Transactions on the Built Environment vol 31, © 1997 WIT Press, www.witpress.com, ISSN 1743-3509

Electrical network formulations of mechanical finite-element models

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Abstract

This paper presents a new and general method to include mechanical finite element models within an electronics simulator. The approach used directly transforms a finite element into an equivalent electrical network. With that, we accomplish integrated simulations of electronics and complicated, irregular mechanical structures which are very typical in microsystem technology. The methodology can be applied to all analog circuit simulators containing capabilities for analog hardware description languages and a procedural language interface.

1 Introduction

In Pelz et al. [1] introduced an integrated approach for the simulation of microsystems by an electronics simulator. They applied the finite-difference method to model one-dimensional partial differential equations. In this way, partial differential equations are transformed into a set of differential equations which in turn can be mapped onto an equivalent electrical network. With that, microsystems containing strong electromechanical interaction – like a pressure sensor system fabricated in surface micromechanics and its read-out electronics, see Pelz et al. [2] – can be handled easily within an standard electronical network simulator. Mrčarica et al. [3] extended this methodology to two-dimensional regions. Klein and Gerlach [4] used a slightly different method whereby generalized finite networks describe the

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spatial derivatives of the mechanical plate deflection equation in two dimensions.
However, the usual modelling technique for structural mechanics is given by the finite-element method. Two major advantages in comparison to the finite differences can be identified:

1. The finite element method (FEM) is well suited to model inhomogenous structures which are typical in microsystems. Etch holes within plates or complicated geometry of hinge structures can not be modelled by the finite difference method (FDM).

2. A very crucial problem connected with the FDM is the stability of the numerical solution. If a fine discretization scheme of the structure is adopted, the simulator runs into numerical problems because the differences are in the order of magnitude of the floating point precision. Moreover, the spatial discretization step can not be decoupled from the time discretization. The FEM overcomes these difficulties: a finer discretization leads to more accurate results of the simulation and the numerical solution remains stable. This is directly due to the integral formulation of the FEM-algorithm.

Up to now the methods to include finite-element models into an electronical circuit simulator are restricted to some special cases. E.g. Popovic [5] carried out numerical analysis of Hall plates with the standard simulator SPICE. The underlying semiconductor equations considering the magnetic influence were reformulated as finite element descriptions which represent a small area of the Hall plate. Only in the present context the electronical relations are calculated by built-in MOS-Transistors models which solve exactly these equations. Hence, two built-in MOS-transistors of the simulator and additional current sources are a good equivalent to compute directly the Hall effect of the regarded area. By a suited connection of these subcircuits the entire structure can be modelled.
Hitherto, a general solution for other disciplines in physics could not be shown because of the more difficult mathematical apparatus which is necessary for modelling finite-elements. Therefore, in this paper we present an new methodology to incorporate mechanical finite-element models into an electronics simulator. This approach to include general finite-elements enables us to model a wide range of physical phenomena. Recently Hsu and Vu-Quoc [6],[7] introduced an FEM-approach for electrothermal simulation which seems to be similar to resemble our method. But there are important differences: The governing parabolic partial differerential equations (PDE) for thermal systems lead always to non-oscillating solutions of the heat flow. In contrast to this, structure mechanics is described by hyperbolic PDEs resulting in periodical behavior. From the numerical point of view this is more critical for a simulation tool. Furthermore, the respective mechanical structure is time-varying and, thus, the FEM-approach

$$\mathbf{S} = \frac{b_i}{l_i^3} \begin{array}{c} \begin{array}{cccc} u_{yk} & r_{zk} & u_{yl} & r_{zl} \end{array} \\ \left[\begin{array}{cccc} 12 & -6l & -12 & -6l \\ -6l & 4l^2 & 6l & 2l^2 \\ -12 & 6l & 12 & 6l \\ -6l & 2l^2 & 6l & 4l^2 \end{array}\right]_i \end{array} \quad \mathbf{M} = \frac{\mu_i l_i}{420} \begin{array}{c} \begin{array}{cccc} u_{yk} & r_{zk} & u_{yl} & r_{zl} \end{array} \\ \left[\begin{array}{cccc} 156 & -22l & 54 & 13l \\ -22l & 4l^2 & -13l & -3l^2 \\ 54 & -13l & 156 & 22l \\ 13l & -3l^2 & 22l & 4l^2 \end{array}\right]_i \end{array}$$

