

## Electrical Resistance of Ferromagnetic Metals

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(Received February 13, 1956)

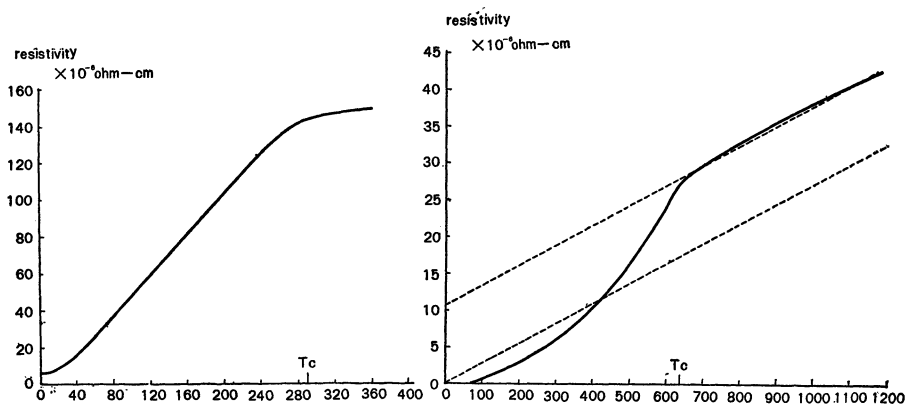
In ferromagnetic metals an anomalous electrical resistance is observed which is characteristic of the magnetization and is additive to the ordinary electrical resistance. The origin of this anomalous electrical resistance has been discussed by several authors, but all such attempts do not seem to have satisfactorily explained this phenomenon either quantitatively or qualitatively.

In this paper we calculate the anomalous electrical resistance from the standpoint of s-d interaction as developed by the author. It is our opinion that the anomalous electrical resistance occurs because the exchange energy between the conduction and the unfilled inner shell electrons depends on the relative direction of the spins of the electrons, and that this interaction is not periodic in finite temperatures.

In numerical values as well as in the temperature dependence, our results are in good agreement with experimental results.

### § 1. Introduction

In transition metals, Ni, Fe, etc., and also in rare earth metals, especially Gd, the electrical resistance depends on temperature as shown in Figs. 1<sup>1)</sup> and 2<sup>2)</sup>. We can divide this resistance into two parts. The first is the normal resistance, proportional to the absolute temperature  $T$  in high temperatures. This temperature dependence suggests that this part of the resistance may be caused by electron-phonon interaction. The second is characteristic of ferromagnetism. The main feature of the temperature dependence of this



part is that when the temperature is higher the Curie temperature  $T_c$ , the resistance is almost constant, and when the temperature is lower than  $T_c$ , the resistance decreases to become zero at absolute temperature zero.

So far the origin of this anomalous electrical resistance has been discussed by Mott, etc.,<sup>3)</sup> from the following standpoint. "In metallic ferromagnetics, one must consider two energy bands, that is, s-band and d-band. The effective mass of the d-band electron is very much larger than that of the s-band electron, and almost the entire electrical current is carried by s-electrons. Nevertheless, as the state density of the d-band is very large, the transition of electrons from s-band into d-band as caused by the electron-phonon interaction, is much more frequent than that from s-band into s-band. Thus the electrical resistance of transition metal is submitted to this s-d interband transition, and such a process depends on the magnetization of the d-band."

This process may actually exist and be important in the transition metals, but we think because of the following facts this should rather be included in the normal part of the electrical resistance, and that it does not correspond to the anomalous part in question. The reasons are as follows. (i) In rare earth metals the unfilled shell, that is the f-shell, does not seem to constitute a band, but behaves as if it were a free ion. Thus the above inter-band transition process does not occur. As is shown in Fig. 2, however, the anomalous resistance does occur in rare earth metals too. (Even in transition metals, it does not seem to be a good approximation to treat d-electron to form a band). (ii) The resistance caused by this process should be nearly proportional to the absolute temperature in high temperatures because this process occurs through the interaction with phonons. Such a temperature dependence, however, does not seem to explain the experimental facts of the anomalous electrical resistance, it rather corresponds to the normal one. (iii) Further, this process seems to be unable to explain satisfactorily the abrupt decrease of electrical resistance at the Curie point.

