# Electro-Hydraulic Piston Control using Neural MRAC Based on a Modified State Observer

Y. Yang, S. N. Balakrishnan, L. Tang, R.G. Landers

Abstract—A new model reference adaptive control design method using neural networks that improves both transient and steady stage performance is proposed in this paper. Stable tracking of a desired trajectory can be achieved for nonlinear systems having significant uncertainties. A modified state observer structure is designed to enable desired transient performance during uncertainty learning. The neural network adaptation rule is derived using Lyapunov theory, which guarantees stability of the error dynamics and boundedness of the neural network weights. An extra term is added in the controller expression to introduce a 'soft switching' sliding mode that can be used to adjust tracking errors. The method is applied to control the velocity of an electro-hydraulic piston, and experimental results demonstrate the desired performance is achieved with smooth control effort.

*Index Terms*—Neural networks, adaptive control, electronic-hydraulic systems

# I. INTRODUCTION

A PPLICATIONS of artificial neural networks in the field of control have been developed for decades. Narendra and Parthasarathy [1] provided a stability proof for the first time, and demonstrated the potential of neural networks in the identification and control of nonlinear systems. Sanner and Slotine [2] developed a direct tracking control method with Gaussian radial basis function (RBF) networks for feedback control of nonlinear systems. Since then, various adaptive control techniques using neural networks were put forward. Calise et al. [3]-[6] introduced neural networks to the dynamic inversion technique in order to cancel the inversion error, and developed model reference adaptive control (MRAC) based on neural network estimation. The neural networks are trained online using a Lyapunov-based approach, similar to the approach followed in [2], [7].

Recently, MRAC has been applied in solving control problems for system with matched unmodeled dynamics [8], [9]. However, although these developments can be employed to improve robustness, tracking accuracy can only be shown to be bounded, and the bound depends on the disturbances itself. In [10], a new MRAC neural network controller named  $\mathcal{L}_1$  adaptive control is proposed, and transient performance of both the system's input and output signals are guaranteed.

At the same time, due to its simplicity and robustness, Sliding Mode Control (SMC) is also often used in adaptive control [11]-[13]. One drawback of SMC is that unavoidable chattering occurs when the control signal switches signs along the sliding surface. A soft-switching sliding mode technique was introduced by Lychevsky [14] in order to avoid oscillations and achieve asymptotic stability at the same time. In [15], by using a method similar to SMC, a novel approach combining an adaptive neural network feedforward controller with a continuous robust integral of sign of error (RISE) feedback controller is introduced. In this method, it is shown using Lyapunov theory that the tracking error is asymptotically stable.

This paper develops a new neural network MRAC with guaranteed transient performance and asymptotic stability. Based on the MRAC neural network controller, the neural network observer structure is modified in the manner of [16]. In this modification, instead of introducing additional filters, a factor of the observer error is added to in the neural network observer structure. As a result, this new method enables further increases in the adaptive gain, leading to better tracking performance. At the same time, the modified term is inactive when the neural network estimation is ideal; therefore, estimation accuracy is guaranteed. In order to achieve improved transient performance and stability, a soft-switching sliding mode modification is combined with the neural network adaptive controller. It is proven, using the Lyapunov method, that it ideally leads to asymptotic stability instead of UUB and, at the same time, is free from chattering that are common for typical sliding mode adaptive controllers. In general, the proposed controller enables higher

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adaptive gains while simultaneously providing improved transient performance and asymptotic stability. This method is applied to an electro-hydraulic test bench [17] for velocity tracking control. The results illustrate that desired tracking performance is achieved with smooth (i.e., non-chattering) control.

The rest of the paper is organized as follows. In Section II, the system and neural network structure are defined. In Section III, the control solution is proposed. Stability proofs of the observer and state error signals are presented in Section IV. Section V includes a description of the electro-hydraulic piston system, and presents the results and analysis of a series of velocity tracking experiments using the proposed control methodology.

