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Electrical and Mathematical Modeling of Supercapacitors: Comparison

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Abstract: Supercapacitors are energy storage devices with high electrical power densities and long spanlife. Therefore, supercapacitor-based energy storage systems have been employed for a variety of applications. The modelling and simulation of SCs have been of great interest to this objective. This paper presents an electrical schema and mathematical modelling of three models of supercapacitors. The first is the RC model, the second is the two-branch model and the third is the multi-branch model. The objective of this modelling is to choose the best model that can respect the same behaviour of the experimental model. These models are compared with an experimental model. This comparison prove that the response voltage of the multi-branch model correctly describes the behaviour of the experimental model of Belhachemi. The disadvantage of this model is the slow simulation duration in MATLAB/Simulink. The RC model represented the faster model in terms of simulation. The choice of 15 branches in parallel in multi-branch models gives good results and correctly describes the reel model. The automatic charge and discharge voltage of SCs reduce by reducing the charge current.

Keywords: supercapacitors; RC model; two-branch model; multi-branch model

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1. Introduction

Research on the development of high-performance technologies and power devices has been extensively pursued by many researchers in recent years due to the global energy crisis and deteriorating pollution [1,2]. Electrochemical energy storage devices are unavoidable parts of a clean energy portfolio [3,4]. Among these devices, supercapacitors (SCs) are electrochemical devices, electrochemical double layer capacitors or ultracapacitors are also common names for energy storage devices, whose storage mechanisms are based on a faradic process [5–8]. SCs are used for fast charging and discharging.

However, SCs are cited between traditional capacitors and batteries [8]. They represent high power densities similar to battery. They are characterized by fast charge/discharge rates and long lifespans similar to capacitors [9–11]. The charge/ discharge cycles of SCs can exceed 100,000 cycles for short durations between 1 and 10 s under high currents that exceed a few hundreds of amps [12–15]. Due to this property, they have various applications such as in smart grids [15,16], electric vehicles, hybrid electric vehicles [17–21], uninterruptible power supplies [22,23] and wireless sensor networks [24,25].

SCs are spatially used in applications that need a high power in a short time such as vehicle acceleration. SCs are widely used in the recovery of energy during breaking vehicles [21]. SCs are used for fast frequency support from hybrid wind power plants [26].

From this perspective, much effort has been devoted to the appropriate design and the creation of new SC models with high energy densities [27–29].

This paper presents the mathematical modelling of three SC models. The first is the RC model, the second is the two-branch model and the third is the multi-branch model. These models are compared with the experimental model of Belhachemi. The electrical schema and simulation model of SCs in MATLAB/Simulink will be presented.

Some systems need a variable voltage ranging between tens and hundreds of volts. However, the output voltage of an SC is between 2.1 V and 2.7 V. To achieve the appropriate voltage for an application that needs a high voltage, SCs should be connected in series. To improve the current, SCs should be connected in parallel [5].

The remainder of this paper is structured as follows: Section 2 develops the modelling of RC model, the two-branch model and the multi-branch model of SCs. A comparison and the simulation test results of the different models of the SC are presented in Section 3. Section 4 provides the conclusions of the study.

2. Modelling of Supercapacitors

2.1. RC Model of the Supercapacitor

An SC can be schematized by a series resistance R_{sc}, a leakage resistance R_f and a storage capacitor C_{sc}, as illustrated in Figure 1a, where R_f describes the behaviour of the component during the self-discharge [30].

