Electromagnetic force and torque on magnetic and negative-index scatterers

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Abstract: We derive the analytic expressions of the electromagnetic force and torque on a dipolar particle, with arbitrary dielectric permittivity and magnetic permeability. We then develop a general framework, based on the coupled dipole method, for computing the electromagnetic force and torque experienced by an object with arbitrary shape, dielectric permittivity and magnetic permeability.

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1. Introduction.

The scattering of an electromagnetic (EM) wave by an arbitrary object can be described using the coupled dipole method or CDM, also called the discrete dipole approximation or DDA [1]. In the CDM, a given object is discretized into a collection of polarizable subunits, usually over a cubic lattice. Provided the lattice constant is small enough compared to the spatial variation of the EM fields inside the object, the dipole approximation holds for each subunit, and the object can thus be treated as a collection of dipoles [2, 3, 4]. The CDM has been used successfully to model not only light scattering, but also spontaneous emission in complex geometries [5, 6], optical tomography [7], optical binding [8], optical trapping and manipulation [9, 10, 11, 12], and optical torques [13, 14]. Traditionally, only electric dipoles are considered in the CDM, however, we emphasize that this is circumstantial rather than a consequence of any limitation of the method, and magnetic dipoles can also be accounted for [15, 16, 17, 18, 19].

For simple shapes, analytical methods can been used to study optical forces on magnetic scatterers such as (2D) cylindrical particles [20, 21] and spherical scatterers [22]. However, to

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study optical forces and torques on arbitrary, magnetic objects a numerical approach needs to be formulated.

Since in the CDM one represents an arbitrary object as a collection of dipoles, the physics of the opto-mechanical coupling between light and the object must first be understood at the dipole level. Previously, one of the authors derived the expression for the time-averaged optical force on an electric dipole [23]. In the present article, we present a general derivation of the electromagnetic force and torque, in the case of a dipolar object with arbitrary dielectric permittivity ε and magnetic permeability μ. We then describe how these results can be incorporated into a more general CDM approach to find the electromagnetic force and torque experienced by an arbitrary object (beyond the dipole approximation).

2. Optical force on a small particle.

We start by considering a small particle with permittivity ε and permeability μ, located at the origin of our coordinate system. We seek to derive the optical force and torque experienced by the particle, treated in the dipole approximation, when illuminated by an arbitrary incident EM field $\{\mathbf{E}_0(\mathbf{r},\omega),\mathbf{H}_0(\mathbf{r},\omega)\}$, where ω is the angular frequency. Gaussian units are used throughout. We assume a time-harmonic dependence (i.e., $e^{-i\omega t}$) and we shall henceforth omit the dependence of the fields on ω.

The time-averaged total force $\mathbf{F}$ on the particle is derived from Maxwell's stress tensor as [24]:

$$\mathbf{F}=\frac{1}{8\pi}\mathrm{Re}\left[\int_S\left[(\mathbf{E}(\mathbf{r}).\mathbf{n})\mathbf{E}^*(\mathbf{r})+(\mathbf{H}(\mathbf{r}).\mathbf{n})\mathbf{H}^*(\mathbf{r})-\frac{1}{2}(|\mathbf{E}(\mathbf{r})|^2+|\mathbf{H}(\mathbf{r})|^2)\mathbf{n}\right]\mathrm{d}S\right], \tag{1}$$

where S is a surface enclosing the particle, the unit vector $\mathbf{n}$ defines the local outward normal to S, $*$ denotes the complex conjugate, and Re represents the real part of a complex number. $\mathbf{E}(\mathbf{r})$ and $\mathbf{H}(\mathbf{r})$ are the total fields, i.e. the sum of the incident EM fields $\{\mathbf{E}_0(\mathbf{r}),\mathbf{H}_0(\mathbf{r})\}$ and the EM fields scattered by the object $\{\mathbf{E}_d(\mathbf{r}),\mathbf{H}_d(\mathbf{r})\}$. Let $\mathbf{p}$ ($\mathbf{m}$) be the electric (magnetic) dipole induced by the electric (magnetic) field of the incident EM wave, and let $\hat{\mathbf{r}}$ be the unit vector in the direction of $\mathbf{r}$. The fields scattered by the object are [25]:

$$\begin{aligned}
\mathbf{E}_d(\mathbf{r}) &= e^{ikr}\left\{[3\hat{\mathbf{r}}(\hat{\mathbf{r}}.\mathbf{p})-\mathbf{p}]\left(\frac{1}{r^3}-\frac{ik}{r^2}\right)+\frac{k^2}{r}(\hat{\mathbf{r}}\times\mathbf{p})\times\hat{\mathbf{r}}-k^2(\hat{\mathbf{r}}\times\mathbf{m})\left(\frac{1}{r}+\frac{i}{kr^2}\right)\right\} && (2)\\
&= \mathbf{T}^{\mathrm{ee}}\mathbf{p}+\mathbf{T}^{\mathrm{em}}\mathbf{m} && (3)\\
\mathbf{H}_d(\mathbf{r}) &= e^{ikr}\left\{[3\hat{\mathbf{r}}(\hat{\mathbf{r}}.\mathbf{m})-\mathbf{m}]\left(\frac{1}{r^3}-\frac{ik}{r^2}\right)+\frac{k^2}{r}(\hat{\mathbf{r}}\times\mathbf{m})\times\hat{\mathbf{r}}+k^2(\hat{\mathbf{r}}\times\mathbf{p})\left(\frac{1}{r}+\frac{i}{kr^2}\right)\right\} && (4)\\
&= \mathbf{T}^{\mathrm{me}}\mathbf{p}+\mathbf{T}^{\mathrm{mm}}\mathbf{m}, && (5)
\end{aligned}$$