Our standpoint for this phenomenon is based on the s-d interaction described in detail in the former paper [I]<sup>4)</sup>. We consider that s-electron (corresponding to s-and d-electrons in the rare earth metals) is nearly free and carries current, but the unfilled inner shell electron (d-electron in the transition metals and f-electron in the rare earth metals) does not carry current, being nearly localized. The Coulomb interaction between the conduction and the unfilled shell electrons has the same periodicity as that of the crystal, but the exchange interaction depends on the relative orientation of the spins of both electrons. Therefore at absolute temperature zero, all the spins of d-electrons (we mean by "d-electron" the unfilled shell electron) being in order, there is no resistance, while in finite temperatures this order is disturbed and the exchange interaction is no more periodic, and thus a resistance appears and increases with temperature. Above the Curie temperature, the direction of d-electron spin becomes perfectly at random, and resistance remains constant. This temperature dependence is exactly the same as that experimentally observed.

In the following sections we calculate qualitatively the electrical resistance caused by the above process.

## § 2. The basic equations

The basic Hamiltonian of the exchange interaction between the conduction and the unfilled shell electrons was discussed in detail in [I] and is written

$$H_{int} = -1/N \cdot \sum_{k_1} \sum_{k_2} \sum_{R_n} J(|\mathbf{k}_1 - \mathbf{k}_2|) \exp[i(\mathbf{k}_1 - \mathbf{k}_2) \mathbf{R}_n] \\ \times \{ (a_{k_1+}^* a_{k_2+} - a_{k_1-}^* a_{k_2-}) S_n^z + a_{k_1+}^* a_{k_2-} S_n^- + a_{k_2-}^* a_{k_1+} S_n^+ \} \quad (1)$$

where  $a_{k\pm}^*$  and  $a_{k\pm}$  are the creation and the annihilation operators of the state of wave vector  $\mathbf{k}$  of  $\pm$  spin,  $S_n$  is the spin operator of the unfilled shell electrons located at the lattice point  $\mathbf{R}_n$ , and  $J(|\mathbf{k}_1 - \mathbf{k}_2|)$  is the generalized exchange energy

$$J(|\mathbf{k}_1 - \mathbf{k}_2|) = N \int \int d\mathbf{r}_1 d\mathbf{r}_2 \varphi_{s\mathbf{k}_1}^*(r_1) \varphi_{a\mathbf{k}_2}^*(r_2) \frac{e^2}{r_{12}} \varphi_{s\mathbf{k}_2}(r_2) \varphi_{a\mathbf{k}_1}(r_1) \exp[i(\mathbf{k}_1 - \mathbf{k}_2) \mathbf{R}_n]. \quad (2)$$

Here,  $\varphi_{s\mathbf{k}}(r)$  and  $\varphi_{a\mathbf{k}}(r)$  are the wave functions of the conduction electron of wave vector  $\mathbf{k}$  and the unfilled shell electron at the point  $\mathbf{R}_n$ , and (2) reduces to the ordinary exchange energy when  $\mathbf{k}_1 = \mathbf{k}_2$ . As has already been mentioned,  $J(|\mathbf{k}_1 - \mathbf{k}_2|)$  depends originally not only on  $|\mathbf{k}_1 - \mathbf{k}_2|$  but also slightly on  $\mathbf{k}_1$  or  $\mathbf{k}_2$  where, however, the wave vector  $\mathbf{k}$  is always a vector on the Fermi surface, and hence we can completely neglect  $\mathbf{k}_1$  dependence. Eq. (1) is not applicable when the orbital moment is not quenched.

In the following calculation, we use the approximation of the molecular field because this method is applicable in the whole temperature range and especially convenient to observe the behavior at the Curie temperature.

Above the Curie temperature, each spin  $S_n$  behaves perfectly independently and the conduction electrons are scattered at each lattice point. This situation somewhat resembles the mechanism of the electrical resistance of a perfectly disordered alloy. When the temperature is lower than the Curie temperature, an ordering appears and the situation becomes quite complicated. We must distinguish between the distribution functions of the conduction electrons of  $\pm$  spins, namely  $n_{\pm}^{\pm}$ . In the approximation that the Fermi energy is much larger than the exchange energy  $J(0)$  and the temperature (in the unit of energy), we can consider that the Fermi energies of  $\pm$  spins,  $\varepsilon_0^{\pm}$  are the same because of the equilibrium between the kinetic energy and the exchange energy, as was discussed in [I]. In the first term of (1), a scattering can be caused by the deviation of  $S_n^z$  from the average orientation  $\sigma$ , namely  $(S_n^z - \sigma)$ . From (1) we can easily see that in perfect ordering there is no scattering due to the first term. The second and the third terms represent the quantum effect and do not appear in alloys. When  $S_n^z$  changes by  $\pm 1$  because of these processes, the change in exchange energy can be written in the approximation of the molecular field as  $\mp \alpha \sigma$ , where  $\alpha$  is related to the Curie temperature  $T_c$  as shown by

$$\alpha = (3/S(S+1)) \kappa T_c. \quad (3)$$

From the above consideration we can derive our Bloch equation. For simplicity, however, we calculate the resistance not by solving the Bloch equation, namely the equation

of the detailed balance, but by solving the equation of the balancing of the total wave vector  $\mathbf{K}$ .

