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ELECTROMAGNETIC WAVE INTERACTION WITH STRATIFIED NEGATIVE ISOTROPIC MEDIA

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Abstract—In this paper we provide a general formulation for the electromagnetic wave interaction with stratified media and then specialize to slabs of negative isotropic media. The field solutions are obtained in all regions of the stratified medium. The characteristic waves in negative isotropic media are backward waves. We derive the field solutions and show that a Gaussian beam is laterally shifted by a negative isotropic slab. The amount of beam center shift is calculated for both cases of transmission and reflection. Guided waves in stratified media are studied. Placing a linear antenna and all types of Hertzian dipole antennas in a stratified medium, we obtained solutions in all regions. We demonstrate and locate the positions of the perfect images of the antenna sources in the presence of negative isotropic slabs.

1. INTRODUCTION

In the constitutive relations for negative isotropic media, both the permittivity ϵ and the permeability μ are negative. Vesolago [1] explored their various electromagnetic properties. He observed that in such media, the wave vector $\overline{k}$, the electric field vector $\overline{E}$, and the magnetic field vector $\overline{H}$ form a left-handed system, and called them left-handed subtances. The left-handed materials (LHM) possess negative refractive indeces. Experimental verifications were made with metamaterials and their use as perfect lenses was suggested [2–6]. In light of their potential new applications, negative isotropic media have received wide attention.

In this paper we provide a general formulation with complete solution for electromagnetic waves interacting with a stratified medium and then specialize to slabs of negative isotropic medium. After showing that the characteristic waves in negative isotropic meia are backward waves, we consider relection and transmission of TE and TM waves by a negative isotropic medium. The solutions for the reflection and transmission by a stratified medium are then derived and listed by using the propagation matrices [7]. Specializing to a slab of negative isotropic medium, we show how wave beam incident on the slab will be laterally shifted.

We then consider guided waves in negative isotropic materials in stratified media. Coupling of guided waves are studied. Next we derive field solutions for a linear antenna situated in a stratified medium. The images and their locations are calculated. Finally we consider Hertzian electric and magnetic dipole sources with different orientation inside stratified media. The complete solutions in all regions are derived and listed [8]. Specializing to slabs of negative isotropic medium, we show positions of its images for various configurations, suggesting that perfect images can be made using flat lenzes made of slab negative isotropic slabs.

2. BACKWARD WAVES IN NEGATIVE ISOTROPIC MEDIA

The constitutive relations for isotropic media have been written as

$$\overline{D} = \epsilon\overline{E} \qquad \text{where } \epsilon = \text{permittivity} \tag{1a}$$

$$\overline{B} = \mu\overline{H} \qquad \text{where } \mu = \text{permeability} \tag{1b}$$

For negative isotropic media, both ϵ and μ are negative. For a plane electromagnetic wave of the form

$$\begin{bmatrix} \overline{E}(\overline{r},t) \\ \overline{H}(\overline{r},t) \end{bmatrix} = \begin{bmatrix} \overline{E} \\ \overline{H} \end{bmatrix} \cos(\overline{k}\cdot\overline{r} - \omega t) \tag{2}$$

The Maxwell equations become

$$\overline{k} \times \overline{E} = \omega\mu\overline{H} \tag{3}$$

$$\overline{k} \times \overline{H} = -\omega\epsilon\overline{E} \tag{4}$$

$$\overline{k} \cdot \overline{E} = 0 \tag{5}$$

$$\overline{k} \cdot \overline{H} = 0 \tag{6}$$

where $k = \omega\sqrt{\mu\epsilon}$.

The Poynting's vector power density is

$$\overline{E} \times \overline{H} = \frac{1}{\omega^2\mu\epsilon}(\overline{k}\times\overline{E}) \times (\overline{k}\times\overline{H}) = \begin{cases} \frac{-1}{\omega\mu}\,(\overline{k}\times\overline{E})\times\overline{E} = \frac{\overline{k}}{\omega\mu}|\overline{E}|^2 \\ \frac{1}{\omega\epsilon}\,\overline{H}\times(\overline{k}\times\overline{H}) = \frac{\overline{k}}{\omega\epsilon}|\overline{H}|^2 \end{cases} \tag{7}$$

When μ and ϵ ar both positive, Poynting's power vector is in the same direction as $\overline{k}$ and so are the group and phase velocities. From (3)–(6), we see that the three vectors $\overline{k}$, $\overline{E}$, and $\overline{H}$ form a right-handed system. When either μ or ϵ is negative, we have evanescent instead of propgating waves.

In negative isotropic media, both μ and ϵ are negative, Poynting's power vector is in the opposite direction of $\overline{k}$ and so are the group and phase velolcities. The three vectiors $\overline{k}$, $\overline{E}$, and $\overline{H}$ form a left-handed Medium (LHM). In this medium, the power propagates in a direction that is opposite to the direction of $\overline{k}$. The plane wave in the negative isotropic medium is thus a backward wave.

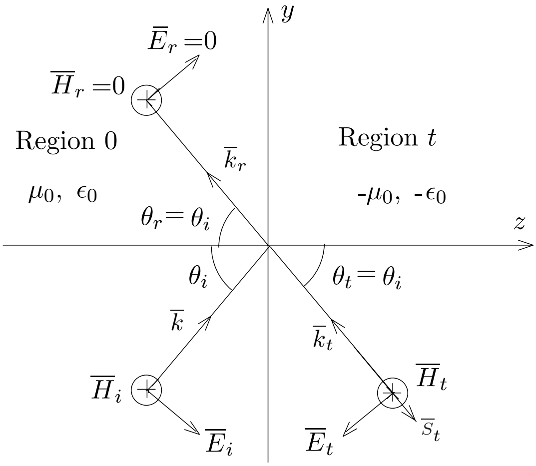

Figure 1. Reflection and transmission of TM waves by negative isotropic media.

3. REFLECTION AND TRANSMISSION BY A NEGATIVE ISOTROPIC MEDIUM

We first study the case of TM wave incidence from free space upon a negative isotropic medium with permittivity $\epsilon_t = -\epsilon_0$ and permeability $\mu_t = -\mu_0$. The incident magnetic field is assumed to have unit amplitude and points in the $\hat{x}$ direction (Fig. 1). We write, omitting the time convention $e^{-i\omega t}$,

$$\overline{H}_i = \hat{x} H_{ix} = \hat{x}\ e^{i(k_y y + k_z z)} \tag{8a}$$

$$\overline{E}_i = [-\hat{y} k_z + \hat{z} k_y] \frac{1}{\omega \epsilon_0} e^{i(k_y y + k_z z)} \tag{8b}$$

$$\overline{S}_i = \overline{E} \times \overline{H}^* = [\hat{y} k_y + \hat{z} k_z] \frac{1}{\omega \epsilon_0} e^{i(\overline{k}_i - \overline{k}_i^*)\cdot \overline{r}} \tag{8c}$$

The reflected field components for the incident TM wave are

$$\overline{H}_r = \hat{x}\ R^{TM} e^{(k_{ry} y + k_{rz} z)} \tag{9a}$$

$$\overline{E}_r = [-\hat{y} k_{rz} + \hat{z} k_{ry}] \frac{1}{\omega \epsilon_0} R^{TM} e^{(k_{ry} y + k_{rz} z)} \tag{9b}$$

$$\overline{S}_r = \overline{E} \times \overline{H}^* = [\hat{y} k_{ry} + \hat{z} k_{rz}] \frac{|R^{TM}|^2}{\omega \epsilon_0} e^{i(\overline{k}_r - \overline{k}_r^*)\cdot \overline{r}} \tag{9c}$$

The incident wave vector $\overline{k}_i = \hat{y} k_y + \hat{z} k_z$ and the reflected wave vector

$\overline{k}_r = \hat{y}k_{ry} + \hat{z}k_{rz}$ are governed by the dispersion relations

$$k_y^2 + k_z^2 = \omega^2\mu_0\epsilon_0 = k^2 \tag{10}$$

$$k_{ry}^2 + k_{rz}^2 = \omega^2\mu_0\epsilon_0 = k^2 \tag{11}$$

The reflection coefficient R^{TM} for the magnetic field component H_{ix} is to be determined by the boundary conditions.

In region t, the transmitted TM field components are

$$\overline{H}_t = \hat{x}\; T^{TM} e^{i(k_{tx}x + k_{tz}z)} \tag{12a}$$

$$\overline{E}_t = [-\hat{y}k_{tz} + \hat{z}k_{ty}]\frac{1}{\omega\epsilon_t} T^{TM} e^{i(k_{ty}y + k_{tz}z)} \tag{12b}$$

$$\overline{S}_t = [\hat{y}k_{ty} + \hat{z}k_{tz}]\frac{|T^{TM}|^2}{\omega\epsilon_t} e^{i(\overline{k}_t - \overline{k}_t^*)\cdot\overline{r}} \tag{12c}$$

where T^{TM} is the transmission coefficient for the magnetic field component H_{ix}. The dispersion relation

$$k_{ty}^2 + k_{tz}^2 = \omega^2\mu_t\epsilon_t = k_t^2 \tag{13}$$

governs the magnitude k_t for the transmitted wave vector $\overline{k}_t = \hat{y}k_{ty} + \hat{z}k_{tz}$.

Let the boundary surface be at $z = 0$. The boundary condition of continuity of tangential $\overline{H}$ field gives

$$e^{ik_y y} + R^{TM} e^{ik_{ry} y} = T^{TM} e^{k_{ty} y} \tag{14}$$

Since (14) must hold for all y and t, it follows that

$$k_y = k_{ry} = k_{ty} \tag{15}$$

Eq. (15) is known as the phase matching condition. Eq. (14) then reduces to

$$1 + R^{TM} = T^{TM} \tag{16}$$

From the dispersion relations (10) and (11) by noting that the reflected wave propagates in the negative $\hat{z}$ direction, we find $k_{rz} = -k_z$.

