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ELECTROMAGNETIC SCATTERING BY A CONDUCTING CYLINDER COATED WITH METAMATERIALS

C. Li and Z. Shen

School of Electrical and Electronic Engineering
Nanyang Technological University
Nanyang Avenue, Singapore 639798

Abstract—The electromagnetic scattering from a conducting cylinder coated with metamaterials, which have both negative permittivity and permeability, is derived rigorously by using the classic separation of variables technique. It is found that a conducting cylinder coated with metamaterials has anomalous scattering cross section compared to that coated with conventional dielectric materials. Numerical results are presented and discussed for the scattering cross section of a conducting cylinder coated with metamaterials.

1. INTRODUCTION

The concept of composite materials possessing negative permittivity and permeability at certain frequencies has recently gained considerable attention and interest [1–8]. The original idea for such media dates back to 1967 when Veselago [9] theoretically studied time-harmonic plane wave propagation in a material whose permittivity and permeability were assumed to be simultaneously negative. His theoretical study showed that for a monochromatic uniform plane wave in such a medium, the direction of the Poynting vector is antiparallel with the direction of the phase velocity, contrary to the case of plane wave propagation in conventional simple media. Shelby, Smith, and Schultz [2] recently constructed such a composite medium for the microwave regime, and experimentally showed the presence of anomalous refraction in this medium. Metamaterials with negative permittivity and permeability, which have been named by several terminologies such as "double-negative" (DNG) media [5], "backward" (BW) media [6], left-handed media [1, 2, 7, 9], and negative-index media (NIM) [7], conceptually possess interesting electromagnetic features such as anomalous refraction.

The anomalous refraction at the boundary of such media and the fact that for a plane wave the direction of the Poynting vector is antiparallel with the direction of the phase velocity, provide us with features that can be advantageous in design of novel devices and components. Recently as a potential application of these metamaterials, an idea was theoretically introduced for compact cavity resonators, in which a combination of a slab of conventional material and a slab of metamaterial with negative permittivity and permeability was inserted [4].

The problem of scattering of electromagnetic waves by cylinders coated concentrically with homogeneous or radially inhomogeneous or plasma media has been extensively studied [10–13]. In the present work, the electromagnetic scattering from a conducting cylinder coated with metamaterials, which have both negative permittivity and permeability, is investigated in order to understand the scattering characteristics of metamaterials and to possibly reduce the radar cross section of a conducting cylinder. The electromagnetic scattering from a conducting cylinder coated with metamaterials is rigorously derived by using the classic separation of variables technique. The far scattered field pattern is calculated after using the large argument approximation of Hankel function. The scattering by a conducting cylinder coated with metamaterials is compared with that coated with a conventional material. Our analysis shows that a conducting cylinder coated with metamaterials exhibits anomalous characteristics compared with that coated with conventional dielectric materials. The metamaterials can also be used to decrease the scattering cross section of a conducting cylinder coated with a conventional dielectric material over a certain frequency range.

2. FORMULATION

In this paper, all fields are considered to be time-harmonic, with the $\exp(-i\omega t)$ time dependence suppressed. When a lossless metamaterial

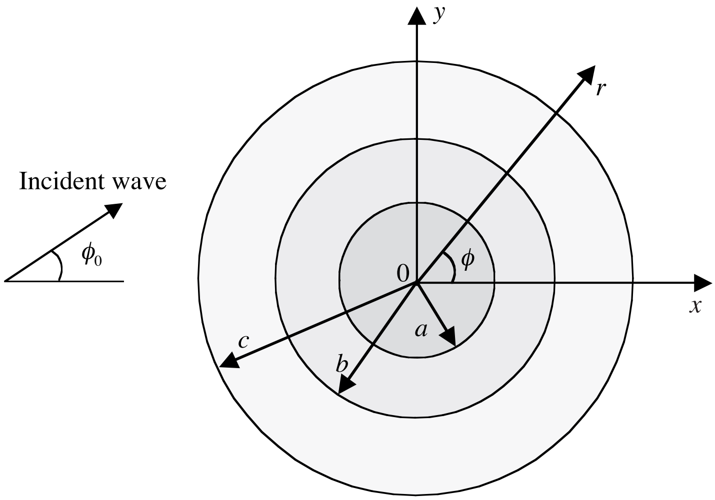

Figure 1. Geometry of the problem.

possesses negative real permittivity and permeability at certain frequencies, the index of refraction in such a medium attains real values. As theoretically predicted by Veselago [9], the electromagnetic wave can propagate in such a medium. However, for a monochromatic uniform plane wave in such a medium the phase velocity is in the opposite direction of the Poynting vector.

