PTEP

Prog. Theor. Exp. Phys. **2021**, 023D02 (15 pages)
DOI: 10.1093/ptep/ptab004

Electric quadrupole form factors of singly heavy baryons with spin 3/2

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Received October 13, 2020; Revised December 24, 2020; Accepted December 24, 2020; Published January 21, 2021

We study the electromagnetic form factors of the lowest-lying singly heavy baryons in a pion mean-field approach, which is also known as the SU(3) chiral quark-soliton model. In the limit of the heavy-quark mass, the dynamics inside a singly heavy baryon is governed by the $N_c - 1$ valence quarks, while the heavy quark remains as a static one. In this framework, a singly heavy baryon is described by combining the $N_c - 1$ soliton with the singly heavy quark. In the infinitely heavy-quark mass limit, we can compute the electric quadrupole form factors of the baryon sextet with spin 3/2, with the rotational $1/N_c$ and linear corrections of the explicit flavor SU(3) symmetry breaking taken into account. We find that the sea-quark contributions or the Dirac-sea level contributions dominate over the valence-quark contributions in the lower Q^2 region. We examined the effects of explicit flavor SU(3) symmetry breaking in detail. The numerical results are also compared with the recent data from the lattice calculation with the unphysical value of the pion mass considered, which was used in the lattice calculation.

Subject Index D32

1. Introduction

Conventional lowest-lying singly heavy baryons consist of a heavy quark and two light valence quarks. In the limit of the infinitely heavy-quark mass ($m_Q \to \infty$), the physics of singly heavy baryons becomes simple: The spin of the heavy quark $\boldsymbol{J}_Q$ is conserved in this limit and hence it leads also to the conservation of the spin of the light-quark degrees of freedom, i.e. $\boldsymbol{J}_L = \boldsymbol{J} - \boldsymbol{J}_Q$. This is known as the heavy-quark spin symmetry [1,2]. In the $m_Q \to \infty$ limit, we do not distinguish a charm quark from a bottom quark, which gives up heavy-quark flavor symmetry. On the other hand, chiral symmetry and its spontaneous breakdown still play an important part in describing the singly heavy baryons because of the presence of the light quarks inside a singly heavy baryon [3]. The singly heavy baryons consisting of two light valence quarks can then be represented in terms of irreducible representations of flavor SU(3) symmetry: $\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}$. Thus we have the two representations for the lowest-lying singly heavy baryons, i.e. the baryon antitriplet and sextet. The baryon antitriplet has the total spin $J = 1/2$ that comes from $J_Q = 1/2$ and $J_L = 0$, whereas the baryon sextet can have either $J = 1/2$ or $J = 3/2$ with $J_L = 1$ and $J_Q = 1/2$.

In a pion mean-field approach, which is also known as the SU(3) chiral quark-soliton model (χQSM), a singly heavy baryon can be viewed as the $N_c - 1$ valence quarks bound by the pion mean fields that are created from the presence of the $N_c - 1$ valence quarks [4,5]. In fact, this idea is taken from Witten's seminal paper on baryons in the large N_c limit [6]. This pion mean-field approach has successfully reproduced the mass spectra of the lowest-lying singly heavy baryons [5] and even explained their nontrivial isospin mass splittings [7]. Interestingly, the corrections from the heavy quark mass are indeed negligible in the description of the isospin mass splittings of the singly heavy baryons, as shown in Ref. [7], although they provide hyperfine interactions which remove the spin degeneracy of the baryon sextet.

Recently, the electromagnetic (EM) form factors of singly heavy baryons have been studied for the first time within lattice quantum chromodynamics (QCD) [8,9]. While there are no experimental data on the EM form factors of the singly heavy baryons to date, the results from the lattice calculation provide a clue to the internal structure of singly heavy baryons. Thus, it is also of great importance to investigate the EM form factors of the singly heavy baryons. In Refs. [10,11], we have studied the electric monopole and magnetic dipole form factors of the singly heavy baryons in detail, based on the χQSM. Since we consider the limit of the infinitely heavy-quark mass, there is no physical difference between the heavy baryons with spin 1/2 and those with 3/2 except for the value of the spin. On the other hand, the baryon sextet with spin 3/2 has yet another structure that arises from its higher spin, which is revealed by the electric quadrupole ($E2$) form factor. The $E2$ form factor of a baryon exhibits how the baryon is deformed. It is also known that the pion clouds play a significant role in understanding this deformation [12]. This will also be discussed in the present work. We will also examine the effects of flavor SU(3) symmetry breaking on the $E2$ form factors of the baryon sextet with spin 3/2. The numerical results for Ω_c^{*0} will be compared with that from the lattice calculation.

The present work is organized as follows: In Sect. 2, we briefly recapitulate the general formalism for the electric quadrupole form factors within the framework of the chiral quark-soliton model. In Sect. 3, we present the numerical results and discuss them in detail. The final section is devoted to the summary and conclusion.

2. Electric quadrupole form factors in the χQSM

We start with the EM current for a singly heavy baryon, which is defined by

$$J^{\mu}(x) = \bar{\psi}(x)\gamma^{\mu}\hat{\mathcal{Q}}\psi(x) + e_Q\bar{\Psi}\gamma^{\mu}\Psi, \tag{1}$$

where $\psi(x)$ stands for the light-quark field $\psi = (u, d, s)$ in SU(3) flavor space and Ψ denotes the heavy-quark field for the charmed or bottom quark. The charge operator $\mathcal{Q}$ is expressed as

$$\hat{\mathcal{Q}} = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix} = \frac{1}{2}\left(\lambda_3 + \frac{1}{\sqrt{3}}\lambda_8\right). \tag{2}$$

The e_Q in the second term in Eq. (1) denotes the charge corresponding to a heavy quark, which has the value $2/3$ for the charm quark and $-1/3$ for the bottom quark. The matrix element of J^{μ} between baryons with spin 3/2 can be parametrized in terms of four different real form factors as follows:

$$\begin{aligned}\langle B(p',s)|J^{\mu}(0)|B(p,s)\rangle = -\overline{u}^{\alpha}(p',s)\Bigg[\gamma^{\mu}\left\{F_1^B(q^2)\eta_{\alpha\beta} + F_3^B(q^2)\frac{q_\alpha q_\beta}{4M_B^2}\right\} \\ + i\frac{\sigma^{\mu\nu}q_\nu}{2M_B}\left\{F_2^B(q^2)\eta_{\alpha\beta} + F_4^B(q^2)\frac{q_\alpha q_\beta}{4M_B^2}\right\}\Bigg]u^{\beta}(p,s),\end{aligned} \tag{3}$$

where M_B denotes the mass of a singly heavy baryon in the baryon sextet with spin 3/2. The metric tensor $\eta_{\alpha\beta}$ of Minkowski space is defined as $\eta_{\alpha\beta} = \mathrm{diag}(1, -1, -1, -1)$. q_α represents the momentum transfer $q_\alpha = p'_\alpha - p_\alpha$ and its square is written as $q^2 = -Q^2$ with $Q^2 > 0$. $u^\alpha(p, s)$ is the Rarita–Schwinger spinor for a singly heavy baryon with spin 3/2, carrying the momentum p and the spin component s projected along the direction of the momentum. $\sigma^{\mu\nu}$ designates the antisymmetric tensor $\sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$. Note that when one takes the limit of the infinitely heavy quark mass ($m_Q \to \infty$), the heavy-quark current given in the second part of Eq. (1) can be safely neglected for the EM form factors. It gives only a constant contribution to the electric form factors as already shown in Ref. [10].