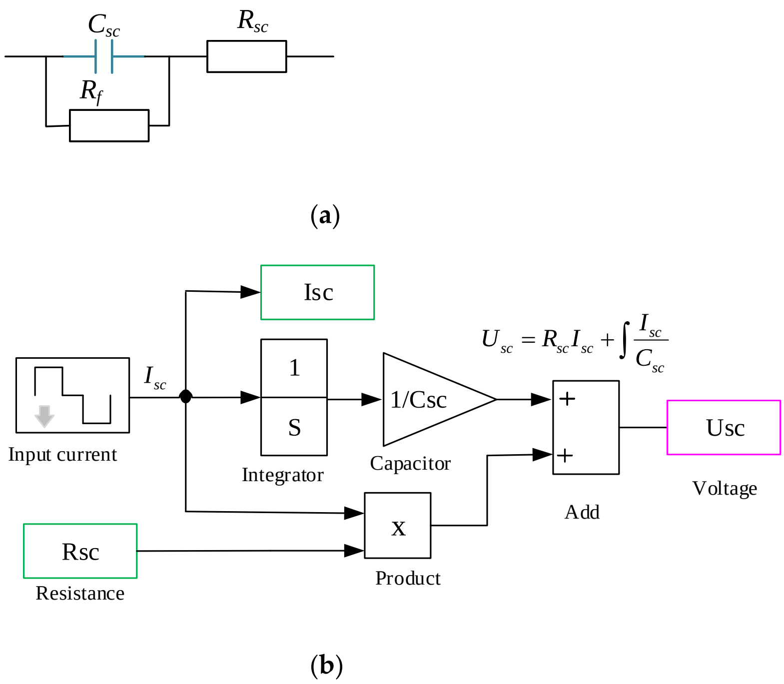


Figure 1. (**a**) RC model of the SC. (**b**) RC model of SC under MATLAB/Simulink by negleging R_f.

This basic representation is important. It provides a first idea about SCs. Calculating the equivalent resistance and capacity at a simple discharge test with a constant current is possible. The difference in voltage level between the end of the discharge phase is five seconds and represents the image of the series resistance. The image of the storage capacity is provided by the voltage drop between the initial state (state of rest before discharge) and the final state (five seconds after the discharge).

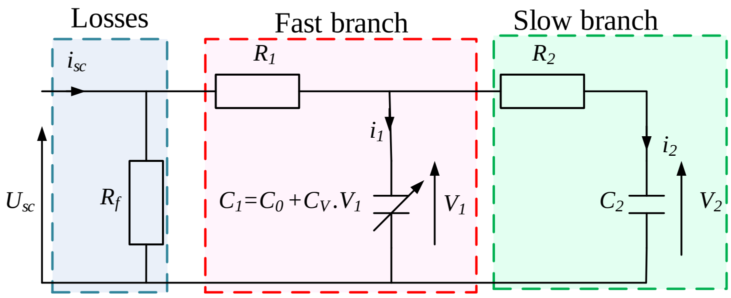
The modelling of the RC model of the SC in the MATLAB/Simulink environment is shown in Figure 1b by neglecting the leakage current.

2.2. Two-Branch Model of SCs

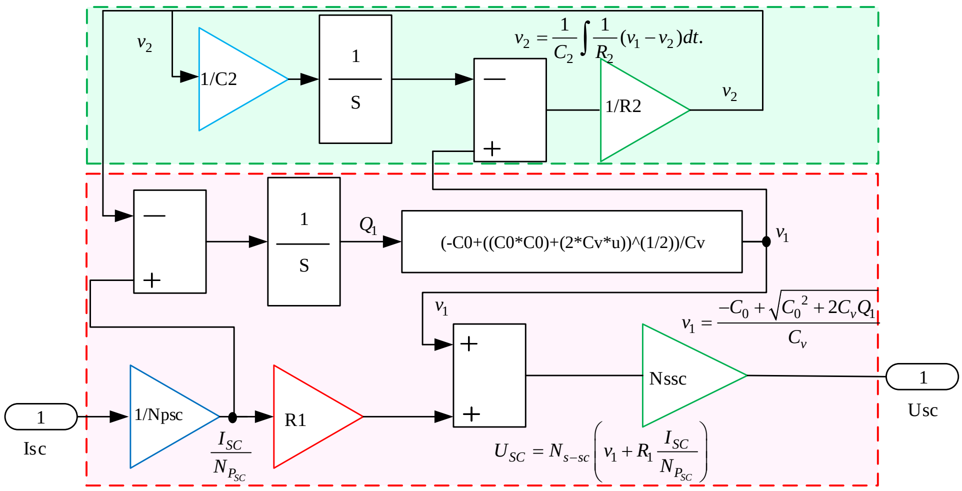
The RC two-branch model is used to describe the behaviour of the system by decomposing the response of the last into several parts. Every part is represented a different constant time.