The continuity of the tangential components of E_x at $z = 0$ for all y and t gives

$$\frac{k_z}{\epsilon_0}(1 - R^{TM}) = \frac{k_{tz}}{\epsilon_t} T^{TM} \tag{17}$$

Note that the boundary conditions of normal $\overline{D}$ and normal $\overline{B}$ components continuous at $z = 0$ are also satisfied. Solving (16) and

(17), we find

$$p_{0t}^{TM} = \frac{\epsilon_0 k_{tz}}{\epsilon_t k_z} \tag{18}$$

$$R^{TM} = \frac{1 - p_{0t}^{TM}}{1 + p_{0t}^{TM}} \tag{19}$$

$$T^{TM} = \frac{2}{1 + p_{0t}^{TM}} \tag{20}$$

for the reflection coefficient R^{TM} and the transmission coefficientT^{TM}.

In the negative isotropic medium, we have $\epsilon_t = -\epsilon_0$ and $\mu_t = -\mu_0$. From Eq. (18), we find $p^{TM} = 1$, and from Eqs. (19) and (20), we determine that $R^{TM} = 0$ and $T^{TM} = 1$. From the dispersion relation (13), we find $k_{tz} = -k_z$. As seen from Fig. 1, the transmitted $\overline{k}_t$ vector is in the same direction as the reflected $\overline{k}_r$ vector.

The Poynting power vector density is, from (23c),

$$\overline{S}_t = [-\hat{y}k_y + \hat{z}k_z]\frac{|T^{TM}|^2}{\omega\epsilon_0}e^{i(\overline{k}_t - \overline{k}_t^*)\cdot\overline{r}} \tag{21}$$

such that the transmitted *power* is directed away from the boundary into the transmitted medium.

When the incident wave is an evanescent wave with $\overline{k}_i = \hat{y}k_y + \hat{z}ik_{Iz}$ and $k_y > k$, we have

$$\overline{H}_i = \hat{x}H_{ix} = \hat{x}\ e^{i(k_y y + k_z z)} \tag{22a}$$

$$\overline{E}_i = [-\hat{y}k_z + \hat{z}k_y]\frac{1}{\omega\epsilon_0}e^{i(k_y y + k_z z)} \tag{22b}$$

$$\overline{S}_i = \overline{E} \times \overline{H}^* = [\hat{y}k_y + \hat{z}k_z]\frac{1}{\omega\epsilon_0}e^{i(\overline{k}_i - \overline{k}_i^*)\cdot\overline{r}} \tag{22c}$$

$$\overline{H}_t = \hat{x}\ T^{TM}e^{i(k_{tx}x + k_{tz}z)} \tag{23a}$$

$$\overline{E}_t = [-\hat{y}k_{tz} + \hat{z}k_{ty}]\frac{1}{\omega\epsilon_t}T^{TM}e^{i(k_{ty}y + k_{tz}z)} \tag{23b}$$

$$\overline{S}_t = [\hat{y}k_{ty} + \hat{z}k_{tz}]\frac{|T^{TM}|^2}{\omega\epsilon_t}e^{i(\overline{k}_t - \overline{k}_t^*)\cdot\overline{r}} \tag{23c}$$

For an incident TE wave, the incident wave vector $\overline{k}_i = \hat{y}k_y + \hat{z}k_z$, the reflected wave vector $\overline{k}_r = \hat{y}k_{ry} + \hat{z}k_{rz} = \hat{y}k_y - \hat{z}k_z$, and the transmitted wave vector $\overline{k}_t = \hat{y}k_y + \hat{z}k_{tz}$ all satisfy the dispersion relations (10), (11), and (13), and the phase match condition (15). The incident electric and magnetic field vectors are

$$\overline{E}_i = \hat{x} e^{i\overline{k}_i \cdot \overline{r}} \tag{24a}$$

$$\overline{H}_i = [\hat{y} k_z - \hat{z} k_y] \frac{1}{\omega \mu_0} e^{i\overline{k}_i \cdot \overline{r}} \tag{24b}$$

$$\overline{S}_i = \overline{E} \times \overline{H}^* = [\hat{y} k_y + \hat{z} k_z^*] \frac{1}{\omega \mu_0} e^{i(\overline{k}_i - \overline{k}_i^*) \cdot \overline{r}} \tag{24c}$$

The reflected electric and magnetic field vectors are

$$\overline{E}_r = \hat{x} R^{TE} e^{i\overline{k}_r \cdot \overline{r}} \tag{25a}$$

$$\overline{H}_r = [-\hat{y} k_z - \hat{z} k_y] \frac{R^{TE}}{\omega \mu_0} e^{i\overline{k}_r \cdot \overline{r}} \tag{25b}$$

$$\overline{S}_r = \overline{E} \times \overline{H}^* = [\hat{y} k_y - \hat{z} k_z^*] \frac{|R^{TE}|^2}{\omega \mu_0} e^{i(\overline{k}_r - \overline{k}_r^*) \cdot \overline{r}} \tag{25c}$$

In region t, the transmitted TE wave solution takes the form

$$\overline{E}_t = \hat{x}\ T^{TE} e^{i\overline{k}_t \cdot \overline{r}} \tag{26a}$$

$$\overline{H}_t = [\hat{y} k_{tz} - \hat{z} k_y] \frac{T^{TE}}{\omega \mu_t} e^{i\overline{k}_t \cdot \overline{r}} \tag{26b}$$

$$\overline{S}_t = [\hat{y} k_y + \hat{z} k_{tz}^*] \frac{|T^{TE}|^2}{\omega \mu_t} e^{i(\overline{k}_t - \overline{k}_t^*) \cdot \overline{r}} \tag{26c}$$

where T^{TE} is the transmission coefficient. From the boundary conditions of continuity of tangential $\overline{E}$ and $\overline{H}$ fields, we find

$$p_{0t}^{TE} = \frac{\mu_0 k_{tz}}{\mu_t k_z} \tag{27}$$

$$R^{TE} = \frac{1 - p_{0t}^{TE}}{1 + p_{0t}^{TE}} \tag{28}$$

$$T^{TE} = \frac{2}{1 + p_{0t}^{TE}} \tag{29}$$

for the reflection coefficient R^{TE} and the transmission coefficientT^{TE}.

In the negative isotropic medium, $\mu_t = -\mu_0$, and $k_{tz} = -k_z$ such that the transmitted *power* is directed away from the boundary into the transmitted medium. From Eq. (27), we find $p^{TE} = 1$. Eqs. (28) and (29) then yield that $R^{TE} = 0$ and $T^{TE} = 1$.

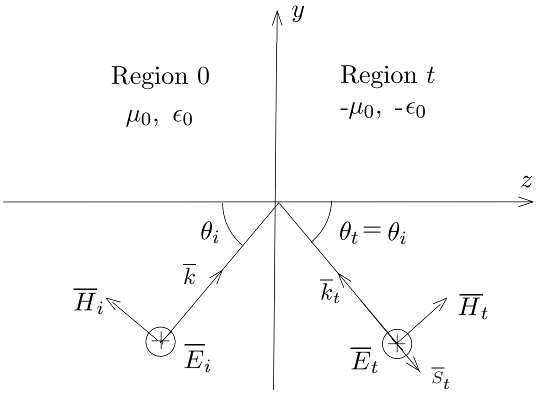

Figure 2. Reflection and transmission of TE waves by negative isotropic media.

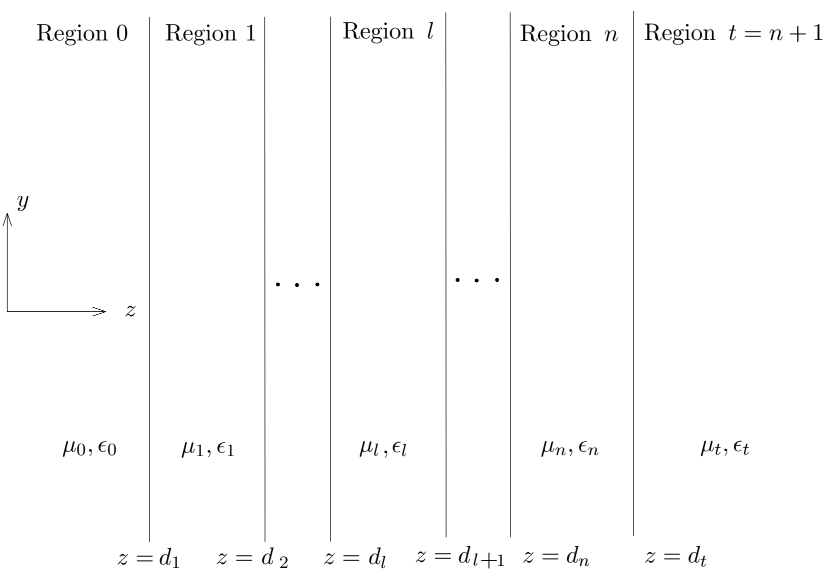

Figure 3. Stratified medium.

4. REFLECTION AND TRANSMISSION BY STRATIFIED MEDIA

Consider a plane wave incident on a stratified isotropic medium with boundaries at $z = d_1,\ d_2, \ldots, d_t$ (Fig. 3). The $(n+1)$th region is semi-infinite and is labeled region t, $t = n+1$. The permittivity and permeability in each region are denoted by ϵ_l and μ_l. The plane wave is incident from region 0 and has the plane of incidence parallel to the y-z plane. All field vectors are dependent on y and z only and independent of x. Since $\partial/\partial x = 0$, the Maxwell equations in any region l can be separated into TE and TM components governed by E_{lx} and H_{lx}. We obtain

$$H_{ly} = \frac{1}{i\omega\mu_l}\frac{\partial}{\partial z}E_{lx} \tag{30}$$

$$H_{lz} = \frac{-1}{i\omega\mu_l}\frac{\partial}{\partial y}E_{lx} \tag{31}$$

$$\Big(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \omega^2\mu_l\epsilon_l\Big)E_{lx} = 0 \tag{32}$$

$$E_{ly} = \frac{-1}{i\omega\epsilon_l}\frac{\partial}{\partial z}H_{lx} \tag{33}$$

$$E_{lz} = \frac{1}{i\omega\epsilon_l}\frac{\partial}{\partial y}H_{lx} \tag{34}$$

$$\Big(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \omega^2\mu_l\epsilon_l\Big)H_{lx} = 0 \tag{35}$$

The TE waves are completely determined by (30)–(32) and the TM waves by (33)–(35). The two sets of equations are duals of each other under the replacements $\overline{E}_l \to \overline{H}_l$, $\overline{H}_l \to -\overline{E}_l$, and $\mu_l \rightleftarrows \epsilon_l$.

