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ELECTROMAGNETIC SCATTERING BY PARALLEL METAMATERIAL CYLINDERS

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Abstract—Electromagnetic Scattering problem by multiple metamaterial cylinders are studied. A formulation for coating conducting cylinders by metamaterial is presented. The formulation is based on the expansion of electromagnetic fields by Bessel functions in local cylindrical coordinates and then transformation from local coordinates to others using addition theorem of Hankel function. Near fields and scattering cross sections for metamaterial and dielectric cylinders are studied and compared to each other. We have shown that an array of metamaterial cylinders increases forward scattering cross section which may use to increase the directivity of antennas.

1. INTRODUCTION

In recent years, there has been an increasing interest in the development of new materials with characteristics which may not be found in nature such as metamaterials and chiral media [1]. Metamaterials are materials with negative permittivity and permeability with certain frequency range. The direction of Poynting vector of a monochromatic uniform plane wave in metamaterial medium is antiparallel with the direction of the phase velocity, contrary to the case of plane wave propagation in a conventional simple media. Metamaterials have many interesting behavior and potential applications in different optical and microwave components [1]. Metamaterial with negative permittivity and permeability have been also named by several terminologies such as "double-negative" (DNG), "backward" BW materials and "negative-index media" (NIM). Different methods have been presented to artificially fabricate metamaterials [1]. In microwave frequencies using of periodic arrangements of split ring resonator (SRR) or other resonator [2] in a host dielectric can pose a negative-index media. However, other realizations exit such as a parallel-plate waveguide filled by a two dimensional (2D) array of broad-side coupled SRRs [3], transmission-line networks loaded with capacitors and inductors [4] and magneto-dielectric spherical particles embedded in a background matrix [5].

The problem of scattering of waves by multiple cylinders coated or non-coated by different artificial materials has been studied in different papers. In reference [6] multiple conducting and dielectric circular cylinders are studied for modeling of other complex cylindrical shapes. In [7] scattering problem by a single circular cylinder coated by metamaterials is studied, and in [8] scattering by elliptically-shaped metamaterials is investigated. In [9] an iterative procedure for electromagnetic scattering from parallel chiral cylinders is developed.

Scattering of electromagnetic waves by complex-shaped metamaterial objects is an interesting research subject to develop potential application of this artificial material. The main task of this paper is to present a useful tool and its numerical results for demonstrations of this potential application. In this paper, we present formulation and the results of multiple scattering by circular-cylindrical objects coated by metamaterials. The details of mathematical formulation are presented in section two. In section three, numerical results are demonstrated in order to show behavior of scattering electromagnetic waves by metamaterials. We also compare the results of near field, back- and forward- scattering for dielectric and metamaterial cylinders. In the present study permittivity and permeability for metamaterials are chosen pure real number with negative sign which indicate a lossless media although loss in metamaterial is a serious and major difficulty for all potential applications.

2. FORMULATION

Figure 1 shows the geometry of M cylinders coated with metamaterial, their coordinates and a TM polarized incident wave with time dependence $\exp(j\omega t)$. The incident electric field is given by:

$$E_z^{inc} = E_0 e^{jk_0\rho\cos(\phi-\phi_0)} \tag{1}$$

where k_0 is the free space wave number, $(\rho,\ \phi,\ z)$ are circular cylindrical coordinates and ϕ_0 is angle of incidence of the plane wave as shown in Fig. 1. The parameters a_i and b_i are the radius of the ith cylinder and radius of coated material. The incidence field can be expressed in local cylindrical coordinates of the ith cylinder $(\rho_i,\ \phi_i)$, whose center is located at (ρ_i', ϕ_i'):

$$\begin{aligned} E_z^{inc} &= E_0 e^{jk_0\rho_i'\cos(\phi_i'-\phi_0)} e^{jk_0\rho_i\cos(\phi_i-\phi_0)} \\ &= E_0 e^{jk_0\rho_i'\cos(\phi_i'-\phi_0)} \sum_{n=-\infty}^{+\infty} j^n J_n(k_0\rho_i)\, e^{jn(\phi_i-\phi_0)} \end{aligned} \tag{2}$$

