Electrical Centrality Measures for Electric Power Grid Vulnerability Analysis

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***Abstract*— This paper investigates measures of centrality that are applicable to power grids. Centrality measures are used in network science to rank the relative importance of nodes and edges of a graph. Here we define new measures of centrality for power grids that are based on its functionality. More specifically, the coupling of the grid network can be expressed as the algebraic equation $YU = I$, where U and I represent the vectors of complex bus voltage and injected current phasors; and Y is the network admittance matrix which is defined not only by the connecting topology but also by the network's electrical parameters and can be viewed as a complex-weighted Laplacian. We show that the relative importance analysis based on centrality in graph theory can be performed on power grid network with its electrical parameters taken into account. In the paper we experiment with the proposed electrical centrality measures on the NYISO-2935 system and the IEEE 300-bus system. We analyze the centrality distribution in order to identify important nodes or branches in the system which are of essential importance in terms of system vulnerability. We also present and discuss a number of interesting discoveries regarding the importance rank of power grid nodes and branches.**

I. INTRODUCTION

The electric power grid is one of the most critical infrastructures. The inter-connectivity of the power grid enables long-distance transmission for more efficient system operation; however, it also allows the propagation of disturbances in the network. The non-decreasing frequency of large cascading blackouts in the United States reveals the existence of intrinsic weakness in the large electric power grids. Studies on the power grid system structures and vulnerability analysis have attracted many research efforts in the past years (see [1] [2] [3] [4] [5] [6]).

It has been observed that the electric power grid network has a distinct topology. In [7] Wang, Scaglione and Thomas (2010) provided a systematic investigation of the topological and electrical characteristics of power grid networks based on both available real-world and synthetic power grid system data. First, power grids have salient "small-world" properties, since they feature much shorter average path length (in hops) and much higher clustering coefficients than that of Erdös-Rényi random graphs with the same network size and sparsity[8]. Second, their average node degree does not scale as the network size increases, which indicates that power grids are more complex than small world graphs; in particular, we know that the node degree distribution is well fitted by a mixture distribution coming from the sum of a truncated Geometric random variable and an irregular Discrete random variable. [4] highlighted that the topology robustness of a network is closely related to its node degree distribution. [6] investigated the deviation of the node degree distribution of power grids from a pure Geometric distribution and concluded that it substantially affects the topological vulnerability of a network under intentional attacks. That is, compared to a network with a pure Geometric node degree distribution, the power grid appears to be more vulnerable to intentional attacks when nodes with large degrees become first targets of the attack. Another less explored but equally important aspect that characterizes a power grid network is its distribution of line impedances, whose magnitude exhibits a heavy-tailed distribution, and is well fitted by a clipped double-Pareto-logNormal (dPlN) distribution [7].

With recent advances in network analysis and graph theory many researchers have applied centrality measures to complex networks in order to study network properties and to identify the most important elements of a network. Various centrality measures have been defined and used to rank the relative importance of nodes and edges in a graph. Girvan and Newman (2002) investigated the property of community structure in many types of networks in which network nodes are joined together in tightly knit groups, between which there are only looser connections [9]. They also proposed a method for detecting such communities based on a generalized centrality measure of "edge betweenness" and experimented with the proposed algorithm on a collaboration network and a food web network. Newman (2005) proposed another centrality measure of vertex by net flow of random walkers which does not flow along the shortest paths [10]. This centrality is known to be particularly useful for finding vertices of high centrality that do not happen to lie on the shortest paths and shown to have a strong correlation with degree and betweenness centrality.

Hines, Blumsack, and *et al.* (2008, 2010) provided insights on the topological and electrical structures of electrical power grids, pointing out the differences of the topology of power grids from that of Erdös-Rényi random graphs, Watts-Strogatz "small-world" networks, or "scale-free" networks (see [11][12]). They proposed an "electrical centrality measure" which is calculated based on the impedance matrix Z^{bus} and used their centrality measure to explain why in power grids a few number of highly connected bus failures can cause cascading effects, which was referred to

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as “scale-free network” vulnerability. However, as we shown in Section IV, the proposed electrical centrality measure is an incorrect one and the corresponding analysis on the vulnerability is misleading.

In [13] Rajashigh, Rajan, and Florence (2009) developed a formula to compute the betweenness centrality for a regular grid network. Torres and Anders (2009) discussed different methods of graph theory, mainly the topology centralities, for ranking the relative importance of substations in a power grid and illustrated the procedure on a synthetic 5-node test system [14]. In [15] Gorton, Huang, Jin, and *et al.* (2009) proposed a new method for contingency selection based on the concept of graph edge betweenness centrality which can be used for the contingency analysis of large scale power grids. In [16] Zio and Piccinelli (2010) improved the model of power flow distribution in the power grid network. That is, the flows are not concentrated only along the shortest paths; instead, they are randomly distributed on all the paths between nodes, as random walks. A centrality measure was defined accordingly for the transmission network analysis and was applied to the IEEE 14-bus system.