This model, developed by the Canadians Bonert and Zubieta, is composed of:

- A leakage resistance;
- Two branches in which capacity is not linear and the voltage is different (Figure 2a) [31,32].



(**a**)



(**b**)

Figure 2. (**a**) SC model with two branches [31]. (**b**) Representation of the two-branch RC model in MATLAB/Simulink.

Zubieta and Bonert used this idea to model the SC. They decomposed the response of a SC into two cells:

- The first cell is the fast branch, which takes into account the charging phases instead of a propagation system. It models this phase by a resistance R_1 and a non-linear capacitance C_1 (no phenomenon of propagation of charges).

The main capacitance C_1 is composed of a constant capacitance C_0 and a constant parameter C_v. This capacity is given in terms of the voltage between its terminals v_1 by the following equation:

$$C_1 = C_0 + C_v.v_1. \tag{1}$$

where v_1 is the voltage of C_1.

- The second cell is the slow branch that represents the redistribution phase of the charges during the rest phase. This phase is modelled by an R_2–C_2 branch with larger time constants than those taken for the fast phase.

The leakage resistance R_f symbolizes the self-discharge of the SC, which takes place after the charge redistribution phase.

By neglecting the leakage current, the voltage across the SC can be described by the following equation [22–32]:

$$U_{SC} = N_{s-sc}v_{sc} = N_{s-sc}\left(v_1 + R_1\frac{I_{SC}}{N_{P_{SC}}}\right) \tag{2}$$

where U_{SC} and I_{SC} are the voltage and current of the SCs, respectively. N_{s-sc} and N_{p-sc} are the number of parallel and serial connections of the SCs, respectively. The voltage v_2 is given by:

$$v_2 = \frac{1}{C_2}\int i_2 dt = \frac{1}{C_2}\int \frac{1}{R_2}(v_1 - v_2)dt. \tag{3}$$

Current i_1 is expressed in terms of instantaneous charge Q_1 and C_1 as follows:

$$i_1 = C_1\frac{dv_1}{dt} = \frac{dQ_1}{dt} = (C_0 + C_v.v_1)\frac{dv_1}{dt} \tag{4}$$

where the charge Q_1 is given by:

$$Q_1 = C_0.v_1 + \frac{1}{2}C_v.{v_1}^2 \tag{5}$$

The voltage v_1 is defined as follows:

$$v_1 = \frac{-C_0 + \sqrt{{C_0}^2 + 2C_v Q_1}}{C_v} \tag{6}$$

The modelling of the two-branch model of SC in the MATLAB/Simulink environment is shown in Figure 2b.

2.3. Multi-Branch Model of SC

The multi-branch model shown in Figure 3 complements the previous two-branch model, including the charge propagation phenomena appearing on the component voltage just after the sudden changes in current. This method uses a simplified model of the transmission line to represent the propagation of charges during the transient (fast phase) and attempts to better take into account the slow behaviour of SCs [32–36].

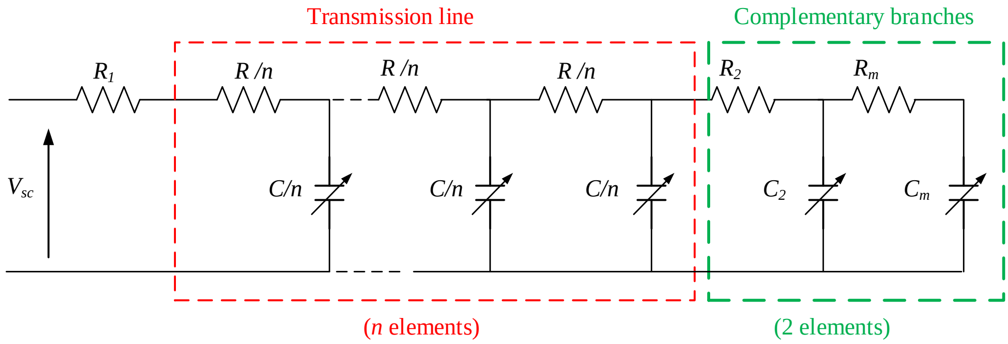


Figure 3. Multi-branch model of the SC.

