the spin of sigma hyperon is $\frac{1}{2}$, the spin of the π-Λ resonance must be $\frac{1}{2}$. It has been suggested[3] that the bound-state hypothesis is most attractive for an S-state π-Λ system. This implies that the Σ-Λ relative parity is odd[7] and that the π-Λ resonance is in the S wave. Given certain assumptions, the bound-state model makes a prediction[7] with regard to the relative sign of the asymmetry parameter, α_Λ, in the decay $\Lambda \to \pi^- + p$, and the asymmetry parameter, α_0, in the decay $\Sigma^+ \to \pi^0 + p$, namely $\alpha_\Lambda \alpha_0 \sim +1$ for an $S_{1/2}$ bound state (a $P_{1/2}$ bound state would give a negative relative sign). The positive relative sign is to be contrasted with the negative relative sign predicted by certain theories[8,9] of strong and weak interactions which assume even Σ-Λ relative parity. These latter theories also predict $\alpha_\Lambda \alpha_\Xi \sim +1$, where α_Ξ is the asymmetry parameter in the decay $\Xi \to \pi^- + \Lambda$. This result is in contradiction to recent experiment.[10]

One of us (S. B.) would like to thank Professor Kenneth Ford for a number of valuable remarks.

*Work partially supported by the National Science Foundation.

†Work partially supported by the Office of Naval Research.

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[2]Private communication from A. Rosenfeld.

[3]S. Barshay and M. Schwartz, Phys. Rev. Letters 4, 618 (1960).

[4]R. H. Dalitz and S. F. Tuan, Ann. Phys. 10, 307 (1960).

[5]The (Σ/Λ) ratio in the isotopic one state is apparently at least 0.2 at threshold, but falls off sharply below the $\bar{K}$-nucleon threshold, the region in which we will be applying formula (1).

[6]R. H. Dalitz, Phys. Rev. Letters 6, 239 (1961).

[7]S. Barshay, Phys. Rev. Letters 1, 97 (1958).

[8]S. Treiman, Nuovo cimento 15, 916 (1960).

[9]A. Pais, Nuovo cimento 18, 1003 (1960).

[10]W. B. Fowler et al., Phys. Rev. Letters 6, 134 (1961).

ELECTRODYNAMIC PROPERTIES OF BARYONS IN THE UNITARY SYMMETRY SCHEME

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(Received March 20, 1961)

Gell-Mann[1] recently introduced a theory of strong interactions involving a new symmetry, called "unitary symmetry." The principal purpose of this note is to use this symmetry to express the magnetic moments of all the baryons in terms of those of the neutron and proton. We also derive a relation among baryon electromagnetic mass splittings.

The unitary symmetry scheme proposes that the elementary particles may be represented as tensors in a three-dimensional (generalized isospin) space, and that the strong interactions are invariant under unitary transformations in this space. In particular, the eight baryons form the components of a traceless matrix ψ,

$$\psi = \begin{pmatrix} -(\frac{2}{3})^{1/2}\Lambda & p & n \\ \Xi^- & (\frac{1}{6})^{1/2}\Lambda + (\frac{1}{2})^{1/2}\Sigma^0 & \Sigma^- \\ \Xi^0 & \Sigma^+ & (\frac{1}{6})^{1/2}\Lambda - (\frac{1}{2})^{1/2}\Sigma^0 \end{pmatrix}, \quad (1)$$

while the seven known pseudoscalar mesons (plus a predicted new pseudoscalar meson, χ^0) form the components of a traceless Hermitian matrix ϕ,

$$\phi = \begin{pmatrix} -(\frac{2}{3})^{1/2}\chi^0 & K^+ & K^0 \\ K^- & (\frac{1}{6})^{1/2}\chi^0 + (\frac{1}{2})^{1/2}\pi^0 & \pi^- \\ \bar{K}^0 & \pi^+ & (\frac{1}{6})^{1/2}\chi^0 - (\frac{1}{2})^{1/2}\pi^0 \end{pmatrix}. \quad (2)$$

The scheme also proposes the existence of eight vector mesons which transform in the same way as ϕ. Although of great importance in Gell-Mann's theory, they will not be described here, for their symmetric interactions do not affect our conclusions.

