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Electromagnetic scattering from uniaxial anisotropic bispheres located in a Gaussian beam

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ABSTRACT

Based on the Generalized Lorenz–Mie Theory (GLMT) and generalized multiparticle Mie-solution (GMM), scattering of two interacting homogeneous uniaxial anisotropic spheres with parallel primary optical axes and arbitrary configuration from a Gaussian beam is investigated. By introducing the Fourier transformation, the electromagnetic fields in the uniaxial anisotropic spheres are expanded in terms of the spherical vector wave functions (SVWFs). The interactive scattering coefficients and the expansion coefficients of the internal fields are derived through the continuous boundary conditions on which the interaction of the bispheres is considered. The influences of the beam waist widths, beam center position, and spheres separation distance on scattering characteristics of uniaxial anisotropic bispheres located in a Gaussian beam are numerically analyzed in detail.

1. Introduction

Due to recent advances in materials science and technology, and the enhancement and reduction of radar cross section (RCS) of various scatterers, the interaction between EM wave and anisotropic media is growning. Many scholars have investigated the scattering of a single anisotropic object by using analytical methods. Using adifferential theory, Stout et al [1] gave the solution of the scattering of an arbitrary-shaped body made of arbitrary anisotropic medium, but he did not give any numerical results. Wong and Chen [2], and Qiu [3] also devoted their endeavors to the scattering of a uniaxial anisotropic dielectric sphere through the expansion in terms of scalar eigen-functions method. By introducing the Fourier transformation method, Geng [4] and Wu [5] studied the scattering of a uniaxial anisotropic sphere illuminated by a plane wave and Gaussian beam, respectively. However, the scatterer in all these studies is limited to a single anisotropic object. The published works on the scattering from multiple anisotropic objects are exiguous.

Since the first presentation of a comprehensive solution of scattering by a two-sphere chain by Bruning and Lo [6], the study of scattering of multiple isotropic spheres has been developed quickly. Fuller and Kattawar [7] introduced the order-of-scattering technique to obtain the consummate solution of electromagnetic (EM) scattering by a cluster of spheres. The T-Matrix approach is also a very effective method and has been applied to the study of this problem by many researchers [8,9]. Xu introduced the generalized multiparticle Mie-solution (GMM) to explore EM scattering by an aggregate of isotropic spheres [10–12]. Recently, the present authors have studied the EM scattering by uniaxial bispheres [13] and multiple uniaxial anisotropic spheres [14]. However, in most of the articles that we have referred, the investigations focus on a plane wave scattering by multiple isotropic spheres.

With the advent of lasers and their growing use in the fields of particle sizing, biomedicine, laser fusion, optical levitation, and so on, the scattering problem from shaped beams has drawn considerable attention. Gouesbet et al. [15] derived three methods to calculate the beam shape coefficient (BSC) based on several relatively simple

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approximate models of Gaussian beam introduced by Davis [16] and put forward the Generalized Lorenz–Mie Theory (GLMT) to solve scattering of spherical particles illuminated by Gaussian beam [17]. Utilizing the addition theorem of the SVWFs, Doicu presented the BSC through different methods [18]. Subsequently, many authors have further studied various cases on particle scattering to a Gaussian beam [19]. But most of the above references considered only a single object scattering to a Gaussian beam. In 1999, Gouesbet and Grehan studied the scattering of an assembly of spherical particles located in an arbitrary electromagnetic shaped beam, but the numerical analysis are not given [20].

In this paper, based on GLMT and GMM, we investigate the scattering of two interacting homogeneous uniaxial anisotropic spheres with parallel primary optical axes along the axis of propagation of a Gaussian beam. The effects of the beam waist widths, spheres separation distance, and beam center positioning on the angular distributions of the radar cross section (RCS) are numerically analyzed in detail.