This model consists of the following:

- An access resistor R_1 for to the transmission line;
- A non-linear transmission line of n branches in parallel, a total resistance R and a total capacitance C for a fine description of the electrical and energetic behaviours of SCs in short times;
- Some RC cells to apprehend the longer times.
- Complementary branches with capacitances C_m and resistances R_m, which will be identified by means of a constant-current partial-charge test, and phases of internal redistribution of energy.

The capabilities of this model vary depending on the voltage at these terminals. The nonlinear capacity model represented in MATLAB/Simulink is depicted in Figure 4.

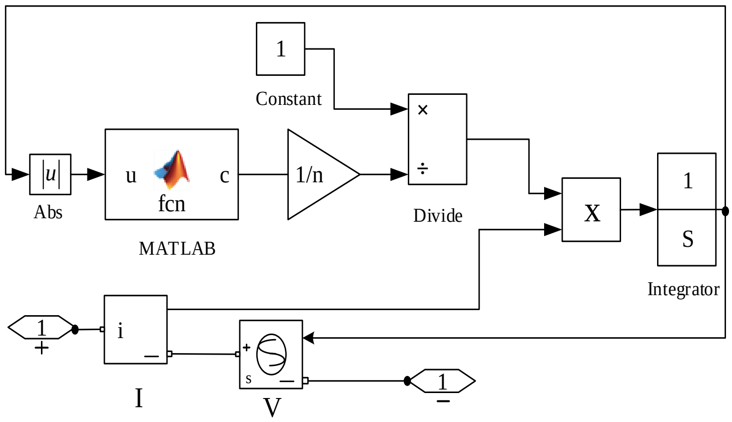


Figure 4. Composition of the block of the variable capacity.

3. Comparison of the Different Models of the SC

The purpose of this section is to validate the modelling of the different models (RC constructor, two-branch and multi-branch models) by comparing the results obtained by these models to those obtained experimentally by [36]. The SC type used in these simulation tests was the 2700 F Maxwell PC7223.

3.1. Parameters of Different Models

- RC model of the constructor:

The characteristics given by the constructor are:
The total capacity is C_{sc} = 2700 F;
The total resistance is R_{sc} = 0.85 mΩ;
The leakage current is Rf = 0 Ω.

- Parameters of the two-branch model

The extraction of the two-branch model parameters for the Maxwell PC7223 SC, based on the fully charged test with a constant current at 100 A, produced the parameters shown in Table 1.

Table 1. Parameters of the two-branch model of the SC.

Parameters	Values
R_1	0.8 mΩ
C_0	2170 F
C_v	520 F/V
R_2	1 Ω
C_2	150 F

- Parameters of the multi-branch model

Fifteen branches (n = 15) were proposed for the simulation of the multi-branch model. The identification parameters of a PC7223 SC are given in Table 2. The MATLAB function program is shown in Figure 5.

Table 2. Identified parameters of the multi-branch model of SC PC7223 [35].