The scattering field is the summation of contribution of all M cylinders and can be expressed as

$$\begin{aligned} E_z^{scatt} &= \sum_{i=1}^{M} E_z^{i\ scatt}(\rho_i, \phi_i) \\ &= E_0 \sum_{i=1}^{M} \sum_{m=-\infty}^{+\infty} c_{im} H_m^2(k_0\rho_i)\, e^{jm\phi_i} \quad \textit{for region } \rho_i > b_i \end{aligned} \tag{3}$$

The field inside metamaterial of ith cylinder is given by

$$E_z^{i\ dielectric} = E_0 \sum_{m=-\infty}^{+\infty} \left[e_{im} J_m(k_i\rho_i) + d_{im} N_m(k_i\rho_i)\right] e^{jm\phi_i} \qquad \textit{for } a_i < \rho_i < b_i \tag{4}$$

where c_{im}, e_{im}, d_{im} are unknown coefficients related to the ith cylinder. The wave number and intrinsic impedance for the coating medium of the ith cylinder are

$$\begin{aligned} k_i &= k_0 n_{ri}, \\ n_{ri} &= \sqrt{\mu_{ri}\varepsilon_{ri}} && \textit{for dielectric}, \\ n_{ri} &= -\sqrt{\mu_{ri}\varepsilon_{ri}} && \textit{for metamaterial}, \\ \eta_i &= \eta_0\sqrt{\mu_{ri}/\varepsilon_{ri}} && \textit{for dielectric and metamaterial} \end{aligned} \tag{5}$$

which k_0 and η_0 are wave number and intrinsic impedance of free space respectively. The refractive index n_{ri} is negative for metamaterial which cause negative wave number, k_i while intrinsic impedance of conventional and metamaterial are positive. Therefore solution of the scattering problem in the presence of metamaterial is reduced to the solution of scattering problem of conventional dielectric by inserting of a negative wave number and positive intrinsic impedance.

Three boundary conditions on the perfect electric conductor and metamaterial surfaces are given by:

$$\begin{aligned} E_Z^{inc} + E_z^{scatt} &= E_z^{dielectric} && \rho_i = b_i,\ 0 \le \phi_i \le 2\pi \\ E_z^{i\ dielectric} &= 0 && \rho_i = a_i,\ 0 \le \phi_i \le 2\pi \\ H_\phi^{inc} + H_\phi^{scatt} &= H_\phi^{i\ dielectric} && \rho_i = b_i,\ 0 \le \phi_i \le 2\pi \end{aligned} \tag{6}$$

From the second boundary condition the coefficients d_{im} is given:

$$d_{im} = -e_{im} \frac{J_m(k_i a_i)}{N_m(k_i a_i)} \tag{7}$$

In order to find unknown expansion coefficients, it is necessary to express the scattering field from other cylinders to local coordinate of the ith cylinder. By using the addition theorem of Hankel function, the transformation from qth to pth coordinates:

$$H_n^{(2)}(k\rho_q)e^{jn\phi_q} = \sum_{m=-\infty}^{+\infty} J_m(k\rho_p)H_{m-n}^{(2)}(kd_{pq})e^{jm\phi_p}e^{-j(m-n)\phi_{pq}} \qquad for\ \rho_q > d_{pq} \tag{8}$$

the boundary conditions can be written in the following matrix form:

$$\begin{bmatrix} \boldsymbol{A}_{1E} & \boldsymbol{A}_{2E} \\ \boldsymbol{A}_{1H} & \boldsymbol{A}_{2H} \end{bmatrix} \begin{bmatrix} [c(j_{mat})] \\ [e(j_{mat})] \end{bmatrix} = \begin{bmatrix} [E^{inc}(i_{mat})] \\ [H^{inc}(i_{mat})] \end{bmatrix} \tag{9}$$