The geometry of the problem considered in the paper is shown in Fig. 1. The cylinders are assumed to be infinite in length and of circular cross section, where a, b and c are radii of a conductor cylinder, the coating dielectric layer I and the coating dielectric layer II, respectively. The external medium is free space, with constitutive parameters (μ_0, ε_0), the wave number $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$ and the intrinsic impedance $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$.

In the following, TM and TE polarizations will be considered simultaneously. When a plane wave is normally incident from ϕ_0, the incident fields may be written as [14]

$$E_z^i = A_{TM} \sum_{n=-\infty}^{\infty} i^n J_n(k_0 r) \exp[in(\phi - \phi_0)] \tag{1a}$$

$$H_z^i = \frac{iA_{TE}}{\eta_0} \sum_{n=-\infty}^{\infty} i^n J_n(k_0 r) \exp[in(\phi - \phi_0)] \tag{1b}$$

$$E_\phi^i = A_{TE} \sum_{n=-\infty}^{\infty} i^n J_n'(k_0 r) \exp[in(\phi - \phi_0)] \tag{1c}$$

$$H_\phi^i = \frac{iA_{TM}}{\eta_0} \sum_{n=-\infty}^{\infty} i^n J_n'(k_0 r) \exp[in(\phi - \phi_0)] \tag{1d}$$

where r and ϕ are the cylindrical coordinates and ϕ_0 defines the direction of incidence; $A_{TE} = 0$, $A_{TM} = 1$ for TM polarization; $A_{TE} = 1$, $A_{TM} = 0$ for TE polarization; and J_n is the nth order Bessel function of the first kind; primes in equation (1) denote derivatives with respect to the entire argument.

From Maxwell's equations, the scattered fields can be expanded as

$$E_z^s = \sum_{n=-\infty}^{\infty} B_{nTM} i^n H_n^{(1)}(k_0 r) \exp[in(\phi - \phi_0)] \tag{2a}$$

$$H_z^s = \frac{i}{\eta_0} \sum_{n=-\infty}^{\infty} B_{nTE} i^n H_n^{(1)}(k_0 r) \exp[in(\phi - \phi_0)] \tag{2b}$$

$$E_\phi^s = \sum_{n=-\infty}^{\infty} B_{nTE} i^n H_n'^{(1)}(k_0 r) \exp[in(\phi - \phi_0)] \tag{2c}$$

$$H_\phi^s = \frac{i}{\eta_0} \sum_{n=-\infty}^{\infty} B_{nTM} i^n H_n'^{(1)}(k_0 r) \exp[in(\phi - \phi_0)] \tag{2d}$$

where $H_n^{(1)}$ is the nth order Hankel function of the first kind, B_{nTM} and B_{nTE} are unknown coefficients to be determined.

In dielectric layer I, no matter they are metamaterial or conventional material, the fields can be expanded as

$$E_z^I = \sum_{n=-\infty}^{\infty} C_{nTM}^I \frac{J_n(k_1 r) Y_n(k_1 a) - J_n(k_1 a) Y_n(k_1 r)}{J_n(k_1 b) Y_n(k_1 a) - J_n(k_1 a) Y_n(k_1 b)} \exp(in\phi) \tag{3a}$$

$$H_z^I = \frac{i}{\eta_1} \sum_{n=-\infty}^{\infty} C_{nTE}^I \frac{J_n(k_1 r) Y_n'(k_1 a) - J_n'(k_1 a) Y_n(k_1 r)}{J_n'(k_1 b) Y_n'(k_1 a) - J_n'(k_1 a) Y_n'(k_1 b)} \exp(in\phi) \tag{3b}$$

$$E_\phi^I = \sum_{n=-\infty}^{\infty} C_{nTE}^I \frac{J_n'(k_1 r) Y_n'(k_1 a) - J_n'(k_1 a) Y_n'(k_1 r)}{J_n'(k_1 b) Y_n'(k_1 a) - J_n'(k_1 a) Y_n'(k_1 b)} \exp(in\phi) \tag{3c}$$