It is more convenient to introduce the Sachs-type form factors or the multipole EM form factors, in particular, when the EM structure of a baryon with spin 3/2 is examined. The electric quadrupole form factor reveals how the shape of a baryon with spin 3/2 is deviated from the rotationally symmetric one. The Sachs-type form factors can be expressed in terms of F_i^B given in Eq. (3)

$$
\begin{aligned}
G^B_{E0}(Q^2) &= \left(1+\frac{2}{3}\tau\right)\left[F^B_1(Q^2)-\tau F^B_2(Q^2)\right]-\frac{1}{3}\tau(1+\tau)\left[F^B_3(Q^2)-\tau F^B_4(Q^2)\right],\\
G^B_{E2}(Q^2) &= \left[F_1(Q^2)-\tau F_2(Q^2)\right]-\frac{1}{2}(1+\tau)\left[F_3(Q^2)-\tau F_4(Q^2)\right],\\
G^B_{M1}(Q^2) &= \left(1+\frac{4}{5}\tau\right)\left[F^B_1(Q^2)+F^B_2(Q^2)\right]-\frac{2}{5}\tau(1+\tau)\left[F^B_3(Q^2)+F^B_4(Q^2)\right],\\
G^B_{M3}(Q^2) &= \left[F^B_1(Q^2)+F^B_2(Q^2)\right]-\frac{1}{2}(1+\tau)\left[F^B_3(Q^2)+F^B_4(Q^2)\right],
\end{aligned}
\tag{4}
$$

where $\tau = Q^2/4M_B^2$. Since G^B_{E0} and G^B_{M1} have already been investigated in Ref. [10], we will focus on the electric quadrupole form factors of the baryon sextet with spin 3/2, i.e., G^B_{E2}, in the present work. At $Q^2 = 0$, $G_{E2}(0)$ yields the electric quadrupole moment

$$
\mathcal{Q}_B = \frac{e}{M_B^2}G^B_{E2}(0) = \frac{e}{M_B^2}\left[e_B - \frac{1}{2}F^B_3(0)\right], \tag{5}
$$

which reveals how much the charge distribution of a baryon is deformed from a spherical shape. If $\mathcal{Q}_B$ has a negative value ($\mathcal{Q}_B < 0$), then the baryon takes a cushion shape, whereas if $\mathcal{Q}_B$ is positive ($\mathcal{Q}_B > 0$), then it looks like a rugby-ball shape.

We want to mention that the $M3$ form factors vanish in the present work. In fact, any chiral solitonic approaches yield the null results of the $M3$ form factors because of the hedgehog structure [13]. However, the experimental data on $M3$ is absent to date and its value should be very tiny even if it is measured. In fact, one could compute the $M3$ form factors if one takes into account the next-to-next-to-leading order in the $1/N_c$ expansion. This means that the $M3$ form factors should be strongly suppressed in the large N_c limit. Thus, we will focus in the present work on the $E2$ form factors of the baryon sextet with spin 3/2.

The SU(3) χQSM is constructed based on the following low-energy effective partition function in Euclidean space, defined by

$$
\mathcal{Z}_{\chi\mathrm{QSM}} = \int \mathcal{D}\psi\mathcal{D}\psi^\dagger\mathcal{D}U\,\exp\left[-\int d^4x\,\psi^\dagger D(U)\psi\right] = \int \mathcal{D}U\,\exp(-S_{\mathrm{eff}}), \tag{6}
$$

where ψ and U denote, respectively, the quark and pseudo-Nambu–Goldstone boson fields. Having integrated over quark fields, we can express the partition function in terms of the effective chiral action S_{eff}, which is defined by

$$S_{\text{eff}}(U) = -N_c \text{Tr} \ln(i\partial\!\!\!/ + iMU^{\gamma_5} + i\hat{m}), \tag{7}$$

where Tr represents the functional trace running over spacetime and all relevant internal spaces. N_c denotes the number of colors. M is the dynamical quark mass that arises from spontaneous symmetry breaking of chiral symmetry. U^{γ_5} represents the chiral field that consists of the pseudo-Nambu–Goldstone (pNG) fields π^a, $a = 1, \ldots 8$, which is expressed as

$$U^{\gamma_5} = \exp(i\gamma_5 \pi^a \lambda^a) = \frac{1+\gamma_5}{2} U + \frac{1-\gamma_5}{2} U^{\dagger} \tag{8}$$

with

$$U = \exp(i\pi^a \lambda^a). \tag{9}$$

We assume isospin symmetry, i.e., $m_{\text{u}} = m_{\text{d}}$. The average mass of the up and down quarks is defined by $\overline{m} = (m_{\text{u}} + m_{\text{d}})/2$. Then, the matrix of the current quark masses is written as $\hat{m} = \text{diag}(\overline{m}, \overline{m}, m_{\text{s}}) = \overline{m} + \delta m$. δm is written as

$$\delta m = \frac{-\overline{m} + m_s}{3} \mathbf{1} + \frac{\overline{m} - m_s}{\sqrt{3}} \lambda^8 = m_1 \mathbf{1} + m_8 \lambda^8, \tag{10}$$

where m_1 and m_8 denote the singlet and octet components of the current quark masses, defined by $m_1 = (-\overline{m} + m_{\text{s}})/3$ and $m_8 = (\overline{m} - m_{\text{s}})/\sqrt{3}$, respectively. The single-quark Hamiltonian $h(U)$ is defined by

$$h(U) = i\gamma_4 \gamma_i \partial_i - \gamma_4 M U^{\gamma_5} - \gamma_4 \overline{m}. \tag{11}$$

Since the pion field has flavor indices, one has to introduce the hedgehog ansatz with which the flavor indices can be coupled to three-dimensional spatial axes. The pion fields are then expressed in terms of a single function $P(r)$, which is called the profile function, as follows:

$$\pi^a(\boldsymbol{x}) = n^a P(r), \tag{12}$$

with $n^a = x^a/r$. Then the SU(2) chiral field is written as

$$U^{\gamma_5}_{\text{SU(2)}} = \exp(i\gamma^5 \hat{\boldsymbol{n}} \cdot \boldsymbol{\tau} P(r)) = \frac{1+\gamma^5}{2} U_{\text{SU(2)}} + \frac{1-\gamma^5}{2} U^{\dagger}_{\text{SU(2)}}, \tag{13}$$

with $U_{\text{SU(2)}} = \exp(i\hat{\boldsymbol{n}} \cdot \boldsymbol{\tau} P(r))$. The SU(3) chiral field can be constructed by Witten's trivial embedding [14]

$$U^{\gamma_5}(x) = \begin{pmatrix} U^{\gamma_5}_{\text{SU(2)}}(x) & 0 \\ 0 & 1 \end{pmatrix}, \tag{14}$$

which preserves the hedgehog ansatz.

Integration over U in Eq. (6) quantizes the pNG fields. In the large N_c limit, the meson mean-field approximation is justified [6,14]. Thus, we can carry out the integration over U in Eq. (6) around the saddle point, where $\delta S_{\text{eff}}/\delta P(r) = 0$ is satisfied. This saddle-point approximation yields the equation of motion that can be solved self-consistently. The solution provides the self-consistent profile function $P_c(r)$ of the chiral soliton. A detailed method of the self-consistent procedure can be found in Ref. [15].