Voltage (V)	Transmission Line R = 1.1 mΩ	Branch R_2C_2 R_2 = 100 mΩ	Branch R_3C_3 R_3 = 1 Ω
	Capacity *C* (F)	**Capacity C_2 (F)**	**Capacity C_3 (F)**
0 V, 0.5 V	$C = 2000 + 700\ v$	$C_2 = 90 + 30\ v$	$C_3 = 31 + 11\ v$
0.5 V, 1 V	$C = 2350 + 700\ (v - 0.5)$	$C_2 = 105 + 30\ (v - 0.5)$	$C_3 = 36.5 + 30\ (v - 0.5)$
1 V, 1.5 V	$C = 2700 + 500\ (v - 1)$	$C_2 = 120 + 22\ (v - 1)$	$C_3 = 42 + 8\ (v - 1)$
1.5 V, 2 V	$C = 2950 + 200\ (v - 1.5)$	$C_2 = 131 + 5\ (v - 1.5)$	$C_3 = 46 + 3\ (v - 1.5)$
$v > 2$ V	$C = 3050$	$C_2 = 133.5$	$C_3 = 51$

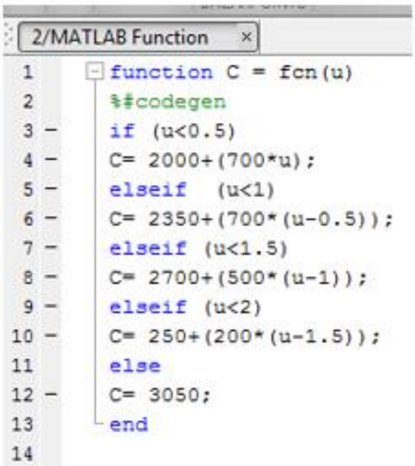


Figure 5. MATLAB function program.

- Parameters of the Belhachemi experimental model

  Table 3 shows the parameters measured several times by Belhachemi [36].

Table 3. Parameters of supercapacitor Maxwell PC 7223 [36].

Voltage (in V)	The Capacity of the Transmission Line	First Complementary Branch	Second Complementary Branch
0	2000	90	31
0.5	2350	105	36.5
1	2700	120	42
1.5	2950	131	46
2	3050	133.5	51
2.5	3050	133.5	51

where:

- The access resistance is R_1 = 0.5 mΩ;
- The total resistance R = 1.4 mΩ;
- The resistance of the first complementary branch R_2 = 100 mΩ;
- The resistance of the second complementary branch R_3 = 100 mΩ.

3.2. Simulation and Validation of the Different Models of the SC

A simulation test with a constant current, 100 A for the charge and −100 A for the discharge were proposed to compare the different models. This comparison is shown in Figure 6. The obtained results indicated that the response voltage of the multi-branch model correctly describes the behaviour of the experimental model of Belhachemi. When we increase the number of branches, the precision increases. Fifteen branches is not a fixed number. The simulation time of this model in MATLAB/Simulink was approximately half that the multi-branch model.