$$H_\phi^I = \frac{i}{\eta_1} \sum_{n=-\infty}^{\infty} C_{nTM}^I \frac{J_n'(k_1 r) Y_n(k_1 a) - J_n(k_1 a) Y_n'(k_1 r)}{J_n(k_1 b) Y_n(k_1 a) - J_n(k_1 a) Y_n(k_1 b)} \exp(in\phi) \tag{3d}$$

where Y_n is the nth order Bessel function of the second kind, C_{nTM}^I and C_{nTE}^I are unknown coefficients to be determined. The boundary condition on the surface of the conducting cylinder ($r = a$) has already been taken into account.

In dielectric layer II, which can also be either metamaterial or conventional material, the fields can be expanded as

$$E_z^{II} = \sum_{n=-\infty}^{\infty} \left[C_{nTM}^{II} \frac{J_n(k_2r)Y_n(k_2c) - J_n(k_2c)Y_n(k_2r)}{J_n(k_2b)Y_n(k_2c) - J_n(k_2c)Y_n(k_2b)} + D_{nTM}^{II} \frac{J_n(k_2b)Y_n(k_2r) - J_n(k_2r)Y_n(k_2b)}{J_n(k_2b)Y_n(k_2c) - J_n(k_2c)Y_n(k_2b)} \right] \exp(in\phi) \quad \text{(4a)}$$

$$H_z^{II} = \frac{i}{\eta_2} \sum_{n=-\infty}^{\infty} \left[C_{nTE}^{II} \frac{J_n(k_2r)Y_n'(k_2c) - J_n'(k_2c)Y_n(k_2r)}{J_n'(k_2b)Y_n'(k_2c) - J_n'(k_2c)Y_n'(k_2b)} + D_{nTE}^{II} \frac{J_n'(k_2b)Y_n(k_2r) - J_n(k_2r)Y_n'(k_2b)}{J_n'(k_2b)Y_n'(k_2c) - J_n'(k_2c)Y_n'(k_2b)} \right] \exp(in\phi) \quad \text{(4b)}$$

$$E_\phi^{II} = \sum_{n=-\infty}^{\infty} \left[C_{nTE}^{II} \frac{J_n'(k_2r)Y_n'(k_2c) - J_n'(k_2c)Y_n'(k_2r)}{J_n'(k_2b)Y_n'(k_2c) - J_n'(k_2c)Y_n'(k_2b)} + D_{nTE}^{II} \frac{J_n'(k_2b)Y_n'(k_2r) - J_n'(k_2r)Y_n'(k_2b)}{J_n'(k_2b)Y_n'(k_2c) - J_n'(k_2c)Y_n'(k_2b)} \right] \exp(in\phi) \quad \text{(4c)}$$

$$H_\phi^{II} = \frac{i}{\eta_2} \sum_{n=-\infty}^{\infty} \left[C_{nTM}^{II} \frac{J_n'(k_2r)Y_n(k_2c) - J_n(k_2c)Y_n'(k_2r)}{J_n'(k_2b)Y_n'(k_2c) - J_n'(k_2c)Y_n'(k_2b)} + D_{nTM}^{II} \frac{J_n(k_2b)Y_n'(k_2r) - J_n'(k_2r)Y_n(k_2b)}{J_n(k_2b)Y_n(k_2c) - J_n(k_2c)Y_n(k_2b)} \right] \exp(in\phi) \quad \text{(4d)}$$

where C_{nTM}^{II}, C_{nTE}^{II}, D_{nTM}^{II} and D_{nTE}^{II} are unknown coefficients to be determined.

In the dielectric layer (I or II), if it is conventional lossless material with real permittivity $\varepsilon_1 > 0$ and real permeability $\mu_1 > 0$, the index of refraction $n_1 \equiv \sqrt{(\varepsilon_1\mu_1)/(\varepsilon_0\mu_0)}$, is taken to be a positive real quantity. The intrinsic impedance of the dielectric material is then $\eta_1 = \sqrt{\mu_1/\varepsilon_1}$, and the wave number k_1 is n_1k_0. Now, let us consider a dielectric layer of lossless metamaterial with negative real permittivity and real permeability, $\varepsilon_2 < 0$ and $\mu_2 < 0$, at certain frequencies. For this layer of metamaterial, the index of refraction is a real quantity denoted by $n_2 \equiv -\sqrt{(\varepsilon_2\mu_2)/(\varepsilon_0\mu_0)}$. It is important to note that we need to specify the negative sign for the square root appearing in the expression of n_2. The intrinsic impedance of the metamaterial is $\eta_2 = \sqrt{\mu_2/\varepsilon_2}$, and the wave number is $k_2 = n_2k_0$.

