Electromagnetic Scintillation

I. Geometrical Optics

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CAMBRIDGE
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE
OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York, NY 10011-4211, USA
10 Stamford Road, Oakleigh, VIC 3166, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa

http://www.cambridge.org

First published 2001

Printed in the United Kingdom at the University Press, Cambridge

Typeface CMR 10/12 *System* LaTeX [HBA]

A catalogue record for this book is available from the British Library

Library of Congress Cataloguing in Publication data

Wheelon, Albert D. (Albert Dewell), 1929–
Electromagnetic scintillation/Albert D. Wheelon.
p. cm.
Includes bibliographical references and index.
Contents: Vol. 1. Geometrical optics.
ISBN 0 521 80198 2
1. Electromagnetic waves–Transmission. I. Title.

QC665.T7.W48 2001
539.2–dc21 00-065167

ISBN 0 521 80198 2 hardback

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1

Introduction

The laws of geometrical optics were known from experiments long before the electromagnetic theory of light was established [1]. Today we recognize that they constitute an approximate solution for Maxwell's field equations. This solution describes the propagation of light and radio waves in media that change gradually with position [2]. The wavelength is taken to be zero in this approximation and diffraction effects are completely ignored. The field is represented by signals that travel along ray paths connecting the transmitter and receiver. In most applications these rays can be approximated by straight lines. These trajectories are uniquely determined by the dielectric constant of the medium and by the antenna pattern of the transmitter. In this approach energy flows along these ray paths and the signal acts locally like a plane wave. Geometrical optics provides a convenient description for a wide class of propagation problems when certain conditions are met.

The assumption that the medium changes gradually means that geometrical optics cannot describe the scattering by objects of dimensions comparable to a wavelength. Similarly, it cannot describe the boundary region of the shadows cast by sharp edges. A further condition is that rays launched by the transmitter must not converge too sharply – as they do for focused beams. These conditions must be refined when ray theory is used to describe propagation in random media.

Geometrical optics is widely used to describe electromagnetic propagation in the nominal atmosphere of the earth, other planets and the interstellar medium. Refractive bending of starlight and microwave signals in the troposphere is accurately described by this approximation. Standard atmospheric-profile models are used to calculate ray paths, radio horizons and angles of arrival for various elevation angles and surface conditions [3]. Ray theory is also the primary tool for describing the reflection of radio signals in the ionosphere [4]. The maximum usable frequency can be estimated for shortwave broadcast and communication services if the electron-density profile of the ionosphere is known from vertical sounding measurements or modeling. The same techniques are used to describe the transmission of acoustic waves in the ocean.

It was initially thought that geometrical optics would provide a valid description for propagation through random media and early studies all relied on this approach [5][6][7][8]. The concept assumed that the signal fluctuations are induced by small dielectric variations located close to the nominal ray trajectory. It was hoped that perturbation solutions of the ray equations would yield valid expressions for phase and amplitude variations. Only the first half of that expectation was realized.

Geometrical optics provides a good description for the phase fluctuations imposed by random media. These are caused by the random *speeding up and slowing down* of the signal as it travels along the nominal ray trajectory. Phase fluctuations computed in this way agree with experiment, even when the path is long and the fluctuations are large. For line-of-sight propagation the predicted phase variance is proportional to the path length and the first moment of the spectrum of turbulent irregularities. This means that phase fluctuations depend primarily on the largest eddies and diffraction effects can be ignored.

Geometrical optics also describes angle-of-arrival fluctuations over a wide range of propagation conditions. Angular errors at the receiver are the result of many small *random refractive bendings* along the ray path. This sets the threshold for astronomical seeing with ground-based telescopes. The angular variance is proportional to distance traveled and to the third moment of the spectrum. Angular errors depend primarily on small eddies. As a practical matter, aperture smoothing suppresses the contributions of eddies smaller than the receiver and such measurements depend primarily on the inertial range of the turbulent spectrum. Again, diffraction effects are relatively unimportant.

By contrast, this method cannot describe amplitude and intensity fluctuations in most situations of practical interest. These scintillations are due to the random *bunching and diverging* of energy-bearing rays in this approximation. The resulting expression for the logarithmic variance of the amplitude is proportional to the third power of distance and the fifth moment of the spectrum. Intensity scintillation therefore depends primarily on the smallest eddies for which diffraction effects play a dominant role. To use this approximation the influential eddies must be larger than the Fresnel length. That condition is seldom met and one cannot use this method to define scintillation levels – unless large receivers and/or transmitters are employed. The geometrical optics description of amplitude fluctuations is primarily of historical interest and one is referred to standard texts for expressions for the variance and correlation [9][10]. Amplitude and intensity fluctuations will be analyzed with diffraction theory in the next volume.

The goal of the second chapter is to describe random media. In the following chapter we adapt geometrical optics to describe propagation through random media and to establish the validity conditions for its application. The single-path phase variance is estimated in Chapter 4. In the following chapter we calculate the phase structure function as a function of the separation between receivers and compare it with results from phase-difference experiments. The temporal correlation of phase and the corresponding power spectrum are addressed in Chapter 6. In the next chapter we describe the angle-of-arrival errors induced by a random medium. We show that the random phase and phase difference are distributed as Gaussian random variables in Chapter 8. Moments of the electric field strength calculated with geometrical optics are presented in the last chapter. Problems are included at the end of each chapter to develop additional insights and to explore related topics. Helpful mathematical relations are summarized in the appendices.

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