Contents

9 Pulsed Electromagnetic and Optical Beam Wavefields in Temporally Dispersive Media 1

9.1 Angular Spectrum Representation 2

9.1.1 Real Direction Cosine Form of the Angular Spectrum of Plane Waves Representation 6

9.1.2 Electromagnetic Energy Flow in the Angular Spectrum Representation 9

9.1.3 Homogeneous and Evanescent Plane Wave Contributions to the Angular Spectrum Representation 10

9.1.4 Paraxial Approximation of the Angular Spectrum of Plane Waves Representation 17

9.2 Angular Spectrum Representation of Multipole Wavefields 23

9.2.1 Multipole Expansion of the Scalar Optical Wavefield due to a Localized Source Distribution 30

9.2.2 Multipole Expansion of the Electromagnetic Wavefield Generated by a Localized Charge–Current Distribution in a Dispersive Dielectric Medium 37

9.3 Stationary Phase Asymptotic Approximations of the Angular Spectrum Representation in Free-Space 50

9.3.1 Approximations Valid Over a Hemisphere 54

9.3.2 Approximations Valid on the Plane $z = z_0$ 58

9.3.3 Asymptotic Approximations of $\tilde{U}_H(r, \omega)$ and $\tilde{U}_E(r, \omega)$ 66

9.3.4 Summary 69

9.4 Separable Pulsed Beam Wavefields 70

9.4.1 Gaussian Beam Propagation 75

9.4.2 Asymptotic Behavior 79

9.5 The Inverse Initial Value Problem 79

9.5.1 The Direct Problem 83

9.5.2 The Inverse Problem 85

9.6 Summary 88

References 88

Problems 91

10 Asymptotic Methods of Analysis using Advanced Saddle Point Techniques 95
10.1 Olver's Saddle Point Method 98
10.1.1 Peak Value of the Integrand at the Endpoint of Integration 98
10.1.2 Peak Value of the Integrand at an Interior Point of the Path of Integration 101
10.1.3 The Application of Olver's Saddle Point Method 103
10.2 Uniform Asymptotic Expansion for Two First-Order Saddle Points at Infinity 105
10.3 Uniform Asymptotic Expansion for Two First-Order Saddle Points 110
10.3.1 The Uniform Asymptotic Expansion for Two Isolated First-Order Saddle Points 111
10.3.2 The Uniform Asymptotic Expansion for Two Neighboring First-Order Saddle Points 113
10.3.3 The Transitional Asymptotic Approximation for Two Neighboring First-Order Saddle Points 125
10.4 Uniform Asymptotic Expansion for a First-Order Saddle Point and a Simple Pole Singularity 125
10.4.1 The Complementary Error Function 131
10.4.2 Asymptotic Behavior for a Single Interacting Saddle Point 135
10.4.3 Asymptotic Behavior for Two Isolated Interacting Saddle Points 139
10.5 Asymptotic Expansions of Multiple Integrals 141
10.5.1 Absolute Maximum in the Interior of the Closure of $\mathcal{D}_\xi$ 141
10.5.2 Absolute Maximum on the Boundary of the Closure of $\mathcal{D}_\xi$ 142
10.6 Summary 144
References 144
Problems 146

11 The Group Velocity Approximation 147
11.1 Historical Introduction 147
11.2 The Pulsed Plane Wave Electromagnetic Field 152
11.2.1 The Delta Function Pulse and the Impulse Response of the Medium 160
11.2.2 The Heaviside Unit Step Function Signal 161
11.2.3 The Double Exponential Pulse 163
11.2.4 The Rectangular Pulse Envelope Modulated Signal 163
11.2.5 The Trapezoidal Pulse Envelope Modulated Signal 165
11.2.6 The Hyperbolic Tangent Envelope Modulated Signal 169