In this work we investigate the measures of centrality that are applicable to power grids and their meanings. We define new measures of centrality for a power grid that are based on its functionality. More specifically, the coupling of the grid network can be expressed as the algebraic equation $YU = I$, where U and I represent the bus voltage and injected current vectors; and Y is the network admittance matrix which is defined not only by the connecting topology but also its electrical parameters and can be viewed as a complex-weighted Laplacian. A simple transformation allows us to compute a weighted adjacency matrix from the weighted Laplacian Y. Therefore, the relative importance analysis based on centrality in graph theory can be performed on power grid network with its electrical parameters taken into account. In the paper we present and discuss some interesting discoveries on the importance rank of the power grid nodes and branches which are obtained from the experiments on the NYISO system and the IEEE 300-bus system. It has been found that when electrical parameters are incorporated into the centrality definition, the distribution of some centrality measures becomes very different from the original ones which were based on the topological structure alone; and with some proposed electrical centrality measures, a large amount of system centrality can reside in a small number of nodes in the system. These findings will help us to identify the electrically critical components of the system for the vulnerability analysis and to search for ways of enhancing system robustness.

The rest of the paper is organized as follows: Section II discusses the system model for power grid networks; Section III examines the definitions of four widely-used centralities and investigate how to extend the definitions to power grid functionality; Section IV gives some probing discussion on a previously proposed definition of electrical centrality and point out its errors; Section V shows some experiments results on the newly proposed centrality measures; and finally Section VI concludes the paper.

II. System Model

The power network dynamics are coupled by its network equation

$$YU = I, \tag{1}$$

where U and I represent the bus voltage and injected current vectors; and Y is the network admittance matrix which is determined not only by the connecting topology but also its electrical parameters. Given a network with n nodes and m links (which may also be referred to as “buses and branches (or lines) ” in power grid analysis; or “vertices and edges” in graph theory and network analysis), each link $l = (s, t)$ between nodes s and t has a line impedance $z_{pr}(l) = r(l) + jx(l)$, where $r(l)$ is the resistance and $x(l)$ the reactance. Usually for high-voltage transmission network, $x(l) \gg r(l)$, i.e., its reactance dominates. The line admittance is obtained from the inverse of its impedance, i.e.,

$$\begin{aligned} y_{pr}(l) &= g(l) + jb(l) \\ &= 1/z_{pr}(l) \end{aligned} \tag{2}$$

Assume that a unit current flows along the link $l = (s, t)$ from node s to t; then the caused voltage difference between the ends of the link equals $\Delta u = U(s) - U(t) = z_{pr}(l)$ or equivalently $\Delta u = 1/y_{pr}(l)$. Therefore $z_{pr}(l)$ can be interpreted as the “electrical” distance between node s and t and $y_{pr}(l)$ reflects the “coupling” strength between the two end nodes.

The line-node incidence matrix of the network A, with size $m \times n$, can be written as

$$A : \begin{cases} A(l, s) = & 1 \\ A(l, t) = & -1 \\ A(l, k) = & 0 \text{ , with } k \neq s \text{ or } t. \end{cases} \tag{3}$$

The Laplacian matrix L of the network, with size $n \times n$, can be obtained as

$$L = A^T A \tag{4}$$

with

$$L(s, t) = \begin{cases} -1, & \text{if there exists link } s - t, & \text{for } t \neq s \\ k, & \text{with } k = -\sum_{t \neq s} L(s, t), & \text{for } t = s \\ 0, & \text{otherewise,} \end{cases} \tag{5}$$

with $s, t = 1, 2, \cdots, n$.

The network admittance matrix Y of the network, with size $n \times n$, can be obtained as

$$Y = A^T diag(\mathbf{y_{pr}}) A \tag{6}$$

where $\mathbf{y_{pr}}$ is the line admittance vector. The entries in Y are as follows:

$$\begin{cases} Y(s, t) = -y_{pr}(s, t), & \text{link } s - t \text{ exists, for } t \neq s \\ Y(s, s) = \displaystyle\sum_{t \neq s} y_{pr}(s, t), & \text{for } t = s \\ Y(s, t) = \ 0, & \text{otherewise.} \end{cases} \tag{7}$$

with $s, t = 1, 2, \cdots, n$.