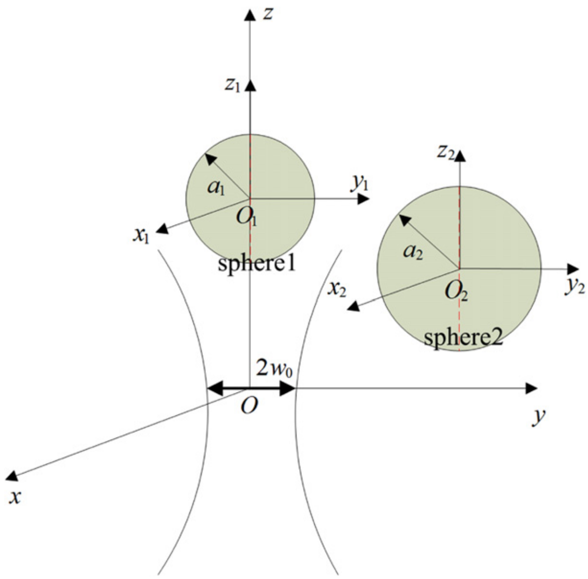

Fig. 1. Geometry for uniaxial anisotropic bispheres illuminated by a Gaussian beam.

2. Formulation

Considering two uniaxial anisotropic spheres with radius a_j $(j=1,2)$ and the primary optical axes coincident with the z-axis in a global coordinate system $Oxyz$, as shown in Fig. 1, the centers of the spheres are located at $(x_1,\ y_1,\ z_1)$ and $(x_2,\ y_2,\ z_2)$. The dashed lines indicate primary optical axes of the uniaxial anisotropic bispheres. The particles are illuminated by a z-propagating and x-polarized Gaussian beam.

In terms of spherical vector wave functions (SVWFs), the incident fields can be expanded in the particle coordinate system $O_jx_jy_jz_j$ as

$$\mathbf{E}_j^i=\sum_{n=1}^{\infty}\sum_{m=-n}^{n}E_{mn}[a_{jmn}^i\mathbf{M}_{mn}^{(1)}(\mathbf{r},k_0)+b_{jmn}^i\mathbf{N}_{mn}^{(1)}(\mathbf{r},k_0)]$$
$$\mathbf{H}_j^i=\frac{k_0}{i\omega\mu_0}\sum_{n=1}^{\infty}\sum_{m=-n}^{n}E_{mn}[a_{jmn}^i\mathbf{N}_{mn}^{(1)}(\mathbf{r},k_0)+b_{jmn}^i\mathbf{M}_{mn}^{(1)}(\mathbf{r},k_0)] \tag{1}$$

where $E_{mn}=E_0i^n\{(2n+1)(n-m)!/[n(n+1)(n+m)!]\}^{1/2}$, and E_0 is the amplitude of electric field at the beam waist center. The expansion coefficients can be derived by introducing the localized approximation of the BSC [18]

$$\begin{bmatrix} a_{jmn}^i \\ b_{jmn}^i \end{bmatrix}=C_{mn}(-1)^{m-1}K_{nm}\overline{\psi}_0^0e^{-ik_0z_j}\frac{1}{2}\left[e^{i(m-1)\varphi_j}J_{m-1}\left(2\frac{\overline{Q}\rho_j\rho_n}{w_0^2}\right)\right.$$
$$\left.\mp e^{i(m+1)\varphi_j}J_{m+1}\left(2\frac{\overline{Q}\rho_j\rho_n}{w_0^2}\right)\right]/E_{mn} \tag{2}$$

where

$$\psi_j=i\overline{Q}_j\exp\left(-i\overline{Q}_j\rho_j^2/w_0^2\right)\exp\left(-i\overline{Q}_j(n+0.5)^2/k_0^2w_0^2\right),$$
$$\overline{Q}_j=(i-2z_j/(k_0w_0^2))^{-1}\rho_j=\sqrt{x_j^2+y_j^2},\ \rho_n=(n+0.5)/k_0,$$
$$\varphi_j=\arctan\left(x_j/y_j\right),\ k_0=2\pi/\lambda \tag{3}$$

$$K_{nm}=\begin{cases}(-i)^{|m|}\dfrac{i}{(n+0.5)^{|m|-1}}, & m\neq 0\\[2mm] \dfrac{n(n+1)}{n+0.5}, & m=0\end{cases} \tag{4}$$

In Eqs. (1)–(4), $j=1$, 2 denotes the correlative parameter of jth sphere.