11.2.7 The Van Bladel Envelope Modulated Pulse 174
11.2.8 The Gaussian Envelope Modulated Pulse 177
11.3 Wave Equations in a Simple Dispersive Medium and the Slowly Varying Envelope Approximation 178
11.3.1 The Dispersive Wave Equations 179
11.3.2 The Slowly Varying Envelope Approximation 180
11.3.3 Dispersive Wave Equations for the Slowly Varying Wave Amplitude and Phase 185
11.4 The Classical Group Velocity Approximation 194
11.5 Failure of the Classical Group Velocity Method 200
11.5.1 Impulse Response of a Double-Resonance Lorentz Model Dielectric 209
11.5.2 Heaviside Unit Step Function Signal Evolution 212
11.5.3 Rectangular Envelope Pulse Evolution 214
11.5.4 Van Bladel Envelope Pulse Evolution 215
11.5.5 Concluding Remarks on the Slowly Varying Envelope and Classical Group Velocity Approximations 222
11.6 Extensions of the Group Velocity Method 223
11.7 Localized Pulsed-Beam Propagation 231
11.7.1 Mathematical Preliminaries 232
11.7.2 Paraxial Asymptotics 234
11.8 The Necessity of an Asymptotic Description 244
References 245
Problems 249

12 Analysis of the Phase Function and Its Saddle Points 251
12.1 General Saddle Point Dynamics for Causally Dispersive Dielectrics 253
12.1.1 The Region About the Origin ($|\omega| \ll \omega_0$) 254
12.1.2 The Region About Infinity ($|\omega| \gg \omega_m$) 260
12.1.3 Summary 263
12.2 The Behavior of the Phase in the Complex ω-Plane for Causally Dispersive Materials 264
12.2.1 Single-Resonance Lorentz Model Dielectrics 264
12.2.2 Multiple-Resonance Lorentz Model Dielectrics 279
12.2.3 Rocard–Powles–Debye Model Dielectrics 293
12.2.4 Drude Model Conductors 306
12.3 The Location of the Saddle Points and the Approximation of the Phase 316
12.3.1 Single-Resonance Lorentz Model Dielectrics 316
12.3.2 Multiple-Resonance Lorentz Model Dielectrics 353
12.3.3 Rocard–Powles–Debye Model Dielectrics 366
12.3.4 Drude Model Conductors 367
12.3.5 Semiconducting Materials 370

12.4 Procedure for the Asymptotic Analysis of the Propagated Field 376
12.5 Synopsis 386
References 386
Problems 387

13 Evolution of the Precursor Fields 389
13.1 The Field Behavior for $\theta < 1$ 391
13.2 The Sommerfeld Precursor Field 393
13.2.1 The Nonuniform Approximation 394
13.2.2 The Uniform Approximation 400
13.2.3 Field Behavior at the Wavefront 405
13.2.4 The Instantaneous Oscillation Frequency 407
13.2.5 The Delta Function Pulse Sommerfeld Precursor 408
13.2.6 The Heaviside Step Function Pulse Sommerfeld Precursor 412
13.3 The Brillouin Precursor Field in Lorentz Model Dielectrics 416
13.3.1 The Nonuniform Approximation 417
13.3.2 The Uniform Approximation 430
13.3.3 The Instantaneous Oscillation Frequency 437
13.3.4 The Delta Function Pulse Brillouin Precursor 439
13.3.5 The Heaviside Step Function Pulse Brillouin Precursor 440
13.4 The Brillouin Precursor Field in Debye Model Dielectrics 445
13.5 The Middle Precursor Field 448
13.6 Impulse Response of Causally Dispersive Materials 454
References 462
Problems 464

14 Evolution of the Signal 467
14.1 The Nonuniform Asymptotic Approximation 468
14.2 Rocard–Powles–Debye Model Dielectrics 471
14.3 The Uniform Asymptotic Approximation 474
14.4 Single Resonance Lorentz Model Dielectrics 478
14.4.1 Frequencies below the Absorption Band 479
14.4.2 Frequencies above the Absorption Band 483
14.4.3 Frequencies within the Absorption Band 485
14.4.4 The Heaviside Unit Step Function Signal 488
14.5 Multiple Resonance Lorentz Model Dielectrics 494
14.6 Drude Model Conductors 499
References 500
Problems 501