A close comparison of the matrix structures of L and Y uncovers some interesting discoveries. It is known that the Laplacian matrix L fully describes the topology of a network; while the network admittance matrix Y not only contains information about the system topology but also information about its electrical coupling. The off-diagonal entries of Y, $Y(s,t)$ equals the line admittance of the link between node s and t (with a '$-$' sign), whose magnitude reflects the coupling strength between the two nodes. The diagonal entries of the Laplacian L represent the total number of links connecting each node with the rest of the network. Whereas a diagonal entry of Y represents the total coupling capability one node has with the rest of the network. Therefore the network admittance matrix Y can be viewed as a complex-weighted Laplacian; and the Laplacian L can be equivalent to a "flat" network admittance matrix, which assumes all the links in the network have the same line impedance (with a common proportional factor). These analogies are very important in the sense that, as shown in the next section, they will help extend the centrality measures which were originally defined on a network topological structure to be more appropriately defined on the electrical structure.

III. CENTRALITY DEFINITIONS AND EXTENSIONS

Centrality measures are used in network science to rank the relative importance of vertices and edges in a graph. Within graph theory and network analysis, there are various measures of the centrality of a vertex or an edge. In the following subsections, we examine the definitions of four widely used measures of centrality, i.e., degree centrality, betweenness, closeness, and eigenvector centrality. Then we discuss how to extend the definitions to corresponding "electrical" measures of centrality for power grids.

A. Degree Centrality

The simplest centrality for a vertex is its node degree, i.e. , the total number of edges incident upon a node. This centrality represents the connectivity of a node to the rest of the network and reflects the immediate chance for a node to exert its influences to the rest of the network or to be exposed to whatever is flowing through the network, such as disturbances, shared information, power or traffic flows, or even a virus. For a graph $G := (V, E)$ wit n vertices, where V represents the set of vertices and E the set of edges, given its Laplacian L, the degree centrality of a vertex or node v is defined as

$$C_d(v) = \frac{deg(v)}{n-1} = \frac{L(v,v)}{n-1}, \tag{8}$$

where $n-1$ is used as a normalization factor.

For a node in the power grid network, its connectivity or "coupling" with the rest of the network is not only related to how many links it connects but also related to the connecting strength of each link; and the admittance of each link just reflects this coupling strength. Therefore, by using the analogy between the Laplacian L and the network admittance matrix Y, we define the electrical degree centrality $C_{d_Y}(v)$ as

$$C_{d_Y}(v) = \frac{\|Y(v,v)\|}{n-1}. \tag{9}$$

B. Eigenvector Centrality

Eigenvector centrality is a measure of the importance of a node in a network according to its adjacency matrix.

Given a network $G := (V, E)$, its adjacency matrix $\mathcal{A}$, one eigenvalue λ, and the corresponding eigenvector x satisfy

$$\lambda x = \mathcal{A} x. \tag{10}$$

The centrality of a node v is defined as the v-th entry of the eigenvector x corresponding the largest eigenvalue $\lambda_{\max}$:

$$C_e(v) = x_v = \frac{1}{\lambda_{\max}} \sum_{j=1}^{n} \mathcal{A}(v,j) x_j. \tag{11}$$

Clearly the centrality of node v is proportional to the sum of the centralities of all its neighboring nodes. The definition chooses the eigenvector corresponding to the largest eigenvalue $\lambda_{\max}$ in order to guarantee all the centrality scores, which are all the entries in the eigenvector, to be positive (see Perron-Frobenius Theorem [17]).

As stated in Section II, the off-diagonal entries in the network admittance matrix Y can be viewed as the connectivity strength between neighboring nodes in the network. Therefore, just as we extract the adjacency matrix from the Laplacian, $\mathcal{A} = -L + D(L)$, we can retrieve the complex-weighted electrical adjacency matrix as

$$\mathcal{A}_Y = -Y + D(Y). \tag{12}$$

where $D(\cdot)$ represent the diagonal matrix retrieved from the original matrix.

After performing the eigen-analysis of $\mathcal{A}_Y$, we take the magnitude of the entries of the eigenvector as the centrality measure.

$$C_{e_Y}(v) = \|x_v\| = \|\frac{1}{\lambda_{\max}} \sum_{j=1}^{n} \mathcal{A}_Y(v,j) x_j\|. \tag{13}$$

