to the assumption that electricity cannot be economically stored in a meaningful capacity. Equation (3.26) bounds each firm's production at each node. The differential equation described in Equation (3.27) defines the ramping rate $r_i^f(t)$ each generator i experiences at each of their facilities as a function of time t. Equation (3.29) imposes a lower and upper bound on $r_i^f(t)$. These constrained dynamics add a level of modeling sophistication that ensures our Stackelberg game follows real world limitations that generators face. Our model is general enough to allow different firms and facility locations to have different ramping bounds. These ramping limitations correspond to electricity generation technology that prevents sudden deviations in the generation plan. In practice, generators use several forecasts at multiple time scales to make both investment and operating decisions concerning generation equipment and deployment of resources. However, as the planning horizon shortens to the period of about one day, physical limitations are imposed on each generator on how quickly they can deviate from the operational plan. These limits on the agility of electricity generation in the short term are referred to as ramping rate bounds.

3.2.3 Complete MPEC Formulation

We present the concatenated bi-level game consisting of both the upper and lower level in equations (3.29) through (3.37).

$$\max_{w_i(\cdot)} Z(c^f, q^f) = \int_{t_0}^{t_f} \sum_{i \in N} \left\{ \int_0^{\sum_{g \in F} c_i^g(t)} \pi_i(x,t)\, dx - \sum_{f \in F} V_i^f(q_i^f(t)) \right\} dt \tag{3.29}$$

subject to

$$\sum_{i \in N} PTDF_{i,a} \cdot \Big[\sum_{f \in F} (c_i^f(t) - q_i^f(t)) \Big] \leq T_a \quad \forall a \in A \tag{3.30}$$

$$\sum_{i \in N} PTDF_{i,a} \cdot \Big[\sum_{f \in F} (c_i^f(t) - q_i^f(t)) \Big] \geq -T_a \quad \forall a \in A \tag{3.31}$$

$$\sum_{i \in N} \sum_{F \in F} w_i(t)[c_i^f(t) - q_i^f(t)] \geq 0 \tag{3.32}$$

where $w_i^*(\cdot)$ is the minimizer of the objective function (3.29) and solves the following Cournot-Nash game:

$$\max J_f(c^f, q^f; c^{-f}, w) = \int_{t_0}^{t_f} \Bigg\{ \sum_{i \in M} \pi_i \left(\sum_{g \in F} c_i^g(t) \right) \cdot c_i^f(t) - V_i^f(q_i^f(t)) - w_i(t)[c_i^f(t) - q_i^f(t)] \Bigg\} dt \tag{3.33}$$

subject to

$$\sum_{i \in N} q_i^f(t) = \sum_{i \in M} c_i^f(t) \quad \forall i \in M \tag{3.34}$$

$$q_i^f(t) \leq q_{i,max}^f \quad \forall i \in M \tag{3.35}$$

$$\frac{dq_i^f(t)}{dt} = r_i^f(t) \quad \forall i \in M \tag{3.36}$$

$$r_{i,min}^f \leq r_i^f(t) \leq r_{i,max}^f \quad \forall i \in M \tag{3.37}$$

The resulting game has the unique feature of having hierarchy relating the market monitor's optimization problem with $|F|$ number of optimal control problems with state constraints. This unique game theoretic approach can be classified as an optimization problem constrained by other optimization problems (OPcOP). Furthermore, the optimization problems consisting of the lower level can also be classified as a set of equilibrium constraints when viewed together with the upper level problem. This classification leads us to the structure of a mathematical program with equilibrium constraints (MPEC). The hierarchal structure is also in fact a Stackelberg game, as seen in microeconomic analysis, since we assume leader-follower relationship. Specifically, the leader's game is modeled in the upper level and has the luxury of deciding its variables before the followers modeled in the lower level.

3.3 Discrete Time Formulations

We first present a general discrete time formulation stated in Section 3.2. We then reformulate the game as a Mathematical Program with Complementarity Constraints (MPCC) in Section 3.3.2.

3.3.1 Discrete MPEC Formulation

In this subsection, we reformulate our continuos-time MPEC into a discrete-time MPEC. A simple subscript t has been substituted for continuous time with T equal to the total number of time periods in the planning horizon. We also rewrite the lower level constraints of the form "less than or equal to zero" in order to be conducive to further mathematical manipulations in later sections. The dual variables of each lower level constraint are presented in parentheses to the right of each constraint. These dual variables represents the marginal increase of the objective function per additional unit of the constraint. This concept is particularity powerful in conjunction with complementarity as it allows for efficient reformulation and subsequent computation of MPECs.

$$\max_{w_{i,t}} Z(c^f, q^f) = \sum_{t \in T} \sum_{i \in M} \left\{ - \int_0^{\sum_g c_{i,t}^g} \left\{ \pi_{i,t}(x) \right\} dx - \sum_{f \in F} V_{i,t}^f(q_{i,t}^f) \right\} \tag{3.38}$$

subject to the upper level constraints

$$\sum_{i \in N} PTDF_{i,a} \cdot \left[\sum_f (c_{i,t}^f - q_{i,t}^f) \right] \leq T_a \quad \forall a \in A, \forall t \in T \tag{3.39}$$

$$\sum_{i \in N} PTDF_{i,a} \cdot \left[\sum_f (c_{i,t}^f - q_{i,t}^f) \right] \geq -T_a \quad \forall a \in A, \forall t \in T \tag{3.40}$$