While the quantum fluctuations of the self-consistent pion fields can be ignored by the large N_c argument, the fluctuations along the direction of both the rotational and translational zero modes cannot be ignored, since they are not at all small. Note that rotational and translational zero modes are related to rotational and translational symmetries. Thus, the zero modes can be taken into account by the following rotational and translational transformations:

$$\tilde{U}(\boldsymbol{x},t) = A(t)U[\boldsymbol{x} - \boldsymbol{Z}(t)]A^{\dagger}, \tag{15}$$

where $A(t)$ is an SU(3) unitary matrix. Thus, the functional integral over U can be approximated by those over zero modes:

$$\int DU[\cdots] \approx \int DAD\boldsymbol{Z}[\cdots]. \tag{16}$$

The integration over translational zero modes will naturally give the Fourier transform of the EM densities. We refer to Ref. [16] for a detailed description of the zero-mode quantization in the present scheme.

Having carried out the zero-mode quantization, we obtain the collective Hamiltonian as

$$H_{\rm coll} = H_{\rm sym} + H_{\rm sb}, \tag{17}$$

where

$$H_{\rm sym} = M_{\rm cl} + \frac{1}{2I_1}\sum_{i=1}^{3} J_i^2 + \frac{1}{2I_2}\sum_{p=4}^{7} J_p^2, \quad H_{\rm sb} = \alpha D_{88}^{(8)} + \beta\hat{Y} + \frac{\gamma}{\sqrt{3}}\sum_{i=1}^{3} D_{8i}^{(8)}\hat{J}_i. \tag{18}$$

I_1 and I_2 denote the moments of inertia for the soliton, the explicit expressions of which can be found in Appendix A. The parameters α, β, and γ for heavy baryons arise from the breaking of flavor SU(3) symmetry, which are defined by

$$\alpha = \left(-\frac{\overline{\Sigma}_{\pi N}}{3m_0} + \frac{K_2}{I_2}\overline{Y}\right)m_{\rm s}, \quad \beta = -\frac{K_2}{I_2}m_{\rm s}, \quad \gamma = 2\left(\frac{K_1}{I_1} - \frac{K_2}{I_2}\right)m_{\rm s}, \tag{19}$$

where $K_{1,2}$ are the anomalous moments of inertia, the expressions of which are found in Appendix A. Note that the number of light valence quarks for a singly heavy baryon is $N_c - 1$. This means that the expression for the valence part of $\overline{\Sigma}_{\pi N}$ also contains $N_c - 1$ in place of N_c. It can be related to the πN sigma term as follows: $\overline{\Sigma}_{\pi N} = (N_c - 1)N_c^{-1}\Sigma_{\pi N}$. The detailed expressions for the moments of inertia and $\overline{\Sigma}_{\pi N}$ are given in Ref. [17].

The presence of the symmetry-breaking part in the collective Hamiltonian, $H_{\rm sb}$, causes baryon wavefunctions mixed with those in higher SU(3) representations. In the present case, the collective wavefunctions for the baryon antitriplet ($J = 0$) and the sextet ($J = 1$) are obtained respectively as [17]

$$|B_{\overline{\mathbf{3}}_0}\rangle = |\overline{\mathbf{3}}_0, B\rangle + p^B_{\overline{15}}|\overline{\mathbf{15}}_0, B\rangle, \quad |B_{\mathbf{6}_1}\rangle = |\mathbf{6}_1, B\rangle + q^B_{\overline{15}}|\overline{\mathbf{15}}_1, B\rangle + q^B_{\overline{24}}|\overline{\mathbf{24}}_1, B\rangle, \tag{20}$$

with the mixing coefficients

$$p^B_{\overline{15}} = p_{\overline{15}}\begin{bmatrix} -\sqrt{15}/10 \\ -3\sqrt{5}/20 \end{bmatrix}, q^B_{\overline{15}} = q_{\overline{15}}\begin{bmatrix} \sqrt{5}/5 \\ \sqrt{30}/20 \\ 0 \end{bmatrix}, q^B_{\overline{24}} = q_{\overline{24}}\begin{bmatrix} -\sqrt{10}/10 \\ -\sqrt{15}/10 \\ -\sqrt{15}/10 \end{bmatrix}, \tag{21}$$

in the basis $\left[\Lambda_Q,\ \Xi_Q\right]$ for the antitriplet and $\left[\Sigma_Q,\ \Xi'_Q,\ \Omega_Q\right]$ for the sextets with both spin 1/2 and 3/2. The parameters $p_{\overline{15}}$, $q_{\overline{15}}$, and $q_{\overline{24}}$ are explicitly written as

$$p_{\overline{15}} = \frac{3}{4\sqrt{3}}\alpha I_2, q_{\overline{15}} = -\frac{1}{\sqrt{2}}\left(\alpha + \frac{2}{3}\gamma\right) I_2, q_{\overline{24}} = \frac{4}{5\sqrt{10}}\left(\alpha - \frac{1}{3}\gamma\right) I_2. \tag{22}$$

The collective wavefunction for the soliton with $(N_c - 1)$ valence quarks is then obtained in terms of the SU(3) Wigner D functions

$$\psi_{(\nu;F),(\overline{\nu};\overline{S})}(R) = \sqrt{\dim(\nu)}(-1)^{Q_S}[D^{(\nu)}_{F\,S}(R)]^*, \tag{23}$$

where $\dim(\nu)$ represents the dimension of the representation ν and Q_S is a charge corresponding to the soliton state S, i.e., $Q_S = J_3 + Y'/2$. F and S stand for the flavor and spin quantum numbers corresponding to the soliton for the singly heavy baryon. Finally, the complete wavefunction for a singly heavy baryon can be derived by coupling the soliton wavefunction to the heavy quark spinor

$$\Psi^{(\mathcal{R})}_{B_Q}(R) = \sum_{J_3, J_{Q3}} C^{J'\,J'_3}_{J, J_3\, J_Q\, J_{Q3}}\ \chi_{J_{Q3}}\ \psi_{(\nu;\,Y,\,T,\,T_3)(\overline{\nu};\,Y',\,J,\,J_3)}(R), \tag{24}$$

where $\chi_{J_{Q3}}$ denote the Pauli spinors for the heavy quark and $C^{J'\,J'_3}_{J, J_3\, J_Q\, J_{Q3}}$ are the Clebsch–Gordan coefficients.

The matrix elements of the EM current (3) can be computed within the χQSM by representing them in terms of the functional integral in Euclidean space,

$$\begin{aligned}\langle B, p'|J_\mu(0)|B, p\rangle = \frac{1}{\mathcal{Z}} \lim_{T\to\infty} \exp\left(ip_4\frac{T}{2} - ip'_4\frac{T}{2}\right) \int d^3x d^3y \exp(-i\boldsymbol{p}'\cdot\boldsymbol{y} + i\boldsymbol{p}\cdot\boldsymbol{x}) \\ \times \int \mathcal{D}U \int \mathcal{D}\psi \int \mathcal{D}\psi^\dagger J_B(\boldsymbol{y},\, T/2)\psi^\dagger(0)\gamma_4\gamma_\mu\hat{Q}\psi(0)J^\dagger_B(\boldsymbol{x},\, -T/2) \\ \exp\left[-\int d^4z \psi^\dagger iD(U)\psi\right],\end{aligned} \tag{25}$$

where the baryon states $|B, p\rangle$ and $\langle B, p'|$ are, respectively, defined by

$$\begin{aligned}|B, p\rangle &= \lim_{x_4\to-\infty} \exp(ip_4x_4)\frac{1}{\sqrt{\mathcal{Z}}} \int d^3x \exp(i\boldsymbol{p}\cdot\boldsymbol{x})J^\dagger_B(\boldsymbol{x},\, x_4)|0\rangle, \\ \langle B, p'| &= \lim_{y_4\to\infty} \exp(-ip'_4y_4)\frac{1}{\sqrt{\mathcal{Z}}} \int d^3y \exp(-i\boldsymbol{p}'\cdot\boldsymbol{y})\langle 0|J^\dagger_B(\boldsymbol{y},\, y_4).\end{aligned} \tag{26}$$