C. Closeness Centrality

Compared to degree centrality, the definition of closeness centrality is more sophisticated. It is the mean geodesic distance (i.e., the shortest path length in hops) between a vertex v and all the other vertices reachable from it:

$$C_c(v) = \frac{\sum_{t \in V \backslash v} d_G(v,t)}{n-1}, \tag{14}$$

with $d_G(v,t)$ being the shortest path length between vertices v and t. Obviously definition (14) in fact measures how "far away" a node is from the rest of the network instead of its closeness. Therefore some researchers define closeness to be the reciprocal of this quantity, to make the name more appropriate [18], that is

$$C_c(v) = \frac{n-1}{\sum_{t \in V \backslash v} d_G(v,t)}. \tag{15}$$

The shortcoming of the definition of closeness centrality is that it does not properly reflect how vulnerable is a network to becoming disconnected. In fact, the shortest path length $d_G(v,t)$ between vertices v and t turns out to be infinity if the network is disconnected and there is not a path between the two vertices. As a result the definitions of (14) and (15) can only be applied to connected networks. In order to incorporate the disconnectivity and to more conveniently measure the network vulnerability, Danalchev modified in [19] the definition of closeness to be:

$$C_c(v) = \sum_{t \in V \setminus v} 2^{-d_G(v,t)}. \tag{16}$$

Because the power grid networks we are interested in are connected topologies, undirected with neither multiple links nor self-loops, definition (15) above is still suitable for our purposes.

In all definitions above, the distance along a path from vertex v to t is measured as in "hops". That is, $d_G(v,t)$ equals the total number of hops along the path:

$$d_G(v,t) = \sum_{(i,j) \in E \cap path(v \to t)} 1. \tag{17}$$

To adapt the definition of closeness centrality to nodes in a power grid network, we define the "electrical distance" between the nodes as $d_Z(v,t)$ which is counted in "electrical hops" as

$$d_Z(v,t) = \| \sum_{(i,j) \in E \cap path(v \to t)} Z_{pr}(i,j) \|, \tag{18}$$

where $Z_{pr}(i,j)$ is the line impedance of the link (i,j). Therefore the corresponding closeness centrality based on electrical distance is defined as

$$C_{c_Z}(v) = \frac{n-1}{\sum_{t \in V \setminus v} d_Z(v,t)}, \tag{19}$$

It is worth noting that the line impedance Z_{pr} in a power grid is a complex number, i.e., $Z_{pr} = R + jX$, where R is the resistance and X the reactance. Therefore the electrical distance $d_Z(v,t)$ is in fact a complex number though one could take the magnitude of $d_Z(v,t)$ to make a more "real" distance measure. According to the definition in (18), the line impedance of each link is in fact used as the edge weight in the search of the shortest path between v and t. However, this will complicate the shortest-path search algorithm because one can not compare or add up two complex-number weights so straightforward as with real-number weights. On the other hand, it is known that for the high-voltage transmission network in a power grid the reactance X is usually the dominant component of a line impedance, whereas R only takes a trivial value which in many cases can even be neglected. Therefore for the purpose of simplicity, we can only take the reactance X as the edge weights and hence the distance evaluation can be approximated by real numbers.

D. Vertex and Edge Betweenness Centrality

Vertex Betweenness is one of the most widely used centrality measure. It was first suggested by Freeman (1977) in [20]. This measure reflects the influence of a node over the flow of information between other nodes, especially in cases where information flow over a network primarily follows the shortest available path.

Given a undirected graph $G(V,E)$, the betweenness of a node v is defined as the number of shortest paths between pairs of other vertices that run through v:

$$C_b(v) = \frac{\sum_{s \neq v \neq t \in V} \sigma_{st}(v)/\sigma_{st}}{(n-1)(n-2)/2} \tag{20}$$

where σ_{st} the number of shortest paths from s to t and $\sigma_{st}(v)$ is the total number from the mentioned paths that pass through vertex v.

In order to find which edges in a network are most between other pairs of vertices, Girvan and Newman [9] generalize Freemans betweenness centrality to edges and define the edge betweenness of an edge as the number of shortest paths between pairs of vertices that run along it. If there is more than one shortest path between a pair of vertices, each path is given equal weight such that the total weight of all of the paths is unity. Note that the normalization factor of edge betweenness is different from that of vertex betweenness.

$$C_b(e) = \frac{\sum_{s \neq t \in V} \sigma_{st}(e)/\sigma_{st}}{n(n-1)/2} \tag{21}$$

Obviously, vertices or edges that occur on many shortest paths have higher betweenness that those that do not. It is found that removal of the nodes or edges with larger betweenness will put the network at higher risk to become disconnected.

Definitions (20) and (21) are based on the shortest path counted in hops. Using the the shortest electrical path counted in electrical hops as (18), we can define the electrical betweenness for nodes and edges in power grid networks, which are denoted as $C_{b_Z}(v)$ and $C_{b_Z}(e)$ respectively.