Applying the boundary conditions at $r = b$ and $r = c$, respectively, a linear matrix equation about unknown coefficients, B_{nTM}, B_{nTE}, C_{nTM}^{I}, C_{nTE}^{I}, C_{nTM}^{II}, C_{nTE}^{II}, D_{nTM}^{II}, and D_{nTE}^{II} can be obtained. Then by solving the set of linear equations, all the unknown coefficients can be determined.

The far scattered field pattern is calculated after using the large argument approximation of Hankel function. For TM and TE incident plane wave, the normalized bistatic echo widths have the following form:

$$\frac{\sigma^{TM}}{\lambda_0} = \frac{2}{\pi A_{TM}^2}\left|\sum_{n=-\infty}^{\infty} B_{nTM}\exp[jn(\phi-\phi_0)]\right|^2, \tag{5a}$$

$$\frac{\sigma^{TE}}{\lambda_0} = \frac{2}{\pi A_{TE}^2}\left|\sum_{n=-\infty}^{\infty} B_{nTE}\exp[jn(\phi-\phi_0)]\right|^2. \tag{5b}$$

3. NUMERICAL RESULTS AND DISCUSSION

This section presents numerical results for scattering echo width of a conducting cylinder coated with metamaterials. In order to verify the formulation described above, the back scattered cross sections of two normal dielectric coated cylinders were computed and compared with experimental results available in [15]. The two problems considered here are two conducting cylinders each coated with one dielectric layer. The conducting cylinders are of radii 4.71 and 6.51 millimeters, respectively, and the relative permittivities of their coating dielectric are 2.54 and 6, respectively. The free space wave number is 200 rad/m. It can be seen from Fig. 2 and Fig. 3 that our theoretical results are in excellent agreement with experimental data [15], which verifies that the formulation described and the computer code written are correct.

Numerical tests indicate that the truncation terms in the series summation of a conducting cylinder coated with metamaterials should be more than the case with a conventional dielectric of the same geometrical dimension in order to obtain the same precision. This is because metamaterial's wave number k has a negative sign and the difference of its wave number from the free space's positive wave number is in turn bigger than that for a conventional dielectric. Table 1 list the convergence behavior of truncation terms of back scattered echo width as a function of the integer N, which is the absolute value of the upper limit of the index of summation in the series expressions. One typical value of truncation index N for the case with a metamaterials coating is

$$N = |kc| + 5 \tag{6}$$

Fig. 4 and Fig. 5 compare the back scattered cross section of a conducting cylinder coated, respectively, with a metamaterial

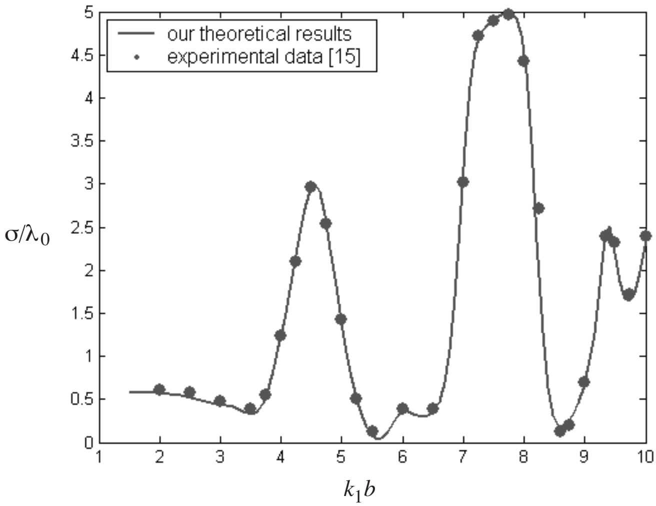

Figure 2. The back scattered cross section of a normal dielectric coated cylinder. ($a = 4.71\,\text{mm},\ k_0 = 200,\ \varepsilon_r = 2.54$).