For a TE plane wave, $E_x = E_0\, e^{ik_z z + ik_y y}$, incident on the stratified medium, the total field in region l can be written as

$$E_{lx} = \left(E_l^+ e^{ik_{lz}z} + E_l^- e^{-ik_{lz}z}\right) e^{ik_y y} \tag{36}$$

$$H_{ly} = \frac{k_{lz}}{\omega\mu_l}\left(E_l^+ e^{ik_{lz}z} - E_l^- e^{-ik_{lz}z}\right) e^{ik_y y} \tag{37}$$

$$H_{lz} = \frac{-k_y}{\omega\mu_l}\left(E_l^+ e^{ik_{lz}z} + E_l^- e^{-ik_{lz}z}\right) e^{ik_y y} \tag{38}$$

Obviously (36) satisfies the Helmholtz wave equation in (32). Substitution of (36) in (32) yields the dispersion relation

$$k_{lz}^2 + k_y^2 = \omega^2 \mu_l \epsilon_l \tag{39}$$

We do not write a subscript l for the y component of $\overline{k}$ as a consequence of the phase-matching conditions. Truly, there are multiple reflections and transmissions in each layer l. The amplitude E_l^+ thus represents all wave components that have a propagating velocity component along the positive $\hat{z}$ direction, and E_l^- represents those with a velocity component along the negative $\hat{z}$ direction.

We note that in region 0 where $l = 0$,

$$E_0^+ = E_0 \tag{40}$$

$$E_0^- = RE_0 \tag{41}$$

In region t where $l = n + 1 = t$, we have

$$E_t^+ = TE_0 \tag{42}$$

$$E_t^- = 0 \tag{43}$$

because region t is semi-infinite and there is no wave propagating with a velocity component in the positive $\hat{z}$ direction. We denote the transmitted amplitude by T.

The wave amplitudes E_l^+ and E_l^- are related to wave amplitudes in neighboring regions by the boundary conditions. At $z = d_{l+1}$, boundary conditions require that E_x and H_y be continuous. We obtain

$$E_l^+ e^{ik_{lz}d_{l+1}} + E_l^- e^{-ik_{lz}d_{l+1}} = E_{l+1}^+ e^{ik_{(l+1)\,z}d_{l+1}} + E_{l+1}^- e^{-ik_{(l+1)\,z}d_{l+1}} \tag{44}$$

$$\begin{aligned} E_l^+ e^{ik_{lz}d_{l+1}} &- E_l^- e^{-ik_{lz}d_{l+1}} \\ &= p_{l(l+1)} \left[E_{l+1}^+ e^{ik_{(l+1)\,z}d_{l+1}} - E_{l+1}^- e^{-ik_{(l+1)\,z}d_{l+1}} \right] \end{aligned} \tag{45}$$

where

$$p_{l(l+1)} = \frac{\mu_l k_{(l+1)\,z}}{\mu_{l+1} k_{lz}} \tag{46}$$

for the TE wave. There are $n+1$ boundaries which give rise to $(2n+2)$ equations. In region 0, we have an unknown reflection coefficient R. In region t, we have an unknown transmission coefficient T. There are two unknowns E_l^+ and E_l^- in each of the regions $l = 1, 2, \ldots, n$. Thus we have a total of $(2n + 2)$ unknowns. To solve for the

$(2n+2)$ unknowns from the $(2n+2)$ linear equations, we can arrange the equations in matrix form with the unknowns forming a $(2n+2)$ column matrix and the coefficients forming a $(2n+2)\times(2n+2)$ square matrix. The solution is then obtained by inverting the square matrix. This procedure is straightforward but tedious. We shall now describe simpler ways to deal with the problem.

For a TM plane wave, $H_x = H_0\, e^{ik_z z + ik_y y}$, incident on the stratified medium, the total field in region l can be written as

$$H_{lx} = \left(E_l^+ e^{ik_{lz}z} + E_l^- e^{-ik_{lz}z}\right) e^{ik_y y} \tag{47}$$

$$E_{ly} = \frac{-k_{lz}}{\omega\epsilon_l}\left(E_l^+ e^{ik_{lz}z} - E_l^- e^{-ik_{lz}z}\right) e^{ik_y y} \tag{48}$$

$$E_{lz} = \frac{k_y}{\omega\epsilon_l}\left(E_l^+ e^{ik_{lz}z} + E_l^- e^{-ik_{lz}z}\right) e^{ik_y y} \tag{49}$$

Matching boundary conditions at the boundaries, identifying

$$p_{l(l+1)} = \frac{\epsilon_l k_{(l+1)\,z}}{\epsilon_{l+1} k_{lz}} \tag{50}$$

for TM waves, we obtain the same equations as in (44) and (45).

4.1. Reflection Coefficients

To find a closed form solution for the reflection coefficient R for the stratified medium, we first solve (44) and (45) for A_l and B_l.

$$\begin{aligned} E_l^+ e^{ik_{lz}d_{l+1}} &= \frac{1}{2}\left(1 + p_{l(l+1)}\right) \\ &\quad \left\{E_{l+1}^+ e^{ik_{(l+1)z}d_{l+1}} + R_{l(l+1)} E_{l+1}^- e^{-ik_{(l+1)z}d_{l+1}}\right\} \end{aligned} \tag{51}$$

$$\begin{aligned} E_l^- e^{-ik_{lz}d_{l+1}} &= \frac{1}{2}\left(1 + p_{l(l+1)}\right) \\ &\quad \left\{R_{l(l+1)} E_{l+1}^+ e^{ik_{(l+1)z}d_{l+1}} + E_{l+1}^- e^{-ik_{(l+1)z}d_{l+1}}\right\} \end{aligned} \tag{52}$$

where

$$R_{l(l+1)} = \frac{1 - p_{l(l+1)}}{1 + p_{l(l+1)}} \tag{53}$$

is the reflection coefficient for waves in region l, caused by the boundary separating regions l and $l+1$. We note from (46) that

$$p_{(l+1)\,l} = \frac{1}{p_{l(l+1)}} \tag{54}$$

which also gives

$$R_{(l+1)\,l} = -R_{l(l+1)} \tag{55}$$

Thus the reflection coefficient in region $l+1$, $R_{(l+1)\,l}$, caused by the boundary separating regions $l+1$ and l, is equal to the negative of $R_{l(l+1)}$.

Forming the ratio of (51) and (52) we obtain

$$\begin{aligned}\frac{E_l^-}{E_l^+} &= \frac{e^{i2k_{lz}d_{l+1}}}{R_{l(l+1)}} + \frac{\left[1-\left(1/R^2_{l(l+1)}\right)\right] e^{i2(k_{(l+1)\,z}+k_{lz})d_{(l+1)}}}{\left[1/R_{l(l+1)}\right] e^{i2k_{(l+1)\,z}d_{(l+1)}} + \left(E^-_{l+1}/E^+_{l+1}\right)} \\ &= \frac{e^{i2k_{lz}d_{(l+1)}}}{R_{l(l+1)}} + \frac{\left[1-\left(1/R^2_{l(l+1)}\right)\right] e^{i2(k_{(l+1)\,z}+k_{lz})d_{(l+1)}}}{\left[1/R_{l(l+1)}\right] e^{i2k_{(l+1)\,z}d_{(l+1)}}}\Bigg| + \frac{E^-_{l+1}}{E^+_{l+1}}\end{aligned} \tag{56}$$

With the second equality we introduce a notation for writing a continued fraction. Equation (56) expresses (E_l^+/E_l^-) in terms of E^-_{l+1}/E^+_{l+1} and so on, until the transmitted region t, where $E_t^-/E_t^+ = 0$, is reached.

The reflection coefficient due to the stratified medium is $R = B_0/A_0$. Making use of the continued fractions, we obtain

$$\begin{aligned}R = &\ \frac{e^{i2k_{0z}d_1}}{R_{01}} + \frac{\left[1-(1/R^2_{01})\right] e^{i2(k_1z+k_{0z})d_1}}{(1/R_{01})e^{i2k_{1z}d_1}}\Bigg| + \frac{e^{i2k_{1z}d_2}}{R_{12}} \\ &+ \frac{\left[1-(1/R^2_{12})\right] e^{i2(k_{2z}+k_{1z})d_2}}{(1/R_{12})e^{i2k_{2z}d_2}}\Bigg| + \cdots + \frac{e^{i2k_{(n-1)\,z}d_n}}{R_{(n-1)\,n}} \\ &+ \frac{\left[1-(1/R^2_{(n-1)\,n})\right] e^{i2(k_{nz}+k_{(n-1)\,z})d_n}}{(1/R_{(n-1)\,n})e^{i2k_{nz}d_n}}\Bigg| + R_{nt}e^{i2k_{nz}d_{(n+1)}}\end{aligned} \tag{57}$$

This is a closed-form solution for the reflection coefficient expressed in continued fractions. Such a solution is very easily programmed for numerical computation.

4.2. Propagation Matrices and Transmission Coefficients

For a plane wave incident on a stratified medium, we have obtained the boundary conditions of continuity of tangential electric and magnetic fields at each interface $z = d_l$, with the two equations (44)–(45) relating wave amplitudes in regions l and $l+1$:

$$E^+_{l+1}e^{ik_{(l+1)\,z}d_{(l+1)}} + E^-_{l+1}e^{-ik_{(l+1)\,z}d_{(l+1)}} = E_l^+e^{ik_{lz}d_{(l+1)}} + E_l^-e^{-ik_{lz}d_{(l+1)}} \tag{58}$$

$$E^+_{l+1}e^{ik_{(l+1)\,z}d_{(l+1)}} - E^-_{l+1}\,e^{-ik_{(l+1)\,z}d_{(l+1)}} = p_{(l+1)\,l}\left[E^+_l\,e^{ik_{lz}d_{(l+1)}} - E^-_l\,e^{-ik_{lz}d_{(l+1)}}\right] \tag{59}$$

where for TE waves

$$p_{(l+1)\,l} = \frac{\mu_{l+1}k_{lz}}{\mu_l k_{(l+1)z}} \tag{60}$$

and for TM waves

$$p_{(l+1)\,l} = \frac{\epsilon_{l+1}k_{lz}}{\epsilon_l k_{(l+1)z}} \tag{61}$$

Equations (60) and (61) follow from duality but bear in mind that E^+_l and E^-_l denote amplitudes of tangential electric fields for TE waves and H^+_l and H^-_l denote amplitudes of tangential magnetic fields for TM waves. In the last section we determined the reflection coefficients $R = E^-_0/E^+_0$ from the $(2n+2)$ boundary conditions. We will now show that the transmission coefficient $T = E^+_t/E^+_0$ can be obtained by the use of propagation matrices.