IV. SOME DISCUSSION ON THE ELECTRICAL CENTRALITY PROPOSED IN [11]

Hines and Blumsack (2008) proposed an "electrical centrality measure" which is calculated based on the Z^{bus} matrix [11]. The Z^{bus} matrix is the inverse of the Y matrix, which, unlike the Y matrix, is a non-sparse (dense) matrix. That is, $Z^{bus} = Y^{-1}$. This centrality measure has also been adopted by other researchers [14].

The principle of assigning this centrality measure is restated as follows (see [11][12]):

It was claimed that the equivalent electrical distance between node k and l is thus given by the magnitude of the (k,l) entry in the Z^{bus} matrix. Smaller $\|Z^{bus}_{k,l}\|$ corresponds to a shorter electrical distances and a stronger coupling between these node hence a larger propensity for power to flow between these nodes. From the non-zero off-diagonal entries in the Z^{bus} matrix, select only the same number of "links" (k, l) as that in Y, which have the shortest electrical distances (corresponding to the entries with the least $\|Z^{bus}_{k,l}\|$'s). With these newly selected "links", one has a different "electrical" topology from the original network. Then node degrees, counted on the newly formed electrical topology, are defined as the electrical betweenness, or the electrical degrees.

With further analysis based on the electrical degrees, it is found that although a power grid topologically does not have the properties of a scale-free network[1]; electrically, there exist a number of highly-connected nodes in the electrical topology (obtained from Z^{bus}) similar to what would be expected from a scale-free network. That is, the electrical node degree distribution has a "fat" tail. This finding, according to [11], explains the relatively high vulnerability of power grid to failures at some "hub" buses (i.e., intentional attacks to high-degree nodes).

However, a deeper analysis of the impedance matrix Z^{bus} reveals that the definition of the electrical betweenness (degree) and the corresponding analysis in [11] and [12] are potentially misleading. The problem is that the entry of the impedance matrix, $Z^{bus}_{k,l}$, does not represent the mutual electrical distance between node k and l. In fact, from the network equation $U = Z^{bus} I$, we can learn the physical meaning of the entry of $Z^{bus}_{k,l}$: when only node k has a unit injected current, the voltage increases occurring at node l and k are $Z^{bus}_{k,l}$ and $Z^{bus}_{k,k}$ respectively. That is

$$\begin{aligned} \Delta U(l) &= Z^{bus}_{k,l} \\ \Delta U(k) &= Z^{bus}_{k,k}. \end{aligned} \tag{22}$$

And usually $\|Z^{bus}_{k,k}\| > \|Z^{bus}_{k,l}\|$, which means the Z^{bus} matrix is diagonally dominant. Only if $Z^{bus}_{k,l}$ is close to $Z^{bus}_{k,k}$, which happens when $Z^{bus}_{k,l}$ has a relatively large magnitude, does there exist a strong coupling between the two nodes. Whereas the mutual electrical distance between node k and l equals the voltage difference caused by an injected unit current at node k and an output unit current from node l. That is

$$Z_{k \leftrightarrow l} = Z^{bus}_{k,k} + Z^{bus}_{l,l} - 2Z^{bus}_{k,l}, \tag{23}$$

which also implies that a large entry of $Z^{bus}_{k,l}$, not a small one as shown in [11], gives a small mutual electrical distance and a strong coupling between the two nodes.

On the other hand, for a connected power grid network with grounding branches, its network admittance matrix Y is non-singular therefore the inverse of Y, which is Z^{bus}, exists. The network equation $YU = I$ and $U = Z^{bus} I$ are equivalent to each other. That is, the matrices Y and Z^{bus} in fact describe the same electrical and topological structure of the power grid. It is, however, misleading to interpret Z^{bus} as something that reveals a structurally different "new" topology with denser connections or with a different node degree distribution.

[1]i.e., its node degree distribution does not have a 'fat' power-law tail.

V. EXPERIMENT RESULTS

Based on defined centrality measures in Section III, we perform experiments on the NYISO-2935 system and the IEEE-300 system. The NYISO-2935 system is a representation of the New York Independent System Operator's transmission network containing 2935 nodes and 6567 links, with an average node degree $\langle k \rangle = 4.47$ and an average shortest path length $\langle l \rangle = 16.43$ (in hops). The IEEE-300 system is a synthesized network from the New England power system and has a topology with 300 nodes and 409 links, with $\langle k \rangle = 2.73$ and $\langle l \rangle = 9.94$. We evaluate the relative importance for the nodes and lines according to different centrality measures and normalize the results to make sure the sum of all the vertex or edge centralities in a system equals 1.0. Then we analyze their distribution and correlation, and identify the most "significant" nodes in each system.