The heavy baryon current J_B can be constructed from the $N_c - 1$ valence quarks

$$J_B(x) = \frac{1}{(N_c-1)!}\epsilon_{i_1\cdots i_{N_c-1}}\Gamma^{\alpha_1\cdots\alpha_{N_c-1}}_{JJ_3TT_3Y}\psi_{\alpha_1 i_1}(x)\cdots\psi_{\alpha_{N_c-1}i_{N_c-1}}(x), \tag{27}$$

where $\alpha_1\cdots\alpha_{N_c-1}$ represent spin-flavor indices and $i_1\cdots i_{N_c-1}$ color indices. The matrices $\Gamma^{\alpha_1\cdots\alpha_{N_c-1}}_{JJ_3TT_3Y}$ are taken to consider the quantum numbers JJ_3TT_3Y of the $N_c - 1$ soliton. The creation operator $J^\dagger_B$ can be constructed in a similar way. The calculation of the baryonic correlation function given in Eq. (25) is a tedious one, so we will present here only the final expressions for the $E2$ form factor. As for the detailed formalism, we refer to Refs. [15,16].

The final expressions for the electric quadrupole form factors of the baryon sextet with spin 3/2 can be written as

$$G_{E2}^{B_6}(Q^2) = 6\sqrt{5}\frac{M_B^2}{|\boldsymbol{q}|^2}\int d^3z\, j_2(|\boldsymbol{q}||\boldsymbol{z}|)\mathcal{G}_{E2}^B(\boldsymbol{z}), \tag{28}$$

where $j_2(|\boldsymbol{q}||\boldsymbol{z}|)$ stands for the spherical Bessel function with order 2 and the corresponding density of the $E2$ form factors is given as

$$\begin{aligned}\mathcal{G}_{E2}^B(\boldsymbol{z}) =& -2\left(\frac{3}{I_1}\langle D_{Q3}^{(8)}J_3\rangle_B - \frac{1}{I_1}\langle D_{Qi}^{(8)}J_i\rangle_B\right)\mathcal{I}_{1E2}(\boldsymbol{z}) \\ &+ 4m_8\left(\frac{K_1}{I_1}\mathcal{I}_{1E2}(\boldsymbol{z}) - \mathcal{K}_{1E2}(\boldsymbol{z})\right)\left(3\langle D_{83}^{(8)}D_{Q3}^{(8)}\rangle_B - \langle D_{8i}^{(8)}D_{Qi}^{(8)}\rangle_B\right).\end{aligned} \tag{29}$$

The densities of $E2$ form factors $\mathcal{I}_{1E2}$ and $\mathcal{K}_{1E2}$ can be found in Appendix A. In the limit of $m_Q \to \infty$, the charge distribution of the heavy quark becomes a point-like static charge given as $\rho_Q(\boldsymbol{r}) = e_Q\delta^{(3)}(\boldsymbol{r})$. This leads to $\mathcal{Q}_{ij} = \int d^3r\rho_Q(\boldsymbol{r})(3r_ir_j - r^2\delta_{ij}) = 0$. This implies that the $E2$ form factors of the singly heavy baryons are solely governed by the light quarks in the $m_Q \to \infty$ limit.

Having calculated the matrix elements of the collective operators in Eq. (29), we arrive at the final expressions for the $E2$ form factors of the baryon sextet with spin 3/2:

$$\mathcal{G}_{E2}^B(\boldsymbol{z}) = \mathcal{G}_{E2}^{B(0)}(\boldsymbol{z}) + \mathcal{G}_{E2}^{B(\mathrm{op})}(\boldsymbol{z}) + \mathcal{G}_{E2}^{B(\mathrm{wf})}(\boldsymbol{z}), \tag{30}$$

where $\mathcal{G}_{E2}^{B(0)}$, $\mathcal{G}_{E2}^{B(\mathrm{op})}$, and $\mathcal{G}_{E2}^{B(\mathrm{wf})}$ denote, respectively, the symmetric terms, the flavor SU(3) symmetry-breaking terms from the effective chiral action, and those from the mixed collective wavefunctions, expressed explicitly as

$$\mathcal{G}_{E2}^{B_6(0)}(\boldsymbol{z}) = \frac{3}{10}\frac{1}{I_1}\mathcal{Q}_B\mathcal{I}_{1E2}(\boldsymbol{z}), \tag{31}$$

$$\mathcal{G}_{E2}^{B_6(\mathrm{op})}(\boldsymbol{z}) = -\frac{1}{405}m_s\left(\frac{K_1}{I_1}\mathcal{I}_{E2}(\boldsymbol{z}) - \mathcal{K}_{E2}(\boldsymbol{z})\right)\begin{pmatrix} 6\mathcal{Q}_{\Sigma_c^*}+1 \\ -24\mathcal{Q}_{\Xi_c^*}-13 \\ 9\end{pmatrix}, \tag{32}$$

$$\mathcal{G}_{E2}^{B_6(\mathrm{wf})}(\boldsymbol{z}) = -\frac{2}{I_1}\left[q_{\overline{15}}\begin{pmatrix} -\frac{2}{9\sqrt{5}}(3\mathcal{Q}_{\Sigma_c^*}-4) \\ -\frac{1}{18\sqrt{5}}(15\mathcal{Q}_{\Xi_c^*}-2) \\ 0\end{pmatrix} + q_{\overline{24}}\begin{pmatrix} -\frac{1}{180}(3\mathcal{Q}_{\Sigma_c^*}+5) \\ -\frac{1}{90}(3\mathcal{Q}_{\Xi_c^*}+5) \\ \frac{3}{40}\mathcal{Q}_{\Omega_c^*}\end{pmatrix}\right]\mathcal{I}_{1E2}(\boldsymbol{z}), \tag{33}$$

where $\mathcal{Q}_B$ stands for the charge of the light-quark components of the corresponding baryons. We can derive similar sum rules for the electric quadrupole moments of singly heavy baryons with spin 3/2 as follows [13]

$$\begin{aligned}&\sum_{B\in\mathrm{sextet}}\mathcal{Q}_B = 0, \\ &\mathcal{Q}_{\Sigma_c^{*0}} = \mathcal{Q}_{\Xi_c^{*0}} = \mathcal{Q}_{\Omega_c^{*0}} = -2\mathcal{Q}_{\Sigma_c^{*+}} = -2\mathcal{Q}_{\Xi_c^{*+}} = -\frac{1}{2}\mathcal{Q}_{\Sigma_c^{*++}}.\end{aligned} \tag{34}$$

Even though the flavor SU(3) symmetry is broken, we still can find the following sum rules

$$\mathcal{Q}_{\Sigma_c^{*++}} - \mathcal{Q}_{\Sigma_c^{*+}} = \mathcal{Q}_{\Sigma_c^{*+}} - \mathcal{Q}_{\Sigma_c^{*0}},$$
$$\mathcal{Q}_{\Sigma_c^{*0}} - \mathcal{Q}_{\Xi_c^{*0}} = \mathcal{Q}_{\Xi_c^{*0}} - \mathcal{Q}_{\Omega_c^{*0}},$$
$$2(\mathcal{Q}_{\Sigma_c^{*+}} - \mathcal{Q}_{\Xi_c^{*0}}) = \mathcal{Q}_{\Sigma_c^{*++}} - \mathcal{Q}_{\Omega_c^{*0}}. \tag{35}$$