# II. PROBLEM DESCRIPTION

Consider the following single input single output (SISO) system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \vdots \\ \dot{x}_n = b(u - f(\mathbf{x})) \end{cases}$$
(1)

where b > 0. The system output is defined as

$$y = cx_1 \tag{2}$$

where c is a non-zero constant. The initial condition is

$$\mathbf{x}(0) = \mathbf{0} \tag{3}$$

The set of equations in (1) can be written in a compact form as

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B(u(t) - f(\mathbf{x})) \tag{4}$$

where  $\mathbf{x} \in \mathbb{R}^n$  is the system state vector,  $u \in \mathbb{R}$  is the control signal, *A* is an  $n \times n$  system matrix, *B* is  $n \times 1$  vector, (A, B) is assumed to be controllable, and  $f : \mathbb{R}^n \to \mathbb{R}$  is an unknown continuous nonlinear function. All of the states are assumed to be measurable.

The control objective is to design a neural adaptive controller which ensures the output tracks a desired bounded continuous trajectory, denoted r(t), and the system behavior follows a nominal linear time-invariant (LTI) system which is designed through standard methods (e.g., through linear quadratic regulator theory). At the same time, the controller should guarantee desired transient and steady state performance in the presence of uncertainties.

Assume the following neural network approximation of  $f(\mathbf{x})$  exists

$$f(\mathbf{x}) = \mathbf{W}^T \phi(\mathbf{x}) + \varepsilon(\mathbf{x}) |\varepsilon(\mathbf{x})| < \varepsilon^*$$
(5)

where  $\phi(\mathbf{x})$  is a set of radial basis functions. Each element of  $\phi(\mathbf{x})$  is defined as

$$\phi(y) = \exp(-(y-z)^{T}(y-z)/\sigma^{2})$$
 (6)

where z is the center location and  $\sigma$  is the 'width.' The vector **W** contains the ideal network weights,  $\varepsilon(\mathbf{x})$  is the network approximation error, and  $\varepsilon^*$  is its uniform bound. Further, it is assumed that a compact convex set  $\Omega$  is known a priori such that

$$\mathbf{W} \in \Omega \tag{7}$$

In order to realize tracking control for this SISO system, a neural network adaptive controller is developed in the next section.

#### **III. CONTROL SOLUTION**

The proposed controller consists of three parts: linear feedback control  $K_1$ **x**, neural network adaptive control  $u_e$ , and soft switching sliding mode control  $\mu$ 

$$u = K_1 \mathbf{x} + u_e + \mu \tag{8}$$

where  $K_1$  is the closed loop feedback gain that ensures the closed-loop system matrix  $A_m = A - BK_1$  is Hurwitz. The linear feedback control ensures stability when there is no uncertainty. The adaptive control is obtained through the neural network observer, which cancels the uncertainty. The soft switching sliding mode control guarantees asymptotic stability in the presence of neural network estimation error.

Substituting (8) into (4)

$$\dot{\mathbf{x}}(t) = A_m \mathbf{x}(t) + B(u_e(t) + \mu - f(\mathbf{x}))$$
(9)

The following state observer structure is defined

$$\dot{\mathbf{x}}(t) = A_m \, \hat{\mathbf{x}}(t) + B(u_e(t) + \mu - \hat{f}) - K_2 \, \tilde{\mathbf{x}}(t)$$
(10)

where  $\hat{\mathbf{x}}(t)$  represents the observer states at time *t*. The initial observer conditions are

$$\hat{\mathbf{x}}(0) = \mathbf{0} \tag{11}$$

Since the uncertainty and the true neural network weights are unknown, they are represented as  $\hat{\mathbf{W}}^T \phi(\mathbf{x})$  where  $\hat{\mathbf{W}}$  represents the estimated neural network weights with a proper weight update law. The observer gain matrix is assumed diagonal for convenience and is expressed as  $K_2 = diag(k_2^1, k_2^2, ..., k_2^n)$ . In the observer structure,  $\hat{f}$  is assumed to be canceled perfectly by the neural network controller, i.e.  $\hat{f} = \hat{\mathbf{W}}^T \phi(\mathbf{x})$ .