Figure 1: Element stiffness matrix **S** and mass matrix **M** of beam i

generates system matrices with time-varying coefficients which have to be considered. In one-, two-, and three-dimensional thermal problems the coefficients remain constant during the simulation run. Finally, more than one degree of freedom per node as in the thermal case (temperature T) exist for mechanical finite elements – up to six degrees of freedom for more complicated shell elements. This produces a substantial increase of expense for implementation and computation of the mechanical elements.
The approach presented here – developed independently from Hsu and Vu-Quoc [6],[7] – takes care of these differences to accomplish mechanical simulations by an electronics simulator.

2 Finite-element modelling by electrical circuits

In the following we show how finite elements can be calculated by equivalent electrical circuits. Two major problems are to be solved: first, the element mass, damping, and stiffness matrices have to be implemented. These matrices describe the solution of each finite element representing a subregion of the considered structure. Second, the transformation has to be carried out from the element matrices to the global system matrix, which represents the behaviour of the whole structure. Let us now assume a 2-node beam-element with 2 degrees of freedom (DOF) per node (deflection u_y, rotation r_z) where structural damping and shear deflection are neglected. We consider constant stiffness $B_i = EI_{zz}$ and constant mass distribution $\mu_i = \rho \cdot A_i$ throughout each beam i with the nodes k, l. Hereby is given young's modulus E, the moment of inertia I_{zz}, the density ρ, and the cross section A of the beam. Then element stiffness and mass matrix are given as depicted in Fig. 1, see Gasch and Knothe [8]. To map this to electronics the nodal admittance matrix derived from electrical networks containing passive devices like capacitances or inductances is an appropriate equivalent for these matrices. Since nodal matrix and stiffness and mass matrix have similar structure: they are symmetric and the elements in the major diagonal of the matrices are positive. In this way, a capacitance-inductance circuit model shall be found which reflects correctly mass and stiffness matrix. Such a model can be seen in Fig. 2. The node voltages at the nodes

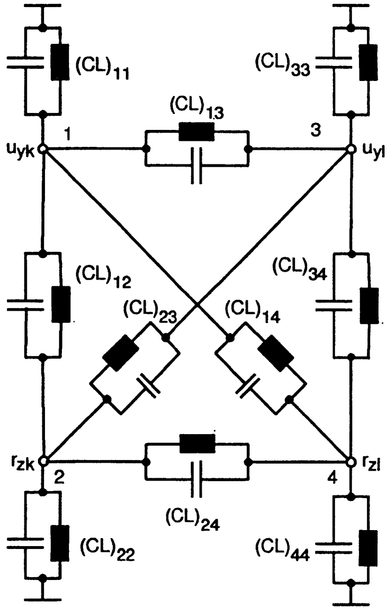

Figure 2: Equivalent electrical circuit model of 2-node beam-element

$1, 2, 3, 4$ in the circuit represent the degrees of freedom $u_{yk}, r_{zk}, u_{yl}, r_{zl}$. With that, capacitance and inductance coefficients can be computed from mass and stiffness coefficients respectively.

The next important step in the modelling procedure is the derivation of the capacitance and inductance coefficients. The circuit of Fig. 2 can be analyzed manually by Kirchhoffs current law. With the theorem of superposition, we formulate the equations for the branch currents of the capacitances:

$$+(C_{11} + C_{12} + C_{13} + C_{14})\dot{\phi}_1 - C_{12}\dot{\phi}_2 - C_{13}\dot{\phi}_3 - C_{14}\dot{\phi}_4 = 0 \ , \qquad (1)$$
$$-C_{12}\dot{\phi}_1 + (C_{22} + C_{12} + C_{23} + C_{24})\dot{\phi}_2 - C_{23}\dot{\phi}_3 - C_{24}\dot{\phi}_4 = 0 \ , \qquad (2)$$
$$-C_{13}\dot{\phi}_1 - C_{23}\dot{\phi}_2 + (C_{33} + C_{13} + C_{23} + C_{34})\dot{\phi}_3 - C_{34}\dot{\phi}_4 = 0 \ , \qquad (3)$$
$$-C_{14}\dot{\phi}_1 - C_{24}\dot{\phi}_2 - C_{34}\dot{\phi}_3 + (C_{44} + C_{14} + C_{24} + C_{34})\dot{\phi}_4 = 0 \ . \qquad (4)$$