$$\sum_{i \in N} \sum_{F \in F} w_{i,t} [c_{i,t}^f - q_{i,t}^f] \geq 0 \quad \forall t \in T \tag{3.41}$$

where $w^*_{i,t}$ is the minimizer of the objective function (3.29) and solves the following Cournot-Nash game:

$$\min J_f(c^f, q^f; c^{-f}, w) = -\sum_t \sum_i \ \pi_{i,t} \cdot c^f_{i,t} - V^f_{i,t} - w_{i,t}[c^f_{i,t} - q^f_{i,t}] \tag{3.42}$$

subject to the lower level equilibrium constraints

$$\sum_{i \in N} q^f_{i,t} - \sum_{i \in M} c^f_{i,t} \le 0 \quad \forall t \in T \quad (\zeta^{+f}_t) \tag{3.43}$$

$$-\sum_{i \in N} q^f_{i,t} + \sum_{i \in M} c^f_{i,t} \le 0 \quad \forall t \in T \quad (\zeta^{-f}_t) \tag{3.44}$$

$$-c^f_{i,t} \le 0 \quad \forall i \in M, \forall t \in T \quad (\phi^f_{i,t}) \tag{3.45}$$

$$-q^f_{i,t} \le 0 \quad \forall i \in M, \forall t \in T \quad (\rho^f_{i,t}) \tag{3.46}$$

$$q^f_{i,t} - q^f_{i,max} \le 0 \quad \forall i \in M, \forall t \in T \quad (\sigma^f_{i,t}) \tag{3.47}$$

$$\frac{q^f_{i,t} - q^f_{i,t-\Delta t}}{\Delta t} - r^f_{i,max} \le 0 \quad \forall i \in M, \forall t = 1, ..., T \quad (\mu^f_{i,t}) \tag{3.48}$$

$$\frac{-q^f_{i,t} + q^f_{i,t-\Delta t}}{\Delta t} + r^f_{i,min} \le 0 \quad \forall i \in M, \forall t = 1, ..., T \quad (\theta^f_{i,t}) \tag{3.49}$$

where Δt is a user-chosen time step parameter used to approximate the derivative $\frac{dq^f_i(t)}{dt}$. Equation (3.28) and (3.29) can be transformed into the set of discrete time constraint as seen in equations (3.48) and (3.49). The term $\frac{dq^f_i(t)}{dt}$ can be expressed as the quantity $\frac{q_t - q_{t-\Delta t}}{\Delta t}$.

3.3.2 Complementarity Conditions for Generating Firms

We can now transform the discrete math program described in equations (3.42) through (3.49) as a complementarity problem by formulating the necessary conditions for the generating firms's game. Our lower level program has the convenient property of containing only linear constraints and thus Abadie's constraint qualification holds. This property allows us to compose the Karush-Kuhn-Tucker(KKT) conditions. These necessary conditions are combined to form a Mixed Complementarity Problem (MCP) or more precisely, a Nonlinear Complementarity Problem (NCP). The KKT identities, with respect to c and q, are found to be respectively

$$0 = -\pi_{i,t} \sum_{g \in F} c^g_{i,t} - c^f_{i,t} \cdot \pi'_{i,t} \sum_{g \in F} c^g_{i,t} + w_{i,t} - \zeta^{+f}_t + \zeta^{-f}_t - \phi^f_{i,t} \tag{3.50}$$

$$0 = -w_{i,t} + \zeta^{+f}_t - \zeta^{-f}_t + \sigma^f_{i,t} - \mu^f_{i,t} + \theta^f_{i,t} - \rho^f_{i,t} \tag{3.51}$$

where $\pi'_{i,t}\left(\sum_{g \in F} c^g_{i,t}\right)$ denotes the derivative of π with respect to $c^f_{i,t}$. The following complementarily slackness conditions accompany the KKT identities

$$0 \leq \left[-\sum_{i \in N} q_{i,t}^f + \sum_{i \in M} c_{i,t}^f \right] \perp \zeta_t^{+f} \geq 0 \tag{3.52}$$

$$0 \leq \left[\sum_{i \in N} q_{i,t}^f - \sum_{i \in M} c_{i,t}^f \right] \perp \zeta_t^{-f} \geq 0 \tag{3.53}$$

$$0 \leq c_{i,t}^f \perp \phi_{i,t}^f \geq 0 \tag{3.54}$$

$$0 \leq q_{i,t}^f \perp \rho_{i,t}^f \geq 0 \tag{3.55}$$

$$0 \leq -q_{i,t}^f + q_{i,max}^f \perp \sigma_{i,t}^f \geq 0 \tag{3.56}$$

$$0 \leq \frac{q_{i,t}^t - q_{i,t-\Delta t}^f}{\Delta t} - r_{i,min}^f \perp \mu_{i,t}^f \geq 0 \tag{3.57}$$

$$0 \leq \frac{-q_{i,t}^t + q_{i,t-\Delta t}^f}{\Delta t} + r_{i,max}^f \perp \theta_{i,t}^f \geq 0 \tag{3.58}$$

4 Numerical Examples

4.1 Data Sources and Methodology

A key feature of any proposed model is how computable it is with near real-world data sets. In this section, we present an illustrative example as well as the stylized 15-node network of the Western European electric grid. All examples are shown to be computable with the commercially available NLPEC solver within GAMS. The illustrative example solved in less than one second while the Western European network solved in eight minutes. We chose $\Delta t = 1$ that resulted in 24 discrete time periods. All computations were performed on the Network-Enabled Optimization System (NEOS) server Czyzyk et al. (1998). NEOS is a free high performance computing platform that allows researchers to use a variety of software packages and optimization solvers.