Which $\boldsymbol{A}_{1E}$, $\boldsymbol{A}_{2E}$, $\boldsymbol{A}_{1H}$, and $\boldsymbol{A}_{2H}$ are of dimension ($M\ N_{\text{tbessel}} \times MN_{\text{tbessel}}$) where M is number of cylinders and ($N_{\text{tbessel}} = 2N_b + 1$) is number of terms in the infinite summation which should be chosen large enough for convergence. The vectors $E^{inc}(i_{mat})$, $H^{inc}(i_{mat})$, $[c(j_{mat})]$, and $[e(j_{mat})]$ are representing of incident electric and magnetic fields and coefficients c_{im} and e_{im} respectively. All these vectors have size MN_{tbessel} and are given by,

$$\begin{aligned} N_{\text{tbessel}} &= 2N_b + 1 \\ i_{mat} &= (i-1)N_{\text{tbessel}} + m + N_b + 1 \\ & \quad i = 1, 2, \ldots, M \quad m = -N_b, \ldots, N_b \\ j_{mat} &= (g-1)N_{\text{tbessel}} + n + N_b + 1 \\ & \quad g = 1, 2, \ldots, M \quad n = -N_b, \ldots, N_b \end{aligned} \tag{10}$$

$$E^{inc}(i_{mat}) = E_0 e^{jk_0\rho_i' \cos(\phi_i' - \phi_0)} j^m J_m(k_0 b_i) e^{jm(\phi_i - \phi_0)} \tag{11}$$

$$H^{inc}(i_{mat}) = (E_0/\eta_i) e^{jk_0\rho_i' \cos(\phi_i' - \phi_0)} j^m J_m(k_0 b_i) e^{jm(\phi_i - \phi_0)} \tag{12}$$

$$\boldsymbol{A}_{1E}(i_{mat}, j_{mat}) = \begin{cases} -J_m(k_0 b_i) H_{m-n}^2(k_0 d_{ig}) \exp(-j(m-n)\varphi_{ig}) & if\ i \neq g \\ -H_m^2(k_0 b_i) & if\ i_{mat} = j_{mat} \\ 0 & otherwise \end{cases} \tag{13}$$

$$\boldsymbol{A}_{2E}(i_{mat}, j_{mat}) = \begin{cases} J_m(k_i b_i) - \frac{J_m(k_i a_i)}{N_m(k_i a_i)} N_m(k_i b_i) & \textit{if } i_{mat} = j_{mat} \\ 0 & \textit{if } i_{mat} \neq j_{mat} \end{cases} \tag{14}$$

$$\boldsymbol{A}_{1H}(i_{mat}, j_{mat}) = \begin{cases} -J'_m(k_0 b_i) H^2_{m-n}(k_0 d_{ig}) \exp(-j(m-n)\varphi_{ig})/(j\eta_0) & \\ & \textit{if } i \neq g \\ -H'^2_m(k_0 b_i)/(j\eta_0) & \textit{if } i_{mat} = j_{mat} \\ 0 & \textit{otherwise} \end{cases} \tag{15}$$

$$\boldsymbol{A}_{2H}(i_{mat}, j_{mat}) = \begin{cases} \left[J'_m\left(k_i b_i\right) - \frac{J_m(k_i a_i)}{N_m(k_i a_i)} N'_m\left(k_i b_i\right)\right]\left(1/j\eta_i\right) & \textit{if } i_{mat} = j_{mat} \\ 0 & \textit{if } i_{mat} \neq j_{mat} \end{cases} \tag{16}$$

The scattering cross section $\sigma(\phi)$ of the multiple cylinder geometry is then given by:

$$\begin{aligned} F(\phi) &= \sum_{g=1}^{M} \sum_{n=-\infty}^{+\infty} \exp(jk_0 \rho'_g \cos(\phi'_g - \phi)) c_{gn} j^n \exp(jn\phi) \\ \sigma(\rho) &= \frac{2\lambda_0}{\pi} |F(\phi)|^2 \end{aligned} \tag{17}$$

The scattering by TE polarization can be obtained in same method and the above equations are slightly changed.

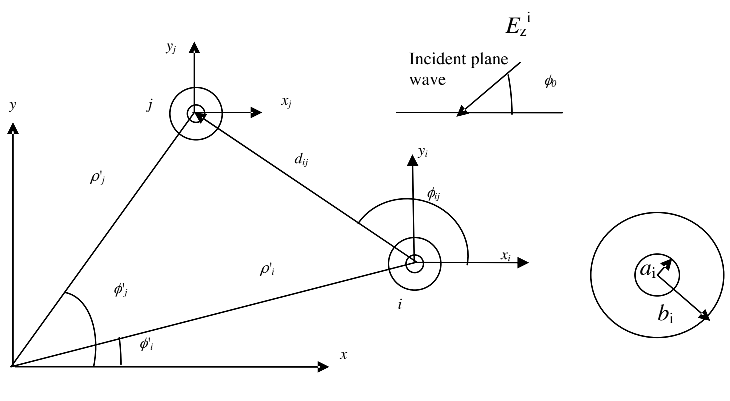

Figure 1. Geometry of M coated cylinders and their local coordinates. Each cylinder defined by radius of conducting cylinder a_i, radius of coated dielectric or meta-material b_i and its permeability and permittivity (μ_i, ε_i). Angle Incident plane wave is ϕ_0.