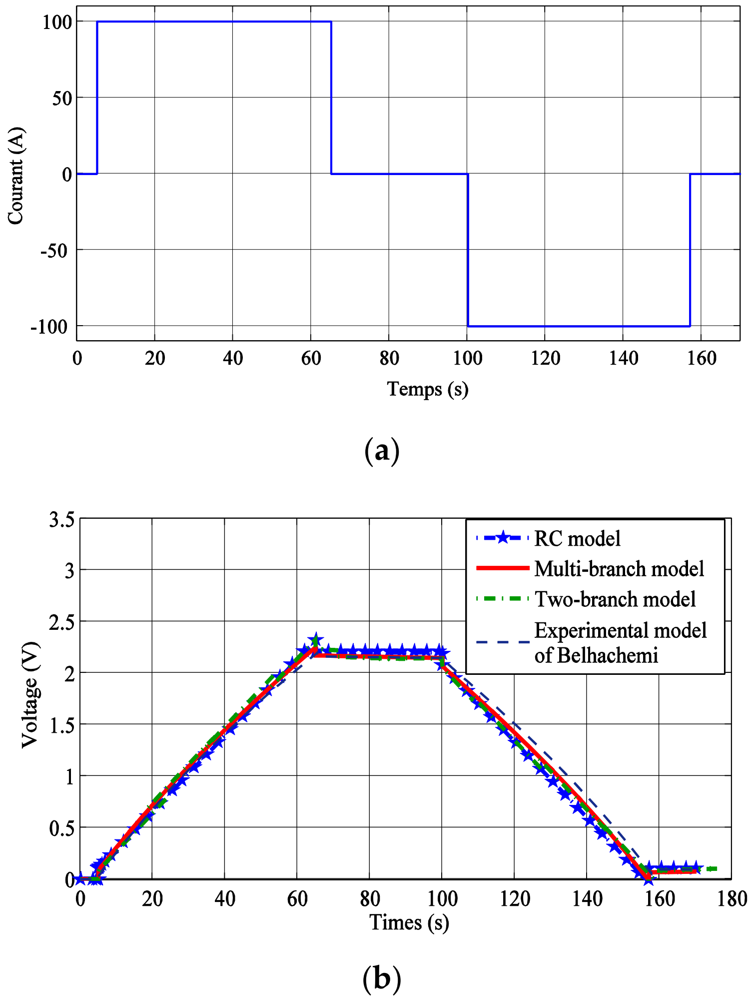


Figure 6. Comparison of the voltage of the different models of the SC type Maxwell PC7223: (**a**) SC current; (**b**) SC voltage.

3.3. Calculation of the Error between Different Models of the SC

The difference in errors between the experimental model and the RC, two-branch and multi-branch models is given by Figures 7–9, respectively. The RC model represents the very high error of 0.125 V. The two-branch model represents a medium error of 0.09 V. The multi-branch model represents the low error of 0.08 V. The jumps at t = 5, t = 65, t = 100 and t = 155 of voltage are caused by the sudden change of current represented in Figure 6a.

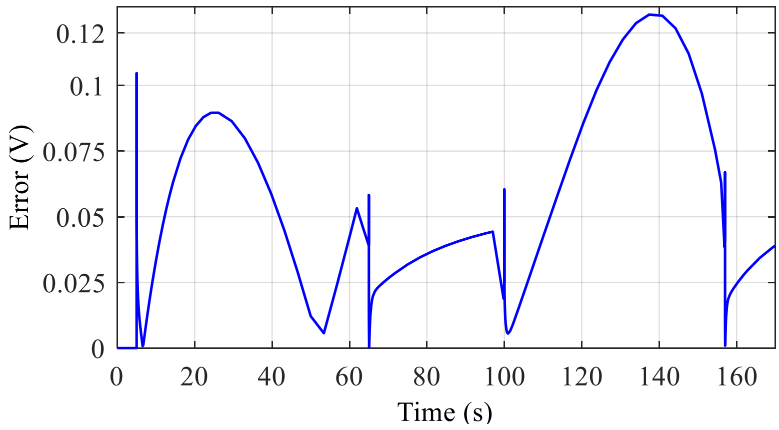


Figure 7. Error between experimental and RC models.

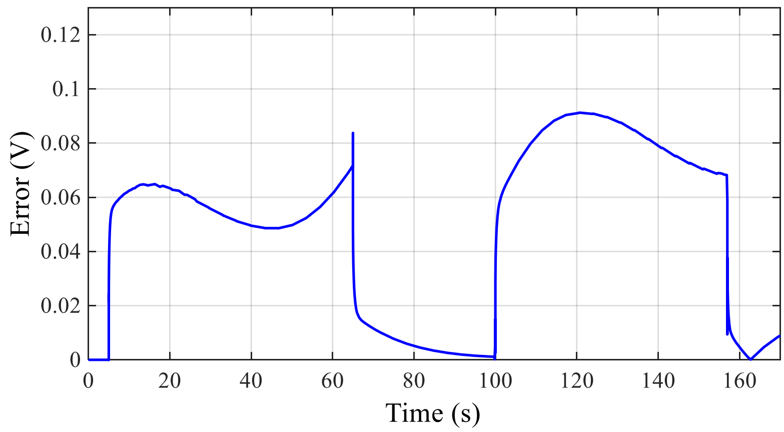


Figure 8. Error between experimental and two-branch models.