The scattered fields of the jth $(j=1,\ 2)$ uniaxial anisotropic sphere can also be expanded in terms of SVWFs in the jth sphere coordinate system $O_jx_jy_jz_j$ as

$$\mathbf{E}_j^s=\sum_{n=1}^{\infty}\sum_{m=-n}^{n}E_{mn}[a_{jmn}^s\mathbf{M}_{mn}^{(3)}(\mathbf{r}_j,k_0)+b_{jmn}^s\mathbf{N}_{mn}^{(3)}(\mathbf{r}_j,k_0)]$$
$$\mathbf{H}_j^s=\frac{k_0}{i\omega\mu_0}\sum_{n=1}^{\infty}\sum_{m=-n}^{n}E_{mn}[a_{jmn}^s\mathbf{N}_{mn}^{(3)}(\mathbf{r}_j,k_0)+b_{jmn}^s\mathbf{M}_{mn}^{(3)}(\mathbf{r}_j,k_0)] \tag{5}$$

The uniaxial anisotropic medium is characterized by a permittivity and permeability with tensor $\overline{\varepsilon}_j$ and $\overline{\mu}_j(j=1,\ 2)$, respectively, which can be expressed as:

$$\overline{\varepsilon}_j=\varepsilon_0\begin{bmatrix}\varepsilon_{jt} & 0 & 0\\ 0 & \varepsilon_{jt} & 0\\ 0 & 0 & \varepsilon_{jz}\end{bmatrix},\ \overline{\mu}_j=\mu_0\begin{bmatrix}\mu_{jt} & 0 & 0\\ 0 & \mu_{jt} & 0\\ 0 & 0 & \mu_{jz}\end{bmatrix} \tag{6}$$

The EM fields in the jth uniaxial anisotropic sphere can be expanded in terms of the SVWFs as [13]

$$\mathbf{E}_j^I(\mathbf{r}_j)=\sum_{q=1}^{2}\sum_{n=1}^{\infty}\sum_{m=-n}^{n}\sum_{n'=1}^{\infty}2\pi G_{jmn'q}\int_0^{\pi}[A_{jmnq}^e\mathbf{M}_{mn}^{(1)}(\mathbf{r}_j,k_{jq})$$
$$+B_{jmnq}^e\mathbf{N}_{mn}^{(1)}(\mathbf{r}_j,k_{jq})+C_{jmnq}^e\mathbf{L}_{mn}^{(1)}(\mathbf{r}_j,k_{jq})]P_{n'}^m(\cos\theta_{k_j})k_{jq}^2\sin\theta_{k_j}d\theta_{k_j}$$
$$\mathbf{H}_j^I(\mathbf{r}_j)=\sum_{q=1}^{2}\sum_{n=1}^{\infty}\sum_{m=-n}^{n}\sum_{n'=1}^{\infty}2\pi G_{jmn'q}\int_0^{\pi}[A_{jmnq}^h\mathbf{M}_{mn}^{(1)}(\mathbf{r}_j,k_{jq})$$
$$+B_{jmnq}^h\mathbf{N}_{mn}^{(1)}(\mathbf{r}_j,k_{jq})+C_{jmnq}^h\mathbf{L}_{mn}^{(1)}(\mathbf{r}_j,k_{jq})]P_{n'}^m(\cos\theta_{k_j})k_{jq}^2\sin\theta_{k_j}d\theta_{k_j} \tag{7}$$

where $A^e_{jmnq}, B^e_{jmnq}, C^e_{jmnq}, A^h_{jmnq}, B^h_{jmnq}$ and C^h_{jmnq} are the expansion coefficients and their expressions can be found in [3], but the expressions should be coincident with the corresponding permittivity tensor $\bar{\varepsilon}_j$ and permeability $\bar{\mu}_j$ for each uniaxial anisotropic sphere. $G_{jmn'q}$ is the unknown expansion coefficient relative to the internal fields and is determined by the boundary conditions of the jth sphere.