We solve for E^+_{l+1} and E^-_{l+1} in terms of E^+_l and E^-_l from (58)–(59) and obtain

$$E^+_{l+1}e^{ik_{(l+1)z}d_{l+1}} = \frac{1}{2}(1+p_{(l+1)l})\left(E^+_l\,e^{ik_{lz}d_{l+1}} + R_{(l+1)l}E^-_l\,e^{-ik_{lz}d_{l+1}}\right)$$

$$E^-_{l+1}e^{-ik_{(l+1)z}d_{l+1}} = \frac{1}{2}(1+p_{(l+1)l})\left(R_{(l+1)l}E^+_l\,e^{ik_{lz}d_{l+1}} + E^-_l\,e^{-ik_{lz}d_{l+1}}\right)$$

Expressing in the form of matrix multiplication, we have

$$\begin{pmatrix} E^+_{l+1} \\ E^-_{l+1} \end{pmatrix} = \overline{\overline{V}}_{(l+1)\,l}\cdot\begin{pmatrix} E^+_l \\ E^-_l \end{pmatrix} \tag{62}$$

where

$$\overline{\overline{V}}_{(l+1)\,l} = \frac{1}{2}[1+p_{(l+1)\,l}] \begin{pmatrix} e^{-i(k_{(l+1)z}-k_{lz})d_{l+1}} & R_{(l+1)\,l}e^{-i(k_{(l+1)z}+k_{lz})d_{l+1}} \\ R_{(l+1)\,l}e^{i(k_{(l+1)z}+k_{lz})d_{l+1}} & e^{i(k_{(l+1)z}-k_{lz})d_{l+1}} \end{pmatrix} \tag{63}$$

is called the forward-propagating matrix. In (63),

$$R_{(l+1)\,l} = \frac{1-p_{(l+1)\,l}}{1+p_{(l+1)\,l}} = -R_{l(l+1)}$$

is the reflection coefficient at the boundary separating regions $l+1$ and l, and the first subscript denotes the region with the incident wave.

It is to be noted for the forward-propagating matrix between layers n and $t = n + 1$,

$$\begin{pmatrix} T \\ 0 \end{pmatrix} = \overline{\overline{V}}_{tn} \cdot \begin{pmatrix} E_n^+ \\ E_n^- \end{pmatrix}$$

with

$$\overline{\overline{V}}_{tn} = \frac{1}{2}(1 + p_{tn}) \begin{pmatrix} e^{-i(k_{tz}-k_{nz})d_t} & R_{tn}e^{-i(k_{tz}+k_{nz})d_t} \\ R_{tn}e^{i(k_{tz}+k_{nz})d_t} & e^{i(k_{tz}-k_{nz})d_t} \end{pmatrix}$$

By the same token, we may express E_l^+ and E_l^- in terms of E_{l+1}^+ and E_{l+1}^- by using (51)–(52) and define a backward-propagating matrix.

The propagation matrices can be used to determine wave amplitudes in any region in terms of those in any other region. For $m > l$, we make use of the forward propagation matrix to obtain

$$\begin{pmatrix} E_m^+ \\ E_m^- \end{pmatrix} = \overline{\overline{V}}_{m(m-1)} \cdot \overline{\overline{V}}_{(m-1)(m-2)} \cdots \overline{\overline{V}}_{(l+1)\,l} \cdot \begin{pmatrix} E_l^+ \\ E_l^- \end{pmatrix}$$

Similarly, backward-propagating matrices can be used to express wave amplitudes in any region j in terms of those in region l for $l > j$.

In particular, the transmission coefficient $T = E_t^-/E_0$ for a stratified medium with $t = n + 1$ layers can be calculated by the multiplication of $n + 1$ propagation matrices. Using the forward-propagating matrices, we have

$$\begin{pmatrix} T \\ 0 \end{pmatrix} = \overline{\overline{V}}_{t0} \cdot \begin{pmatrix} 1 \\ R \end{pmatrix}$$

where

$$\overline{\overline{V}}_{t0} = \overline{\overline{V}}_{tn} \cdot \overline{\overline{V}}_{n(n-1)} \cdots \overline{\overline{V}}_{10}$$

includes all information about the stratified medium. Once $\overline{\overline{V}}_{t0}$ is known, both the reflection and transmission coefficients can be calculated from its matrix elements.

4.3. Reflection and Transmission by a Slab Medium

For a slab medium with boundary surfaces at $z = d_1$ and $z = d_2$, we find from (57), with $t = 2$ and $n = 1$, the reflection coefficient

$$\begin{aligned} R &= \frac{e^{i2k_{0z}d_1}}{R_{01}} + \frac{\left[1 - (1/R_{01}^2)\right] e^{i2(k_{1z}+k_{0z})d_1}}{(1/R_{01})e^{i2k_{1z}d_1} + R_{12}e^{i2k_{1z}d_2}} \\ &= \frac{R_{01} + R_{12}e^{i2k_{1z}(d_2-d_1)}}{1 + R_{01}R_{12}e^{i2k_{1z}(d_2-d_1)}} e^{i2k_{0z}d_1} \end{aligned} \tag{64}$$

Making use of propagation matrix $\overline{\overline{V}}_{(l+1)\,l}$ as shown in (62)

$$\begin{pmatrix} E_1^+ \\ E_1^- \end{pmatrix} = \frac{1}{2}(1+p_{10}) \begin{pmatrix} e^{-i(k_{1z}-k_{0z})d_1} & R_{10}e^{-i(k_{1z}+k_{0z})d_1} \\ R_{10}e^{i(k_{1z}+k_{0z})d_1} & e^{i(k_{1z}-k_{0z})d_1} \end{pmatrix} \begin{pmatrix} 1 \\ R \end{pmatrix}$$
$$\begin{pmatrix} T \\ 0 \end{pmatrix} = \frac{1}{2}(1+p_{t1}) \begin{pmatrix} e^{-i(k_{tz}-k_{1z})d_t} & R_{t1}e^{-i(k_{tz}+k_{1z})d_t} \\ R_{t1}e^{i(k_{tz}+k_{1z})d_t} & e^{i(k_{tz}-k_{1z})d_t} \end{pmatrix} \begin{pmatrix} E_1^+ \\ E_1^- \end{pmatrix}$$

we find the amplitudes inside the slab medium to be

$$E_1^+ = \frac{2e^{-i(k_{1z}-k_{0z})d_1}}{(1+p_{01})(1+R_{01}R_{1t}e^{i2k_{1z}(d_2-d_1)})} \tag{65}$$

$$E_1^- = \frac{2R_{12}e^{-i(k_{1z}-k_{0z})d_1}e^{i2k_{1z}d_2}}{(1+p_{01})(1+R_{01}R_{1t}e^{i2k_{1z}(d_2-d_1)})} \tag{66}$$

The transmission coefficient is

$$T = \frac{4e^{ik_{0z}d_1}e^{ik_{1z}(d_2-d_1)}e^{-ik_{2z}d_2}}{(1+p_{01})(1+p_{1t})(1+R_{01}R_{1t}e^{i2k_{1z}(d_2-d_1)})} \tag{67}$$

For an electromagnetic wave incident on a negative isotropic slab in free space, we let $\mu_1 = -\mu_0$, $\epsilon_1 = -\epsilon_0$, and $\mu_t = \mu_0$, $\epsilon_t = \epsilon_0$. We find $k_{1z} = -k_{0z}$, $k_{2z} = k_{0z}$, $p_{01} = p_{12} = 1$, $R_{01} = R_{12} = R = 0$, $E_1^- = 0$, $E_1^+ = e^{i2k_{0z}d_1}$, $T = e^{-i2k_{0z}(d_2-d_1)}$.

Assuming a Gaussian beam with beamwidth g incident on the negative isotropic slab with incident angle θ_i and incident vector $\overline{k}_i = \hat{y}k_{iy} + \hat{z}k_{iz}$, where $k_{iy} = k_0 \sin\theta_i$, $k_{iz} = k_0 \cos\theta_i$. For an incident TM wave field, we write

$$H_x = \int_{-\infty}^{\infty} dk_y\, e^{i(k_y y + k_{0z} z)}\Psi(k_y) \tag{68}$$

$$\Psi(k_y) = \frac{g^2}{4\pi} e^{-g^2|k_y - k_{iy}|^2/4} \tag{69}$$

Then for the transmitted magnetic field, we have

$$H_x = \int_{-\infty}^{\infty} dk_y\, e^{i(k_y y + k_{0z}[z-2(d_2-d_1)])}\Psi(k_y) \tag{70}$$

Thus on the plane at a constant z in the transmitted region, the Gaussian beam spot is at y_{out}, which is shifted by a distance of $2(d_2 - d_1)\tan\theta_i$ in the negative $\hat{y}$ direction in the presence of the negative isotropic slab as compare to the location of the spot y_{in} in the absence of the slab.

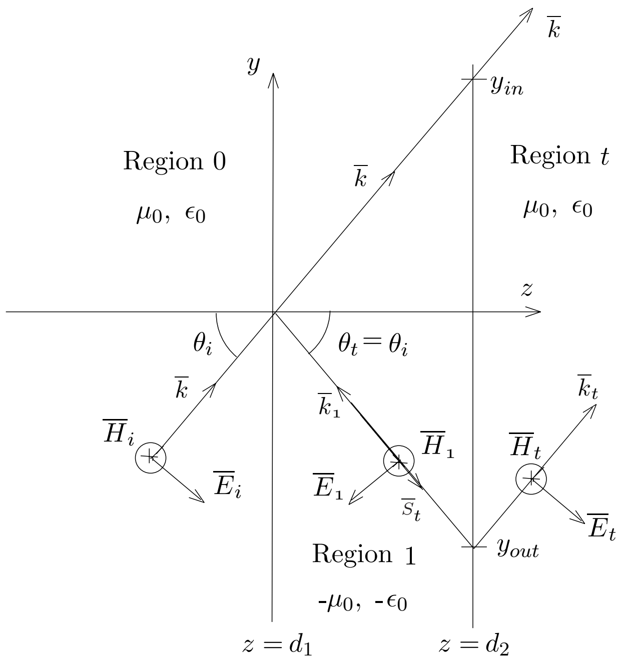

Figure 4. Beam center shifted by $2(d_2 - d_1)\tan\theta_i$.