The observer error is defined as

$$\tilde{\mathbf{x}}(t) \equiv \hat{\mathbf{x}}(t) - \mathbf{x}(t) \tag{12}$$

The adaptive weight update law is defined as

$$\hat{\mathbf{W}}(t) = \Gamma_c \operatorname{Pr}\operatorname{oj}(\hat{\mathbf{W}}(t), \phi(\mathbf{x})\tilde{\mathbf{x}}(t)^T PB)$$
(13)

where *P* is found by solving  $A_m^T P + PA_m = -Q$ , where *Q* is a positive definite matrix and  $\Gamma_c$  is the neural network learning rate. The projection operator property guarantees the boundedness of the neural network weights error

$$\tilde{\mathbf{W}}^T \tilde{\mathbf{W}} \le W_{\max} \tag{14}$$

where  $W_{\max} \equiv \max_{\mathbf{W} \in \Omega} 4 \|\mathbf{W}\|^2$ ,  $\tilde{\mathbf{W}} \equiv \hat{\mathbf{W}} - \mathbf{W}$  [18]. With the neural network weights, the adaptive control expression becomes

$$u_e = k_g r(t) + \hat{\mathbf{W}}^T \phi(\mathbf{x}) \tag{15}$$

where

$$k_g \equiv \frac{1}{CA_m^{-1}B} \tag{16}$$

is the reference system open loop gain.

Subtracting (9) from (10), and substituting (15) into the resulting equation, the observer error dynamics are

$$\dot{\tilde{\mathbf{x}}}(t) = (A_m - K_2)\tilde{\mathbf{x}} + B(\tilde{\mathbf{W}}^T \phi(\mathbf{x}) - \varepsilon)$$
(17)

By using the Lyapunov method [9], it will be shown that the neural network estimation error and the observer error are bounded. By introducing the observer gain  $K_2$ , the learning process is smoothed, and the modified term decreases as  $\tilde{\mathbf{x}}$  decreases; therefore, learning accuracy is guaranteed. As a result, the modified observer structure enables increasing adaptation gain.

#### IV. STABILITY ANALYSIS

In this section, the Lyapunov method is used to prove the boundedness of the observer error dynamics. In order to assure asymptotic convergence of the reference error, the soft-switching sliding mode controller is derived. Details of the proofs are provided in the following subsections.

# A. Observer error

To derive the error bound for the neural network observer, consider the Lyapunov function:  $V(\tilde{\mathbf{x}}, \tilde{\mathbf{W}}) = \tilde{\mathbf{x}}^T P \tilde{\mathbf{x}} + \Gamma_c^{-1} \tilde{\mathbf{W}}^T \tilde{\mathbf{W}}$ . Differentiating V

$$\dot{V} = \dot{\tilde{\mathbf{x}}}^T P \tilde{\mathbf{x}} + \tilde{\mathbf{x}}^T P \dot{\tilde{\mathbf{x}}} + \Gamma_c^{-1} (\tilde{\mathbf{W}}^T \dot{\tilde{\mathbf{W}}} + \dot{\tilde{\mathbf{W}}}^T \tilde{\mathbf{W}})$$
(18)

Substituting the weight update law in (13) and the observer dynamics in (17), (18) becomes

$$\vec{V} = -\tilde{\mathbf{x}}^{T} Q \tilde{\mathbf{x}} - 2 \tilde{\mathbf{x}}^{T} K_{2} P \tilde{\mathbf{x}} - 2 \tilde{\mathbf{x}}^{T} P B(\tilde{\mathbf{W}}^{T} \phi(\mathbf{x}) - \varepsilon(\mathbf{x})) 
+ 2 \tilde{\mathbf{W}}^{T} \operatorname{Pr} \operatorname{oj}(\hat{\mathbf{W}}, \phi(\mathbf{x}) \tilde{\mathbf{x}}^{T} P B) 
\leq 2 \| P B \varepsilon \| \| \tilde{\mathbf{x}} \| - [\lambda_{\min}(Q) + 2\lambda_{\min}(K_{2} P)] \| \tilde{\mathbf{x}} \|^{2}$$
(19)

therefore  $\dot{V} \leq 0$  when

$$\|\tilde{\mathbf{x}}\| \ge \frac{2 \|Pb\varepsilon^*\|}{\lambda_{\min}(Q) + 2\lambda_{\min}(K_2P)}$$
(20)