On the spherical boundary at $r_j = a_j$, the tangential components (designated by the subscript t) of the EM fields continue as:

$$\mathbf{E}^I_j|_t = \mathbf{E}^{it}_j|_t + \mathbf{E}^s_j|_t, \quad \mathbf{H}^I_j|_t = \mathbf{H}^{it}_j|_t + \mathbf{H}^s_j|_t \quad (r_j = a_j) \tag{8}$$

where $\mathbf{E}^{it}_j$ and $\mathbf{H}^{it}_j$ represent the total EM fields incident on the jth($j=1, 2$) uniaxial anisotropic sphere. The total incident field can be derived applying the addition theorem of the SVWFs, then substituting the expressions of the incident, internal and scattered fields into Eq. (8), we can derive the scattering coefficients as [13]:

$$a^s_{jmn} = \frac{1}{h_n^{(1)}(k_0a_j)} \left[\frac{1}{E_{mn}}\sum_{q=1}^{2}\sum_{n'=1}^{\infty} 2\pi G_{jmn'q}\int_0^\pi A^e_{jmnq} j_n(k_{jq}r_j) P^m_{n'}(\cos\theta_{jk}) k^2_{jq}\sin\theta_{jk} d\theta_{jk} - f^{it}_{jmn} j_n(k_0a_j)\right] \tag{9}$$

$$b^s_{jmn} = \frac{1}{h_n^{(1)}(k_0a_j)}\frac{i\omega\mu_0}{k_0} \left[\frac{1}{E_{mn}}\sum_{q=1}^{2}\sum_{n'=1}^{\infty} 2\pi G_{jmn'q}\int_0^\pi A^h_{jmnq} j_n(k_q r_j) P^m_{n'}(\cos\theta_{jk}) k^2_{jq}\sin\theta_{jk} d\theta_{jk} - g^{it}_{jmn} j_n(k_0a_j)\right] \tag{10}$$

where f^{it}_{jmn} and g^{it}_{jmn} are the expansion coefficients of the total incident field illuminating the jth ($j=1, 2$) uniaxial anisotropic bispheres, they can be derived by applying the addition theorem of the SVWFs. [13].

Based on the scattering coefficients above, the total scattered fields which are the vector superposition of the scattered field of every uniaxial anisotropic sphere can be derived by applying the addition theorem of the SVWFs again. Then according to the definition of the RCS, we can calculate the angular distribution of the RCS of the uniaxial anisotropic bispheres illuminated by a Gaussian beam.

3. Numerical results and discussion

The effect of the beam waist width on the RCS of uniaxial anisotropic bispheres with both electric and magnetic anisotropy in the E- and H- planes is shown by

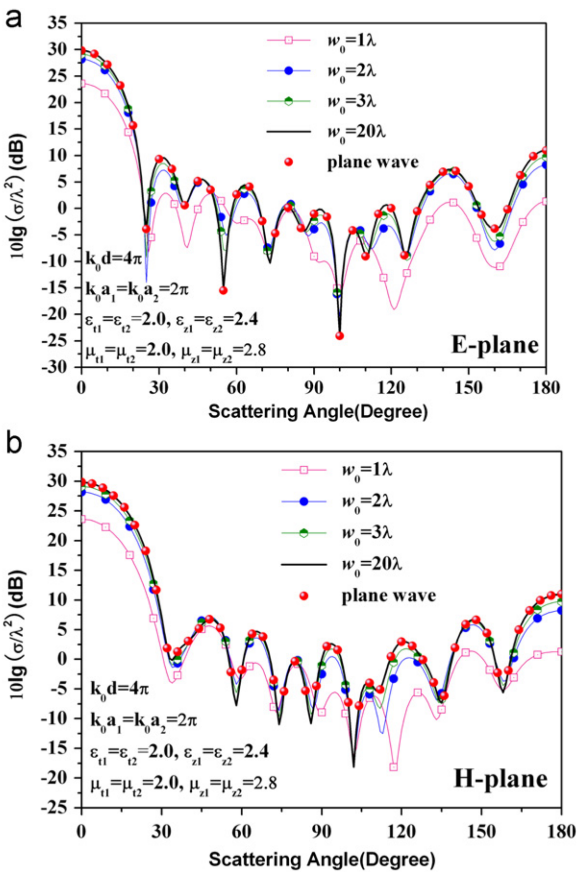

Fig. 2. Angular distribution of the RCS of two uniaxial anisotropic spheres placed along z-axis illuminated by a Gaussian beam with different beam waist widths.