where $\mathbf{k}$ is the wave vector. The quantities $\mathbf{T}$ are field susceptibility tensors [26] and the superscripts relate to the electric or magnetic nature of the field and the source. We emphasize that the surface of integration S can be chosen arbitrarily as long as it encloses the object under consideration. Because we treat the particle as a point particle we can take the surface S arbitrarily close to it. Specifically, we can choose a spherical surface centered on the particle and with a radius $r\ll\lambda$, where λ is the wavelength of the incident field. We can thus expand the incident fields in Taylor series:

$$\mathbf{E}_0(\mathbf{r})=\mathbf{E}_0+r(\hat{\mathbf{r}}.\nabla)\mathbf{E}_0+\cdots \quad\text{and}\quad \mathbf{H}_0(\mathbf{r})=\mathbf{H}_0+r(\hat{\mathbf{r}}.\nabla)\mathbf{H}_0+\cdots \tag{6}$$

If we insert the total fields into the expression of the stress tensor we obtain three types of terms: those involving the incident field only, those involving the fields scattered by the dipoles only,

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and terms involving both incident and scattered fields. The terms involving the incident field only (as if the particle were not there) give no net contribution to the force. Therefore we are left with the contribution to the force from the fields scattered by the dipole only ($\mathbf{F}_{\text{self}}$), and the contribution due to the cross terms involving both the scattered and incident fields ($\mathbf{F}_{\text{mix}}$). The force can be written as:

$$\begin{aligned}\mathbf{F} &= \frac{1}{8\pi}\text{Re}\left[\int_S (\mathbf{E}_d(\mathbf{r}).\mathbf{n})\mathbf{E}_0^*(\mathbf{r}) + (\mathbf{E}_0^*(\mathbf{r}).\mathbf{n})\mathbf{E}_d(\mathbf{r}) + (\mathbf{H}_d(\mathbf{r}).\mathbf{n})\mathbf{H}_0^*(\mathbf{r})\right.\\ &+ (\mathbf{H}_0^*(\mathbf{r}).\mathbf{n})\mathbf{H}_d(\mathbf{r}) - [\mathbf{E}_d(\mathbf{r}).\mathbf{E}_0^*(\mathbf{r}) + \mathbf{H}_d(\mathbf{r}).\mathbf{H}_0^*(\mathbf{r})]\mathbf{n}\\ &+ \left.(\mathbf{E}_d(\mathbf{r}).\mathbf{n})\mathbf{E}_d^*(\mathbf{r}) + (\mathbf{H}_d(\mathbf{r}).\mathbf{n})\mathbf{H}_d^*(\mathbf{r}) - \frac{1}{2}\left[|\mathbf{E}_d(\mathbf{r})|^2 + |\mathbf{H}_d(\mathbf{r})|^2\right]\mathbf{n}\mathrm{d}S\right]. \end{aligned} \tag{7}$$

Using Eq. (6) in Eq. (7), and retaining the near-field terms only in the expression of the scattered fields, the i^{th} Cartesian component of the force due to the cross terms is (repeated indices are summed over):

$$\begin{aligned}F^i_{\text{mix}} &= \frac{1}{6}\text{Re}\left[2p^j\partial^j E_0^{*i} - p^i\partial^j E_0^{*j} + p^j\partial^i E_0^{*j} + 2ik\varepsilon^{ijk}H_0^{*j}p^k\right.\\ &- \left. 2ik\varepsilon^{ijk}E_0^{*j}m^k + 2m^j\partial^j H_0^{*i} - m^i\partial^j H_0^{*j} + m^j\partial^i E_0^{*j}\right],\end{aligned} \tag{8}$$

where ε^{ijk} is the Levi-Civita tensor, and i, j, or k stands for either x, y or z. To simplify further the expression of the force we can use Maxwell's equations. We have $\nabla\cdot\mathbf{E}_0 = 0$ and $\nabla\cdot\mathbf{B}_0 = 0$. Furthermore, using $\nabla\times\mathbf{E}_0 = ik\mathbf{H}_0$ in the expression involving the electric dipole, and $\nabla\times\mathbf{H}_0 = -ik\mathbf{E}_0$ in the expression involving the magnetic dipole, the contribution of cross terms to the optical force on a dipolar particle becomes:

$$F^i_{\text{mix}} = \frac{1}{2}\text{Re}\left[p^j\partial^i E_0^{*j} + m^j\partial^i H_0^{*j}\right]. \tag{9}$$