4.2 Specific Formulation

We assume π follows a well studied linear inverse demand where

$$\pi_{i,t} = a_{i,t} - b_{i,t} \cdot \sum_f c_{i,t}^f \tag{4.59}$$

The parameter $a_{i,t}$ represents that amount of power (MW) that consumers at node i would use if prices were set to zero, and $b_{i,t}$ signifies the slope of the inverse demand curve. We have previously defined $V(q)$ as an explicit function. A realistic generation cost function typically exerts a quadratic or nonlinear property as a result of decreasing returns to scale (Varian, 2006). The model is general enough to handle higher order functions that would model to scale both increasing and decreasing returns. We approximate a quadratic cost curve assuming a piecewise linear function as described in (4.60).

$$V_{i,t}^f = \max \left\{ m_{1,i}^f q_{i,t}^f + b_{1,i}^f \ , \ m_{2,i}^f q_{i,t}^f + b_{2,i}^f \right\} \tag{4.60}$$

We can further express $V_{i,t}^{f}$ as a variable subject to the following inequalities

$$V_{i,t}^{f} \geq m_{1,i}^{f} q_{i,t}^{f} + b_{1,i}^{f} \tag{4.61}$$

$$V_{i,t}^{f} \geq m_{2,i}^{f} q_{i,t}^{f} + b_{2,i}^{f} \tag{4.62}$$

This clever reformulation, including V as a variable and additional inequalities, all us to remove any nonlinear effects that a quadratic V function might have imposed. The two constraints represented by (4.61) and (4.62) form a feasible region bounded below. The term $V_{i,t}^{f}$ is always minimized by each generator and therefore is guaranteed to be on the lower boundary of the feasible region set. This property allows $V_{i,t}^{f}$ to be equal the piecewise linear segment, approximating a quadratic cost function. In our example, we approximated the quadratic cost function with two affine segments. However, the model is general enough to handle greater number of affine segments. A greater number of segments increases the quality of the approximation.

We substitute V and π into the upper level market monitor problems in Equations (3.38) through (3.40) along with the constraints (4.61) and (4.62). Equations (4.68) and (4.70) have been modified accordingly to handle the addition of V as a variable.

$$\max_{w_{i,t}} Z(c^f, q^f) = \sum_{t \in T} \sum_{i \in M} \left\{ -a_{i,t} \sum_{f} c_{i,t}^{f} + \frac{b_{i,t}}{2} \Big[\sum_{f} c_{i,t}^{f} \Big]^2 + \sum_{f} V_{i,t}^{f} \right\} \tag{4.63}$$

subject to

$$\sum_{i \in N} PTDF_{i,a} \cdot \Big[\sum_f (c^f_{i,t} - q^f_{i,t})\Big] \leq T_a \quad \forall a \in A, \forall t \in T \tag{4.64}$$

$$\sum_{i \in N} PTDF_{i,a} \cdot \Big[\sum_f (c^f_{i,t} - q^f_{i,t})\Big] \geq -T_a \quad \forall a \in A, \forall t \in T \tag{4.65}$$

$$\sum_{i \in N} \sum_{F \in F} w_{i,t}[c^f_{i,t} - q^f_{i,t}] \geq 0 \quad \forall t \in T \tag{4.66}$$

$$0 \leq c^f_{i,t} \perp \left[2b_{i,t}c^f_{i,t} - a_{i,t} + b_{i,t} \sum_{g \in F, g \neq f} c^g_{i,t} + w_{i,t} - \zeta^{+f}_t + \zeta^{-f}_t\right] = \phi^f_{i,t} \geq 0 \tag{4.67}$$

$$0 \leq q^f_{i,t} \perp \Big[-w_{i,t} + \zeta^{+f}_t - \zeta^{-f}_t + m^f_{1,i}\gamma^{+f}_{i,t} + m^f_{2,i}\gamma^{-f}_{i,t} + \sigma^f_{i,t} - \mu^f_{i,t} + \theta^f_{i,t}\Big] = \rho^f_{i,t} \geq 0 \tag{4.68}$$

$$0 \leq V^f_{i,t} \perp \Big[1 + \gamma^{+f}_{i,t} - \gamma^{-f}_{i,t}\Big] = \delta^f_{i,t} \geq 0 \tag{4.69}$$

$$0 \leq \left[-\sum_{i \in N} q^f_{i,t} + \sum_{i \in M} c^f_{i,t}\right] \perp \zeta^{+f}_t \geq 0 \tag{4.70}$$

$$0 \leq \left[\sum_{i \in N} q^f_{i,t} - \sum_{i \in M} c^f_{i,t}\right] \perp \zeta^{-f}_t \geq 0 \tag{4.71}$$

$$0 \leq \Big[V^f_{i,t} - m^f_{1,i}q^f_{i,t} - b^f_{1,i}\Big] \perp \gamma^{+f}_{i,t} \geq 0 \tag{4.72}$$

$$0 \leq \Big[V^f_{i,t} - m^f_{2,i}q^f_{i,t} - b^f_{2,i}\Big] \perp \gamma^{-f}_{i,t} \geq 0 \tag{4.73}$$

$$0 \leq c^f_{i,t} \perp \phi^f_{i,t} \geq 0 \tag{4.74}$$

$$0 \leq V^f_{i,t} \perp \delta^f_{i,t} \geq 0 \tag{4.75}$$

$$0 \leq q^f_{i,t} \perp \rho^f_{i,t} \geq 0 \tag{4.76}$$

$$0 \leq -q^f_{i,t} + q^f_{i,max} \perp \sigma^f_{i,t} \geq 0 \tag{4.77}$$

$$0 \leq q^t_{i,t} - q^f_{i,t-1} - r^f_{i,min} \perp \mu^f_{i,t} \geq 0 \tag{4.78}$$

$$0 \leq -q^t_{i,t} + q^f_{i,t-1} + r^f_{i,max} \perp \theta^f_{i,t} \geq 0 \tag{4.79}$$