The reflected beam shift will be twice when we place the negative isotropic slab in front of a perfect conductor. We find

$$\mu_1 = -\mu_0,\ \epsilon_1 = -\epsilon_0,\ k_{1z} = -k_{0z},$$

$$T = 0,\ E_1^+ = e^{i2k_{0z}d_1},\ E_1^- = R_{12}e^{-i2k_{0z}(d_2-d_1)},$$

$$p_{01} = 1,\ R_{01} = 0.$$

For TE waves

$$R_{12}^{TE} = -1,\ R^{TE} = -e^{-i2k_{0z}(d_2-d_1)}e^{i2k_{0z}d_1}.$$

For TM waves

$$R_{12}^{TM} = 1,\ R^{TM} = e^{-i2k_{0z}(d_2-d_1)}e^{i2k_{0z}d_1}.$$

In the absence of the negative isotropic slab, the reflection coefficient is

$$R_0^{TE} = -e^{i2k_{0z}d_2}.\ R_0^{TM} = e^{i2k_{0z}d_2}.$$

Thus on the plane at a constant z in Region 0, the Gaussian beam spot is at y_{out}, which is shifted by a distance of $4(d_2 - d_1)\tan\theta_i$ in the negative $\hat{y}$ direction in the presence of the negative isotropic slab as compare to the location of the spot y_{in} in the absence of the slab.

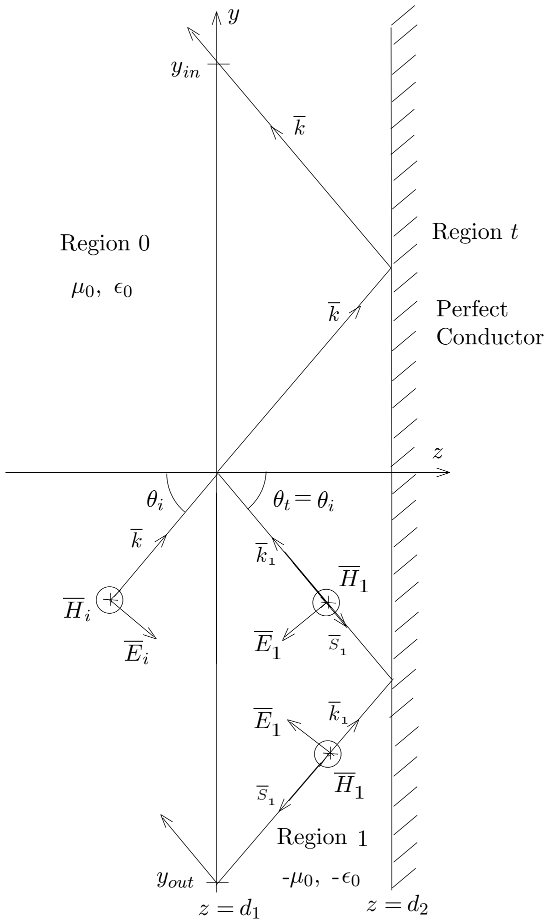

Figure 5. Beam center shifted by $4(d_2 - d_1)\tan\theta_i$.

5. GUIDED WAVES IN STRATIFIED MEDIA

5.1. Guidance Conditions

The geometrical configuration of the problem is shown in Figure 6. There are t layers at $z = d_1, d_2, \ldots d_t$, and $s + 1$ layers at $z = d_0, d_{-1}, \ldots, d_{-s}$. We shall first assume that all regions contain isotropic media. In region l, we denote the permittivity and permeability by ϵ_l and μ_l. Notice that in region 0, ϵ_0 and μ_0 are not necessarily equal to the free space permittivity ϵ_o and permeability μ_o.

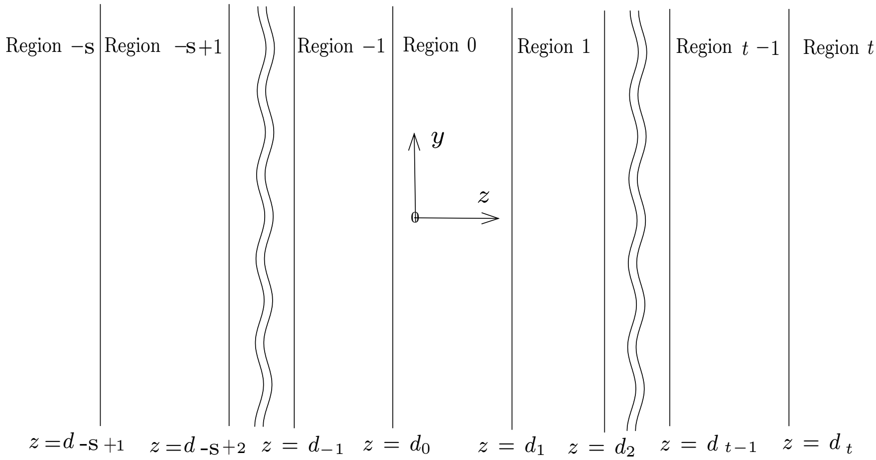

Figure 6. Guided Waves in stratified media.

For TE waves, the solutions in region l take the following form:

$$E_{lx} = \left[E_l^+ e^{ik_{lz}z} + E_l^- e^{-ik_{lz}z}\right] e^{ik_y y} \tag{71}$$

$$H_{ly} = \frac{k_{lz}}{\omega\mu_l}\left[E_l^+ e^{ik_{lz}z} - E_l^- e^{-ik_{lz}z}\right] e^{ik_y y}$$

$$H_{lz} = \frac{-k_y}{\omega\mu_l}\left[E_l^+ e^{ik_{lz}z} + E_l^- e^{-ik_{lz}z}\right] e^{ik_y y} \tag{72}$$

For TM waves we invoke duality with the replacement $\overline{E} \to \overline{H}$, $\overline{H} \to -\overline{E}$, $\mu_0 \rightleftarrows \epsilon_0$.

The boundary conditions of the interfaces require that tangential electric and magnetic field components be continuous for all x and y. At $z = d_{l+1}$, we obtain

$$E_l^+ e^{ik_{lz}d_{l+1}} + E_l^- e^{-ik_{lz}d_{l+1}} = E_{l+1}^+ e^{ik_{(l+1)z}d_{l+1}} + E_{l+1}^- e^{-ik_{(l+1)z}d_{l+1}} \tag{73}$$

$$E_l^+ e^{ik_{lz}d_{l+1}} - E_l^- e^{-ik_{lz}d_{l+1}} = p_{l(l+1)}\left[E_{l+1}^+ e^{ik_{(l+1)z}d_{l+1}} - E_{l+1}^- e^{-ik_{(l+1)z}d_{l+1}}\right] \tag{74}$$

where

$$p_{l(l+1)} = \frac{\mu_l k_{(l+1)z}}{\mu_{l+1}k_{lz}} = \frac{1}{p_{(l+1)l}} \tag{75}$$

We now determine the wave amplitudes in region l. From (73)–(74) we can express E_l^+ and E_l^- in terms of E_{l+1}^+ and E_{l+1}^- or express E_{l+1}^+ and E_{l+1}^- in terms of E_l^+ and E_l^- . We find

$$\begin{aligned} E_l^+ e^{ik_{lz} d_{l+1}} &= \frac{1}{2}\Big(1 + p_{l(l+1)}\Big) \\ &\quad \Big\{E_{l+1}^+ e^{ik_{(l+1)z} d_{l+1}} + R_{l(l+1)} E_{l+1}^- e^{-ik_{(l+1)z} d_{l+1}}\Big\} \quad (76) \\ E_l^- e^{-ik_{lz} d_{l+1}} &= \frac{1}{2}\Big(1 + p_{l(l+1)}\Big) \\ &\quad \Big\{R_{l(l+1)} E_{l+1}^+ e^{ik_{(l+1)z} d_{l+1}} + E_{l+1}^- e^{-ik_{(l+1)z} d_{l+1}}\Big\} \quad (77) \end{aligned}$$

for E_l^+ and E_l^- in terms of E_{l+1}^+ and E_{l+1}^-, and

$$\begin{aligned} E_{l+1}^+ e^{ik_{(l+1)z} d_{l+1}} &= \frac{1}{2}(1 + p_{(l+1)l}) \\ &\quad \Big\{E_l^+ e^{ik_{lz} d_{l+1}} + R_{(l+1)\,l} E_l^- e^{-ik_{lz} d_{l+1}}\Big\} \quad (78) \\ E_{l+1}^- e^{-ik_{(l+1)z} d_{l+1}} &= \frac{1}{2}(1 + p_{(l+1)l}) \\ &\quad \Big\{R_{(l+1)\,l} E_l^+ e^{ik_{lz} d_{l+1}} + E_l^- e^{-ik_{lz} d_{l+1}}\Big\} \quad (79) \end{aligned}$$

for E_{l+1}^+ and E_{l+1}^- in terms of E_l^+ and E_l^- . In (76)–(**??**), the Fresnel reflection coefficient

$$R_{(l+1)l} = \frac{1 - p_{(l+1)l}}{1 + p_{(l+1)l}} = -R_{l(l+1)} \quad (80)$$

where

$$R_{l(l+1)} = \frac{1 - p_{l(l+1)}}{1 + p_{l(l+1)}} \quad (81)$$

is the reflection coefficient for waves in region l, caused by the boundary separating regions l and $l+1$. The reflection coefficient in region $l+1$, $R_{(l+1)\,l}$, caused by the boundary separating regions $l+1$ and l , is equal to the negative of $R_{l(l+1)}$.