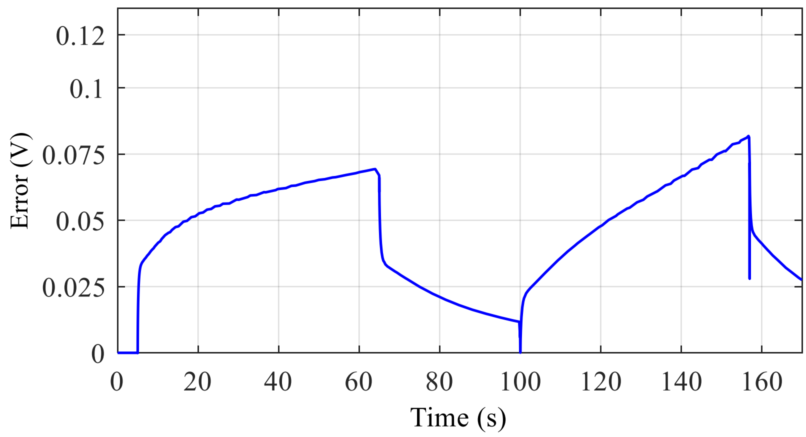
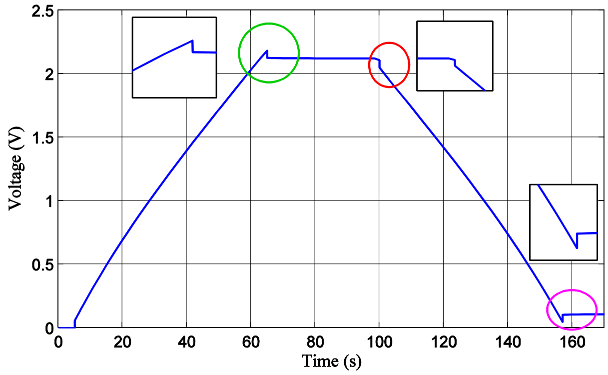


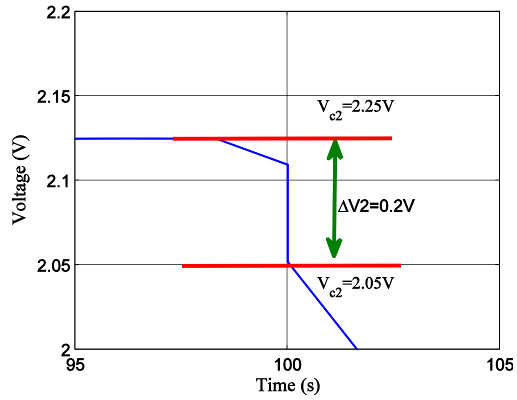
Figure 9. Error between experimental and multi-branch models.

3.4. Influence of the Charge Current on the Voltage

The charge and discharge of the SC with a current of 100 A and 10 A is given in Figures 10 and 11, respectively. With the discharge current of 100 A, the automatic discharge and charge have an important value at t = 65 s, t = 100 s and t = 155 s. The charge and discharge of an SC with a current of 10 A represent a very low automatic charge and discharge. The automatic charge and discharge voltage reduce by reducing the current.



(**a**)