Now if we consider the terms involving the scattered fields only in Eq. (7), one can show that the integral of $(\mathbf{E}_d(\mathbf{r}).\mathbf{n})\mathbf{E}_d^*(\mathbf{r}) + (\mathbf{H}_d(\mathbf{r}).\mathbf{n})\mathbf{H}_d^*(\mathbf{r})$ gives no net contribution and there remain only the terms involving the modulus of the electric and magnetic fields:

$$\mathbf{F}_{\text{self}} = \frac{1}{8\pi}\text{Re}\left[\int_S \left[-\frac{1}{2}\left[|\mathbf{E}_d(\mathbf{r})|^2 + |\mathbf{H}_d(\mathbf{r})|^2\right]\mathbf{n}\mathrm{d}S\right], \tag{10}$$

If we express the fields in terms of the electric and magnetic dipole moments of the small particle we get :

$$\begin{aligned}\mathbf{F}_{\text{self}} &= -\frac{k^4}{8\pi}\text{Re}\left[\int_S \left\{[(\hat{\mathbf{r}}\times\mathbf{p}^*)\times\hat{\mathbf{r}}].(\hat{\mathbf{r}}\times\mathbf{m}) - [(\hat{\mathbf{r}}\times\mathbf{m}^*)\times\hat{\mathbf{r}}].(\hat{\mathbf{r}}\times\mathbf{p})\right\}\frac{\mathbf{n}}{r^2}\mathrm{d}S\right]\\ &= -\frac{k^4}{3}\text{Re}(\mathbf{p}\times\mathbf{m}^*).\end{aligned} \tag{11}$$

The total force experienced by the particle can be now written as:

$$F^i = \frac{1}{2}\text{Re}\left[p^j\partial^i E_0^{*j} + m^j\partial^i H_0^{*j} - \frac{2k^4}{3}\varepsilon^{ijk}p^j m^{*k}\right]. \tag{12}$$

Note that the term $\mathbf{F}_{\text{self}}$ is important for a single particle but is negligible when describing a large object as a collection of small polarizable subunits. If we introduce the electric and magnetic polarizabilities of a dipolar sphere of radius a, we have $\mathbf{p} = \alpha_0^{\mathrm{e}}\mathbf{E}_0$ and $\mathbf{m} = \alpha_0^{\mathrm{m}}\mathbf{H}_0$ with the polarizabilities written as [25]:

$$\alpha_0^{\mathrm{e}} = a^3\frac{\varepsilon - 1}{\varepsilon + 2} \qquad \text{and} \qquad \alpha_0^{\mathrm{m}} = a^3\frac{\mu - 1}{\mu + 2}. \tag{13}$$

However, these expressions do not satisfy the optical theorem as they do not account for radiation reaction (i.e., interaction of the dipole with its own field) [2, 27]. In the case of an electric dipole, it has previously been shown that the radiation reaction term must be accounted for in order to derive the correct expression for the optical force [23]. The same requirement applies for a magnetic dipole. With this correction the polarizabilities become:

$$\alpha^{\mathrm{e}} = \alpha_0^{\mathrm{e}}/\left(1 - \frac{2}{3}ik^3\alpha_0^{\mathrm{e}}\right) \qquad \text{and} \qquad \alpha^{\mathrm{m}} = \alpha_0^{\mathrm{m}}/\left(1 - \frac{2}{3}ik^3\alpha_0^{\mathrm{m}}\right), \tag{14}$$

and the net force can be written as:

$$F^i = \frac{1}{2}\mathrm{Re}\left[\alpha^{\mathrm{e}}E_0^j\partial^i E_0^{*j} + \alpha^{\mathrm{m}}H_0^j\partial^i H_0^{*j} - \frac{2k^4}{3}\varepsilon^{ijk}\alpha^{\mathrm{e}}E_0^j\left(\alpha^{\mathrm{m}}H_0^k\right)^*\right]. \tag{15}$$

We can notice that the first term on the right-hand-side of Eq. (15), pertaining to the electric dipole contribution to the force, is the same as the optical force experienced by an electric dipole that was derived in Ref. [23] using the Lorentz force. Obviously, the two approaches (Maxwell stress tensor and Lorentz force) are equivalent [20, 21, 28]. Compared to the case of a single electric dipole, we now also have a contribution to the optical force that comes from the magnetic dipole and also from a self-interaction term involving the electric and magnetic dipole moments.

3. Optical torque on a small particle.

Beside the optical force we can also derive the optical torque. From the expression of the force in Eq. (1) the intrinsic optical torque can be written as:

$$\mathbf{\Gamma}^{\mathrm{int}} = \frac{1}{8\pi}\mathrm{Re}\left[\int_S \mathbf{r}\times\left[(\mathbf{E}(\mathbf{r}).\mathbf{n})\mathbf{E}^*(\mathbf{r}) + (\mathbf{H}(\mathbf{r}).\mathbf{n})\mathbf{H}^*(\mathbf{r}) - \frac{1}{2}(|\mathbf{E}(\mathbf{r})|^2 + |\mathbf{H}(\mathbf{r})|^2)\mathbf{n}\right]\mathrm{d}S\right], \tag{16}$$

which, since $\mathbf{r}$ and $\mathbf{n}$ are collinear, can be simplified into:

$$\mathbf{\Gamma}^{\mathrm{int}} = \frac{1}{8\pi}\mathrm{Re}\left[\int_S \mathbf{r}\times\left[(\mathbf{E}(\mathbf{r}).\mathbf{n})\mathbf{E}^*(\mathbf{r}) + (\mathbf{H}(\mathbf{r}).\mathbf{n})\mathbf{H}^*(\mathbf{r})\right]\mathrm{d}S\right]. \tag{17}$$