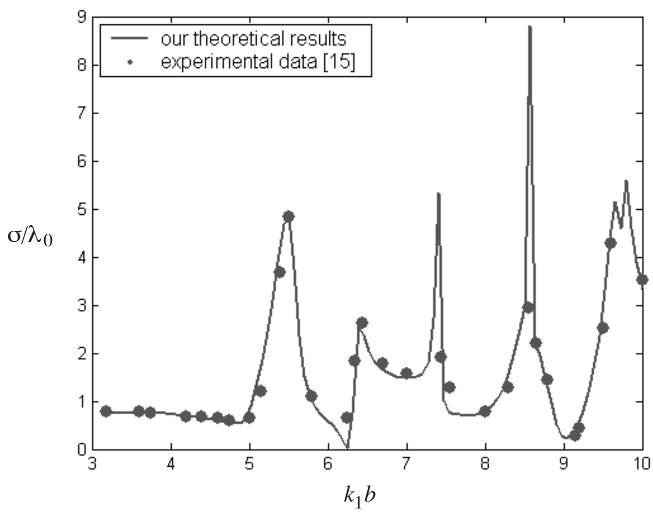

Figure 3. The back scattered cross section of a normal dielectric coated cylinder. ($a = 6.51\,\text{mm},\ k_0 = 200,\ \varepsilon_r = 6$).

Table 1. Back scattered echo width against N for $a = 50\,\text{mm}$, $b = 60\,\text{mm}$ and $f = 5\,\text{GHz}$.

	Conventional material $\varepsilon_{r1} = 2.2,\ \mu_{r1} = 1$	Metamaterial $\varepsilon_{r1} = -2.2,\ \mu_{r1} = -1$
N	σ^{TM}/λ_0	σ^{TM}/λ_0
1	1.71019794771201	1.89707676713716
5	2.76605738697248	5.10213064469963
10	2.63518412930355	4.46675094253025
15	2.63509817456894	4.46721343769394
20	2.63509817461576	4.46721343723989

and a conventional dielectric material. The metamaterial and the conventional dielectric have the same value of permittivity and permeability but in opposite sign. In Fig. 4, the back scattered cross section for TM-wave incidence increases with frequency linearly below 7 GHz, while the back scattered cross section of TE-wave incidence oscillates with frequency from 1 GHz to 10 GHz. Fig. 5 shows that the back scattered cross section for TE-wave increases with frequency linearly in the frequency range from 1 to 10 GHz, while the back scattered cross section for TM-wave sharply decreases between 4 GHz to 5 GHz, and dramatically increases between 5 GHz to 7 GHz. It is seen that the conducting cylinder coated with a metamaterial exhibits anomalous characteristics compared to that coated with a conventional dielectric, and over some frequency band, the back scattered cross section of the cylinder coated with a metamaterial is smaller.

The back scattered cross section of a conducting cylinder coated with a one-layer dielectric versus the coating material's permittivity and permeability is examined in Fig. 6 and Fig. 7, respectively. In Fig. 6, when both the permittivity and permeability of the coating layer are negative, the back scattered cross section of TM and TE polarizations is bigger than that coated with a conventional material, which has the same value of permittivity and permeability, but positive sign. The back scattered cross section also has resonant points when the coating material's permittivity and permeability are both negative. In Fig. 7, when the permittivity and permeability of the coating layer are both negative, the back scattered cross section of TM polarization has a sharp increase at μ between -1 and -2. The back scattered echo

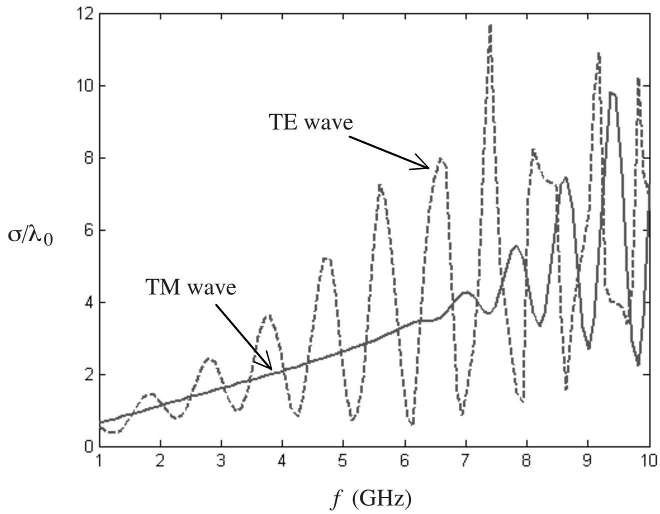

Figure 4. The back scattered cross section of a normal dielectric coated cylinder. ($a = 50\,\mathrm{mm}$, $b = 60\,\mathrm{mm}$, $\varepsilon_{r1} = 2.2$ and $\mu_{r1} = 1$).