As a result,

$$V(\tilde{\mathbf{x}}, \tilde{\mathbf{W}}) \leq \tilde{\mathbf{x}}^{T} P \tilde{\mathbf{x}} + \Gamma_{c}^{-1} W_{\max}$$

$$\leq \lambda_{\max}(P) \left( \frac{2 \left\| PB\varepsilon^{*} \right\|}{\lambda_{\min}(Q) + 2\lambda_{\min}(K_{2}P)} \right)^{2} + \Gamma_{c}^{-1} W_{\max}$$
(21)

and

$$V(\tilde{\mathbf{x}}, \tilde{\mathbf{W}}) \ge \tilde{\mathbf{x}}^T P \tilde{\mathbf{x}} \ge \lambda_{\min}(P) \|\tilde{\mathbf{x}}\|^2$$
(22)

Equations (21) and (22) lead to  

$$\|\tilde{\mathbf{x}}\| \leq \sqrt{\frac{\left(\lambda_{\max}(P)\left(\frac{2\|Pb\varepsilon^*\|}{\lambda_{\min}(Q) + 2\lambda_{\min}(K_2P)}\right)^2 + \Gamma_c^{-1}W_{\max}\right)}{\lambda_{\min}(P)}}$$
(23)

In (23), by increasing the adaptation gain  $\Gamma_c$  and the observer gain  $K_2$ ,  $\|\tilde{\mathbf{x}}\|$  can be made arbitrarily small; therefore, precise uncertainty estimation using an online neural network is guaranteed.

#### B. Reference error

Note that with adaptive control and linear feedback control alone (i.e.,  $\mu = 0$ ), the controller is able to track the reference system. However, it is only able to do so with bounded tracking errors. With the addition of a soft-switching sliding mode controller, the tracking error can be made asymptotic stable. The reference LTI system dynamics are characterized by

$$\dot{\mathbf{x}}_r = A_m \mathbf{x}_r + b(u_r + k_g r - W^T \phi(\mathbf{x}_r))$$
(24)

where  $u_r \equiv \mathbf{W}^T \phi(\mathbf{x}_r)$  is the reference controller, which cancels the uncertainty. By subtracting the reference dynamics (24) from the actual system dynamics (9), the tracking error dynamics are expressed as

$$\mathbf{e} \equiv \mathbf{x}(t) - \mathbf{x}_{r}(t)$$
  
=  $A_{m}\mathbf{e} + b(\mu + (\hat{\mathbf{W}}^{T} - \mathbf{W}^{T})\phi(\mathbf{x}) - \varepsilon(\mathbf{x}))$  (25)

Recalling the definition of system dynamics as given in (1), (25) can be written as

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = e_3 \\ \cdots \\ \dot{e}_n = bK_1 \mathbf{e} + b(\mu + \tilde{\mathbf{W}}^T \phi(\mathbf{x}) - \varepsilon(\mathbf{x})) \end{cases}$$
(26)

The sliding surface is defined as

$$s \equiv \sum_{p=0}^{n-1} \lambda_p e_1^{(n-p-1)}$$
(27)

where  $\lambda_p > 0$ , p = 0, 1, ..., n - 1. In most cases, the designer can set  $\lambda_0 = 1$ . For example, when n = 3, the sliding manifold is  $s = e_3 + \lambda_1 e_2 + \lambda_2 e_1$ .