This time only the first term in the Taylor series for the incident field is needed, and after simplification the torque becomes:

$$\mathbf{\Gamma}^{\mathrm{int}} = \frac{1}{2}\mathrm{Re}(\mathbf{p}\times\mathbf{E}_0^* + \mathbf{m}\times\mathbf{H}_0^*). \tag{18}$$

This expression represents the *intrinsic* part of the optical torque and does not depend on the position of the particle (aside from a trivial spatial dependence through the incident field). However, as emphasized in Refs. [14, 29, 30, 31], this expression should be modified in order to satisfy the conservation of angular momentum. The modification consists in adding the radiative reaction term to the electromagnetic field $\{\mathbf{E}_0, \mathbf{H}_0\}$ which leads to:

$$\mathbf{\Gamma}^{\mathrm{int}} = \frac{1}{2}\mathrm{Re}\left[\mathbf{p}\times(\mathbf{p}/\alpha_0^{\mathrm{e}})^* + \mathbf{m}\times(\mathbf{m}/\alpha_0^{\mathrm{m}})^*\right]. \tag{19}$$

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As an example, let us consider a sphere that is small enough, compared to the wavelength of illumination, to be treated as a dipole. The sphere is illuminated by a circularly polarized plane wave. The electric field $\mathbf{E}_0 = E_0(1,i,0)e^{ikz}$ induces an electric dipole moment $\mathbf{p} = \alpha^{\mathrm{e}} E_0(1,i,0)e^{ikz}$ and the magnetic field $\mathbf{H}_0 = E_0(-i,1,0)e^{ikz}$ induces a magnetic dipole moment $\mathbf{m} = \alpha^{\mathrm{m}} E_0(-i,1,0)e^{ikz}$. Then the optical torque experienced by the sphere is

$$\boldsymbol{\Gamma}^{\mathrm{int}} = E_0^2 \mathrm{Im}\,[\alpha_0^{\mathrm{e}} + \alpha_0^{\mathrm{m}}]\,\hat{\mathbf{z}}. \tag{20}$$

This result corresponds to the optical torque given in Ref. [30] for a sphere, in the limit of small radius compared to the wavelength, i.e. the optical torque is proportional to the absorption cross section.

4. Optical force and torque on an arbitrary magnetic particle.

The expression of the optical force we just derived can be used in the CDM to find the optical forces on an arbitrary object. Consider an object with dielectric permittivity ε and magnetic permeability μ. We emphasize that although for the sake of brevity we assume here that ε and μ are scalars, the method still applies if they are tensors and/or functions of position [32]. The object is discretized into N polarizable units, each characterized by an electric polarizability α^{e} and a magnetic polarizability α^{m}. The polarizabilities are still given by Eqs. (13) and (14) with the exchange of a^3 by $3d^3/(4\pi)$ where d is the spacing of the CDM grid. Notice that when a sphere is discretized into N subunits, d is chosen such that the total volume represented by the N subunits is equal to the volume of the actual sphere. The local-fields at subunit l can be written as

$$\mathbf{E}_l = \mathbf{E}_{0l} + \sum_{n=1}^{N} [\mathbf{T}_{ln}^{\mathrm{ee}} \alpha_n^{\mathrm{e}} \mathbf{E}_n + \mathbf{T}_{ln}^{\mathrm{em}} \alpha_n^{\mathrm{m}} \mathbf{H}_n] \tag{21}$$

$$\mathbf{H}_l = \mathbf{H}_{0l} + \sum_{n=1}^{N} [\mathbf{T}_{ln}^{\mathrm{me}} \alpha_n^{\mathrm{e}} \mathbf{E}_n + \mathbf{T}_{ln}^{\mathrm{mm}} \alpha_n^{\mathrm{m}} \mathbf{H}_n], \tag{22}$$

where the terms $\mathbf{T}$ are the field susceptibility tensors defined in Eqs. (2)-(4). If we write the equations for the local fields for all N subunits forming the object, we get a linear system of size $6N \times 6N$ which can be solved for the electric and magnetic fields inside the object. Once the fields inside the object are known, the fields anywhere outside the object can be calculated simply by adding the contributions of all the subunits. We now have the fields, however we also need their spatial derivatives to derive the optical forces. The spatial derivatives of the fields at any subunit l are obtained through:

$$\nabla\mathbf{E}_l = \nabla\mathbf{E}_{0l} + \sum_{n=1}^{N} [\nabla\mathbf{T}_{ln}^{\mathrm{ee}} \alpha_n^{\mathrm{e}} \mathbf{E}_n + \nabla\mathbf{T}_{ln}^{\mathrm{em}} \alpha_n^{\mathrm{m}} \mathbf{H}_n] \tag{23}$$

$$\nabla\mathbf{H}_l = \nabla\mathbf{H}_{0l} + \sum_{n=1}^{N} [\nabla\mathbf{T}_{ln}^{\mathrm{me}} \alpha_n^{\mathrm{e}} \mathbf{E}_n + \nabla\mathbf{T}_{ln}^{\mathrm{mm}} \alpha_n^{\mathrm{m}} \mathbf{H}_n]. \tag{24}$$

From this point, the optical force on the object can be computed using a procedure similar to the one presented in [33], i.e. once the fields and their spatial derivatives are known at all subunits, the force on each subunit is derived using Eq. (15). The total net force on the object is then the sum of the force over all subunits.