The increase of  $\mathbf{K}$  due to an external field is

$$(d\mathbf{K}/dt)_{\text{field}} = n(e/\hbar) \mathbf{F}. \quad (4)$$

The increase of  $\mathbf{K}$  due to the scattering is

$$\begin{aligned} \left(\frac{d\mathbf{K}}{dt}\right)_{\text{scatt}} &= \frac{2\pi}{\hbar} N \sum_{\mathbf{k}_1} \sum_{\mathbf{k}_2} \sum_{S_z=-S}^S (\mathbf{k}_1 - \mathbf{k}_2) \\ &\times [ |W_{\mathbf{k}_1\mathbf{k}_2}|^2 \{ (1 - n_{\mathbf{k}_1}^+) n_{\mathbf{k}_2}^+ \delta(\varepsilon_{\mathbf{k}_1}^+ - \varepsilon_{\mathbf{k}_2}^+) + (1 - n_{\mathbf{k}_1}^-) n_{\mathbf{k}_2}^- \delta(\varepsilon_{\mathbf{k}_1}^- - \varepsilon_{\mathbf{k}_2}^-) \} (S_z - \sigma)^2 w_{S_z} \\ &+ |V_{\mathbf{k}_1\mathbf{k}_2}|^2 (1 - n_{\mathbf{k}_1}^-) n_{\mathbf{k}_2}^+ (S - S_z) (S + S_z + 1) w_{S_z} \delta(\varepsilon_{\mathbf{k}_1}^- - \varepsilon_{\mathbf{k}_2}^+ - \alpha\sigma) \\ &- |V_{\mathbf{k}_1\mathbf{k}_2}|^2 (1 - n_{\mathbf{k}_2}^+) n_{\mathbf{k}_1}^- (S - S_z) (S + S_z + 1) w_{S_z+1} \delta(\varepsilon_{\mathbf{k}_2}^+ - \varepsilon_{\mathbf{k}_1}^- + \alpha\sigma) ] + (d\mathbf{K}/dt)_0, \end{aligned} \quad (5)$$

where

$$|W_{\mathbf{k}_1\mathbf{k}_2}|^2 = |V_{\mathbf{k}_1\mathbf{k}_2}|^2 = (1/N^2) J^2 (|\mathbf{k}_1 - \mathbf{k}_2|) \quad (6)$$

and  $w_{S_z}$  is the probability that  $S_n^z$  has the value  $S_z$ , and assuming the thermal equilibrium of the spin system,

$$w_{S_z} = \exp\left(\frac{\alpha\sigma}{\kappa T} S_z\right) / \sum_{S_z=-S}^S \exp\left(\frac{\alpha\sigma}{\kappa T} S_z\right). \quad (7)$$

Further, at temperature lower than  $T_c$ , we have

$$\sigma = \sum_{S_z=-S}^S S_z w_{S_z} \quad (8)$$

and  $(d\mathbf{K}/dt)_0$  means the change due to other processes, for example, interactions with the phonons and impurities. As we are now interested in the anomalous resistance, and as the resistances due to each process are additive in our approximation method, we neglect the term  $(d\mathbf{K}/dt)_0$  in the following calculations.

Next, we assume the distribution function  $n_{\mathbf{k}}^{\pm}$  as Kramer's distribution

$$n_{\mathbf{k}}^{\pm} = [\exp((\varepsilon_{\mathbf{k}}^{\pm} - \varepsilon_0)/\kappa T - \mathbf{c} \cdot \mathbf{k}) + 1]^{-1}. \quad (9)$$

This assumption may be satisfactory when the interaction with the phonon is strong enough.

By the equation of the balancing of the total wave vector

$$(d\mathbf{K}/dt)_{\text{field}} + (d\mathbf{K}/dt)_{\text{scatt}} = 0, \quad (10)$$

we can determine  $\mathbf{c}$ .

### § 3. The calculations

To solve the equation of § 2, we use the following assumptions; the energy spectrum of the conduction electron can be written by using the effective mass  $m^*$ ; the Fermi energy is

much larger than the exchange energy  $J(0)$  and the temperature; the crystal is isotropic, and, as usual, the summation  $\sum_k$  may be replaced by an integration over  $k$ .