4.3 Toy Problem

Our first numerical example is a simple 3-node network based on the small network presented in Gabriel and Leuthold (2010). All three nodes have demand, but the primary load resides at node 3. There are two firms that act as followers, while the market monitor is modeled as the only leader. Firm 1 possess facilities at nodes 1 and 2 . Firm 2 has a plant only at node 2. We present numerical results for two congestion scenarios; one with no congestion and the other in which the transmission line connecting node 2 to 3 is congested.

4.3.1 Formulation

We assumed a planning horizon of one time period. Ramping bounds $r_{i,min}$ and $r_{i,max}$ were not applicable since ramping only applies when more than one discrete time period is under consideration. A summary of the model topology and parameters of the network can be seen in Figure 1 and Tables 2 through 5 respectively.

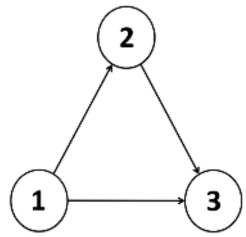

Figure 1: The 3-node network

Table 2: Inverse Demand data π_i

Node	a_i	b_i
1	5	1
2	1	1
3	10	1

Table 3: Plant Production and Capacity data V and q_{max}

Firm	Node	m_1	b_1	m_2	b_2	q_{max}
1	1	2	0	2	0	10
1	2	1	0	1	0	10
2	2	3	0	3	0	10

Table 4: Transmission Arc Capacity T_a

Arc	No Congestion T_a	Congestion T_a
1-2	10	10
2-3	10	3
1-3	10	10

Table 5: $PTDF_{i,a}$ data with Node 3 as Hub

Node	Arc 1-2	Arc 2-3	Arc 1-3
1	-1/3	-1/3	-2/3
2	1/3	-2/3	-1/3
3	0	0	0

The direction arrows in Figure 1 describe the sign of electricity flow. For example, a MW flowing from node 1 to node 2 would be +1 MW while a MW flowing from node 3 to node 1 would be -1 MW. The inverse demand functions described by table 2 assert that the primary load demand resides in node 3, a moderate amount in node 1 and small demand in node 2. This simple example is equivalent to node 3 residing in a high population area while the other nodes represent rural generation nodes. Table 3 describes the cost of electricity generation and capacity. For illustrative purposes, the piecewise production V is transformed into a simple linear function. Firm 1 has a low-cost generation facility located at node 2 in addition to a moderately low-cost facility at node 2. Firm 2 has a high priced facility at node 2. Each facility has a capacity limit of 10 MWs. Table 4 describes the two congestion scenarios with the transmission line connection node 2 and 3 limited to 4 MW in the congestion phase.

We compare our Stackelberg game detailed in section 3 with perfect competitive, Cournot-Nash equilibrium and our dynamic Stackelberg game. Perfect competition serves as an ideal comparison of what any central planner or market monitor hopes to achieve in a market. In a perfectly competitive market, producers are forced to generate electricity with a profit equal to zero. Thus, all economic surpluses reside with the consumer and market monitor. As a consequence, the price of electricity at each node is equal to lowest marginal cost to produce an additional unit of electricity. Furthermore, the concept of a Nash equilibrium between generators is dissolved. Generators must sell and generate electricity such that the consumer's and market monitor's surpluses are maximized. However, it is not practically obtainable or reasonable in real world electricity markets. The scenarios serves merely as a comparison with other market structures. It also serves as a pseudo-upper bound of the possible improvement of market efficiency. Is noted that we do interchange perfectly competitive and *perfect competition*. Perfect competition is a specific market structure that we do not possess since we have limited buyers and sellers. Firms have an impact on price and we do not allow additional firms to enter the market within our planning time horizon. The Cournot-Nash equilibrium for this specific scenario is a game originated in Mookherjee et al. (2010), in which the ISO determines the wheeling simultaneously with the Cournot-Nash game played by the generators. In their model, the ISO's only concern is to efficiently allocate scarce transmission line capacity among the generators. Our Stackelberg equilibrium describes the market monitor possess the ability to impose access charges on the network to incentivize generators to use transmission line capacity efficiently while simultaneously maximizing social welfare.