Beside the optical force, we can also use the CDM to compute the optical torque experienced by an arbitrary object. The total torque on the object will be the sum of the individual torques experienced by each polarizable subunit forming the object. However, since we are now dealing

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with a rigid body represented as a collection of dipoles, were we to merely use Eq. (18) to calculate the *intrinsic* torque on each subunit and sum these contributions, we would not get the correct result. In order to obtain the total torque the *extrinsic* torque experienced by each subunit must be added ($\mathbf{\Gamma}^{\text{ext}} = \mathbf{r} \times \mathbf{F}$). This term obviously depends on the position of the subunit within the object. Hence, when calculating the optical torque on an object using the CDM, the net torque on the object must be written as:

$$\mathbf{\Gamma} = \sum_{n=1}^{N} \left(\mathbf{\Gamma}_n^{\text{ext}} + \mathbf{\Gamma}_n^{\text{int}} \right) = \sum_{n=1}^{N} \left(\mathbf{r}_n \times \mathbf{F}_n + \frac{1}{2} \text{Re} \left[\mathbf{p}_n \times (\mathbf{p}_n / \alpha_{0,n}^{\text{e}})^* + \mathbf{m}_n \times (\mathbf{m}_n / \alpha_{0,n}^{\text{m}})^* \right] \right). \quad (25)$$

5. Computational remarks

The computation of optical force and torque using the CDM is done in two major steps: first we compute the local fields and second, we compute the spatial derivatives of the local fields. The computation of the local field requires us to solve the $6N \times 6N$ linear system corresponding to Eqs. (21)-(22). This is done using an iterative method (quasi minimal residual for example [34]) and fast Fourier transform (FFT) to perform the matrix vector product [35]. The same strategy can be used to calculate the spatial derivatives of the local fields. The sums in Eqs. (23)-(24) can be evaluated directly (method A), however, this process is quite slow. If we note that the derivatives of the field susceptibility tensors, like the tensors themselves, depend only on the difference of the position vectors $\mathbf{r}_i - \mathbf{r}_j$, the sums over the lattice can be viewed as convolution products which can be computed very efficiently using a FFT (method B). We can further improve the computation of the derivatives of the tensors by noting that once the spatial derivatives of the x and y components of the fields are known, the derivative of the z component of the electric and magnetic fields can be found using Maxwell's equations (method C).

6. Results

We use the exact Mie theory for a spherical scatterer to illustrate the validity of the CDM approach.

6.1. Electromagnetic force and torque on a magnetic scatterer

We start by considering a sphere of radius $a = \lambda/2$, where λ is the wavelength, in vacuum, of the incident field. The permittivity and permeability of the sphere are $\varepsilon = \mu = 2.25$. In Fig. 1(a) we plot (on a log scale) the computation time for calculating the derivatives of the local fields, for the three methods outlined in the previous section, versus the number of dipole used to discretize the sphere. Obviously, method A takes a very long time and using a FFT drastically improves (by several orders of magnitude) the speed of the computation (method B). We can also see that method C provides a further, albeit modest, increase in the speed of the computation.

Using the exact Mie theory as a reference, we plot in Fig. 1(b) the relative error, in percent, on the optical force calculated with the CDM, versus the number of dipoles N, for different prescriptions of the polarizability: Clausius-Mossotti formula with the addition of radiation reaction [2] noted as CR, the prescription by Lakthakia [36] noted as LA, the prescription introduced by Dungey and Bohren [37], based on the first Mie coefficient, is labeled DB. Note that there exists other forms of the polarizability [32, 38] however, the three discussed here are the most common ones. Figure 1(b) shows, quite logically, a decrease of the relative error with the number of dipoles. We can also observe that the DB polarizability yields the best result.

In Fig. 2(a) we plot the optical force for a sphere of radius $a = \lambda/4$ versus $\varepsilon = \mu$ for the different forms of the polarizabilities. The sphere is discretized into $N = 113104$ elements. One can see that there is an excellent agreement between the CDM and Mie theory (the relative

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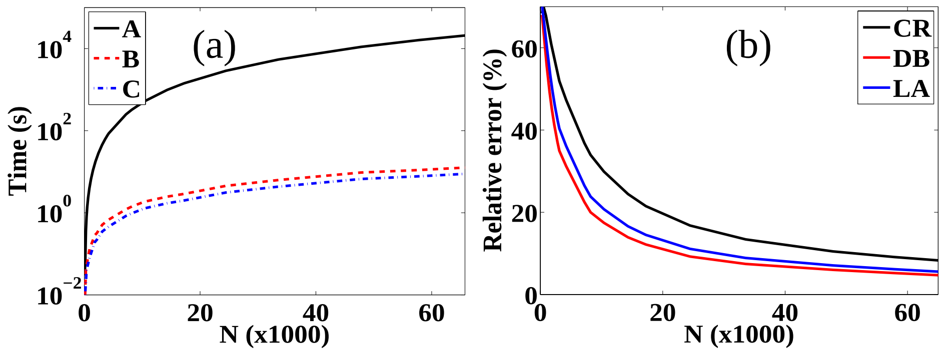

Fig. 1. Sphere of radius $a = \lambda/2$ with $\varepsilon = \mu = 2.25$. (a) Computation time versus number N of dipoles for the calculation of the spatial derivatives of the local fields, for the three different methods outlined in the text. (b) Relative error, in percent, between the optical forces computed using the CDM and using Mie theory versus the number of dipoles. Different forms of the polarizability are considered: (CR) Clausius-Mossotti with radiation reaction [2]; (DB) first Mie coefficient [37]; (LA) Lakthakia's prescription [36].