With the Lyapunov function  $V_s = \frac{1}{2}s^2$ , its derivative is

$$\dot{V}_{s} = s\dot{s} = s(\sum_{p=1}^{n-1} \lambda_{p} e_{1}^{(n-p)} + bK_{1}\mathbf{e} + b(\mu + D))$$
(28)

where

$$D \equiv \tilde{\mathbf{W}}^T \boldsymbol{\phi}(\mathbf{x}) - \boldsymbol{\varepsilon}(\mathbf{x}) \tag{29}$$

Recalling the neural network approximation property in (5) and the error boundedness of the weights in (14), the bound for *D* is

$$\left\|D\right\| = \left\|\tilde{\mathbf{W}}^{T}\phi(\mathbf{x}) - \varepsilon(\mathbf{x})\right\| \le W_{\max} + \varepsilon^{*} \equiv D^{*}$$
(30)

Now the soft switching sliding manifold control term is formulated as

$$\mu = -\sum_{p=1}^{n-1} \lambda_p e_1^{(n-p)} / b - K_1 \mathbf{e} - \beta \tanh(\alpha s)$$
(31)

Substituting (31) into (28)

$$\dot{V}_s = bs(-\beta \tanh(\alpha s) + D)$$
 (32)

When s > 0

$$\dot{V}_s \le bs(-\beta \tanh(\alpha s) + D^*)$$
(33)

Therefore,  $\dot{V}_s \ge 0$  only when

$$\tanh(\alpha s) \le D^* / \beta \tag{34}$$

which leads to

$$0 < s \le \frac{1}{2\alpha} \ln \frac{1 + \frac{D}{\beta}}{1 - \frac{D^*}{\beta}}$$
(35)

When s < 0

$$\dot{V}_s \le b(-s)(\beta \tanh(\alpha s) + D^*)$$
(36)

Therefore,  $\dot{V}_s \ge 0$  only when

$$\tanh(\alpha s) > -D^* / \beta \tag{37}$$

which leads to

$$0 > s \ge \frac{1}{2\alpha} \ln \frac{1 - \frac{D^2}{\beta}}{1 + \frac{D^*}{\beta}}$$
(38)

From (35) and (38), it can be observed that the bound for the sliding manifold is

**D**\*

$$\|s\| < \frac{1}{2\alpha} \ln \frac{1 + \frac{D}{\beta}}{1 - \frac{D^*}{\beta}} \equiv \gamma_1$$
(39)

As long as  $\alpha > 0$  and  $\beta > D^*$ , the sliding manifold will remain bounded. By increasing  $\alpha$  and  $\beta$ , the bound of the sliding manifold will converge to 0. The closed-loop system is asymptotically stable when  $\gamma_1 = 0$ .

With (8), (15), and (31), the final expression for the proposed controller is

$$u = K_1 \mathbf{x}_r + k_g r(t) + \hat{\mathbf{W}}^T \phi(\mathbf{x}) - \sum_{p=1}^{n-1} \lambda_p e_1^{(n-p)} / b - \beta \tanh(\alpha s) (40)$$

Note that since no discontinuous function is introduced, this controller is *smooth* and capable of driving the tracking error asymptotically to zero.

## V. EXPERIMENTAL RESULTS

The test bed for the proposed control method is a Caterpillar Electro-Hydraulic Test Bench, which was a gift from Caterpillar to the Missouri University of Science and Technology as part of a laboratory dedicated to electro-hydraulics and mechatronics. The test bench consists of five distinct physical components which affect the system operation and dynamics: control electronics, pilot solenoid valve, spool valve, piston, and sensors. Additionally, specialized computer hardware and software interfaces for actuators and sensors provide for real-time control. A system diagram is shown in Fig. 1.