There are altogether $s+t$ boundaries which give rise to $2(s+t)$ equations as shown above. There are altogether $s+t+1$ regions. In regions t and $-s$ we have $E_t^- = 0$ and $E_{-s}^+ = 0$ because there are no waves originating from infinity. Thus we have a total of $2(s+t+1) - 2 = 2(s+t)$ unknowns to be solved from the $2(s+t)$ equations.

For $z \geq 0$, we let $l = 0$, and obtain the reflection coefficients $R_{0+} = E_{0+}^-/E_{0+}^+$ in the form of continued fractions. We find

$$R_{0+} = \frac{E_{0+}^-}{E_{0+}^+} = \frac{e^{i2k_{0z}d_1}}{R_{01}} + \frac{\left[1 - (1/R_{01})^2\right] e^{i2(k_{0z}+k_{1z})d_1}}{(1/R_{01})e^{i2k_{1z}d_1} + (E_1^-/E_1^+)} \tag{82}$$

where E_1^-/E_1^+ can be expressed in terms of E_2^-/E_2^+ and so on until region t where $E_t^-/E_t^+ = 0$.

For $z \leq 0$, we let $l = 0$, and obtain the reflection coefficients $R_{0-} = E_{0-}^+/E_{0-}^-$ in the form of continued fractions. We find

$$R_{0-} = \frac{E_{0-}^+}{E_{0-}^-} = \frac{e^{-i2k_{0z}d_0}}{R_{0(-1)}} + \frac{\left[1 - \left(1/R_{0(-1)}\right)^2\right] e^{-i2(k_{0z}+k_{-1z})d_0}}{(1/R_{0(-1)})e^{-i2k_{-1z}d_0} + (E_{-1}^+/E_{-1}^-)} \tag{83}$$

where E_{-1}^+/E_{-1}^- are expressible in terms of E_{-2}^+/E_{-2}^- and so on until region $-s$, where $E_{-s}^+/E_{-s}^- = 0$.

In region 0, the guidance condition is determined from

$$R_{0+}R_{0-} = 1 \tag{84}$$

For $s = 2$ and $t = 2$, we find

$$R_{0+} = \frac{R_{01} + R_{12}e^{i2k_{1z}(d_2-d_1)}}{1 + R_{01}R_{12}e^{i2k_{1z}(d_2-d_1)}} e^{i2k_{0z}d_1} \tag{85}$$

$$R_{0-} = \frac{R_{0(-1)} + R_{(-1)(-2)}e^{-i2k_{-1z}(d_{-1}-d_0)}}{1 + R_{0(-1)}R_{(-1)(-2)}e^{-i2k_{-1z}(d_{-1}-d_0)}} e^{-i2k_{0z}d_0} \tag{86}$$

Consider guidance in Region 0 with evanescence in Regions 1 and -1 such that $k_{1z} = \sqrt{k_1^2 - k_y^2} = i\alpha_{1z}$ and $k_{-1z} = \sqrt{k_{-1}^2 - k_y^2} = i\alpha_{-1z}$. We then have

$$p_{01} = i\frac{\mu_0\alpha_{1z}}{\mu_1 k_{0z}}; \quad R_{01} = e^{i2\phi_{01}}; \quad \phi_{01} = -tan^{-1}\frac{\mu_0\alpha_{1z}}{\mu_1 k_{0z}} \tag{87}$$

$$p_{0(-1)} = i\frac{\mu_0\alpha_{-1z}}{\mu_{-1} k_{0z}}; \; R_{0(-1)} = e^{i2\phi_{0(-1)}}; \; \phi_{0(-1)} = -tan^{-1}\frac{\mu_0\alpha_{-1z}}{\mu_{-1} k_{0z}} \tag{88}$$

where ϕ_{01} and $\phi_{0(-1)}$ are Goos-Haschen shifts at the boundaries $z = d_1$ and $z = d_0$.

5.2. Fields of Guided Waves

For a slab waveguide with $s = 1$ and $t = 1$, we have

$$R_{0+} = R_{01}e^{i2k_{0z}d_1} \tag{89}$$

$$R_{0-} = R_{0(-1)}e^{-i2k_{0z}d_0} \tag{90}$$

The guidance condition becomes

$$\phi_{0(-1)} + \phi_{01} + k_{0z}d_1 - k_{0z}d_0 = m\pi \tag{91}$$

for the mth order guided mode.For an asymmetric slab waveguide, we have

$$E_{-1}^+ = 0 \tag{92}$$

$$E_{-1}^- = \frac{1}{2}\left(1 + p_{(-10)}\right)\left\{R_{(-1)0}E_0^+ e^{i(k_{0z}+k_{-1z})d_0} + E_0^- e^{-i(k_{0z}-k_{-1z})d_0}\right\} \tag{93}$$

$$E_1^+ = \frac{1}{2}(1 + p_{10})\left\{E_0^+ e^{i(k_{0z}-k_{1z})d_1} + R_{10}E_0^- e^{-i(k_{0z}+k_{1z})d_1}\right\} \tag{94}$$

$$E_1^- = 0 \tag{95}$$

The field in Region -1 is

$$E_{-1x} = \left[E_{-1}^- e^{\alpha_{-1z}z}\right] e^{ik_y y} \tag{96}$$

$$H_{-1y} = \frac{k_{-1z}}{\omega\mu_l}\left[-E_{-1}^- e^{\alpha_{-1z}z}\right] e^{ik_y y} \tag{97}$$

$$H_{-1z} = \frac{-k_y}{\omega\mu_{-1}}\left[E_{-1}^- e^{\alpha_{-1z}z}\right] e^{ik_y y} \tag{98}$$

$$< \overline{S}_{-1} > = \hat{y}\frac{k_y}{2\omega\mu_{-1}}\left[|E_{-1}^-|^2 e^{2\alpha_{-1z}z}\right] \tag{99}$$

The field in Region 0 is

$$E_{0x} = \left[E_0^+ e^{ik_{0z}z} + E_0^- e^{-ik_{0z}z}\right] e^{ik_y y} \tag{100}$$

$$H_{0y} = \frac{k_{0z}}{\omega\mu_0}\left[E_0^+ e^{ik_{0z}z} - E_0^- e^{-ik_{0z}z}\right] e^{ik_y y} \tag{101}$$

$$H_{lz} = \frac{-k_y}{\omega\mu_0}\left[E_0^+ e^{ik_{0z}z} + E_0^- e^{-ik_{0z}z}\right] e^{ik_y y} \tag{102}$$

$$< \overline{S}_0 > = \hat{y}\frac{k_y}{2\omega\mu_0}\left[|E_0^+|^2 + |E_0^-|^2 + 2Re\left\{E_0^+ (E_0^-)^* e^{i2k_{0z}z}\right\}\right] \tag{103}$$

The field in Region 1 is

$$E_{1x} = \left[E_1^+ e^{-\alpha_{lz} z}\right] e^{ik_y y} \tag{104}$$

$$H_{1y} = \frac{k_{1z}}{\omega\mu_1}\left[E_1^+ e^{-\alpha_{1z} z}\right] e^{ik_y y} \tag{105}$$

$$H_{1z} = \frac{-k_y}{\omega\mu_1}\left[E_1^+ e^{-\alpha_{1z} z}\right] e^{ik_y y} \tag{106}$$

$$< \overline{S}_1 > = \hat{y}\frac{k_y}{2\omega\mu_1}\left[|E_1^+|^2 e^{-2\alpha_{1z} z}\right] \tag{107}$$

When the waveguide is a negative isotropic slab with $\mu_0 = -\mu_o$ and $\epsilon_0 = -\epsilon_o$, we observe that the Poynting power inside the wave guide is flowing in the negative $\hat{y}$ direction while the Poynting power outside the slab flow in the positive $\hat{y}$ direction.

5.3. Coupling of Guided Waves

For the case of $s = 2$ and $t = 2$, we find

$$E_{-2}^+ = 0 \tag{108}$$

$$E_{-2}^- = \frac{1}{2}\left(1 + p_{-2(-1)}\right) \left\{R_{-2(-1)} E_{-1}^+ e^{i(k_{-1z}+k_{-2z})d_{-1}} + E_{-1}^- e^{-i(k_{-1z}-k_{-2z})d_{-1}}\right\} \tag{109}$$

$$E_{-1}^+ = \frac{1}{2}\left(1 + p_{-10}\right)\left\{E_0^+ e^{i(k_{0z}-k_{-1z})d_0} + R_{-10} E_0^- e^{-i(k_{0z}+k_{-1z})d_0}\right\} \tag{110}$$

$$E_{-1}^- = \frac{1}{2}\left(1 + p_{-10}\right)\left\{R_{-10} E_0^+ e^{i(k_{0z}+k_{-1z})d_0} + E_0^- e^{-i(k_{0z}-k_{-1z})d_0}\right\} \tag{111}$$

$$E_1^+ = \frac{1}{2}(1 + p_{10})\left\{E_0^+ e^{i(k_{0z}-k_{1z})d_1} + R_{10} E_0^- e^{-i(k_{0z}+k_{1z})d_1}\right\} \tag{112}$$

$$E_1^- = \frac{1}{2}(1 + p_{10})\left\{R_{10} E_0^+ e^{i(k_{0z}+k_{1z})d_1} + E_0^- e^{-i(k_{0z}-k_{1z})d_1}\right\} \tag{113}$$

$$E_2^+ = \frac{1}{2}(1 + p_{21})\left\{E_1^+ e^{i(k_{1z}-k_{2z})d_2} + R_{21} E_1^- e^{-i(k_{1z}+k_{2z})d_2}\right\} \tag{114}$$

$$E_2^- = 0 \tag{115}$$

We let waves be guided in Region -1 and Region 1, thus $k_{0z} = i\alpha_{0z}$, $k_{-2z} = i\alpha_{-2z}$, and $k_{2z} = i\alpha_{2z}$. The electromagnetic fields in all regions take the following forms.
In Region -2:

$$E_{-2x} = \left[E_{-2}^- e^{\alpha_{-2z} z}\right] e^{ik_y y} \tag{116}$$

$$H_{-2y} = \frac{k_{-2z}}{\omega\mu_{-2}} \left[-E_{-2}^{-} e^{\alpha_{-2z} z} \right] e^{ik_y y}$$

$$H_{-2z} = \frac{-k_y}{\omega\mu_{-2}} \left[E_{-2}^{-} e^{\alpha_{-2z} z} \right] e^{ik_y y} \tag{117}$$

$$< \overline{S}_{-2} > = \hat{y} \frac{k_y}{2\omega\mu_{-2}} \left[|E_{-2}^{-}|^2 e^{2\alpha_{-2z} z} \right] \tag{118}$$