3. Results and discussion

In the χQSM, there are several parameters to fix. Since the sea-quark or Dirac-sea contributions contain divergent integrals, one has to introduce a regularization to tame the divergences. In the present work, we introduce the proper-time regularizations with the cutoff mass. This can be fixed by using the pion decay constant $f_\pi = 93$ MeV. The average mass of the up and down current quarks $\overline{m}$ is determined by the physical pion mass $m_\pi = 140$ MeV (see Appendix B for details). While the mass of the strange current quark $m_{\rm s}$ can also be fixed by reproducing the kaon mass, which gives $m_{\rm s} = 150$ MeV, we prefer to use $m_{\rm s} = 180$ MeV, since this value of $m_{\rm s}$ yields the best results for the hyperon mass splittings [15,18]. The remaining parameter is the dynamical quark mass M, which is the only free parameter of the model. However, $M = 420$ MeV is known to be the best value in reproducing various observables in the light baryon sector [15]. Thus, we will also use this value in the present calculation.

It was shown that in the calculation of the $E2$ form factors of the baryon decuplet the sea-quark contributions turn out to be rather important; we will first examine the valence- and sea-quark contributions separately. In Fig. 1, we draw the numerical results for the $E2$ form factors of the baryon sextet with spin 3/2. As expected, the general behaviors of the valence- and sea-quark contributions to the $E2$ form factors of the heavy singly baryons are rather similar to those of the baryon decuplet. As shown in Fig. 1, the valence-quark contributions decrease mildly as Q^2 increases, whereas the sea-quark or Dirac-sea contributions fall off drastically in the smaller Q^2 region, so that they govern the Q^2 dependence of the $E2$ form factors. In particular, the magnitudes of the sea-quark contributions are considerably larger than in the region of smaller Q^2. Thus, they provide the main contributions to the electric quadrupole moments of the baryon sextet with spin 3/2. Considering the fact that the electric quadrupole moment shows how the corresponding baryon is deformed, the present results provide certain physical implications. Recent investigations into the gravitational form factors of baryons within the χQSM indicate that the valence quarks are mainly located in the inner part of a baryon, while the sea quarks lie in its outer part [19,20]. Thus, the sea-quark contributions, which can also be interpreted as pion clouds, mainly describe how a singly heavy baryon with spin 3/2 is deformed. The present results are in line with what was discussed in Ref. [12], where the significance of the pion clouds in the electric quadrupole moment of the Δ isobar was studied.

In Fig. 2, we show how much the effects of flavor SU(3) symmetry breaking contribute to the $E2$ form factors of the baryon sextet with spin 3/2. As expressed in Eqs. (32) and (33), there are two different $m_{\rm s}$ corrections to the $E2$ form factors. The first one, $\mathcal{G}_{E2}^{B_6({\rm op})}(Q^2)$, arises from the current-quark mass term in the effective chiral action given in Eq. (7), whereas the second one comes from the wavefunction corrections (20). Each correction affects $E2$ form factors in a different way, as shown in Fig. 3. The wavefunction corrections to the $E2$ form factor of Σ_c^{*++} are negligibly tiny and the corrections from the current-quark mass term are also small. As a result, the $m_{\rm s}$ corrections turn out to be negligible, as shown in the upper left-hand panel of Fig. 2. On the other hand, the wavefunction corrections contribute noticeably to the $E2$ form factors of Σ_c^{*+}, while those from the

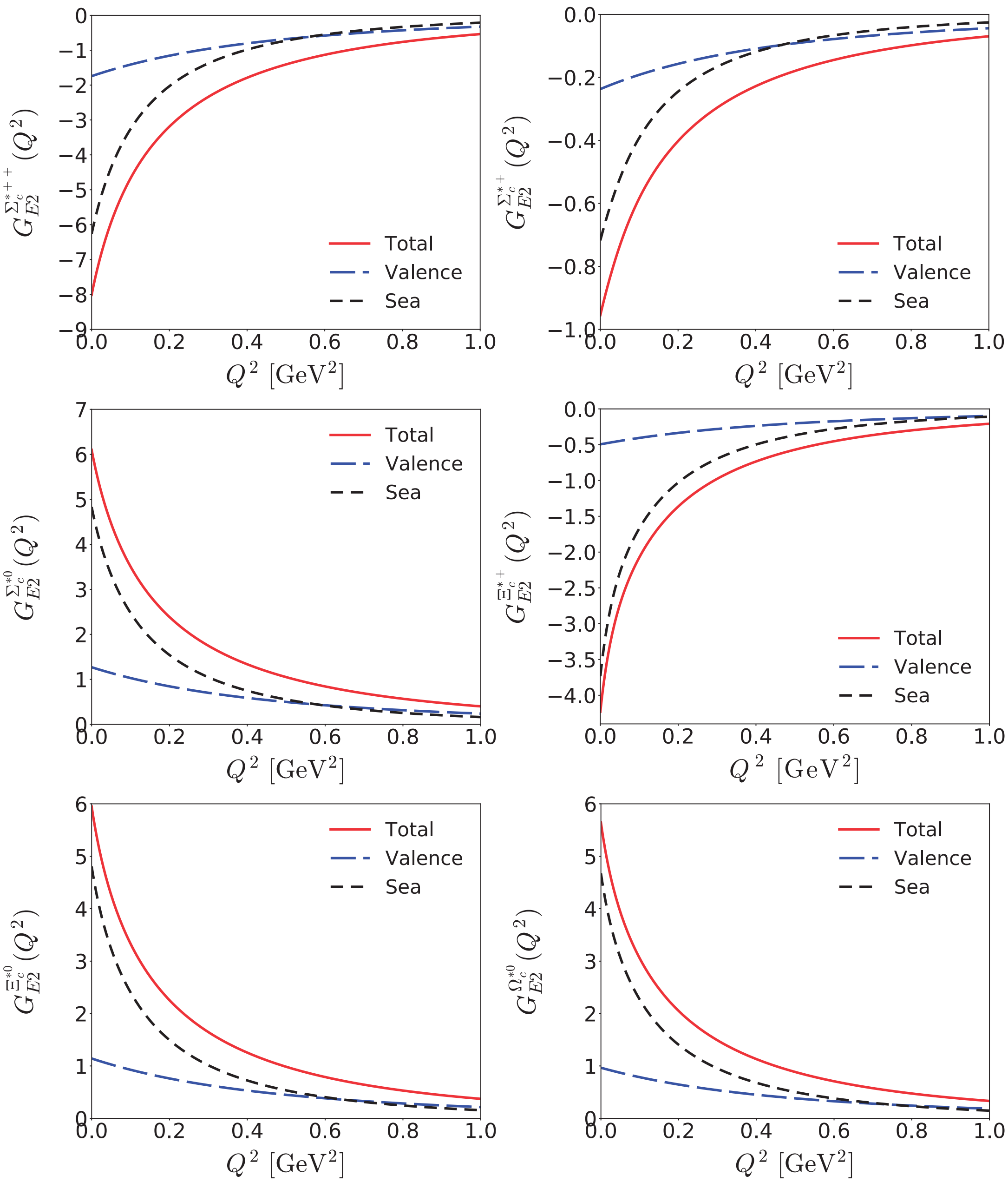

Fig. 1. Valence- and sea-quark contributions to the electric quadrupole form factors of the baryon sextet with spin 3/2. The long-dashed curves draw the valence-quark contributions to the $E2$ form factors, whereas the short-dashed ones depict the sea-quark contributions. The solid ones represent the total results for the $E2$ form factors.

current-quark mass term are of the same order as in the case of Σ_c^{*++}. In the case of Σ_c^{*0} and Ξ_c^{*0}, the wavefunction corrections to $G_{E2}^{\Sigma_c^{*0},\Xi_c^{*0}}$ are even larger than those from the mass term. This can be understood by examining Eqs. (32) and (33).