# Fig. 1. Electro-Hydraulic System Diagram.

The electro-hydraulic system considered in this study cannot be well described by a linear, time-invariant model since the characteristics such as nonlinear friction, dead band, and nonlinear valve gains cannot be neglected. The system includes pressure sensors that measure the pressures in both chambers and an encoder that measures the piston displacement. In this system, the spool valve is contained in a sealed housing with no integrated sensor; therefore, it is impossible to measure its position either in real-time or offline. Additionally, it is subject to significant and unpredictable stiction effects and flow forces, so its position cannot be accurately predicted based solely on the control input and measured states. The pilot valve input will determine the direct input into the forward and reverse valves. The relationship between the variables is

$$\begin{cases} I_{cf} = I_c + I_{cf0}, & I_{cr} = 0, & \text{if } I_c > 0 \\ I_{cf} = 0, & I_{cr} = I_c + I_{cf0}, & \text{if } I_c < 0 \\ I_{cf} = 0 & I_{cr} = 0, & \text{if } I_c = 0 \end{cases}$$
(41)

where  $I_c$  (A) is the input current,  $I_{cf}$  (A) is the input current to the forward valve,  $I_{cr}$  (A) is the input current to the reverse valve and  $I_{cf0} = I_{cr0} = 0.4$  A are the estimated dead band values for the forward and reverse directions, respectively.

From previous work [17], a simple input-output model for the piston response was developed based on experimental data. In order to remove noise from the encoder, a low-pass filter is utilized. The filtered piston position is numerically differentiated to calculate the (approximate) piston velocity. It is done online using a first order backwards finite difference scheme. The relationship between the estimated velocity and the encoder position output is

$$v_e = \dot{x}_f = -5x_f + 5x_e \tag{42}$$

where  $v_e$  (mm/s) is the estimated velocity,  $x_f$  (mm) is the filtered position, and  $x_e$  (mm) is the encoder position measurement.

As a result, instead of using a high order nonlinear system, in the following experiments a simple linear model with matched uncertainties is used

$$\dot{v} = \frac{1}{m} (-Bv + b(I_c - f(x, v, P_1, P_2, I_c)))$$
(43)

where x (mm) is the piston displacement, v (mm/s) is the piston velocity, B = 2 (kg/s) is the estimated value of the viscous friction coefficient, and b = 1 (N/A) is the estimated value of the control gain. The parameters  $P_1$  (kPa) and  $P_2$ (kPa) are the measured pressure from the first and second chambers, respectively, f is the unknown nonlinear dynamics, and m = 3.85 kg is the measured piston mass. The sample period is 0.01 s.

With the feedback control gain  $K_1$ , the closed loop reference velocity dynamics are picked as

$$\dot{v}_r = \frac{1}{3.85} (-3v_r + 3r) \tag{44}$$

A series of open loop experiments are conducted to ensure reference system is realizable.

The control law is

$$I_c = K_1 v + k_g r + \hat{W}^T \mathbf{\phi} - K_1 e - \beta \tanh(\alpha s)$$
(45)

where  $e = v - v_r$ . The radial basis function  $\mathbf{\phi} \in \mathbb{R}^{12}$  used for neural network structure is

$$\mathbf{\phi} = \left[ \mathbf{\phi}_{1}(v), \phi_{2}(x), \mathbf{\phi}_{3}(P_{1}), \mathbf{\phi}_{4}(P_{2}), \phi_{5}(I_{c}), 1 \right]^{T}$$
(46)

where

$$\begin{split} \boldsymbol{\phi}_{1}(v) &= \left[ e^{-(v-z_{1})^{2}/\sigma_{1}^{2}}, e^{-(v-z_{2})^{2}/\sigma_{1}^{2}}, e^{-(v-z_{3})^{2}/\sigma_{1}^{2}} \right]^{I} \\ \boldsymbol{\phi}_{2}(x) &= e^{-(x-z_{4})^{2}/\sigma_{2}^{2}} \\ \boldsymbol{\phi}_{3}(P_{1}) &= \left[ e^{-(P_{1}-z_{5})^{2}/\sigma_{3}^{2}}, e^{-(P_{1}-z_{6})^{2}/\sigma_{3}^{2}}, e^{-(P_{1}-z_{7})^{2}/\sigma_{3}^{2}} \right]^{T} \\ \boldsymbol{\phi}_{4}(P_{2}) &= \left[ e^{-(P_{2}-z_{8})^{2}/\sigma_{4}^{2}}, e^{-(P_{2}-z_{9})^{2}/\sigma_{4}^{2}}, e^{-(P_{2}-z_{10})^{2}/\sigma_{6}^{2}} \right]^{T} \\ \boldsymbol{\phi}_{5}(I_{c}) &= e^{-(I_{c1}-z_{11})^{2}/\sigma_{7}^{2}} \end{split}$$