3. NUMERICAL RESULTS AND DISCUSSION

In order to verify the formulation and written computer program, the results of five cylinders with $0.1\,\lambda_0$ radius, with separation of $0.5\,\lambda_0$ and incident field angle $\phi_0 = 180°$ are shown in Fig. 2. The geometry of these five scatterers is also shown in Figure 2. Scattering Cross Sections (SCS) are for three different cases metamaterial with $\varepsilon_r = -2.2$ and $\mu_r = -1$, perfect electric conductor and conventional dielectric with $\varepsilon_r = 2.2$ and $\mu_r = 1$. The results for conductor case and conventional dielectric are published in [6] with good agreement with our results. The fact that the back-scattering cross section (BSCS), $\sigma(180°)$, for metamaterial is more than BSCS for PEC and conventional dielectric is interesting. There is no simple theory to predict this behavior.

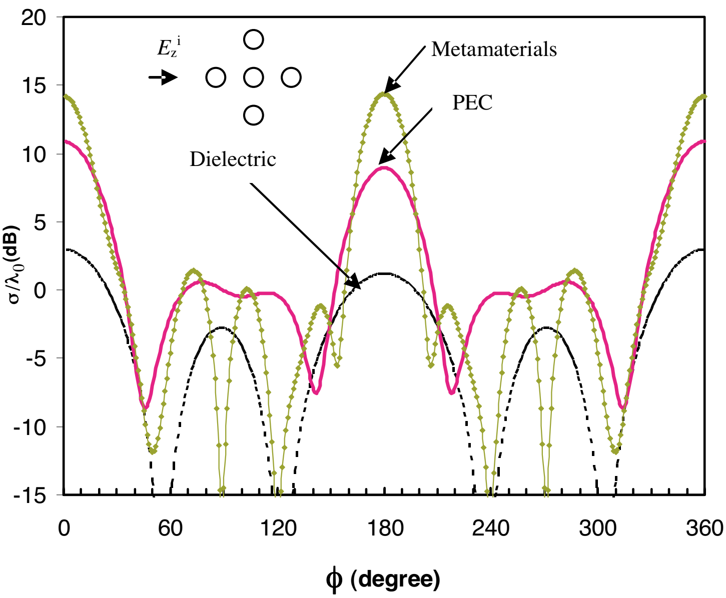

Figure 2. The scattering cross section (SCS) for TM polarized of three types of five cylinders.

$a = 0.1\,\lambda_0$, centers separation by $0.5\,\lambda_0$, $\phi_0 = 180°$

$\varepsilon_r = 2.2$, $\mu_r = 1$ Dielectric cylinders for dashed line - - - - -

perfect conducting for solid line ———

$\varepsilon_r = -2.2$, $\mu_r = -1$ metamaterials cylinders $- \triangleright$——

Figure 3 shows the scattering cross section of five coated conducting cylinders for the same geometry of Fig. 2. The radius of conducting cylinder and coated dielectric/meta-materials are $0.05\,\lambda_0$ and $0.1\,\lambda_0$ respectively. BSCS, $\sigma(180°)$ for metamaterial is lower than BSCS for conventional dielectric coated on conducting cylinders which is not predictable. This fact is also valid for many cases with coated conducting cylinders.