Figure 1 compares the distribution of degree centrality and eigenvector centrality for the nodes in the NYISO system; and shows the correlation between different centrality measures. It can be seen that when electrical parameters are incorporated into the centrality definition, the distribution of the degree centrality and the eigenvector centrality become very different from the original ones which are based on the topological structure alone. The correlation between the "topology" centralities and the corresponding electrical centralities is very weak. However, there is a strong correlation between the two electrical centralities C_{d_Y} and C_{e_Y}.

Table I ranks the first 10 most important nodes in the NYISO system according to different type of centrality measures. The bottom row gives the total centrality of each group of most important nodes. It shows that the degree centrality distributes quite "flatly" among the nodes, because the first 10 most important nodes sum only to 2% of system's total centrality. However, when the electrical parameter is taken into account, a large amount of centrality can be shifted into in a small number of nodes in the system, e.g., the first 10 most important nodes based the electrical eigenvector centrality take more than 99.2% of the system's total centrality. This reveals that the eigenvector centrality vector is in fact a very sparse one. It also shows that C_{d_Y} and C_{e_Y} are kind of consistent with each other in the sense that they are able to locate a very similar group of most important nodes in the system (a 60% overlap) which however, is quite different from the group identified by the topological centralities C_d and C_e.

Figure 2 compares the distribution of different types of centrality for the nodes in the IEEE-300 system. It can be seen that including electrical parameters causes a large change in the distribution of the degree centrality and the eigenvector centrality (see (a) and (b)). However, the effect of electrical parameters is not so evident in the distribution of the closeness and betweenness centrality (see (c), (d) and (e)). Figure 3 displays the correlation between each pair of topological and electrical centrality measures. It shows that strong correlations exist between the electrical and

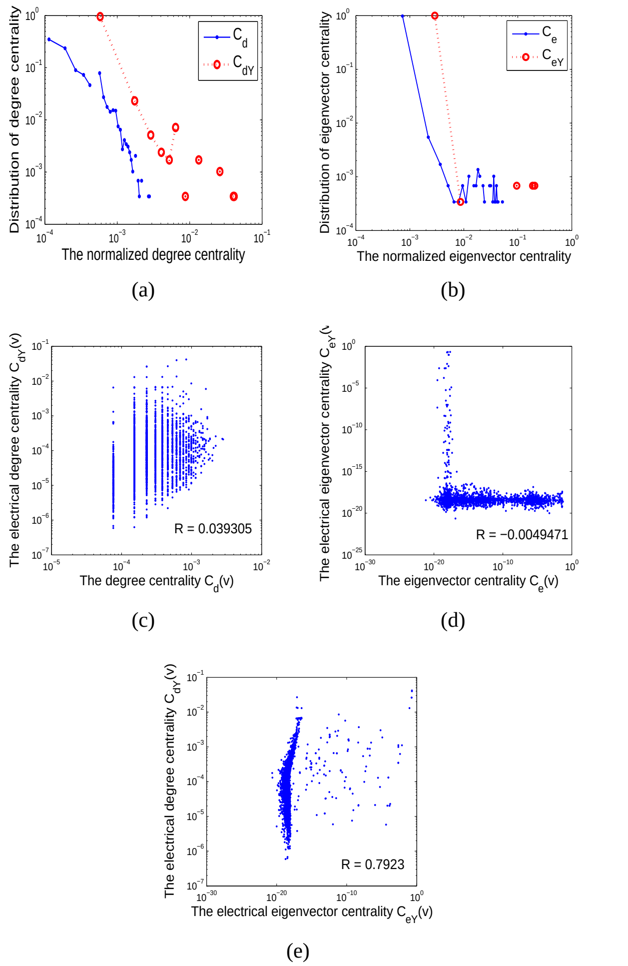

Fig. 1. The Distribution of Centrality Measures and the Correlation Between Different Centrality Measures – the NYISO-2935 System: (a) degree centrality, C_d and C_{d_Y}; (b) eigenvector centrality, C_e and C_{e_Y}; (c) correlation between C_d and C_{d_Y}; (d) correlation between C_e and C_{e_Y} (e) correlation between C_{d_Y} and C_{e_Y}.

TABLE I

THE LIST OF NODES IN THE NYISO-2935 SYSTEM THAT CARRY THE MOST SIGNIFICANT CENTRALITIES

ranking order	C_d	C_e	C_{d_Y}	C_{e_Y}
1	2773	2622	9	9
2	2622	2614	8	8
3	2516	2606	1312	15
4	2511	2619	17	17
5	2894	2605	15	11
6	2728	2613	234	12
7	2435	2608	233	84
8	2614	2601	12	29
9	2481	2610	11	27
10	2409	2609	1518	26
total centrality	0.021471	0.40277	0.22631	0.99227

topological closeness centrality; and between the electrical and topological betweenness centrality as well.