In the left-hand panel of Fig. 4, we compare the results for the $E2$ form factors of the Ω_c^{*0} baryon with that from the lattice calculation. We employ for this comparison the unphysical pion mass $m_\pi = 156$ MeV that is used in the lattice calculation. Note that there is only one lattice data with large uncertainty.

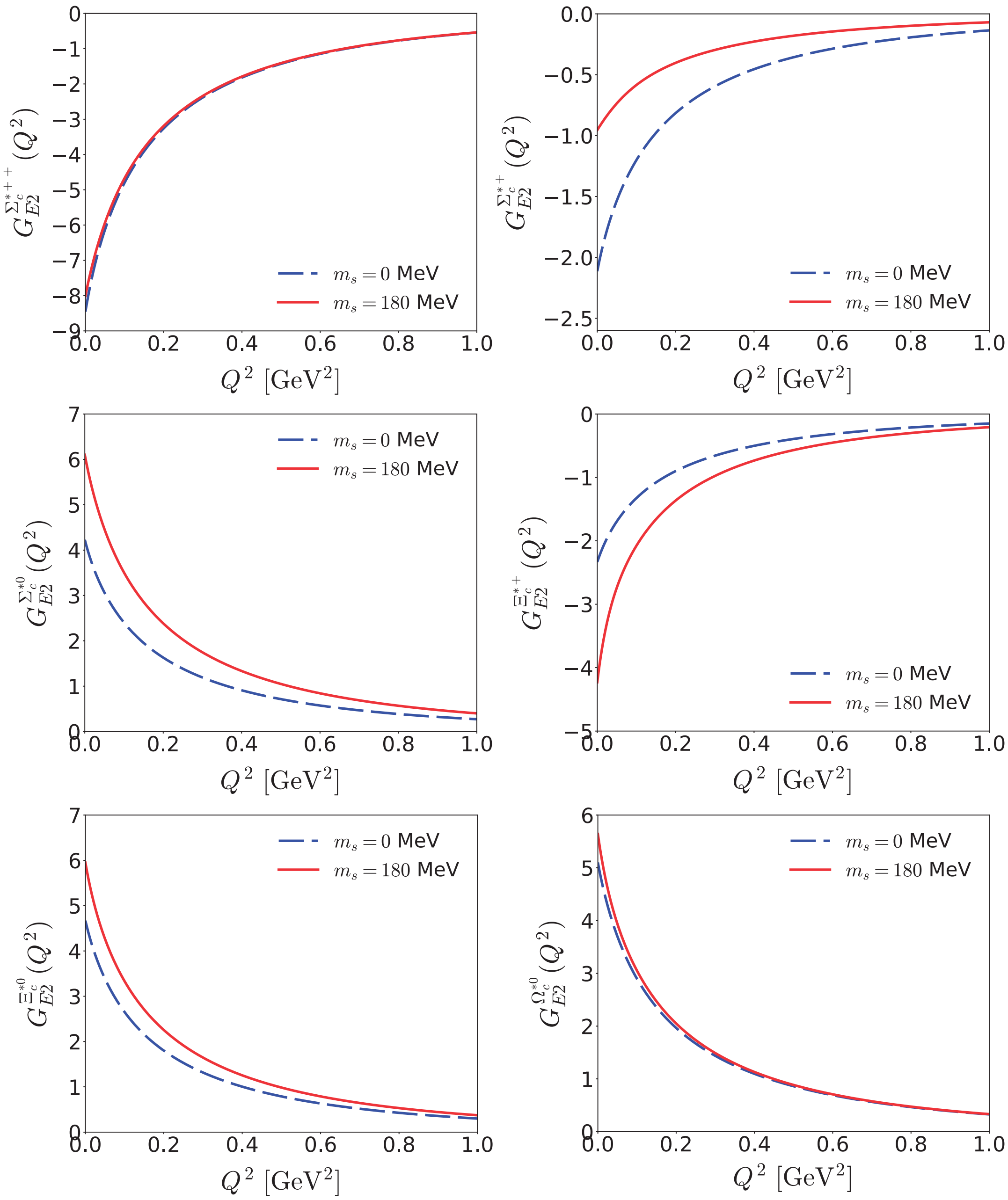

Fig. 2. The effects of flavor SU(3) symmetry breaking on the electric quadrupole form factors of the baryon sextet with spin 3/2. The dashed curves draw the results for the $E2$ form factors without the m_s corrections, whereas the solid curves depict the results with the effects of flavor SU(3) symmetry breaking taken into account.

We anticipate more accurate lattice data in the near future, so that one can draw a clear conclusion. In the right-hand panel of Fig. 4, we depict the results of $G_{E2}^{\Omega_c^{*0}}$ as a function of the pion mass m_π with $Q^2 = 0.183$ GeV^2 fixed. As expected, the present results fall off slowly as m_π increases.

For completeness, we present the results for the electric quadrupole moments of the baryon sextet with spin 3/2. Table 1 lists those of the $\mathcal{Q}_B$ in the second and third rows, which correspond to the SU(3) symmetric and breaking cases, respectively. As already shown in Fig. 2, those of the charged baryon sextet have negative values of $\mathcal{Q}_B$, which indicates that the positively charged singly heavy