If the strong interactions are to be invariant under unitary symmetry, the possible forms of the Lagrangian density are (assuming ps-ps meson-nucleon coupling)

$$L = \mathrm{tr}\bar{\psi}(i\partial_\mu \gamma^\mu - m_0)\psi + \mathrm{tr}(\tfrac{1}{2}\partial_\mu \phi \partial^\mu \phi - \tfrac{1}{2}M_0^2 \phi^2) + g\,\mathrm{tr}(\bar{\psi}\gamma_5 \psi\phi) + g'\,\mathrm{tr}(\bar{\psi}\gamma_5 \phi\psi) + L', \quad (3)$$

where $\bar{\psi}$ is $\psi^\dagger\gamma_0$, tr is the ordinary matrix trace over the generalized isospin space, m_0 is a common baryon bare mass, and M_0 is a common meson bare mass.

There are two terms in L':

(1) A mysterious unknown interaction, weaker than the strong interactions, that breaks the unitary symmetry and causes us to observe the baryons as a singlet, two doublets, and a triplet, rather than as a completely degenerate octet. We know nothing about this interaction—it may be something as simple as a difference in baryon bare masses—and can predict nothing about its effects. Our conclusions are approximations which are exact only in the absence of this mass-splitting interaction. We hope that the mass-splitting interaction is sufficiently weak so that our approximations are good ones; this is the same hope that is fundamental to any quantitative prediction made on the basis of any "higher symmetry" scheme, like global symmetry.

(2) Electromagnetism. In contrast to the mass-splitting interaction, we know the form of the electromagnetic interaction exactly: In our notation it is

$$L_{\text{em}}' = eA^\mu\,\text{tr}(\bar{\psi}\gamma_\mu[\psi, Q] + i\partial_\mu\phi[\phi, Q]) - \tfrac{1}{2}e^2A^\mu A_\mu\,\text{tr}([\phi, Q][\phi, Q]), \quad (4)$$

where Q is the real traceless diagonal matrix,

$$Q = \tfrac{1}{3}\begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (5)$$

Without the mass-splitting interactions, the only departure from unitary symmetry is through the appearance of Q in the electromagnetic interaction. To first order in electromagnetism, but to all orders in the symmetric interactions, the electromagnetic vertex function must be a linear homogeneous invariant function of Q; thus,

$$\langle\bar{\psi}|A_\lambda|\psi\rangle = \mu_1(q^2)q_\nu\,\text{tr}(\bar{u}\sigma_{\lambda\nu}vQ) + \mu_2(q^2)q_\nu\,\text{tr}(\bar{u}\sigma_{\lambda\nu}Qv) + e_1(q^2)\,\text{tr}(\bar{u}\gamma_\lambda vQ) + e_2(q^2)\,\text{tr}(\bar{u}\gamma_\lambda Qv), \quad (6)$$

where u and v are the incoming and outgoing baryon spinors (they are also traceless 3×3 matrices in generalized isospin space), and q_μ is the 4-momentum transfer. Terms in tr$Q$ are absent from (6) because we have chosen Q to be traceless.

The values of e_1 and e_2 at $q^2=0$ are determined by the baryon charges; their higher derivatives yield hyperon form factors which are not likely to be measured soon. Therefore, we fix our attention on the μ_i at $q^2=0$. These give, in principle, nine quantities, the eight baryon magnetic moments and the matrix element for $\Sigma^0\to\Lambda+\gamma$.

Expanding (6) in terms of (1), we find for the magnetic moments,

$$\mu(\Sigma^+) = \mu(p), \quad (7)$$

$$\mu(\Lambda) = \tfrac{1}{2}\mu(n), \quad (8)$$

$$\mu(\Xi^0) = \mu(n), \quad (9)$$

$$\mu(\Xi^-) = \mu(\Sigma^-) = -[\mu(p) + \mu(n)], \quad (10)$$

$$\mu(\Sigma^0) = -\tfrac{1}{2}\mu(n), \quad (11)$$

and for the mixed moment (the multiple of $\bar{\Sigma}^0\sigma_{\mu\nu}\Lambda F_{\mu\nu}$ + H. c.) responsible for the decay $\Sigma^0\to\Lambda+\gamma$,

$$\mu_m = \tfrac{1}{2}\sqrt{3}\mu(n). \quad (12)$$

These equations contain one relation that is independent of the unitary symmetry scheme,

$$\mu(\Sigma^0) = \tfrac{1}{2}[\mu(\Sigma^+) + \mu(\Sigma^-)].$$

This is known to be a consequence of isospin invariance alone.[2]

In the same manner we may consider the electromagnetic corrections to the baryon masses. In the absence of the principal mass-splitting interactions, but in the presence of all symmetric strong interactions, the induced electromagnetic mass difference must be an invariant function of Q. The most general such expression is

$$\langle\bar{\psi}|\delta m|\psi\rangle = \delta_1\,\text{tr}(\bar{u}Qu) + \delta_2\,\text{tr}(\bar{u}uQ) + \delta_3\,\text{tr}(\bar{u}QuQ) + \delta_4\,\text{tr}(\bar{u}u). \quad (13)$$