Table II ranks the first 10 most important nodes in the IEEE-300 system according to eigenvector centrality and closeness centrality. The bottom row gives the total centrality of each group of most important nodes. The same concentration of the eigenvector centrality can be observed here as is observed with the NYISO system. It is interesting to notice that both the topological and electrical closeness centrality distribute very "flatly" among the nodes and the groups of important nodes located by the pair have a 60% overlap. Table III ranks the first 10 most important nodes and branches in the IEEE-300 system according to betweenness centrality. The bottom row gives the total centrality of each group of most important nodes or branches. Similarly we see that both the topological and electrical betweenness centrality distribute quite "flatly" among the nodes or edges and each pair of centrality measures are very consistent with each other. This indicates that the inclusion of electrical parameters in the betweenness centrality does not cause much difference in identifying the most important nodes or branches. In fact there are 80% overlap between the nodes groups and 70% overlap between the branch groups respectively, according to the the topological and electrical betweenness measures. The reason for this strong consistence in identification of most important components or the strong correlations observed in the closeness or betweenness centrality distribution can be interpreted as follows: both closeness and betweenness centrality measures are defined based on the shortest path count; the transmission network of power grids is sparsely connected, therefore the shortest path between any two nodes tends to include a large number of hops (e.g., on average about 10 hops in the IEEE-300 system and 16 hops in the NYISO system); as a result the differences among individual line impedances average out in the evaluation of centrality measure based on the shortest path count.

TABLE II

THE LIST OF NODES IN THE IEEE-300 SYSTEM THAT CARRY THE MOST SIGNIFICANT CENTRALITIES

ranking order	C_e	C_{e_Y}	C_c	C_{c_Z}
1	31	266	36	36
2	35	31	40	40
3	32	270	16	16
4	15	35	39	33
5	43	32	4	28
6	27	34	35	4
7	75	43	15	3
8	74	75	3	7
9	34	15	68	129
10	44	74	31	39
total centrality	0.51401	0.99818	0.046687	0.046551

VI. CONCLUSIONS AND FUTURE WORKS

This paper investigates measures of centrality that are applicable to power grids. We define new measures of centrality for a power grid that are based on its functionality rather than

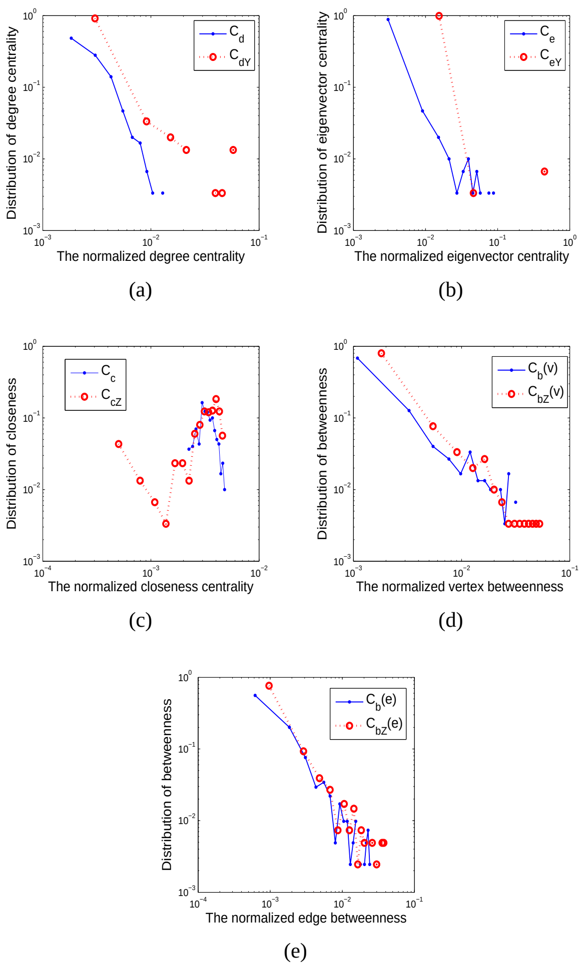

Fig. 2. The Distribution of Centrality Measures – the IEEE-300 System: (a) degree centrality, C_d and C_{d_Y}; (b) eigenvector centrality, C_e and C_{e_Y}; (c) closeness centrality, C_c and C_{c_Z}; (d) vertex betweenness centrality, $C_b(v)$ and $C_{b_Z}(v)$; (e) edge betweenness centrality, $C_b(e)$ and $C_{b_Z}(e)$.