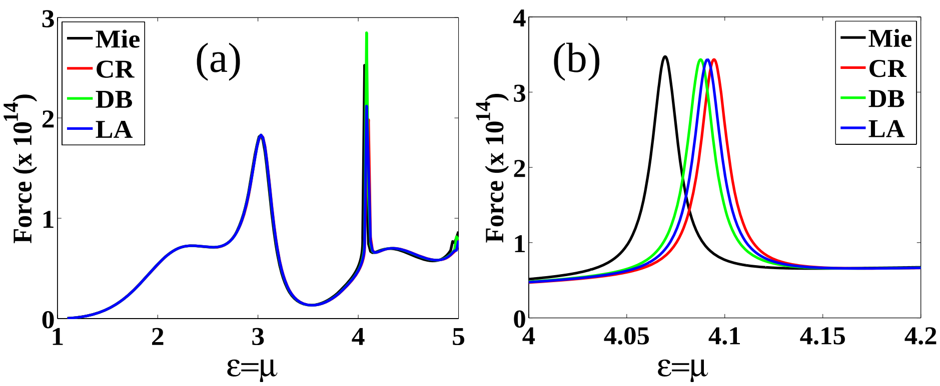

Fig. 2. (a) Optical force on a sphere of radius $a = \lambda/4$ versus $\varepsilon = \mu$ for $N = 113104$. (b) zoom on the resonance around $\varepsilon = \mu \approx 4$.

error is not shown but once again the DB prescription for the polarizabilities works best). If we zoom in on the sharp resonance around $\varepsilon = \mu \approx 4$ [Fig. 2(b)] we find that, of the three forms of polarizability considered here, the DB prescription yields the most accurate position of the resonance. A likely reason for the good performance of the DB prescription in the case of magnetic materials is that since both the electric and the magnetic polarizabilities are based on the corresponding first coefficient in the Mie series expansion, they both account for the fact that we have $\varepsilon \neq 1$ and $\mu \neq 1$. By contrast, CR and LA are based on the Clausius-Mossotti relation which is derived in the static case where electric and magnetic effects are decoupled.

6.2. *Influence of material losses*

We now study the optical force and torque generated by a circularly polarized plane wave on to a sphere with radius $a = \lambda/4$. The sphere is discretized into $N = 113104$ elements. The material parameters of the sphere are $\mathrm{Re}(\varepsilon) = \mathrm{Re}(\mu) = 2.25$. We are interested in the influence of material losses on the electromagnetic force and torque. We start by assuming $\mathrm{Im}(\mu) = 0$