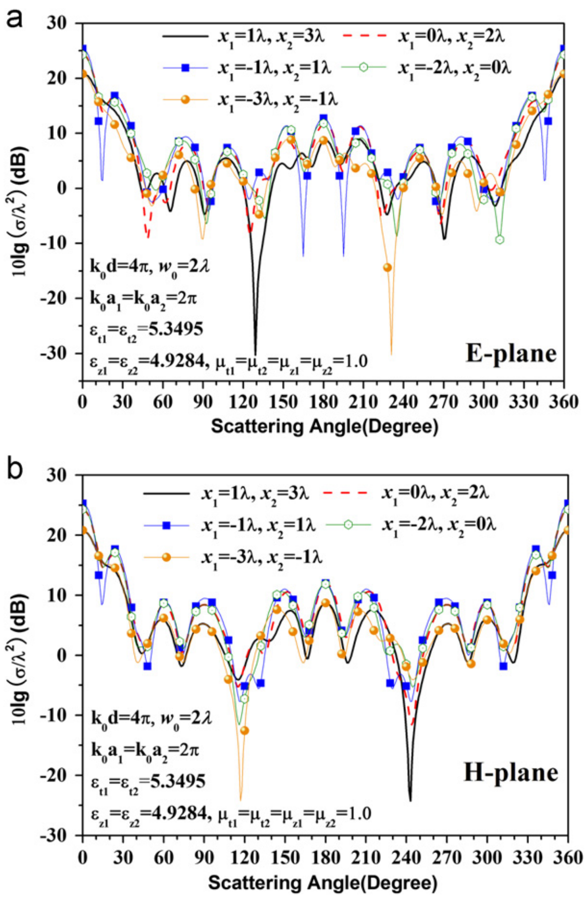

Fig. 3. Effect of beam center position on the RCS of two uniaxial anisotropic spheres placed along x-axis.

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Fig. 2(a) and (b), respectively. It can be found that the RCS for Gaussian beams is smaller than that for a plane wave because of the influence of the beam shape coefficients. As the beam waist width increases, the RCS approaches that for plane wave incidence, but the angular distribution has a little subtle change. As expected, our results in the case of Gaussian beam incidence with a relatively large waist radius of $w_0=20\lambda$ are in excellent agreement with the results in the case of a plane wave incidence provided in [13]. This agreement can verify the validity and correctness of our theory and codes. Note that the *E*-plane and *H*-plane indicate the *xOz* plane and *yOz* plane, respectively.

Fig. 3 shows the angular distributions of the RCS of closely packed uniaxial anisotropic bispheres placed along *x*-axis with different separation distances from the beam center. The other coordinates of the sphere centers are $y_1=z_1=0$, $y_2=z_2=0$. As bispheres offset from the beam-center along *x*-axis,the RCS decreases because the beam waist illuminate on the objects finitely. And the offset symmetrical axes of the Gaussian beam and the bispheres also cause the angular distributions of the RCS to be asymmetrical. But the angular distributions of the RCS in 0–180° when the offset of the bispheres is along the positive *x*-axis from the beam center are the same those in 180–360° when the offset of the bispheres is along the negative *x*-axis from the beam center. The distribution of amplitude is also the same.

As in Fig. 3, Fig. 4 shows that, the angular distributions of the RCS of closely packed uniaxial anisotropic bispheres placed along *z*-axis with different separation distances from the beam center. The other coordinates of the sphere centers are $x_1=y_1=0$, $x_2=y_2=0$. As the bispheres move away from the beam center along *z*-axis, the RCS decreases integrally but the form of the curve exhibits little change. In Fig. 3 the angular distribution of RCS show obvious difference when bispheres move away from the beam center along *x*-axis. This may be attributed to the fact that the incident beam also propagates along *z*-axis. Moreover, the angular distributions of the RCS when the bias of the bispheres is along the positive *z*-axis from the beam center are the same with those when the bias of the bispheres is along the negative *z*-axis from the beam center and the bias is the same. This property can be observed from the BSC in Eq. (2).