4.3.2 Discussion

Figure 2 demonstrates a comparison of all the models described above for consumer, producer and congestion surplus, social welfare for the no congestion scenario. We remind the reader that consumer's surplus is defined as utility derived from the difference of the maximum price consumers are willing to pay and the actual price the pay for electricity. Producer's surplus is simply the net profit producers receive from revenue less generation cost and access charges paid to the market monitor. Congestion surplus is defined as the total revenue the market monitor receives from imposing access charges to the generators.[1]

The goal of the market monitor is to achieve economic surpluses as close as possible to perfectly competitive scenario we described previously. This serves as a benchmark to compare all models. Analyzing the perfectly competitive scenario for our illustrative example yielded the highest consumer surplus and social welfare; by definition, producer surplus was zero while consumer and congestion surpluses were maximized. The Cournot-Nash and Stackelberg equilibrium models produced identical economic surpluses. These results suggest that the market monitor did not utilize equilibrium congestion pricing to increase social welfare. We posit that the Stackelberg equilibrium model is nearly identical to the Cournot-Nash game when little or no congestion exists as socially optimal electricity flows are not encumbered to transmission line constraints. It is only in the presence of congestion that there exists an opportunity to incentivize generators to alter their electricity flows assuming our proposed

[1]Note, we distinguish congestion rent from the term "No congestion" describing the scenario.

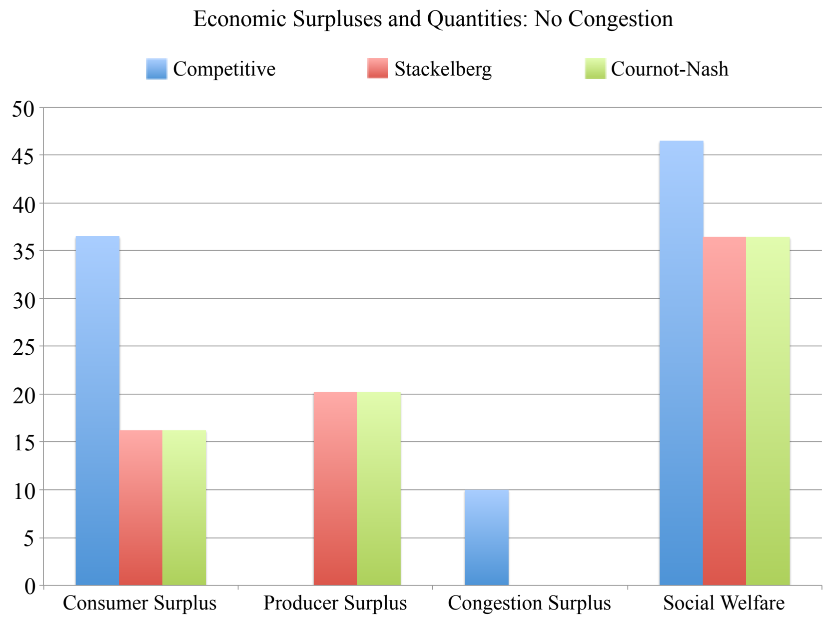

Figure 2: Model comparison under no congestion: Economic surplus and quantities

hypothetical market.

An interesting phenomena is observed in comparing the perfectly competitive solution with both equilibrium solutions. In the perfectly competitive scenario, the market monitor forces generators to produce electricity although they produce zero profit. From a generator's perspective, producing a social welfare-optimal generation yields the same profit as not producing at all. In realistic market structures, generators cannot be forced to generate electricity and thus have to be incentivized by profit to generate power. A producer's surplus of 20 units is required for the generators to produce power at socially optimal levels. Any restriction placed on the producer's surplus would result in lower social welfare, assuming the realistic market structure described by Cournot-Nash and Stackelberg equilibrium.

Figure 5 displays the access fees, sales and production for nodes and firms for the no congestion scenario. The Stackelberg equilibrium demonstrated all equal access fees. Our computational results indicate multiple arcs with equal access fees. This is a result of the directionality of the transmission lines we have defined. The Cournot-Nash equilibrium solution produced identical sales and generation to the Stackelberg equilibrium without the use of access charges/wheeling fees.

We now turn to the congested version of the network as demonstrated by Figure 4. The transmission line connecting nodes 2 and 3 was reduced from a capacity of 10 to 3 units. The remaining transmission lines were unchanged.

The congestion reduced social welfare and consumer surplus of the perfect competition model by 8.5 units and 7.5 units, respectively. Interestingly, the congestion surplus increased from zero to 1 unit in the presence of congestion. The Stackelberg equilibrium did achieve a higher social welfare than the Cournot-Nash equilibrium as expected since its objective function was directly stipulated to maximize social welfare. This provides evidence that our model has the potential to achieve higher social welfare compared to the Cournot-Nash equilibrium, given all else equal. Social welfare was increased by employing equilibrium congestion pricing even for such a small network presented. Furthermore, the increase was achieved without

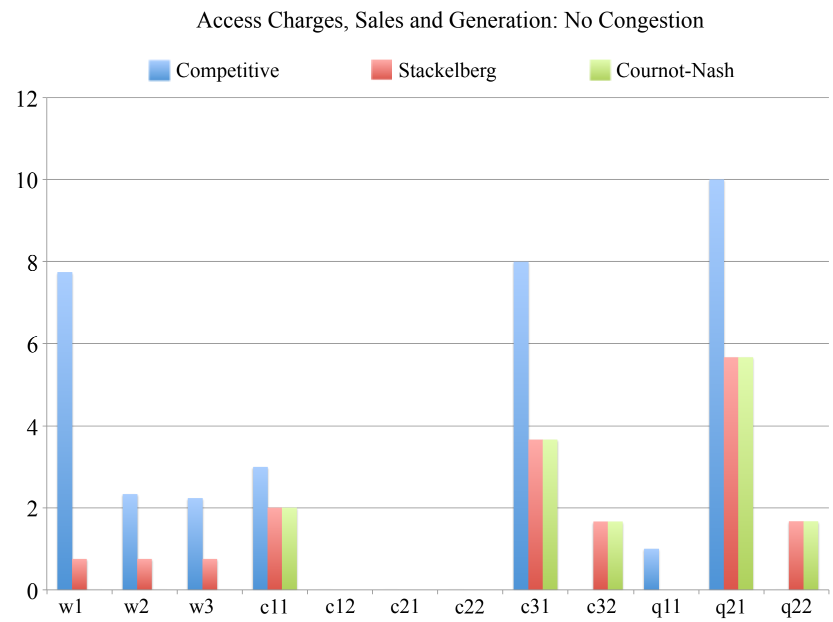

Figure 3: Model comparison under no congestion: Access charges, sales and generation