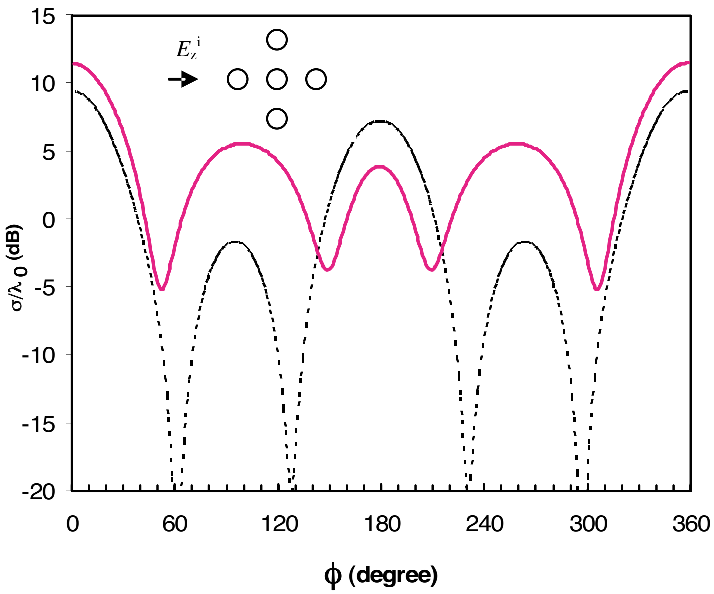

Figure 3. SCS of five coated conducting cylinders. Parameters a and b indicate the radius of conducting cylinders and coated dielectric/meta-materials. (TM polarized)

$a = 0.05\,\lambda_0$, $b = 0.1\,\lambda_0$, centers separation by $0.5\,\lambda_0$, $\phi_0 = 180°$

$\varepsilon_r = 2.2$, $\mu_r = 1$ for dashed line - - - - -

$\varepsilon_r = -2.2$, $\mu_r = -1$ for solid line ———

Figure 4 shows the SCS for dielectric cylinders (without conducting cylinders). The result is different from coated one. BSCS for the metamaterial is larger than BSCS for conventional dielectric cylinders. This observation is also valid for other cases of non-coated conducting cylinders.

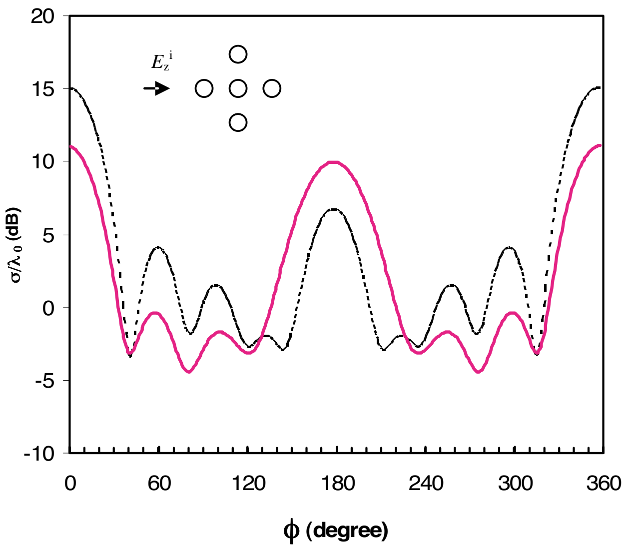

Figure 4. The scattering cross section of five dielectric/metamaterial cylinders.

$a = 0.1\,\lambda_0$, centers separation by $0.5\,\lambda_0$, $\phi_0 = 180°$

$\varepsilon_r = 9.8$, $\mu_r = 1$ for dashed line - - - - -

$\varepsilon_r = -9.8$, $\mu_r = -1$ for solid line ———

Figure 5 shows the back-scattering cross section as a function of angle of incidence for two cylinders located on the x-axis with two different center separation $0.4\,\lambda_0$ and $0.5\,\lambda_0$ and both with $0.1\,\lambda_0$ radius for two cases metamaterial with $\varepsilon_r = -4$, $\mu_r = -1$ and conventional dielectric with $\varepsilon_r = 4$ and $\mu_r = 1$. Due to the symmetry only angles from 0 to 90 degrees are shown. Figure 6 shows the forward-scattering cross section (FSCS) of the same structure in Figure 5. As these figures show the sensitivity of SCS to center separation for metamaterial objects are much larger than dielectric ones. The figures shows that SCSs for metamaterial object are generally larger than dielectric objects.