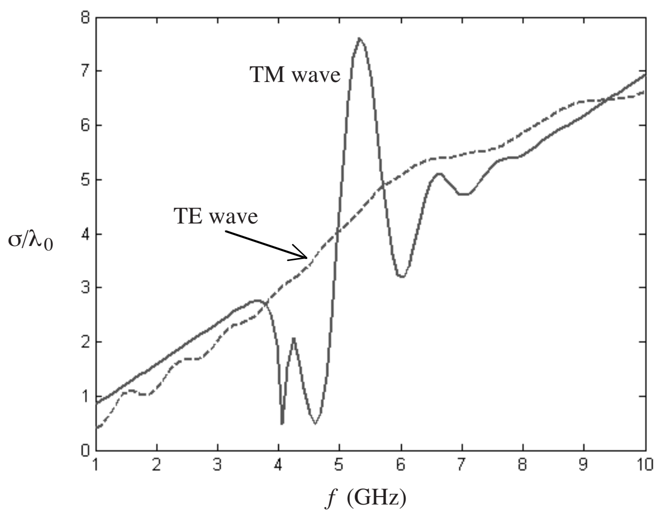

Figure 5. The back scattered cross section of a metamaterial coated cylinder. ($a = 50\,\mathrm{mm}$, $b = 60\,\mathrm{mm}$, $\varepsilon_{r1} = -2.2$ and $\mu_{r1} = -1$).

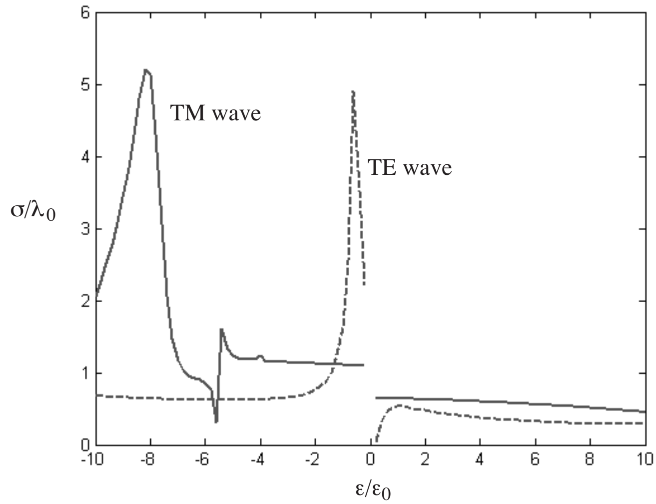

Figure 6. The back scattered cross section of a dielectric coated cylinder. (a = 50 mm, b = 70 mm, f = 1 GHz, $\mu/\mu_0 = -1$ when $\varepsilon/\varepsilon_0 < 0$ and $\mu/\mu_0 = 1$ when $\varepsilon/\varepsilon_0 > 0$).

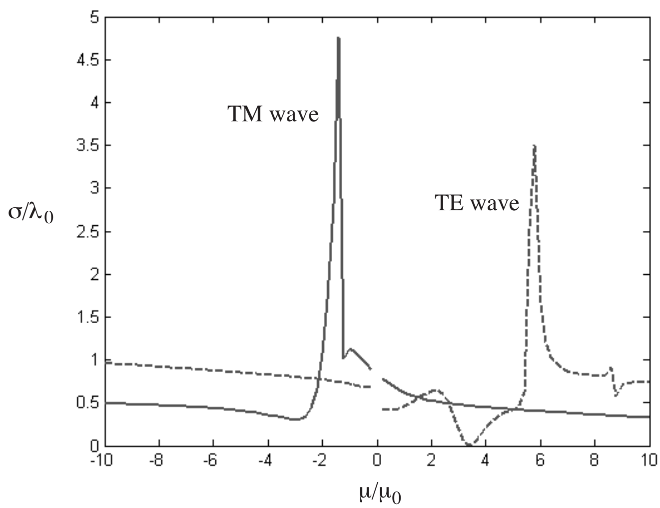

Figure 7. The back scattered cross section of a dielectric coated cylinder. (a = 50 mm, b = 70 mm, f = 1 GHz, $\varepsilon/\varepsilon_0 = -2.2$ when $\mu/\mu_0 < 0$ and $\varepsilon/\varepsilon_0 = 2.2$ when $\mu/\mu_0 > 0$).