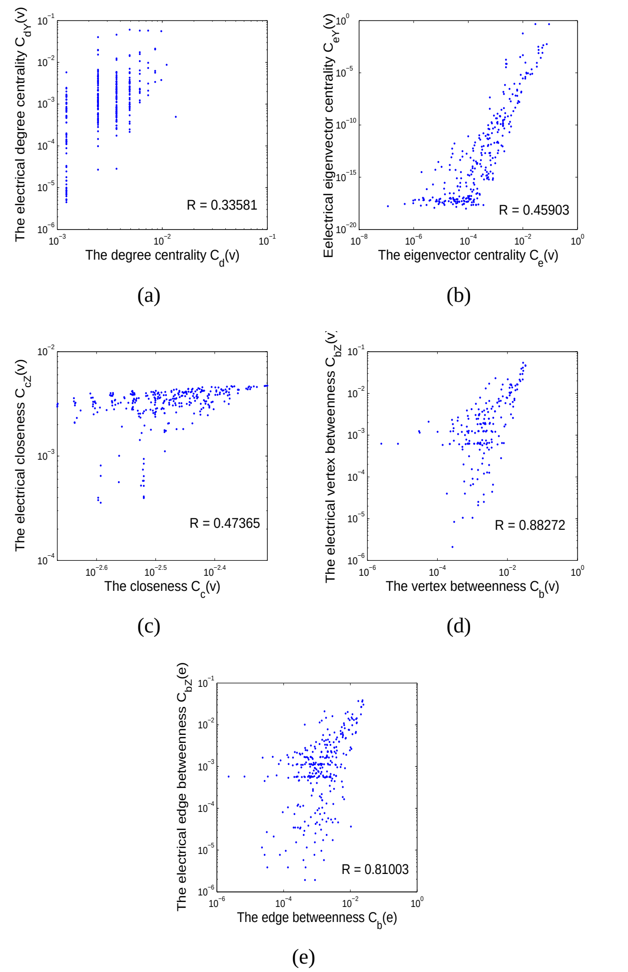

Fig. 3. The Correlation Between Different Centrality Measures – the IEEE-300 System: (a) degree centrality, C_d and C_{d_Y}; (b) eigenvector centrality, C_e and C_{e_Y}; (c) closeness centrality, C_c and C_{c_Z}; (d) vertex betweenness centrality, $C_b(v)$ and $C_{b_Z}(v)$; (e) edge betweenness centrality, $C_b(e)$ and $C_{b_Z}(e)$.

TABLE III

THE LIST OF NODES AND BRANCHES IN THE IEEE-300 SYSTEM THAT CARRY THE MOST SIGNIFICANT BETWEENNESS CENTRALITIES

ranking order	$C_b(v)$	$C_{b_Z}(v)$	$C_b(e)$	$C_{b_Z}(e)$
1	3	36	68→40	40→36
2	40	40	40→36	36→16
3	68	3	16→4	16→4
4	36	16	4 →3	4→3
5	16	4	129→3	68→40
6	31	68	129→109	129→3
7	109	109	266→31	129→109
8	4	129	36→16	7→3
9	266	7	52→39	36→35
10	129	35	173→68	35→31
total centrality	0.27490	0.36570	0.20520	0.29115

just its topology. More specifically, the coupling of the grid network can be expressed as the algebraic equation $YU = I$, where U and I represent the bus voltage and injected current vectors; and Y is the network admittance matrix which is defined not only by the connecting topology but also by its electrical parameters and can be seen as a complex-weighted Laplacian. We show that the relative importance analysis based on centrality in graph theory can be performed on power grid network with its electrical parameters taken into account.

Based on defined centrality measures, we have performed experiments on the NYISO-2935 system and the IEEE-300 system and obtained some interesting discoveries on the importance rank of the power grid nodes and lines. It has been found that when the electrical parameters are incorporated into the centrality definition, the distribution of the degree centrality and the eigenvector centrality become very different from the original ones which are based on the topological structure alone. With the electrical degree centrality and the electrical eigenvector centrality a large amount of centrality can reside in a small number of nodes in the system and help locate of a quite different group of important nodes. From the experimental results of IEEE-300 system, it is shown that the effect of including electrical parameters is not so evident in changing the distribution of the closeness and betweenness centrality. And strong correlations exist between the electrical and topological closeness centrality; and between the electrical and topological betweenness centrality as well.

Although the proposed electrical centrality measures can locate a quite different group of "important" nodes or vertices in the system from the topological centrality measures. However, more tests and analysis need to be done in order to validate the proposed measures, to further understand their physical meaning, and to apply the findings to search for ways of enhancing system robustness.

VII. ACKNOWLEDGMENTS

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