In Region −1:

$$E_{-1x} = \left[E_{-1}^{+} e^{ik_{-1z} z} + E_{-1}^{-} e^{-ik_{-1z} z} \right] e^{ik_y y} \tag{119}$$

$$H_{-1y} = \frac{k_{-1z}}{\omega\mu_{-1}} \left[E_{-1}^{+} e^{ik_{-1z} z} - E_{-1}^{-} e^{-ik_{-1z} z} \right] e^{ik_y y}$$

$$H_{-1z} = \frac{-k_y}{\omega\mu_{-1}} \left[E_{-1}^{+} e^{ik_{-1z} z} + E_{-1}^{-} e^{-ik_{-1z} z} \right] e^{ik_y y} \tag{120}$$

$$< \overline{S}_{-1} > = \hat{y} \frac{k_y}{2\omega\mu_{-1}} \left[|E_{-1}^{+}|^2 + |E_{-1}^{-}|^2 - 2Re \left\{ E_{-1}^{+} (E_{-1}^{-})^* e^{i2k_{1z} z} | \right\} \right] \tag{121}$$

In Region 0:

$$E_{0x} = \left[E_0^{+} e^{ik_{0z} z} + E_0^{-} e^{-ik_{0z} z} \right] e^{ik_y y} \tag{122}$$

$$H_{0y} = \frac{k_{0z}}{\omega\mu_0} \left[E_0^{+} e^{ik_{0z} z} - E_0^{-} e^{-ik_{0z} z} \right] e^{ik_y y}$$

$$H_{0z} = \frac{-k_y}{\omega\mu_0} \left[E_0^{+} e^{ik_{0z} z} + E_0^{-} e^{-ik_{0z} z} \right] e^{ik_y y} \tag{123}$$

$$< \overline{S}_0 > = \hat{y} \frac{k_y}{2\omega\mu_0} \left[|E_0^{+}|^2 + |E_0^{-}|^2 + 2Re \left\{ E_0^{+} (E_0^{-})^* e^{i2k_{0z} z} \right\} \right] \tag{124}$$

In Region 1:

$$E_{1x} = \left[E_1^{+} e^{ik_{1z} z} + E_1^{-} e^{-ik_{1z} z} \right] e^{ik_y y} \tag{125}$$

$$H_{1y} = \frac{k_{1z}}{\omega\mu_1} \left[E_1^{+} e^{ik_{1z} z} - E_1^{-} e^{-ik_{1z} z} \right] e^{ik_y y}$$

$$H_{1z} = \frac{-k_y}{\omega\mu_1} \left[E_1^{+} e^{ik_{1z} z} + E_1^{-} e^{-ik_{1z} z} \right] e^{ik_y y} \tag{126}$$

$$< \overline{S}_1 > = \hat{y} \frac{k_y}{2\omega\mu_1} \left[|E_1^{+}|^2 + |E_1^{-}|^2 + 2Re \left\{ |E_1^{+} (E_1^{-})^* e^{i2k_{1z} z} | \right\} \right] \tag{127}$$

In Region 2:

$$E_{2x} = \left[E_2^{+} e^{-\alpha_{2z} z} \right] e^{ik_y y} \tag{128}$$

$$H_{2y} = \frac{k_{2z}}{\omega\mu_2}\left[E_2^+ e^{-\alpha_{2z}z}\right] e^{ik_y y}$$

$$H_{2z} = \frac{-k_y}{\omega\mu_2}\left[E_2^+ e^{-\alpha_{2z}z}\right] e^{ik_y y} \tag{129}$$

$$< \overline{S}_2 > = \hat{y}\frac{k_y}{2\omega\mu_2}\left[|E_2^+|^2 e^{-2\alpha_{2z}z}|\right] \tag{130}$$

We observe that if Region -1 is a slab medium with positive μ and ϵ, while Region 1 is a negative slab, the guided wave in Region -1 is propagating in the positive $\hat{y}$ direction, and the guided wave in Region 1 is propagating in the negative $\hat{y}$ direction. Thus the guided wave direction is reversed through evanescent coupling with Region 0.

6. LINEAR ANTENNAS IN STRATIFIED MEDIA

6.1. Integral Formulation

The geometrical configuration of the problem is shown in Figure 7. The origin of the coordinate system is placed in the location of the linear antenna, which is in the $\hat{x}$-direction.

$$\overline{J}(\overline{r}') = \hat{x}I\,\delta(y')\delta(z') \tag{131}$$

There are t layers at $z = d_1, d_2, \ldots d_t$ and $s+1$ layers at $z = d_0, d_{-1}, \ldots, d_{-s}$. We shall first assume that all regions contain isotropic media. In region l, we denote the permittivity and permeability by ϵ_l and μ_l. Notice that in region 0, ϵ_0 and μ_0 are not necessarily equal to the free space permittivity and permeability which we denote by ϵ_o and μ_o .

The solution of the electric field vector for the linear antenna in unbounded medium with permittivity ϵ_0 and μ_0 is

$$\begin{aligned}
\overline{E} &= i\omega\mu_0 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dy\,dz\,\overline{\overline{G}}(\overline{r},\overline{r}')\cdot\overline{J}(\overline{r}') \\
&= \hat{x}\; i\omega\mu_0 I\left\{\frac{i}{4}H_0^{(1)}(k\rho)\right\} \\
&= \hat{x}\; i\omega\mu_0 I\left\{\frac{i}{4\pi}\int_{-\infty}^{\infty} dk_y\,\frac{1}{k_{0z}}e^{ik_y y + ik_{0z}|z|}\right\} \\
&= \hat{x}\int_{-\infty}^{\infty} dk_y\,\frac{-\omega\mu_0 I}{4\pi k_{0z}}e^{ik_y y}\begin{Bmatrix} e^{ik_{0z}z} \\ e^{-ik_{0z}z}\end{Bmatrix} \qquad \begin{matrix} z\ge 0 \\ z\le 0\end{matrix}
\end{aligned} \tag{132}$$

We have from the Maxwell equations in source-free regions

$$\overline{H} = \frac{1}{i\omega\mu_0}\nabla\times\overline{E} = \frac{iI}{4\pi}\nabla\times\hat{x}\int_{-\infty}^{\infty} dk_y\,\frac{1}{k_{0z}}e^{ik_y y + ik_{0z}|z|}$$

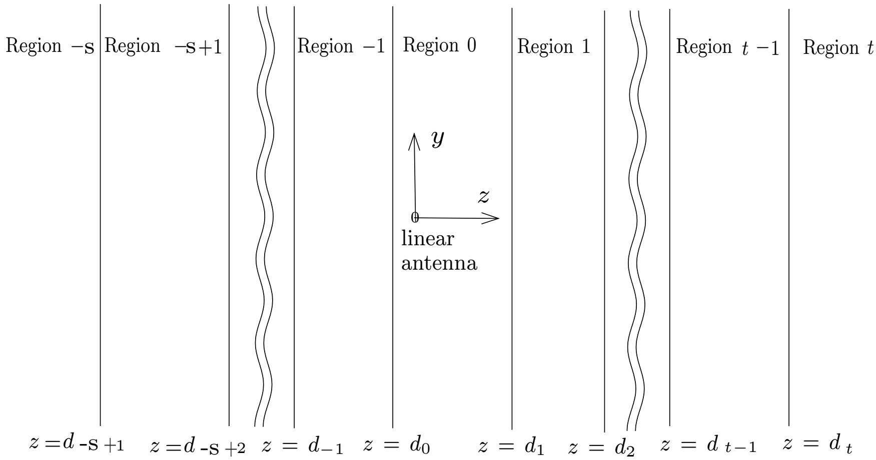

Figure 7. Linear antennas in stratified media.

$$= \hat{y}\frac{-I}{4\pi}\int_{-\infty}^{\infty} dk_y e^{ik_y y}\begin{Bmatrix} e^{ik_{0z}z} \\ -e^{-ik_{0z}z} \end{Bmatrix} \qquad \begin{matrix} z \geq 0 \\ z \leq 0 \end{matrix} \tag{133}$$

$$+\hat{z}\frac{I}{4\pi}\int_{-\infty}^{\infty} dk_y \frac{k_y}{k_{0z}} e^{ik_y y}\begin{Bmatrix} e^{ik_{0z}z} \\ e^{-ik_{0z}z} \end{Bmatrix} \qquad \begin{matrix} z \geq 0 \\ z \leq 0 \end{matrix} \tag{134}$$

The solutions in region l take the following form:

$$E_{lx} = \int_{-\infty}^{\infty} dk_y \left[E_l^+ e^{ik_{lz}z} + E_l^- e^{-ik_{lz}z}\right] e^{ik_y y} \tag{135}$$

$$H_{ly} = \int_{-\infty}^{\infty} dk_y \frac{k_{lz}}{\omega\mu_l}\left[E_l^+ e^{ik_{lz}z} - E_l^- e^{-ik_{lz}z}\right] e^{ik_y y}$$

$$H_{lz} = \int_{-\infty}^{\infty} dk_y \frac{-k_y}{\omega\mu_l}\left[E_l^+ e^{ik_{lz}z} + E_l^- e^{-ik_{lz}z}\right] e^{ik_y y} \tag{136}$$

The boundary conditions of the interfaces require that tangential electric and magnetic field components be continuous for all x and y. At $z = d_{l+1}$, we obtain

$$E_l^+ e^{ik_{lz}d_{l+1}} + E_l^- e^{-ik_{lz}d_{l+1}} = E_{l+1}^+ e^{ik_{(l+1)z}d_{l+1}} + E_{l+1}^- e^{-ik_{(l+1)z}d_{l+1}} \tag{137}$$

$$E_l^+ e^{ik_{lz}d_{l+1}} - E_l^- e^{-ik_{lz}d_{l+1}} = p_{l(l+1)}\left[E_{l+1}^+ e^{ik_{(l+1)z}d_{l+1}} - E_{l+1}^- e^{-ik_{(l+1)z}d_{l+1}}\right] \tag{138}$$

where

$$p_{l(l+1)} = \frac{\mu_l k_{(l+1)\,z}}{\mu_{l+1} k_{lz}} = \frac{1}{p_{(l+1)l}} \tag{139}$$