15 Continuous Evolution of the Total Field 503
15.1 The Total Precursor Field 504
15.2 Resonance Peaks of the Precursors and the Signal Contribution 507
15.3 The Signal Arrival and the Signal Velocity 509
15.3.1 Transition from the Precursor Field to the Signal 509
15.3.2 The Signal Velocity 515
15.4 Comparison of the Signal Velocity with the Phase, Group, and Energy Velocities 526
15.5 The Heaviside Step Function Modulated Signal 532
15.5.1 Signal Propagation in a Single Resonance Lorentz Model Dielectric 533
15.5.2 Signal Propagation in a Double Resonance Lorentz Model Dielectric 554
15.5.3 Signal Propagation in a Drude Model Conductor 561
15.5.4 Signal Propagation in a Rocard–Powles–Debye Model Dielectric 566
15.5.5 Signal Propagation along a Dispersive Transmission Line 569
15.6 The Rectangular Pulse Envelope Modulated Signal 572
15.6.1 Rectangular Envelope Pulse Propagation in a Single Resonance Lorentz Model Dielectric 574
15.6.2 Rectangular Envelope Pulse Propagation in a Rocard–Powles–Debye Model Dielectric 591
15.6.3 Rectangular Envelope Pulse Propagation in Triply Distilled Water 597
15.6.4 Rectangular Envelope Pulse Propagation in Saltwater 603
15.7 Noninstantaneous Rise-Time Signals 605
15.7.1 Hyperbolic Tangent Envelope Signal Propagation in a Single Resonance Lorentz Model Dielectric 606
15.7.2 Raised Cosine Envelope Signal Propagation in a Single Resonance Lorentz Model Dielectric 617
15.7.3 Trapezoidal Envelope Pulse Propagation in a Rocard–Powles–Debye Model Dielectric 619
15.8 Infinitely Smooth Envelope Pulses 621
15.8.1 Gaussian Envelope Pulse Propagation in a Single Resonance Lorentz Model Dielectric 621
15.8.2 Van Bladel Envelope Pulse Propagation in a Double Resonance Lorentz Model Dielectric 638
15.8.3 Brillouin Pulse Propagation in a Rocard–Powles–Debye Model Dielectric; Optimal Pulse Penetration 638

15.9 The Pulse Centroid Velocity of the Poynting Vector 643
15.9.1 Mathematical Formulation 643
15.9.2 Numerical Results 645
15.9.3 The Instantaneous Centroid Velocity 649
15.10 Dispersive Pulse Propagation in the Singular and Weak Dispersion Limits 651
15.10.1 The Singular Dispersion Limit 652
15.10.2 The Weak Dispersion Limit 653
15.11 Comparison with Experimental Results 656
15.12 The Myth of Superluminal Pulse Propagation 669
References 672
Problems 676

16 Physical Interpretations of Dispersive Pulse Dynamics 679
16.1 Energy Velocity Description of Dispersive Pulse Dynamics 681
16.1.1 Approximations Having a Precise Physical Interpretation 683
16.1.2 Physical Model of Dispersive Pulse Dynamics 689
16.2 Extension of the Group Velocity Description 701
16.3 Signal Model of Dispersive Pulse Dynamics 702
16.4 Summary and Conclusions 708
References 711
Problems 712

17 Applications 713
17.1 On the Use and Application of Precursor Waveforms 713
17.2 Electromagnetic Energy Dissipation in Causally Dispersive Media 716
17.2.1 General Formulation 717
17.2.2 Evolved Heat in Lorentz Model Dielectrics 719
17.2.3 Numerical Results 720
17.3 Reflection and Transmission Phenomena 728
17.3.1 Reflection and Transmission at a Dispersive Half-Space 728
17.3.2 The Goos–Hänchen Shift 738
17.3.3 Reflection and Transmission at a Dispersive Layer: The Question of Superluminal Tunneling 751
17.4 Optimal Pulse Penetration through Dispersive Bodies 751
17.4.1 Ground Penetrating Radar 752
17.4.2 Foliage Penetrating Radar 756
17.4.3 Undersea Communications using the Brillouin Precursor 759
17.5 Ultrawideband Pulse Propagation through the Ionosphere 761
17.6 Health and Safety Issues Associated with Ultrashort Pulsed Electromagnetic Radiation 767
17.7 Future Prospects 771
References 773
Problems 775

Appendix 777

F Asymptotic Expansion of Single Integrals 777
F.1 Foundations 780
F.2 Asymptotic Sequences, Series and Expansions 783
F.3 Integration by Parts 790
F.4 The Method of Stationary Phase 792
F.5 Watson's Lemma 794
F.6 Laplace's Method 800
F.7 The Method of Steepest Descents 805
References 810

G Proof of Theorem 1 811
References 818

H The Radon Transform 819
References 823

Index 825