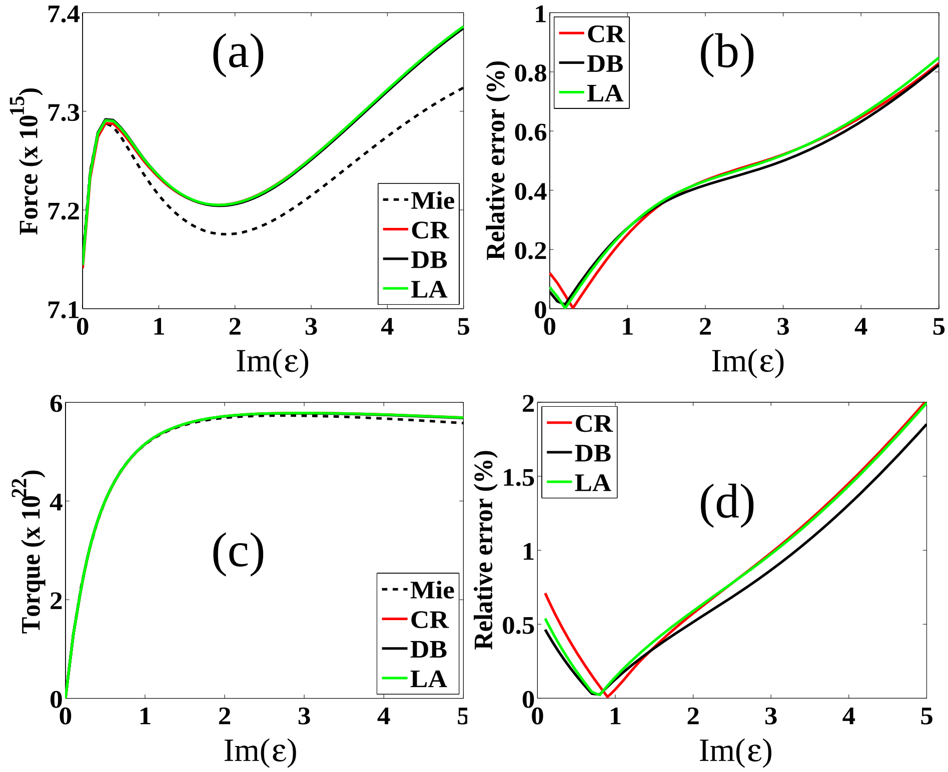

Fig. 3. Sphere of radius $a = \lambda/4$ with $\mathrm{Re}(\varepsilon) = \mathrm{Re}(\mu) = 2.25$ and $\mathrm{Im}(\mu) = 0$. (a) Optical force versus $\mathrm{Im}(\varepsilon)$. Note that the three CDM plots corresponding to the three forms of the polarizabilities are superimposed on the scale of the figure. (b) Relative error in percent between the optical force obtained from the CDM and the Mie series. (c) Optical torque versus versus $\mathrm{Im}(\varepsilon)$. (d) Relative error in percent between the optical torque obtained from the CDM and the Mie series.

and we vary the imaginary part of the permittivity between 0 and 5. Figures 3 shows the optical force [Fig. 3(a)] and torque [Fig. 3(c)] calculated by Mie theory and by the CDM, along with their respective relative errors [Figs. 3(b) and 3(d)]. Note that the three CDM plots of the force and the torque, corresponding to the three forms of the polarizabilities are superimposed on the scale of the figures. A very good agreement is observed as confirmed by the plots of the relative errors. Notice that the optical torque is zero for a lossless sphere in agreement with Eq. (20) and Ref. [29]: a lossless, homogeneous spherical particle does not rotate when illuminated by a circularly polarized plane wave. We emphasize that had we not accounted for radiation reaction in the polarizabilities, we would have obtained a non-zero torque. This highlights the fact that radiation reaction is not only required to satisfy the optical theorem in a scattering configuration, it is also required to satisfy the conservation of angular momentum [14].

We now consider the case where $\mathrm{Im}(\varepsilon) = \mathrm{Im}(\mu)$ while still keeping $\mathrm{Re}(\varepsilon) = \mathrm{Re}(\mu) = 2.25$. The optical force and torque are plotted in Figs. 4(a) and 4(c), and the corresponding relative errors in Figs. 4(b) and 4(d). Once again a very good agreement between the CDM and Mie is observed over the range of absorption considered here.

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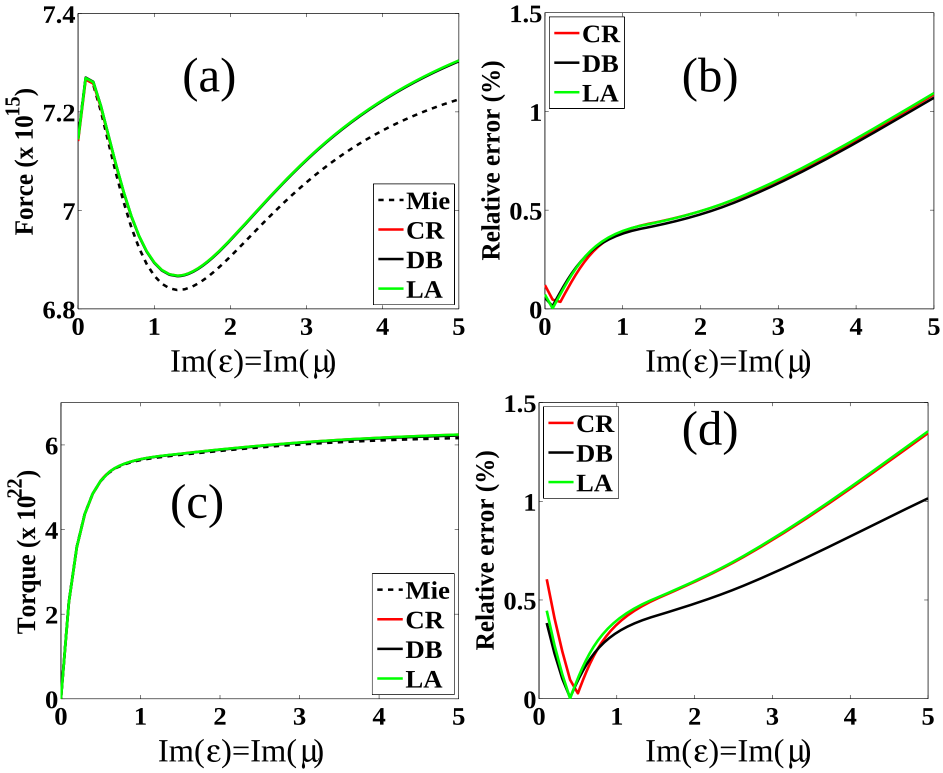

Fig. 4. Sphere of radius $a = \lambda/4$ with $\mathrm{Re}(\varepsilon) = \mathrm{Re}(\mu) = 2.25$. (a) Optical force versus $\mathrm{Im}(\varepsilon) = \mathrm{Im}(\mu)$. (b) Relative error in per cent between the optical force obtained from the CDM and the Mie series. (c) Optical torque versus versus $\mathrm{Im}(\varepsilon) = \mathrm{Im}(\mu)$. (d) Relative error in percent between the optical torque obtained from the CDM and the Mie series.