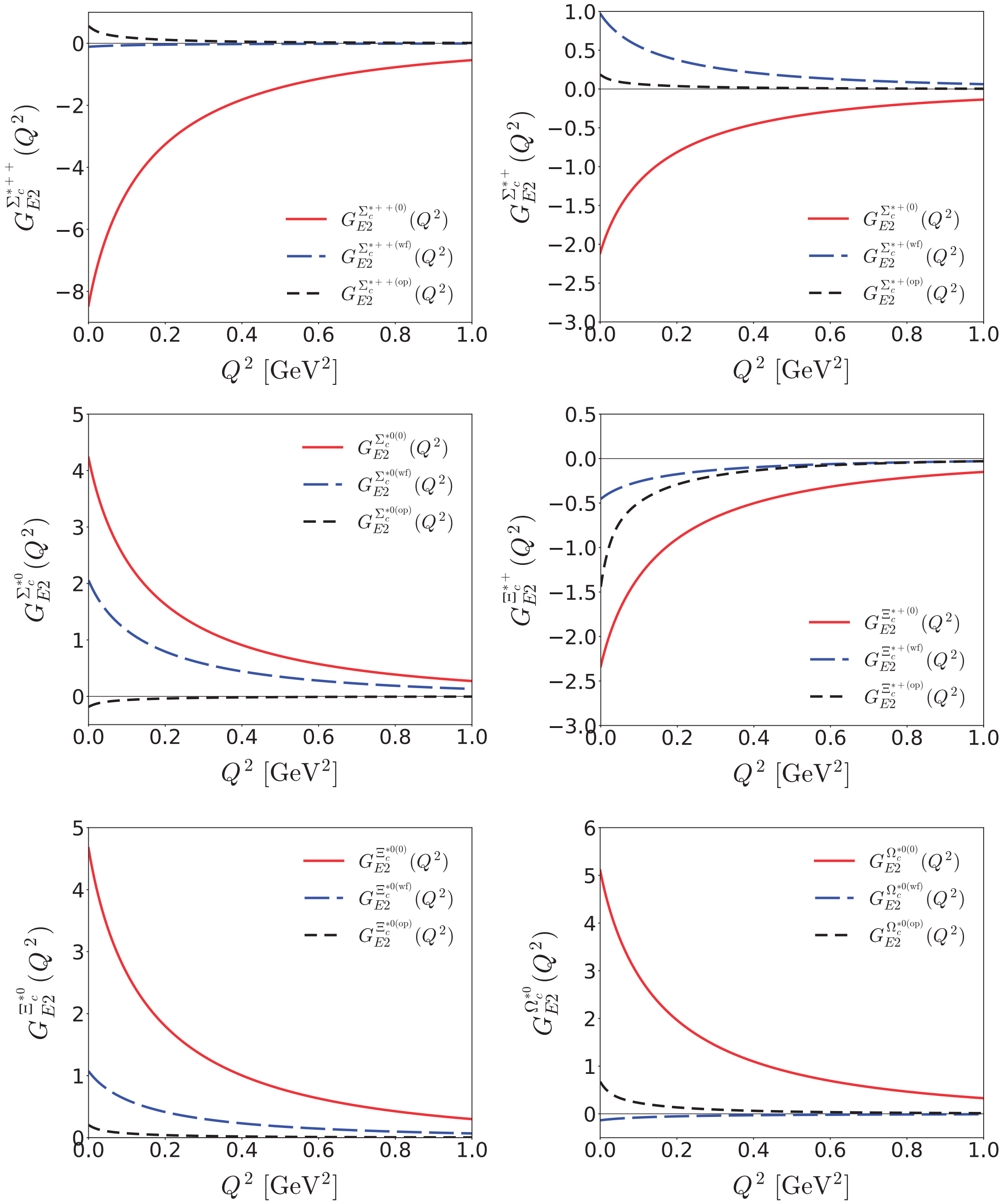

Fig. 3. Linear m_s corrections from the current-quark mass term in the effective chiral action $G_{E2}^{B_c^*(\mathrm{op})}$ and from the collective wavefunctions $G_{E2}^{B_c^*(\mathrm{wf})}$, which are drawn using short-dashed and long-dashed curves, respectively.

baryons with spin 3/2 take oblate shapes. On the other hand, those of the neutral ones get positive values, so they are distorted in prolate forms. It is interesting to see that the $\mathcal{Q}_B$ of the doubly positive-charged Σ_c^* is approximately 8 times larger than that of the singly positive-charged one. This can be understood by examining Eq. (30).

4. Summary and conclusion

In the present work, we have investigated the electric quadrupole form factors of the lowest-lying singly heavy baryons with spin 3/2 in a pion mean-field approach, also known as the SU(3) chiral

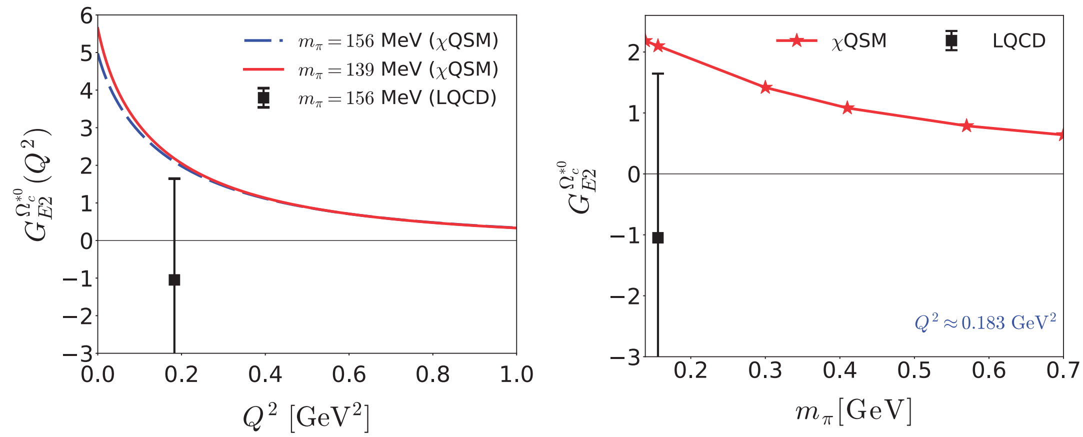

Fig. 4. Electric quadrupole form factors of the baryon sextet with spin 3/2 in comparison with the data from the lattice QCD. The data of the lattice QCD is taken from Ref. [9].

Table 1. Electric quadrupole moments of the baryon sextet.

$\mathcal{Q}_B$ [$e\cdot$ fm^2]	Σ_c^{*++}	Σ_c^{*+}	Σ_c^{*0}	Ξ_c^{*+}	Ξ_c^{*0}	Ω_c^{*0}
$m_s = 180$ MeV	-0.0490	-0.0058	0.0373	-0.0234	0.0330	0.0286
$m_s = 0$ MeV	-0.0518	-0.0129	0.0259	-0.0129	0.0259	0.0259

quark-soliton model. In the limit of an infinitely heavy quark, a heavy quark inside a singly heavy baryon can be regarded as a mere static one. This means that the $N_c - 1$ light valence quarks govern the quark dynamics inside a heavy baryon. The presence of the $N_c - 1$ light valence quarks make the vacuum polarized, which produces the pion mean fields. The $N_c - 1$ valence quarks are bound by the attraction provided by the pion mean fields self-consistently, from which a soliton consisting of the $N_c - 1$ valence quarks arises. We call this soliton an $N_c - 1$ soliton. The singly heavy baryon can then be constructed by coupling the $N_c - 1$ soliton with a heavy quark. This is called the pion mean-field approach for the singly heavy baryons. Based on this pion mean-field approach, we computed the electric qudrupole form factors of the baryon sextet with spin 3/2, taking into account the rotational $1/N_c$ and linear m_s corrections.

We first examined the valence- and sea-quark contributions separately. As in the case of the baryon decuplet, the contributions from the sea quarks or the Dirac-sea level quarks govern the electric quadrupole form factors, in particular, in the smaller Q^2 region. Considering the fact that the electric quadrupole moment of a baryon provides information on how the baryon is deformed, we can draw the following physical implications: the deformation of a singly heavy baryon is also mainly governed by the sea-quark contributions or the pion cloud effects. We found a similar feature in the case of the baryon decuplet. The effects of the explicit flavor SU(3) symmetry breaking are also sizable except for the case of the Σ_c^{*++} and Ω_c^{*0}. Since there are two different linear m_s corrections, we have scrutinized each effect in detail. To compare the present results with those from the lattice calculation, we have computed the electric quadrupole form factor with the adopted unphysical value $m_\pi = 156$ MeV, which was used by the lattice work. We also showed how the value of the form factor at a fixed Q^2 is changed as the m_π increases. As expected from previous works, the value of the form factor falls off as m_π increases. We also presented the results for the electric quadrupole moment. The charged singly-heavy baryons have consistently negative values of electric quadrupole moments. This indicates that the charged baryons take oblate shapes. On the other hand, the neutral baryons take prolate shapes, having positive values of electric quadrupole moments.