Figure 7 shows back-scattering cross-section of the same structure of Figures 5 and 6 for conducting cylinders with four different coating materials, metamaterial ($\varepsilon_r = -4$, $\mu_r = -1$) and ($\varepsilon_r = -2.2$, $\mu_r = -1$) and conventional dielectric ($\varepsilon_r = 4$, $\mu_r = 1$) and ($\varepsilon_r = 2.2$, $\mu_r = 1$). The scattering cross section for metamaterials and dielectric are slightly different from each other and the sensitivity to ε_r for the two cases is also not significantly different. This shows that for this case conducting cylinder has a significant effect on the scattering cross-section regardless of coating materials.

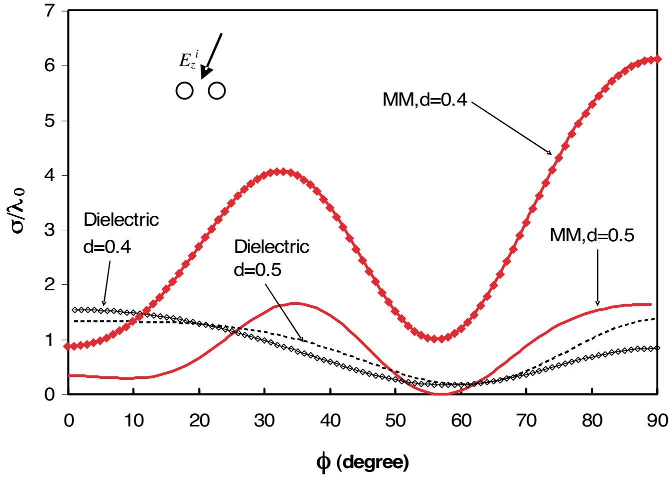

Figure 5. Back-scattering cross section versus incidence angle. The configuration includes two cylinders both with $0.1\,\lambda_0$ radius, located on the x-axis with two different center separation $0.4\,\lambda_0$ and $0.5\,\lambda_0$ and for two cases metamaterial with $\varepsilon_r = -4$, $\mu_r = -1$ and conventional dielectric with $\varepsilon_r = 4$ and $\mu_r = 1$.

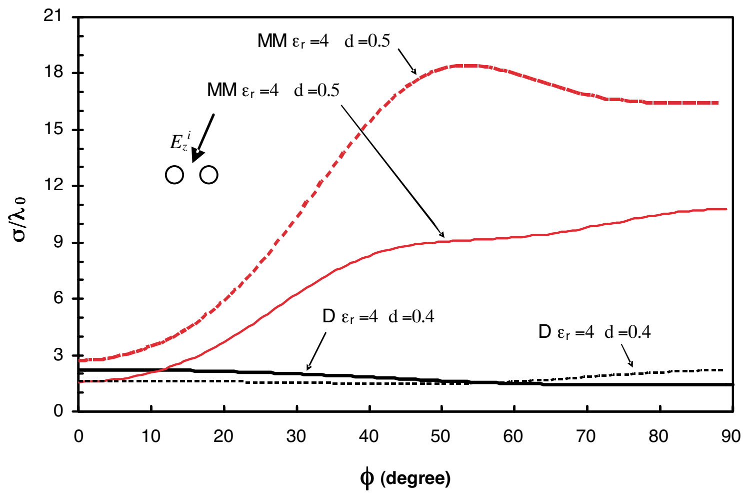

Figure 6. Forward-scattering cross sections (FSCS) versus incidence angle. (Same configuration as Fig. 5).

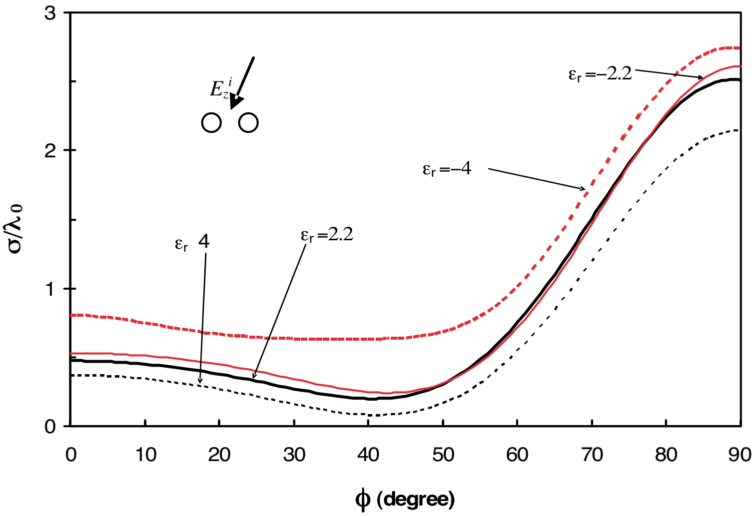

Figure 7. BSCS of two conducting cylinders located on x-axis with center separation $0.4\,\lambda_0$ and a=$0.05\,\lambda_0$ and $b = 0.1\,\lambda_0$.