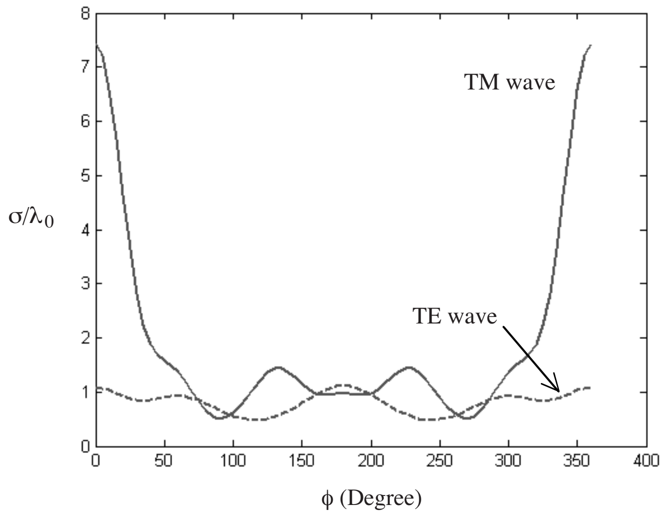

Figure 8. Normalized bistatic echo width of a normal dielectric coated cylinder. ($a = 50\,\text{mm}$, $b = 100\,\text{mm}$, $\varepsilon_{r1} = 9.8$, $\mu_{r1} = 1$, $\phi_0 = 0^\circ$ and $f = 1\,\text{GHz}$).

width of TE polarization changes with μ linearly for metamaterial coating. When the coating material's permittivity and permeability are both positive, the back scattered cross section of TE polarization have a sharp increase at μ near 6, and the back scattered cross section of TM polarization varies with μ linearly. It is shown that the back scattered cross section of a cylinder coated with a metamaterial has a very different performance versus $|\varepsilon/\varepsilon_0|$ and $|\mu/\mu_0|$ compared to that coated with a conventional material.

The variation of the back scattered cross section of a conducting cylinder coated with a dielectric versus the observation angle is shown in Fig. 8 and Fig. 9. Comparing the back scattered cross section of a cylinder coated with a metamaterial and the same cylinder coated with a conventional material, we find that the back scattered cross sections of TM incidence are of very similar behavior: they both have large forward scattering. However, a cylinder coated with a conventional material has smaller forward scattering for TE incidence compared to a cylinder coated with a metamaterial.

In order to utilize the anomalous scattering characteristics of metamaterials for the purpose of decreasing the back scattered cross section of a normal dielectric coated cylinder, we can choose a suitable frequency band of metamaterial. Fig. 10 shows the back scattered

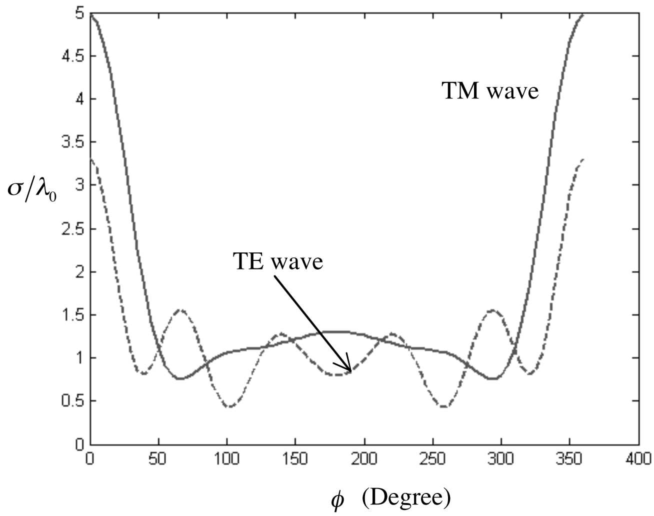

Figure 9. Normalized bistatic echo width of a metamaterial dielectric coated cylinder. ($a = 50\,\text{mm}$, $b = 100\,\text{mm}$, $\varepsilon_{r1} = -9.8$, $\mu_{r1} = -1$, $\phi_0 = 0^\circ$ and $f = 1\,\text{GHz}$).