Terms involving Q^2 do not explicitly appear because they are eliminated by the relation $9Q^2 = 2+3Q$. The last term in (13) is an addition to the common baryon mass and may be ignored. So, although there are four independent electromagnetic baryon mass splittings, there are only three significant independent parameters in (13). Thus we may obtain a condition on the baryon masses,[3]

$$m(\Xi^-) - m(\Xi^0) = m(\Sigma^-) - m(\Sigma^+) + m(p) - m(n). \quad (14)$$

Gell-Mann[1] has suggested that, in addition to invariance under unitary symmetry, the strong interactions may be invariant under a discrete symmetry, called R. In our notation, R corresponds to replacing every matrix by its transpose. This does not affect the free Lagrangian, but interchanges the two kinds of ps-ps couplings. Thus R invariance demands that $g=g'$.

The electric current is odd under R. Thus we can make the electromagnetic interactions R invariant by extending the definition of R so that A_μ is also odd under R. Should the strong interactions be R invariant as well, it follows in our approximation that $\mu_1=-\mu_2$ and $\delta_1=\delta_2$. This yields the manifestly unacceptable results that both the neutron magnetic moment and the charged Σ mass difference vanish.[4] We consider this sufficient grounds for abandoning the R invariance.

*National Science Foundation Predoctoral Fellow.

[1]M. Gell-Mann, Phys. Rev. (to be published). Although our notation differs from that of Gell-Mann, we study the same symmetries as he does. Gell-Mann emphasizes that his symmetry scheme may be seriously broken; experiment indicates that $K\bar{N}\Lambda$ and $K\bar{N}\Sigma$ interactions are much weaker than $\pi\bar{N}N$ interactions, in conflict with the requirements of unitary symmetry.

[2]R. Marshak, S. Okubo, and G. Sudarshan, Phys. Rev. 106, 599 (1957).

[3]Equation (14) gives a predicted value for $m(\Xi^-)-m(\Xi^0)$ of 5.3 ±0.2 Mev. The observed value is 7.4 ±8.0 Mev. We use the data tabulated by W. H. Barkas and A. H. Rosenfeld, Proceedings of the Tenth Annual International Rochester Conference on High-Energy Physics, 1960 (Interscience Publishers, Inc., New York, 1960), p. 877.

[4]It is easy to see this is not true if we abandon R invariance. In this case, we can, for example, set $g'=0$. Then the only interaction of the neutron with charged mesons is through $g(\bar{n}\gamma_5 p\pi^- + \bar{p}\gamma_5 n\pi^+)$. This gives a nonvanishing moment in order eg^2.

POSSIBLE EFFECT OF COLLECTIVE CORRELATION BETWEEN VACUUM NUCLEONS IN PION PHYSICS

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Recently, investigations into interacting systems of many particles have made great advances, and for Fermi systems it has been found that, if the interaction is predominantly attractive, a strong collective correlation arises and the system exhibits properties very different from the free one.[1,2] It seems possible that this effect can play an important role in pion physics also, since the internucleon force between vacuum nucleons due to pions is predominantly attractive.[3] Thus, it is expected that the structure of the vacuum would be very much altered by taking this effect into account, and that the processes in which the properties of the vacuum appear explicitly, such as the formation or the annihilation of nucleon pairs, would be greatly modified.[4]

The vacuum is usually defined as the eigenstate of the total Hamiltonian with the lowest energy eigenvalue. For the pion-nucleon system, if there is no interaction, this is realized by filling all negative-energy levels of the nucleons with all positive levels unoccupied, as given by Dirac. If an attractive interaction exists, however, this is no longer the lowest energy state. Instead, the lowest energy state is realized by moving some of the nucleons in negative-energy states to positive-energy states. This is because although the energy of the single nucleon is increased by 2κ when the nucleon is moved from a negative-energy state to a positive-energy state, where κ is the nucleon mass,[5] the potential energy of the total system can be decreased by doing so because attractive potentials, which had no effect when all negative levels were filled due to the Pauli principle, become effective. Thus, the total energy can be decreased by moving some of the nucleons in negative-energy states to positive-energy states, and can take a minimum value at a distribution different from the free one. This is exactly the same as the situation in which a superconducting state is realized by moving some of the electrons below the Fermi energy above it. So, if we want to define the vacuum as the eigenstate of the total system with the lowest energy eigenvalue, we must take this state as the true vacuum for the pion-nucleon system.

Of course, in order that such a situation be realized, the attractive potential must be sufficiently strong that the decrease of the potential