Figure 8 and 9 shows the near field of total electric fields (scattering and incident fields) for two structure a single and two metamaterial cylinders with $\varepsilon_r = -4$, $\mu_r = -1$ respectively. The cylinders are located at $x = 0$ and $x = 3\,\lambda_0$. The radius of cylinders is $1\,\lambda_0$ and incidence angle is 90°. From Figures 8 and 9, mutual interactions are observed specially in middle of two cylinders. Figure 10 shows the scattering cross section of these two structure. Far fields and near fields shows the same behavior such as multiple null angles. Figure 10 also shows SCS for three metamaterial cylinders. Strong forward SCS in Figure 10 is observed specially for three cylinders. We expect that FSCS increases with number of cylinders in figure 10. The side lobes in Figure 10 may reduced by controlling the center separation. Therefore an array of metamaterial in front of an antenna may increase directivity. More study is required to design an optimized of this kind of array to improve directivity.

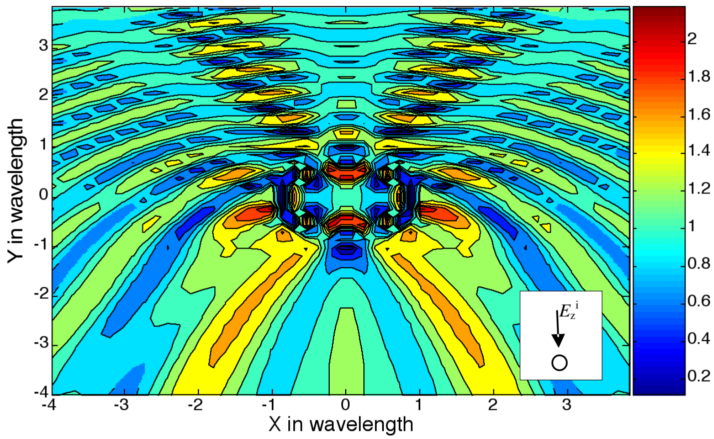

Figure 8. Total electric field of one metamaterial cylinder located on x-axis $x=0$ with radius $a = 1\,\lambda_0$, metamaterial ($\varepsilon_r = -4$, $\mu_r = -1$). The angle of incidence is 90 degrees as the small figure in right hand shows.

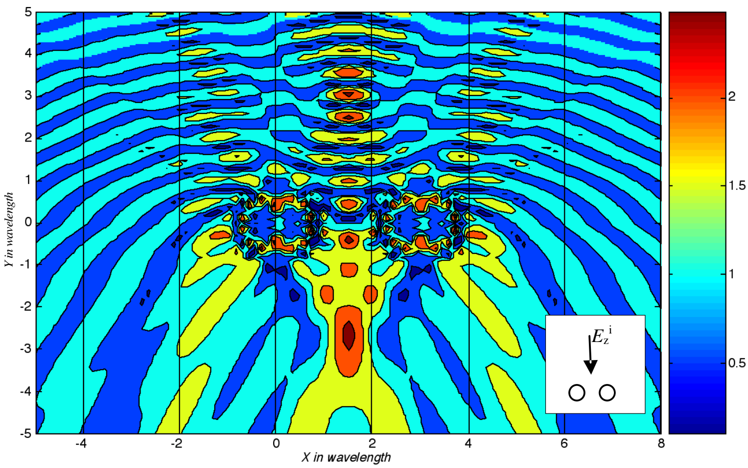

Figure 9. Total electric field of two dielectric cylinders located on x-axis $x = 0, 3\,\lambda_0$ and $a = 1\,\lambda_0$ radius, metamaterial ($\varepsilon_r = -4$, $\mu_r = -1$). The angle of incidence is 90 degrees as the small figure in right hand shows.