Note that $C_{ij} = C_{ji}$ and $L_{ij} = L_{ji}$. By simple comparison of the capacitance coefficients with the mass coefficients m_{ij} of the element mass matrix $\mathbf{M}$ we can derive the defining equations:

$$C_{ij} = -m_{ij}, \quad i,j = 1(1)4, \quad i \neq j \ , \qquad (5)$$
$$C_{ii} = \sum_{j=1}^{4} m_{ij}, \quad i = 1(1)4 \ . \qquad (6)$$

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In analog manner the inductance coefficients can be formulated by suited comparison with the stiffness coefficients s_{ij} of the element stiffness matrix **S**:

$$L_{ij} = -1/s_{ij}, \quad i,j = 1(1)4, \quad i \neq j \,, \tag{7}$$

$$L_{ii} = 1/\left(\sum_{j=1,}^{4} s_{ij}\right), \quad i = 1(1)4 \,. \tag{8}$$

The subcircuit of Fig. 2 representing a beam element is built using the MAST® (Analogy Inc.) analog hardware description language. Within this subcircuit a C-function is called each time step which calculates the coefficients in dependence of material constants and geometry of the beam. The coefficients are propagated back into the subcircuit where the relations between currents and voltages are calculated to

$$I_{ij} = \frac{\mathrm{d}}{\mathrm{d}t}(V_{ij} \cdot C_{ij}) \,, \tag{9}$$

$$V_{ij} = \frac{\mathrm{d}}{\mathrm{d}t}(I_{ij} \cdot L_{ij}) \,. \tag{10}$$

This offers an enormous advantage: If the mass and stiffness coefficients become time-dependent – as it is the case for the most finite elements – the proposed way of solution remains applicable.
The transformation from the element matrices into the global matrix is accomplished for free by simply connecting the finite element subcircuits according to the device geometry. The handling of these subcircuits is exactly the same as for each other standard device like transistor, resistor, etc. Boundary conditions are applied by simply connecting the respective nodes to ground, see Bielefeld et al. [9].
Up to now, we have implemented a 4-node shell-element (DOF/node: u_z, r_x, r_y, r_z) and the beam-element described above. Especially the shell-element is important for microsystems to model multidimensional structures. Note that the modelling procedure for the shell-element results in 272 coefficients for each element and their processing works pretty well.

3 Simulation

3.1 Cantilever beam

As a general example we simulate dynamically a cantilever beam, see Fig. 3(a), consisting of 40 beam-elements modelled by MAST and performed by the electronics simulator SABER®, Analogy Inc. For verification purposes we model and simulate the same structure with the FEM-tool ANSYS®. A pulsed force is applied with a pulse width of 8 s and a rise and fall time of 1 s. The results are shown in Fig. 4(a) wherein the dynamic deflection is depicted for the spatial positions $x = 0.25l$, $0.5l$, $0.75l$, l.

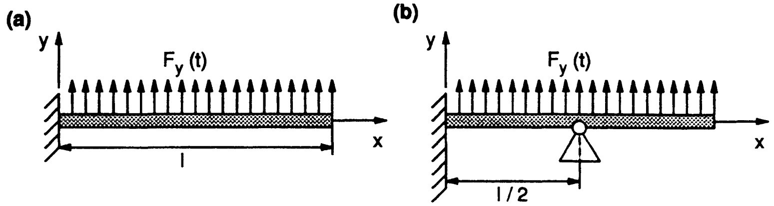

Figure 3: Cantilever beam without (a) and with (b) supporting point at $x_1 = 0.5l \Rightarrow u_y(x_1) = 0$