Acknowledgements

The authors are grateful to Gh.-S. Yang for valuable discussions. They want to express the gratitude to M. Oka and K. U. Can for providing us with the lattice data. The present work was supported by Basic Science Research Program through the National Research Foundation of Korea funded by the Ministry of Education, Science and Technology (2018R1A2B2001752 and 2018R1A5A1025563). J.-Y. Kim acknowledges partial support by the Deutscher Akademischer Austauschdienst (DAAD) doctoral scholarship.

Funding

Open Access funding: SCOAP³.

Appendix A. Densities for the $E2$ form factor and moments of inertia

In this Appendix, we provide the explicit expressions for the $\mathcal{I}_{1E2}$ and $\mathcal{K}_{1E2}$ densities of the electric quadrupole form factors in Eq. (29):

$$\begin{aligned}\mathcal{I}_{1E2}(z) &= -\frac{(N_c-1)}{2\sqrt{10}}\sum_{n\neq\mathrm{val}}\frac{1}{E_n-E_{\mathrm{val}}}\langle\mathrm{val}|\boldsymbol{\tau}|n\rangle\cdot\langle n|z\rangle\{\sqrt{4\pi}Y_2\otimes\tau_1\}_1\langle z|\mathrm{val}\rangle\\&\quad+\frac{N_c}{4\sqrt{10}}\sum_{n,m}\mathcal{R}_3(E_n,E_m)\langle n|\boldsymbol{\tau}|m\rangle\cdot\langle m|z\rangle\{\sqrt{4\pi}Y_2\otimes\tau_1\}_1\langle z|n\rangle,\\\mathcal{K}_{1E2}(z) &= -\frac{(N_c-1)}{2\sqrt{10}}\sum_{n\neq\mathrm{val}}\frac{1}{E_n-E_{\mathrm{val}}}\langle\mathrm{val}|\gamma^0\boldsymbol{\tau}|n\rangle\cdot\langle n|z\rangle\{\sqrt{4\pi}Y_2\otimes\tau_1\}_1\langle z|\mathrm{val}\rangle\\&\quad-\frac{N_c}{4\sqrt{10}}\sum_{n,m}\mathcal{R}_5(E_n,E_m)\langle n|\gamma^0\boldsymbol{\tau}|m\rangle\cdot\langle m|z\rangle\{\sqrt{4\pi}Y_2\otimes\tau_1\}_1\langle z|n\rangle,\end{aligned}\tag{A.1}$$

where the regularization functions are defined by

$$\begin{aligned}\mathcal{R}_3(E_n,E_m) &= \frac{1}{2\sqrt{\pi}}\int_0^\infty\phi(u)\frac{du}{\sqrt{u}}\left[\frac{e^{-uE_m^2}-e^{-uE_n^2}}{u(E_n^2-E_m^2)}-\frac{E_me^{-uE_m^2}+E_ne^{-uE_n^2}}{E_n+E_m}\right],\\\mathcal{R}_5(E_n,E_m) &= \frac{\mathrm{sign}(E_n)-\mathrm{sign}(E_m)}{2(E_n-E_m)},\end{aligned}\tag{A.2}$$

with the proper-time regulator $\phi(u)$ [15]. Here, $|\mathrm{val}\rangle$ and $|n\rangle$ denotes the state of the valence and sea quarks with the corresponding eigenenergies E_{val} and E_n of the single-quark Hamiltonian $h(U_c)$, respectively.

The moments of inertia (I_1, I_2) and anomalous moments of inertia (K_1, K_2) are expressed respectively as

$$\begin{aligned}I_1 &= \frac{(N_c-1)}{6}\sum_{n\neq\mathrm{val}}\frac{1}{E_n-E_{\mathrm{val}}}\langle\mathrm{val}|\boldsymbol{\tau}|n\rangle\cdot\langle n|\boldsymbol{\tau}|\mathrm{val}\rangle+\frac{N_c}{12}\sum_{n,m\neq n}\langle m|\boldsymbol{\tau}|n\rangle\cdot\langle n|\boldsymbol{\tau}|m\rangle\mathcal{R}_3(E_n,E_m),\\I_2 &= \frac{(N_c-1)}{4}\sum_{n^0}\frac{1}{E_{n^0}-E_{\mathrm{val}}}\langle\mathrm{val}|n^0\rangle\langle n^0|\mathrm{val}\rangle+\frac{N_c}{4}\sum_{n^0,m}\langle m|\boldsymbol{\tau}|n^0\rangle\langle n^0|m\rangle\mathcal{R}_3(E_{n^0},E_m),\\K_1 &= \frac{(N_c-1)}{6}\sum_{n\neq\mathrm{val}}\frac{1}{E_n-E_{\mathrm{val}}}\langle\mathrm{val}|\boldsymbol{\tau}|n\rangle\cdot\langle n|\gamma^0\boldsymbol{\tau}|\mathrm{val}\rangle+\frac{N_c}{12}\sum_{n,m\neq n}\langle m|\boldsymbol{\tau}|n\rangle\cdot\langle n|\gamma^0\boldsymbol{\tau}|m\rangle\mathcal{R}_5(E_n,E_m),\end{aligned}$$

$$K_2 = \frac{(N_c-1)}{4}\sum_{n^0}\frac{1}{E_{n^0}-E_{\rm val}}\langle {\rm val}|n^0\rangle\langle n^0|\gamma^0|{\rm val}\rangle + \frac{N_c}{4}\sum_{n^0,m}\langle m|\boldsymbol{\tau}|n^0\rangle\langle n^0|\gamma^0|m\rangle\mathcal{R}_5(E_{n^0},E_m). \tag{A.3}$$

Appendix B. Fixing the model parameters

The chiral condensate and the pion decay constant can be derived from the effective chiral action given in Eq. (7). The chiral condensates are written as

$$\langle\overline{\psi}\psi\rangle = -\int\frac{d^4p_E}{(2\pi)^4}\frac{8N_cM}{p_E^2+M^2}\bigg|_{\rm reg} = M\frac{N_c}{2\pi^2}\int_0^\infty\phi(u)\frac{du}{u^2}e^{-uM^2}, \tag{B.1}$$

and the pion decay constants are given by

$$f_\pi^2 = -\int\frac{d^4p_E}{(2\pi)^4}\frac{4N_cM^2}{(p_E^2+M^2)^2}\bigg|_{\rm reg} = M^2\frac{N_c}{4\pi^2}\int_0^\infty\phi(u)\frac{du}{u}e^{-uM^2}, \tag{B.2}$$

with proper-time regulator $\phi = c\theta(u-\Lambda_1^{-2}) + (1-c)\theta(u-\Lambda_2^{-2})$. The pion mass is determined by the pole position of the pion propagator that is obtained by a low-energy effective chiral theory given by Eq. (7):

$$m_\pi^2 = \frac{\overline{m}\langle\overline{\psi}\psi\rangle}{f_\pi^2} + \mathcal{O}(\overline{m}^2). \tag{B.3}$$

The above expressions satisfy the Gell-Mann–Oakes–Renner (GMOR) relation. With Eqs. (B.1), (B.2) and (B.3), one can determine the cut-off mass. The average value of the up and down current quark masses is obtained as $\overline{m} = 6.13$ MeV. The strange current quark mass m_s is fixed by the hyperon mass splittings, by treating m_s perturbatively up to the second-order corrections [15,17,18]. The preferable value of m_s is found to be $m_s = 180$ MeV.

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