The technique of the calculation is the same as that of ordinary electrical resistance. The calculation is straightforward and (10) becomes

$$n \frac{e}{\hbar} \mathbf{F} = c \frac{2\pi}{\hbar} \frac{3}{2} \frac{n^2}{N} \frac{m}{\hbar^2} \frac{\kappa T}{\varepsilon_0} (S - \sigma) (S + \sigma + 1) J_{eff}^2, \quad (11)$$

where  $k_0$  is the wave vector of the Fermi surface, and

$$J_{eff}^2 = 4 \int_0^1 J^2(2k_0 x) x^3 dx. \quad (12)$$

On the other hand, the electric current  $\mathbf{j}$  is

$$\mathbf{j} = \mathbf{F} / \rho = (e/m) n \kappa T C \quad (13)$$

and thus the electrical resistance  $\rho$  is

$$\rho = (3\pi m^{*2} / N e^2 \hbar^2) (S - \sigma) (S + \sigma + 1) J_{eff}^2 / \varepsilon_0 \quad (14)$$

or introducing the ratio of the effective mass and the mass of electron,

$$\mu = m^* / m \quad (15)$$

and putting values of the physical constants,

$$\rho = 4.3 \times 10^{-4} \left( \frac{10^{23}}{N} \right) \mu^2 (S - \sigma) (S + \sigma + 1) \frac{J_{eff}^2}{\varepsilon_0} \text{ ohm} \quad (16)$$

where  $J_{eff}$  and  $\varepsilon_0$  are measured in the unit of e.V. and  $N$  per c.c..

#### § 4. Discussions

Equation (16) corresponds fairly well to the temperature dependence, as we expected. At absolute temperature zero,  $\sigma = S$  and thus the electrical resistance  $\rho = 0$ . When the temperature is raised,  $\sigma$  decreases, and  $\rho$  increases. Above the Curie temperature,  $\sigma = 0$ , and  $\rho$  remains constant. To examine the temperature dependence more in detail, we should proceed to a higher order of the approximation. At sufficiently low temperatures, the method of the molecular field is not appropriate and we should use the spin-wave method. The situation is not simple unless we use the hypothesis that the spin-wave system is always in thermal equilibrium corresponding to Bloch's hypothesis for electron-phonon interaction. In the temperature near Curie temperature too, we should consider the short range ordering. For example, the fact that the curving of the resistance-temperature curve begins at a temperature slightly above the magnetic Curie point may be the effect of the short range ordering. Above the Curie temperature, the electrical resistance (16) is proportional to  $S(S+1)$ , thus in the metal of large spin,  $\rho$  would be very large. This tendency is seen also in the experimental results. For example, the anomalous electrical resistance of Fe is larger than that of Ni, and in Gd ( $S=7/2$ ) particularly the anomalous electrical resistance is very large, and the ordinary resistance due to the electron-phonon interaction is rather small at room temperature. In rare earth metals, the anoma-

lous electrical resistance decreases when the spin value decreases, but in this case, except for Gd, there remains an orbital moment and our basic Hamiltonian (1) is not applicable. The calculation for such cases is now in progress.

Finally, we calculate the numerical values of the resistance at a temperature higher than  $T_c$  for two cases, that is, Ni and Gd. In Ni, by Zener<sup>5)</sup>

$$N=0.9 \times 10^{23} \text{ per c.c.}, \quad S=0.3,$$

$$J_{eff} = J(0) = 0.48 \text{ e.V.}, \quad \epsilon_0 = 3.14/\mu \text{ e.v.}, \quad (17)$$

and

$$\rho = 14 \times 10^{-6} \mu^3 \text{ ohm.} \quad (18)$$

The observed value is about  $10 \times 10^{-6}$  ohm, as shown in Fig. 1,

In Gd,

$$N=3.07 \times 10^{22} \text{ per c.c.}, \quad S=7/2,$$

$$J_{eff} = J(0) = 0.157 \text{ e.v.}, \quad \epsilon_0 = 4.4/\mu \text{ e.v.}, \quad (19)$$

and

$$\rho = 120 \times 10^{-6} \mu^3 \text{ ohm} \quad (20)$$

where, to estimate  $J(0)$ , we have used the mean value of the results of the optical spectrum<sup>6)</sup>, multiplying by the same factor as in Ni. The experimental value is about  $130 \times 10^{-6}$  ohm.

In both cases, the theoretical and the experimental values coincide fairly well in the order of magnitude.

The author expresses his cordial thanks to Prof. Ariyama and also to Prof. Matsubara for their many valuable discussions.

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