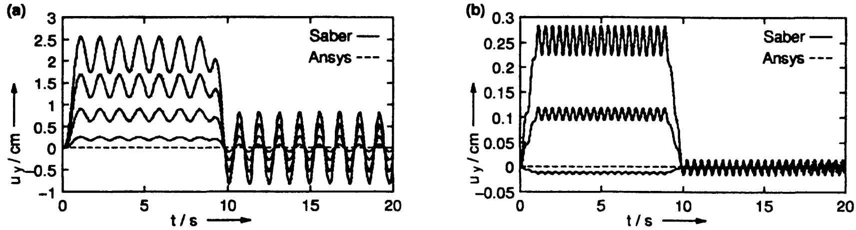

Figure 4: Simulation of cantilever beam without (a) and with (b) supporting point at $x_1 = 0.5l \Rightarrow u_y(x_1) = 0$

The configuration of Fig. 3(b) includes a supporting point at the half length of the considered beam resulting in an additional constraint of the form $u_y = 0$. With the FEM this is easy to model and simulate, but with the FDM-approach serious numerical problems arise. The dynamic bending behavior can be studied in Fig 4(b). The traces of the positions $x = 0.25l$, $0.75l$, l are given. The differences in amplitude and frequency of the deflection for the two beam configurations are below 0.1% between SABER and ANSYS.

3.2 Deformable-mirror-device-system

The second example investigates a micromachined, electrostatically excited deformable mirror device with a pixel area of $20 \cdot 20 \mu\mathrm{m}^2$. Fig. 5 displays the geometry and the deflection of the DMD, if a constant voltage of 30 V is applied. The simulation is performed by SABER. Modelling the four rectangular shaped hinges of the DMD is simple using the introduced FEM-approach; doing the same by the FDM is nearly impossible. The DMD is modelled by 69 shell-elements; the same discretization scheme is used for both SABER and ANSYS. In Fig. 6 a static simulation of the DMD is depicted, wherein the excitation voltage is rising from 0 V to 30 V. As can be recognized a difference of 1.0% is achieved for the maximum deflection of

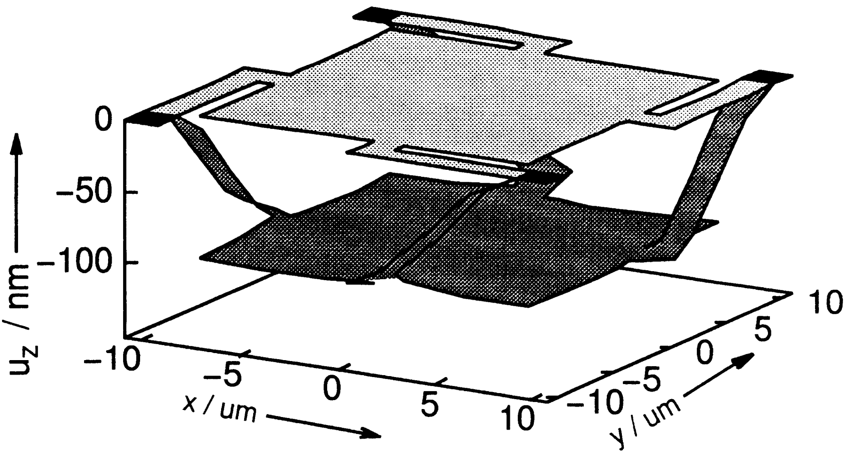

Figure 5: Undeformed and deformed shape of deformable mirror device, bending results from SABER

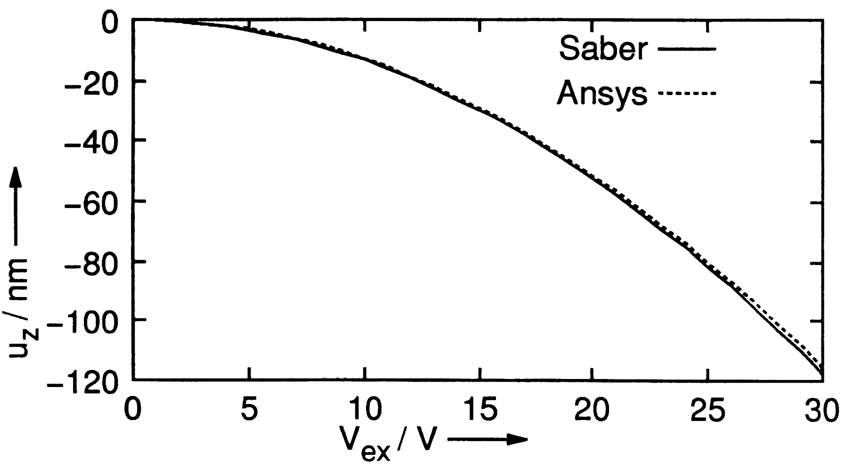

Figure 6: Maximum z-deflection at the center of the DMD caused by electrostatic force