TiO_2 characterized by $\varepsilon_t=5.913$ and $\varepsilon_z=7.197$ is a typical uniaxial anisotropic medium. In Fig. 5, the angular distributions of the RCS of uniaxial anisotropic bispheres placed along *x*-axis with different spheres separation distances illuminated by an off-axis Gaussian are shown. The other coordinates of the sphere centers are $y_1=z_1=0$,

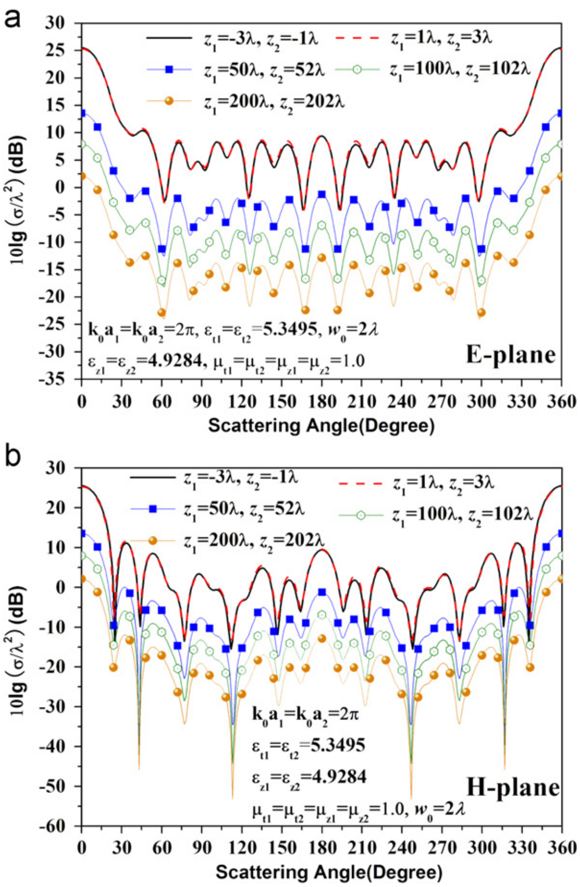

Fig. 4. Effect of beam center position on the RCS of two uniaxial anisotropic spheres placed along *z*-axis.

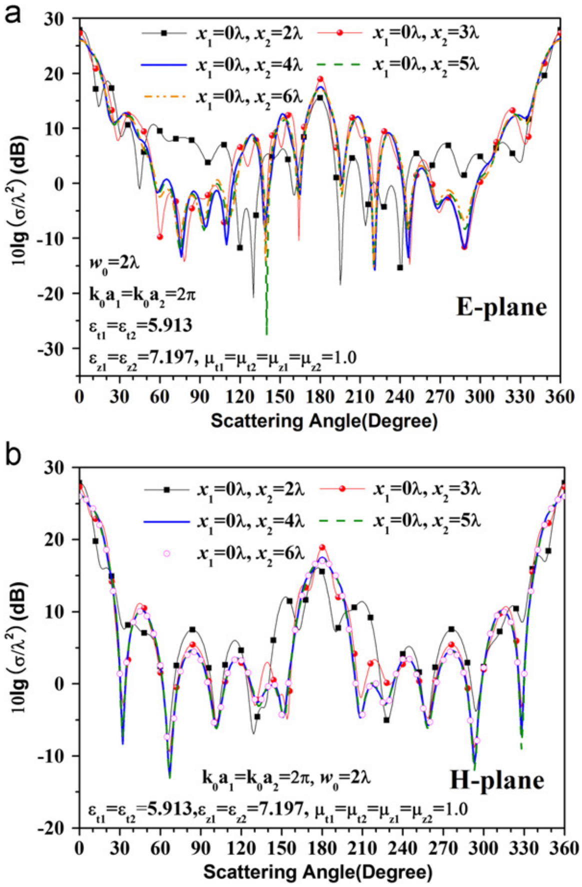

Fig. 5. Effect of the spheres separation distance on the RCS of two uniaxial anisotropic spheres placed along *x*-axis.