Here  $z_1 = 0$  mm/s,  $z_2 = 15$  mm/s,  $z_3 = -15$  mm/s,  $z_4 = 0$  mm/s,  $z_5 = z_8 = 0$  kPa,  $z_6 = z_9 = 40$  kPa,  $z_7 = z_{10} = 40$  kPa,  $z_{11} = 0$  A,  $\sigma_1 = 1$ ,  $\sigma_2 = 20$ ,  $\sigma_3 = \sigma_4 = \sigma_5 = \sigma_6 = 2$ , and  $\sigma_7 = 0.05$ . The centers and widths of the Radial Basis Functions (RBFs) are selected so that the neural network can estimate uncertainty over the entire working region of the system with similar sensitivity. Notice that as long as f is a continuous function of x, v,  $P_1$ ,  $P_2$ , and  $I_c$ , the neural network approximation assumption (3) is valid. While friction is not continuous at origin of velocity, two separate networks can be used for positive and negative direction to enable precise approximation.

The learning rate for the adaptive controller is selected as  $\Gamma_c = 100$ , the observer gain is  $K_2 = 100$ , and the sliding surface is  $s = e = v - v_r$  since the modeled system is first order. The soft switching sliding mode control parameters are  $\alpha = 150$  and  $\beta = 150$ . Increasing  $\Gamma_c$  causes larger overshoot, while decreasing it increases the error bound. Increasing  $K_2$  increases the error bound, while decreasing it increases overshoot. The parameters are tuned in order to decrease the steady state error bound and obtain the best possible transient performance. Increasing  $\alpha$  will increase the feedback controller's sensitivity to the tracking error, and increasing  $\beta$  can increase the controller's response speed; however, when it is too large there will be significant overshoot. Neural network weights are updated by the adaptive law (10), with P = 1.

To verify the feasibility of the proposed controller, take the command input as

$$r = 18 \operatorname{sign}(\sin(2\pi/32)t) \operatorname{mm/s}$$
 (47)

The results, with comparison to previous work using a well tuned PI controller, are shown in Fig. 2.



Fig. 2. Experimental results for  $r = 18 \operatorname{sign}(\sin(2\pi/32)t) \operatorname{mm/s}$ .

There is an initial inevitable delay (approximately 0.8 s) for each experiment, due to the flow filling process of the test bench. Neglecting the first peak, the controller keeps the velocity error bounded within 4.5 mm/s during both steady and transient stages. Disregarding the first step, the 5% settling times for the second, third and fourth steps are 1.5, 2.7, and 1.5 s respectively. The difference is due to the nonlinearity and asymmetry of the system. As comparison, for PI controller, the steady stage errors are greater than 1mm/s for all steps, which means it doesn't settle under 5%. During the transient stage, the tracking error increases to a peak value of 4.1 mm/s for the forward direction and 4.4 mm/s for the reverse direction, while for PI they are 4.7 mm/s and 4.8 mm/s. To sum up, proposed method improved response speed, and provide better tracking performance during both transient and steady stage.

### VI. SUMMARY AND CONCLUSION

A new model reference adaptive control method has been created in this paper. Bounds on the transient response error have been derived. A novel sliding mode term has been added, resulting in guaranteed asymptotic stability of the errors, as opposed to upper bound guarantees. The controller is designed and applied for velocity tracking of an electro–hydraulic system. Experimental results illustrate that precise tracking of the reference model output is realized using the adaptive controller for different cases. At the expense of relatively complex structure, the controller makes it possible to achieve asymptotic stability. Future research will focus on further increasing robustness under significant disturbance, and more convenient tuning techniques.

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