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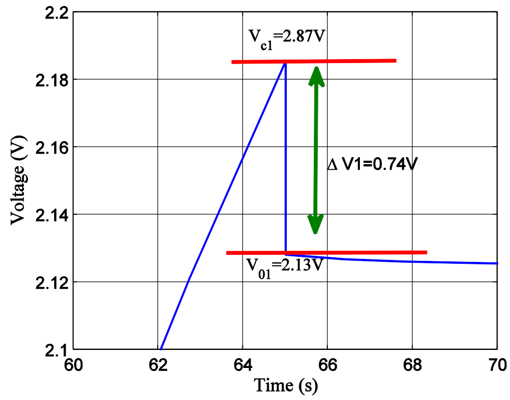
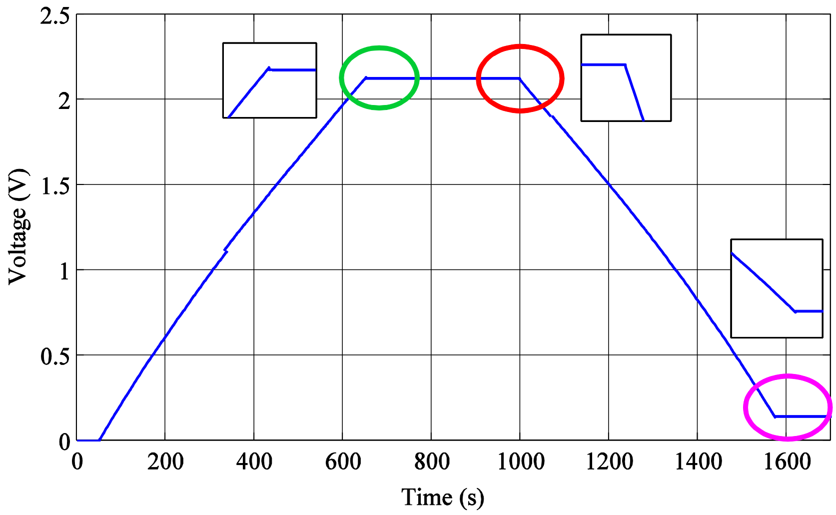


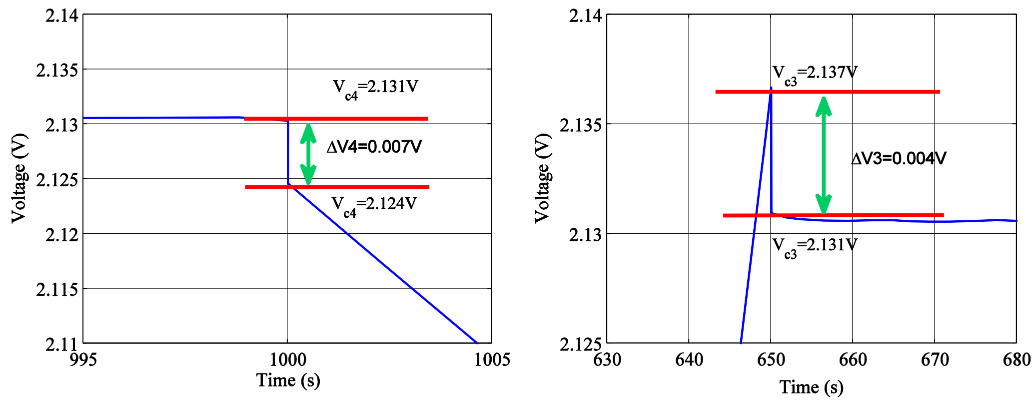
(**c**)

Figure 10. Charge and discharge of SC with a current of 100 A: (**a**) Charge and discharge curve of SC with a current of 100 A. (**b**) Self-discharge of SC with a current of 100 A after full charge. (**c**) Self-discharge of SC with a current of 100 A before charging.



(**a**)

Figure 11. *Cont.*



(**b**) (**c**)

Figure 11. Charge and discharge of SC with a current of 10 A: (**a**) Charge and discharge curve of SC with a current of 10 A. (**b**) Self-discharge of SC with a current of 10 A in the start of discharging. (**c**) Self-discharge of SC with a current of 10 A in the end of discharging.

4. Conclusions

The modelling of the RC, two-branch and multi-branch model of SCs are presented in this paper and compared with the experimental model of Belhachemi. This comparison demonstrates that the response voltage of the multi-branch model correctly describes the behaviour of the experimental model. the multi-branch model represents the best accuracy model and gives more precision. The disadvantage of this model is the slow simulation duration in MATLAB/Simulink. The RC model represented the faster model in terms of simulation. The choice of 15 branches in parallel in the multi-branch model gives good results and correctly describes the reel model. The automatic charge and discharge voltage of the SCs reduce by reducing the current.

This paper also presents the modelling of SCs in a MATLAB/Simulink environment of these models.

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