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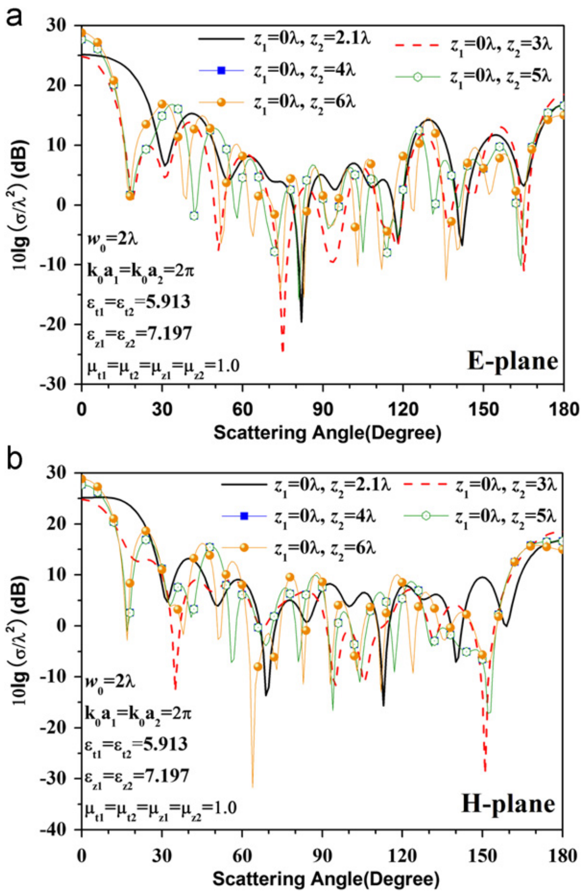

Fig. 6. Effect of the spheres separation distance on the RCS of two uniaxial anisotropic spheres placed along z-axis.

$y_2 = z_2 = 0$. Because the second sphere moves away from the first sphere along positive x-axis, the RCS in 180–360° in E-plane oscillates more drastically with the increase of the spheres separation distance. The effect of the spheres separation distance on the RCS in E-plane is more sensitive than that in H-plane due to the fact that the uniaxial anisotropic bispheres are placed along x-axis. Moreover, it can be found that as the spheres separation distance increases, the angular distribution of the RCS tends to be consistent. It is obvious that the curves of the RCS in H-plane are almost the same for $x_2 = 5\lambda$ and 6λ. This is because the beam waist width is finite and the Gaussian beam can be incident on only some part of the bispheres, which is different from plane wave incidence.

The effect of the spheres separation distance on the RCS of uniaxial anisotropic bispheres placed along z-axis is shown in Fig. 6. The other parameters are the same as inFig. 5. The other coordinates of the sphere centers are $x_1 = y_1 = 0$, $x_2 = y_2 = 0$. It can be observed that the effect of the separation distance on the RCS when the bispheres placed along z-axis is distinct from that when the bispheres are placed along x-axis. The angular distributions of the RCS of the bispheres with larger spheres separation distance oscillate sharper than that of bispheres with smaller spheres separation distance, resulting from the fact that both the interaction scattering and the interference scattering act.

4. Conclusion

In summary, an analytical solution of the scattering of a Gaussian beam by homogeneous uniaxial anisotropic bispheres with parallel primary optical axes and arbitrary configuration is derived. The accuracy of the theory is verified by comparing the numerical results reduced to some special cases of a plane wave incidence given by existing references. The effects of the beam waist widths, beam center position, and spheres separation distance on the RCS are numerically analyzed. Due to the length restriction, the scattering characteristics of the uniaxial anisotropic bispheres placed along y-axis are not calculated, but the characteristics may be similar to those of the uniaxial anisotropic bispheres placed along y-axis due to the symmetry. The scattering of an aggregate of uniaxial anisotropic spheres illuminated by an arbitrary direction Gaussian beam may be investigated in our future work.

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