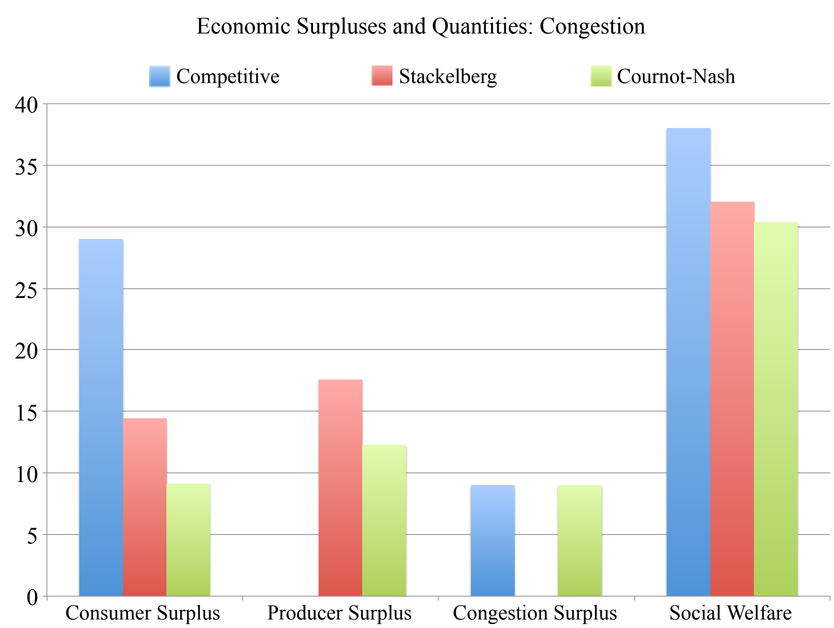

Figure 4: Model comparison under congestion: Economic surpluses and quantities

directly coercing generators to produce electricity. Simply, the access charges incentivized generators to sell and produce electricity in a manner consistent with social welfare optimal electricity flows. It is also noteworthy to state that all production and demand characteristics were held constant between the Stackelberg equilibrium and Cournot-Nash equilibrium scenarios.

Our analysis has shown the existence of multiple optimal solutions in presence of congestion. Specifically, the access charges vary among solutions, suggesting the market monitor has a certain amount of latitude in determining the set of access charges to use within the network. It makes sense that congestion allows access charges to influence the flow of electricity since congestion creates price differentials between nodes.

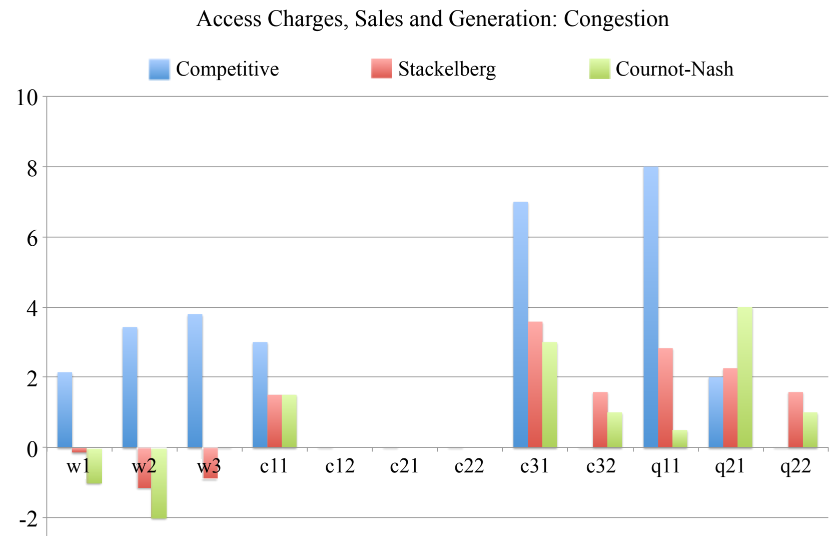

Figure 5: Model comparison under congestion: Access charges, sales and generation

Figure 5 shows the access fees, sales and production for nodes and firms for the no congestion scenario. The first subscript each quantities refers to nodes and the second subscript indicates firms. It is clear that the Stackelberg equilibrium had higher sales and production than the Cournot-Nash since social welfare was improved. Production and sales were more diverse among the equilibrium models than the perfectly competitive model.

4.4 Western European Electric Grid

We now present a stylized network based upon the Western European electric grid presented in Neuhoff et al. (2005). Analysis and results are presented similarly to the previous illustrative example but further demonstrate the ability of the computational framework to solve larger network problems.

4.4.1 Data Formulation

The network consists of 15 nodes, 28 arcs and 12 generating firms. Specifically, the set of firms is $F = \{1, ..., 12\}$, the set of nodes is $N = \{1, ..., 15\}$, the set of nodes in which there are markets is $M = \{4, 5, 6, 8, 9, 14, 15\}$ and the set of arcs is $A = \{1, ..., 28\}$. A total of 24 discrete time periods were considered in our planning horizon used to mimic each hour of a day.