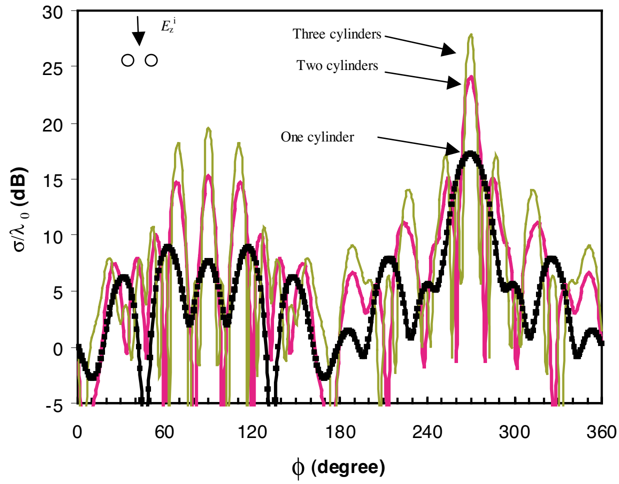

Figure 10. Scattering cross section of three different number: one, two, three metamaterial cylinders located on x-axis with $3\,\lambda_0$ center separation and $a = 1\,\lambda_0$ radius, metamaterial ($\varepsilon_r = -4$, $\mu_r = -1$). The angle of incidence is 90 degrees.

4. CONCLUSION

A formulation for multiple scattering by dielectric or metamaterial conducting cylinders were presented. Far fields and near fields behavior of metamaterial and dielectric cylinders were studied. Back and forward scattering cross section for metamaterial cylinders generally larger than dielectrics ones while BSCS and FSCS for coated conducting cylinders for metamaterial generally lower than dielectric ones. Near field patterns show mutual interaction between two cylinders and the shadows in near fields indicates null in far field patterns. The numerical results also show that SCS for metamaterial objects are more sensitive to physical parameter comparing to dielectric cases. We have shown that an array of metamaterial cylinders increases FSCS which may use to increase the directivity of antennas.

REFERENCES

1. *IEEE Transaction on Antennas and Propagation Special Issue on Metamaterials*, Vol. 51, No. 10, Part I, Oct. 2003.
2. Ishimaru, A., S. W. Lee, Y. Kuga, and V. Jandhyala, "Generalized constitutive relations for metamaterials based on the quasi-static

Lorenz theory," *IEEE Transaction on Antennas and Propagation*, Vol. 51, No. 10, Part I, 2550–2557, Oct. 2003.

3. Shelby, R. A., D. R. Smith, S. C. Nemat-Nasser, and S. Schultz, "Microwave transmission through a two dimensional isotropic, left-handed metamaterial," *App. Phys. Lett.*, Vol. 78, No. 4, 489–491, Jan. 2001.
4. Pendry, J. B., A. J. Holden, W. J. Stewart, and I. Youngs, "Extremely low frequency plasmons in metallic mesostructure," *Phys. Rev. Lett.*, Vol. 76, No. 4, 4773–4776, June 1996.
5. Shelby, R. A., D. R. Smith, and S. Schultz, "Experimental verification of a negative index of refraction," *Science*, Vol. 292, No. 5514, 77–79, Apr. 2001.
6. Elsherbeni, A. Z. and A. A. Kishk, "Modeling of cylindrical objects by circular dielectric and conducting cylinders," *IEEE Trans. on Antennas and Propagation*, Vol. 40, No. 1, 96–99, Jan. 1992.
7. Li, C. and Z. Shen, "The electromagnetic scattering by a conducting cylinder coated with metamaterials," *Progress in Electromagnetic Research*, Vol. 42, 91–105, 2003.
8. Pastorini, M., M. Raffetto, and A. Randazzo, "Interactions between electromagnetic waves and elliptically-shaped metamaterials," *Applied Electromagnetic Laboratory*, No. 1.6.4, 1–14, 2004.
9. Sharkawy, M. A., A. Z. Elsherbeni, and S. F. Mahmoud, "Electromagnetic scattering from parallel chiral cylinders of circular cross-sections using an iterative procedure," *Progress in Electromagnetic Research*, Vol. 47, 87–110, 2004.