6.3. Electromagnetic force and torque on a scatterer with negative index

We now turn our attention to the case where the scatterer has material parameters $\varepsilon = \mu = -1$, i.e, a negative index of refraction. The optical force and torque are plotted in Figs. 5(a) and 5(c), and the corresponding relative errors in Figs. 5(b) and 5(d). Because the CDM is based on a representation of an object as a collection of dipoles, as one gets close to the dipole resonance ($\varepsilon = -2$, $\mu = -2$), the polarizabilities become very large in magnitude. Note that although the Clausius-Mossotti polarizabilities are singular for $\varepsilon = -2$, or $\mu = -2$, the final polarizabilities are not [$\alpha^{\mathrm{e}} = \alpha^{\mathrm{m}} = 3i/(2k^3)$] owing to the radiation reaction term. However, if the material forming the object is not lossy, the convergence of the CDM becomes very slow around a dipole resonance. Accordingly, we expect that, for a comparable level of discretization, the relative error on the optical force and torque computed by the CDM will be larger for $\varepsilon = \mu = -1$ than it was in the cases considered in the previous paragraph.

This is indeed confirmed in Figs. 5(b) and 5(d) where we see that if the imaginary part of ε and μ is zero or very small, the relative error increases up to about 10% for the optical force. For the optical torque the relative error at low loss is even more significant, reaching over 100%. However, the seemingly bad result for the optical torque is mitigated by the fact that the torque tends to zero for vanishing levels of absorption, which yields a significant relative error even if the absolute error is quite small. Therefore, the overall performance of the CDM for the

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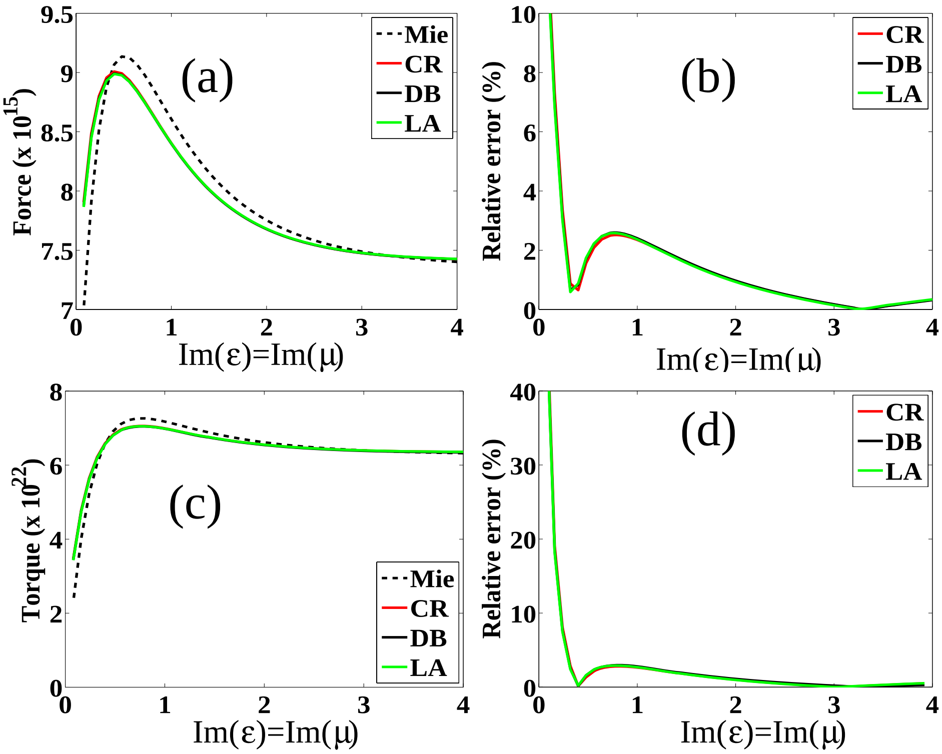

Fig. 5. Sphere of radius $a = \lambda/4$ with $\varepsilon = \mu = -1$. (a) Optical force versus $\mathrm{Im}(\varepsilon) = \mathrm{Im}(\mu)$. (b) Relative error in percent between the optical force obtained from the CDM and the Mie series. (c) Optical torque versus versus $\mathrm{Im}(\varepsilon) = \mathrm{Im}(\mu)$. (d) Relative error in percent between the optical torque obtained from the CDM and the Mie series.

calculation of optical forces and torques on an object with negative refraction is still excellent. Note that for the case $\varepsilon = -2$ and/or $\mu = -2$ a similar result would be obtained, however a larger number of discretization cells would be required in order to achieve convergence, as discussed in Ref. [19].

7. Conclusion

We have derived the analytic expressions of the force and torque induced by an arbitrary electromagnetic wave on a magnetic Rayleigh particle. From these expression we have developed a formalism based on the coupled dipole method (CDM) to compute optical forces and torques on arbitrary objects with an arbitrary dielectric permittivity and magnetic permeability, and we have illustrated the method by comparing it to the exact Mie theory. The present approach can be used to extend the study of the opto-mechanical coupling between light and matter to magnetic and meta-materials.

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