the DMD. Since such devices are used for optical data processing the maximum deflection is the most important parameter which has to be controlled.

3.3 Performance of the approach

In Table 1, the simulation performance can be studied. As can be recognized, the presented electrical network approach based on analog hardware description languages is at least as good as than those of a comprehensive FEM-simulator like ANSYS. The simulations were performed on a SUN SPARC 20.

Simulation of	SABER	ANSYS
cantilever beam	91 s	94 s
cantilever beam with support	155 s	270 s
DMD, static	58 s	63 s

Table 1: Simulation performance of SABER and ANSYS

4. Conclusions

A new approach was developed to overcome the problems of modelling complicated and inhomogenous mechanical structures within an electronics simulator. The proposed approach transforms mechanical finite elements onto an equivalent electrical network. Based on the electrical circuit theory a capacitance-inductance model can be formulated matching correctly the mass and stiffness matrix of the finite elements which describe the mechanical behavior. This general method leads to the implementation of a beam and shell element which can be used like a typical electrical or electronical device of state-of-the-art circuit simulators.
As shown, useful simulations are carried out of a cantilever beam and a micromachined deformable mirror device. The achieved simulation performance equals those of specialized FEM-tools.

5 References

[1] **Pelz, G., J. Bielefeld, F.-J. Zappe and G. Zimmer** 1994. MEXEL : simulation of microsystems in a circuit simulator using automatic electromechanical modeling. In: Micro System Technologies '94, Reichl, H., A. Heuberger, (Ed.), 651-657. Berlin, Offenbach: vde-Verlag.

[2] **Pelz, G., J. Bielefeld and G. Zimmer** 1995. Model transformation for coupled electro-mechanical simulation in an electronics simulator. *Springer J. Microsystem Technologies* , 1, 4, 173-177.

[3] **Mrčarica, Ž., D. Glozić, V. Litovski and H. Detter** 1995. Simulation of microsystems using a behavioural hybrid simulator Alecsis. In: *Simulation and Design of Microsystems and Microstructures*, Adey, R. A., A. Lahrmann, C. Leßmöllmann, (Ed.), 129-136. Southampton: Computational Mechanics Publications.

[4] **Klein, A., G. Gerlach** 1996. System modelling of microsystems containing mechanical bending plates using an advanced network description method. In: *Micro System Technologies '96*, Reichl, H., A. Heuberger, (Ed.), 299-304. Berlin, Offenbach: VDE-Verlag.

[5] **Popović, R. S.** 1985. Numerical analysis of MOS magnetic field sensors. In: *Solid-State Electronics*, 28, 7, 711-716.

[6] **Hsu, J. T., L. Vu-Quoc** 1996a. A rational formulation of thermal circuit models for electrothermal simulation – part I: finite element method. *IEEE Trans. Circuits & Systems – I: Fund. Theory & Appl.* 43, 9,721-732.

[7] **Hsu, J. T., L. Vu-Quoc** 1996b. A rational formulation of thermal circuit models for electrothermal simulation – part II: model reduction techniques. *IEEE Trans. Circuits & Systems – I: Fund. Theory & Appl.* 43, 9,733-744.

[8] **Gasch, R., K. Knothe** 1989. *Strukturdynamik: Kontinua und ihre Diskretisierung.* Berlin: Springer (in german).

[9] **Bielefeld, J., G. Pelz, G. Zimmer** 1996. Theoretical foundations of the model transformation approach. In: *Micro System Technologies '96*, Reichl, H., A. Heuberger, (Ed.), 133-136. Berlin, Offenbach: VDE-Verlag.