All data is formulated in the style of Mookherjee et al. (2010), in which the authors state the inverse demand was created synthetically, based upon demand patterns obtained from California Independent System Operator (CAISO). The data successfully represents the daily load profiles of a typical node in California. This double hump profile matches the peak profiles of consumers utilizing electricity in the middle of the day and again returning home from work. The ramping bounds were created synthetically. Additionally, the piecewise linear generation cost, PTDF and transmission line capacities originate from the Energy research

Centre of the Netherlands (ECN). Figure 6 provides a topology of our network of interest at time period 19.

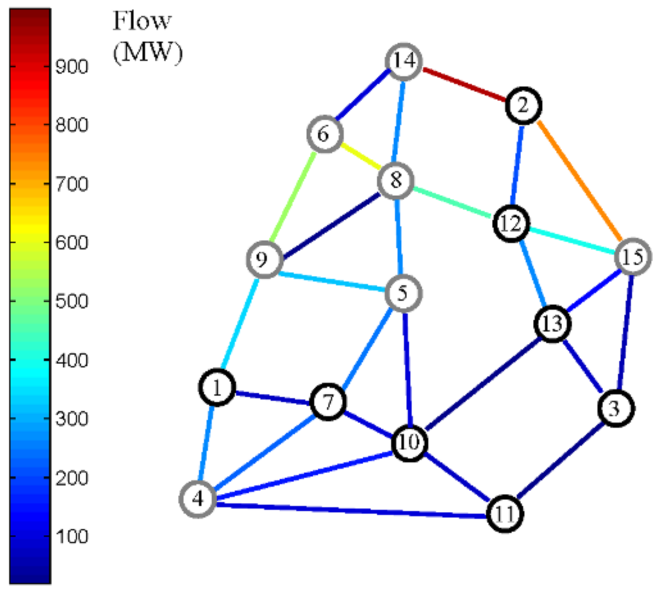

Figure 6: Transmission flows for dynamic Stackelberg game (t=19)

4.4.2 Discussion

Figure 6 provides a clever manner of displaying the flow in MW of our optimal solution found for $t = 19$. Nodes {4, 5, 6, 8, 9, 14, 15} represents markets while the color of the transmission arc corresponds to the scale of flow on the left. The figure shows a large difference between arcs as well as the phenomenon that a majority of the load resides in the upper portion of the network. The figure demonstrates the intricacies that real world electric networks exhibit. For example, two of the three lines connected to node 2 have extremely large flows while, the third transmission line contains barely any flow.

Figure 8 shows the generation profile at node 4 for all 24 hours in the planning horizon. Firm 7 produces a nearly constant generation profile as it has a zero marginal cost while firms 2 and 6 have traditional upward sloping cost curves. The sudden drop in generation at time period 14 marks a point in which congestion fees rose and demand shifted to another node.

Figure 8 displays the power sales at node 15 across all time periods. The upper double humped curve corresponds to the total sales of the node. This profile mimics the traditional load profile experienced in California and many areas with similar daily consumption patterns. The interaction among firms is particularly interesting in the first time period, as significant market-share changes are experienced. This could be explained, in part, by the time periods required for the ramping dynamics to exit a warmup period affected by the initial conditions.

Figure 9 displays the economic surpluses and quantities for our dynamic Stackelberg game and the Cournot-Nash equilibrium model. All quantities were scaled to a value of 1 for display purposes, since large social welfare numbers are difficult to compare. All economic quantities were relatively similar with the slight exception of consumer and producer surplus. It appears that our dynamic Stackelberg game shifted some economic benefits from producers to consumers without increasing social welfare by a large amount. The production quantities on the right hand axis did vary. The congestion surplus for both models was extremely small