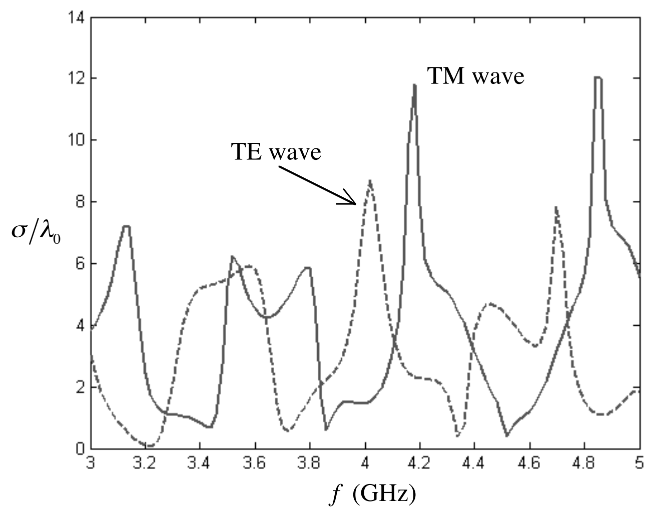

Figure 10. The backscattered cross section of a normal dielectric coated cylinder. ($a = 50\,\text{mm}$, $b = 100\,\text{mm}$, $\varepsilon_{r1} = 2.54$ and $\mu_{r1} = 1$).

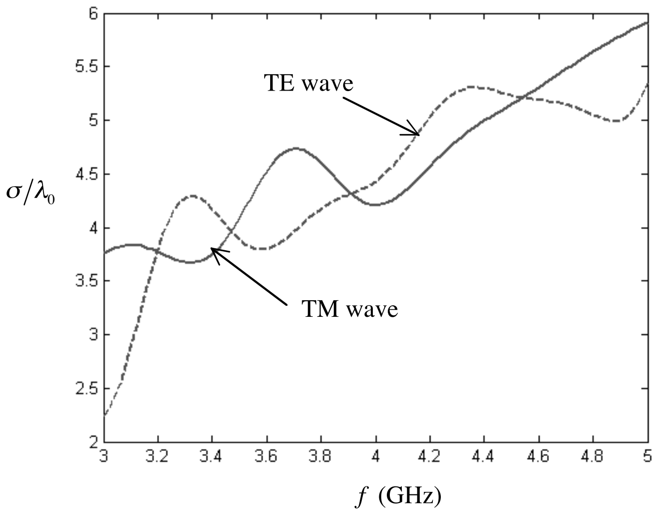

Figure 11. The back scattered cross section of a conducting cylinder coated with a two-layer dielectric, where the first layer is a conventional material, and the second layer is a metamaterial. ($a = 50\,\mathrm{mm}$, $b = 100\,\mathrm{mm}$, $c = 150\,\mathrm{mm}$, $\varepsilon_{r1} = 2.54$, $\mu_{r1} = 1$, $\varepsilon_{r2} = -2.54$ and $\mu_{r2} = -1$).

cross section of a normal dielectric coated cylinder from 3 GHz to 5 GHz. The radii of the conducting cylinder and the conventional dielectric coating are 50 and 100 millimeters, respectively, and the permittivity and permeability of the coating material are 2.54 and 1, respectively. In Fig. 11, a second layer of metamaterial is added to the dielectric coated cylinder of Fig. 10, with the same thickness and possessing equally absolute value and opposite sign of permittivity and permeability. It is seen that the back scattered cross section can be successfully smoothened and decreased compared to that in Fig. 10.

4. CONCLUSION

This paper has presented a rigorous analysis of scattering by a conducting cylinder coated with metamaterials for a TM or TE normal incident plane wave. It has been found that a conducting cylinder coated with metamaterials has different scattering cross section performance compared to a conducting cylinder coated with conventional dielectric materials. The metamaterials can also be used to decrease the radar cross section of a conducting cylinder coated with usual conventional dielectric material over a certain frequency range.

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