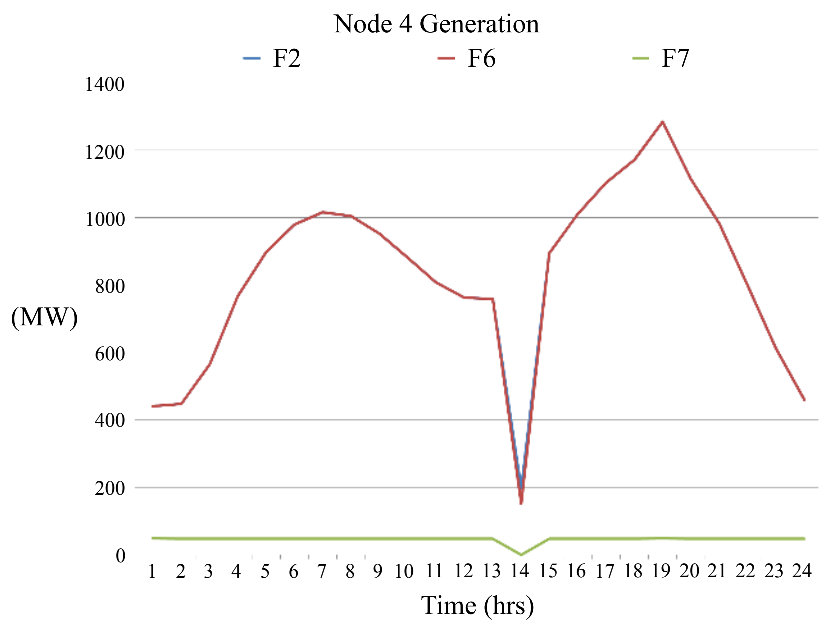

Figure 7: Generation at node 4 for all 24 hours

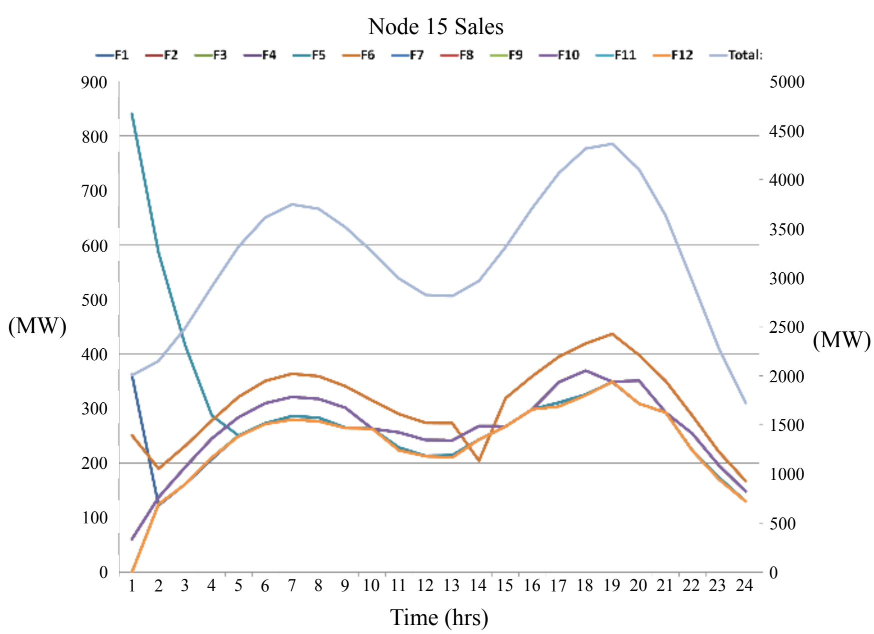

Figure 8: Sales at node 15 for all 24 hours

in comparison to other economic surpluses. This suggests that even for a large network, large access charges may not be needed for the market monitor to influence electricity flows and subsequently social welfare.

However, a comparison between our Stackelberg model to the Cournot-Nash game is not completely relevant due to the large difference of each games' information structure. Our dynamic Stackelberg game anticipates and calculates the lower level equilibrium in tandem. Hence, all agents execute their solutions simultaneously and in open-loop equilibrium. This

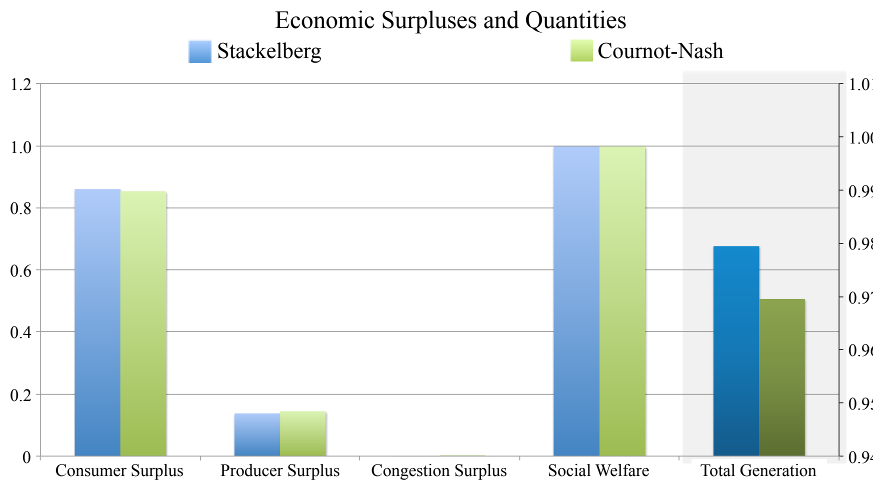

Figure 9: Economic surpluses and generation between models

differs from the Cournot-Nash game where the generation firms use the model as a decision support tool for what their generation and sales will be given their forecasts of access charges. Our dynamic Stackelberg game has the feature in which the optimal access charges are announced to all generators and then an equilibrium is found among the firms.

5 Conclusions

In this article, we have presented a hypothetical new market represented by a dynamic Stackelberg game of an electric power oligopoly. Our single leader is represented by the market monitor using equilibrium congestion pricing to increase social welfare. Our model may be used as a decision support tool for the existing operations the market monitor conducts. The followers are generators that play a Cournot-Nash game with other generators to sell and transmit power over an electric network assuming an oligopoly market structure. The congestion pricing determined by the market monitor solves the Cournot-Nash equilibrium problem in the lower level. We described the evolution of ramping rates over time for each generator via a differential equation as well employed a multi-period time horizon.

The dynamic Stackelberg model considers a new market design mechanism that also includes the following numerous realistic and computable features: oligopolistic competition, inter-temporal constraints, dynamic production constraints, time-varying demand, transmission constrained network and multi-generator assets. We were able to compute the dynamic Stackelberg game by first reformulating it as a Mathematical Program with Complementarity Constraints (MPCC) and utilizing the commercially available NLPEC solver within GAMS. We solved a 15-node Western European Electric Network in approximately eight minutes. Our numerical experimentation concluded that equilibrium congestion pricing can increase social welfare in our proposed market.

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