We now determine the wave amplitudes in region l. From (137)–(138) we can express E_l^+ and E_l^- in terms of E_{l+1}^+ and E_{l+1}^- or express E_{l+1}^+ and E_{l+1}^- in terms of E_l^+ and E_l^-. We find

$$\begin{aligned} E_l^+ e^{ik_{lz}d_{l+1}} &= \frac{1}{2}\left(1 + p_{l(l+1)}\right) \\ &\qquad \left\{E_{l+1}^+ e^{ik_{(l+1)z}d_{l+1}} + R_{l(l+1)} E_{l+1}^- e^{-ik_{(l+1)z}d_{l+1}}\right\} \quad (140) \\ E_l^- e^{-ik_{lz}d_{l+1}} &= \frac{1}{2}\left(1 + p_{l(l+1)}\right) \\ &\qquad \left\{R_{l(l+1)} E_{l+1}^+ e^{ik_{(l+1)z}d_{l+1}} + E_{l+1}^- e^{-ik_{(l+1)z}d_{l+1}}\right\} \quad (141) \end{aligned}$$

for E_l^+ and E_l^- in terms of E_{l+1}^+ and E_{l+1}^-, and

$$\begin{aligned} E_{l+1}^+ e^{ik_{(l+1)z}d_{l+1}} &= \frac{1}{2}(1 + p_{(l+1)l}) \\ &\qquad \left\{E_l^+ e^{ik_{lz}d_{l+1}} + R_{(l+1)\,l} E_l^- e^{-ik_{lz}d_{l+1}}\right\} \quad (142) \\ E_{l+1}^- e^{-ik_{(l+1)z}d_{l+1}} &= \frac{1}{2}(1 + p_{(l+1)l}) \\ &\qquad \left\{R_{(l+1)\,l} E_l^+ e^{ik_{lz}d_{l+1}} + E_l^- e^{-ik_{lz}d_{l+1}}\right\} \quad (143) \end{aligned}$$

for E_{l+1}^+ and E_{l+1}^- in terms of E_l^+ and E_l^-. In (140)–(143), the Fresnel reflection coefficient

$$R_{(l+1)l} = \frac{1 - p_{(l+1)l}}{1 + p_{(l+1)l}} = -R_{l(l+1)} \tag{144}$$

where

$$R_{l(l+1)} = \frac{1 - p_{l(l+1)}}{1 + p_{l(l+1)}} \tag{145}$$

is the reflection coefficient for waves in region l, caused by the boundary separating regions l and $l+1$. The reflection coefficient in region $l+1$, $R_{(l+1)\,l}$, caused by the boundary separating regions $l+1$ and l, is equal to the negative of $R_{l(l+1)}$.

There are altogether $s+t$ boundaries which give rise to $2(s+t)$ equations as shown above. There are altogether $s+t+1$ regions. In regions t and $-s$ we have $E_t^- = 0$ and $E_{-s}^+ = 0$ because there are no waves originating from infinity. Thus we have a total of

$2(s+t+1)-2 = 2(s+t)$ unknowns to be solved from the $2(s+t)$ equations. The wave amplitudes are related to the configurations and the excitation amplitudes of the dipole antenna in region 0. Thus field amplitudes in Region 0 require special attention.

For $z \geq 0$, we notice that $E_t^- = 0$. Letting $l = 0$, we obtain the reflection coefficients $R_{0+} = E_{0+}^-/E_{0+}^+$ in the form of continued fractions. We find

$$R_{0+} = \frac{E_{0+}^-}{E_{0+}^+} = \frac{e^{i2k_{0z}d_1}}{R_{01}} + \frac{\left[1-(1/R_{01})^2\right] e^{i2(k_{0z}+k_{1z})d_1}}{(1/R_{01})e^{i2k_{1z}d_1} + (E_1^-/E_1^+)} \tag{146}$$

where E_1^-/E_1^+ can be expressed in terms of E_2^-/E_2^+ and so on until region t where $E_t^-/E_t^+ = 0$.

For $z \leq 0$, we notice that $E_{-s}^+ = 0$. Letting $l = 0$, we obtain the reflection coefficients $R_{0-} = E_{0-}^+/E_{0-}^-$ in the form of continued fractions. We find

$$R_{0-} = \frac{E_{0-}^+}{E_{0-}^-} = \frac{e^{-i2k_{0z}d_0}}{R_{0(-1)}} + \frac{\left[1-\left(1/R_{0(-1)}\right)^2\right] e^{-i2(k_{0z}+k_{-1z})d_0}}{(1/R_{0(-1)})e^{-i2k_{-1z}d_0} + (E_{-1}^+/E_{-1}^-)} \tag{147}$$

where E_{-1}^+/E_{-1}^- are expressible in terms of E_{-2}^+/E_{-2}^- and so on until region $-s$, where $E_{-s}^+/E_{-s}^- = 0$.

Once the wave amplitudes in region 0 are found, wave amplitudes in other regions can be determined by the use of propagation matrices, and from a set of dual equations for TE waves.

In region 0 it becomes necessary that we distinguish the wave amplitudes in region 0 for $z \geq 0$ from those in region 0 for $z < 0$. For $z > 0$ we use E_{0+}^+, E_{0+}^-; and for $z < 0$ we use E_{0-}^+, E_{0-}^-. Thus we have

$$\left.\begin{array}{ll} E_{0-}^+ = E_0^{+x} & E_{0+}^+ = E_0^{+x} + E_{lin} \\ E_{0-}^- = E_0^{-x} + E_{lin} & E_{0+}^- = E_0^{-x} \\ E_{lin} = -\omega\mu_0 I/4\pi k_{0z} & \end{array}\right\} \tag{148}$$

where E_0^{-x} and E_0^{+x} characterize contributions due to the stratified medium. Let

$$R_{0+} = \frac{E_{0+}^-}{E_{0+}^+} = \frac{E_0^{-x}}{E_0^{+x} + E_{lin}} \tag{149}$$

$$R_{0-} = \frac{E_{0-}^+}{E_{0-}^-} = \frac{E_0^{+x}}{E_0^{-x} + E_{lin}} \tag{150}$$

we find

$$E_0^{+x} = \frac{R_{0-}(1+R_{0+})}{1-R_{0-}R_{0+}} E_{lin} \tag{151}$$

$$E_0^{-x} = \frac{R_{0+}(1+R_{0-})}{1-R_{0-}R_{0+}} E_{lin} \tag{152}$$

$$\left.\begin{array}{ll} E_{0-}^{+} = \dfrac{R_{0-}(1+R_{0+})}{1-R_{0-}R_{0+}} E_{lin} & E_{0+}^{+} = \dfrac{1+R_{0-}}{1-R_{0-}R_{0+}} E_{lin} \\ E_{0-}^{-} = \dfrac{1+R_{0+}}{1-R_{0-}R_{0+}} E_{lin} & E_{0+}^{-} = \dfrac{R_{0+}(1+R_{0-})}{1-R_{0-}R_{0+}} E_{lin} \end{array}\right\} \tag{153}$$

We have expressed the solution in region 0 where the linear antenna is located in terms of superpositions of the primary excitations in the absence of the stratified medium and the homogeneous solutions of the stratified medium in the absence of the source.

6.2. Linear Antenna Between Two Negative Isotropic Slabs

For a linear antenna situated in-between two negative isotropic slabs, we let $\mu_{-2} = \mu_0$, $\epsilon_{-2} = \epsilon_0$; $\mu_{-1} = -\mu_0$, $\epsilon_{-1} = -\epsilon_0$; $\mu_1 = -\mu_0$, $\epsilon_1 = -\epsilon_0$, and $\mu_t = \mu_0$, $\epsilon_t = \epsilon_0$. We find $k_{-2z} = k_{2z} = k_{0z}$, $k_{-1z} = k_{1z} = -k_{0z}$, $p_{(-2)(-1)} = p_{(-1)0} = p_{01} = p_{12} = 1$, $R_{(-2)(-1)} = R_{(-1)0} = R_{01} = R_{12} = R_{0-} = R_{0+} = 0$.

$$\left.\begin{array}{ll} E_{0-}^{+} = 0 & E_{0+}^{+} = E_{lin} \\ E_{0-}^{-} = E_{lin} & E_{0+}^{-} = 0 \end{array}\right\} \tag{154}$$

From (140) to (143), we find

$$E_l^{+} = E_{l+1}^{+} e^{i(k_{(l+1)z} - k_{lz})d_{l+1}} \tag{155}$$

$$E_l^{-} = E_{l+1}^{-} e^{-i(k_{(l+1)z} - k_{lz})d_{l+1}} \tag{156}$$

It follows that

$$\left.\begin{array}{l} E_{-1}^{+} = 0 \\ E_{-1}^{-} = E_{lin} e^{-i(k_{0z} - k_{-1z})d_0} = E_{lin} e^{-i2k_{0z}d_0} \end{array}\right\} \tag{157}$$

$$\left.\begin{array}{l} E_{-2}^{+} = 0 \\ E_{-2}^{-} = E_{lin} e^{-i(k_{-1z} - k_{-2z})d_{-1}} e^{-i2k_{0z}d_0} = E_{lin} e^{-i2k_{0z}(d_0 - d_{-1})} \end{array}\right\} \tag{158}$$

$$\left.\begin{array}{l} E_1^{+} = E_{lin} e^{-i(k_{1z} - k_{0z})d_1} = E_{lin} e^{i2k_{0z}d_1} \\ E_1^{-} = 0 \end{array}\right\} \tag{159}$$

$$\left.\begin{array}{l} E_2^+ = E_{lin} e^{-i(k_{2z}-k_{1z})d_2} e^{i2k_{0z}d_1} = E_{lin} e^{-i2k_{0z}(d_2-d_1)} \\ E_2^- = 0 \end{array}\right\} \tag{160}$$

The field in Region -2 is

$$E_{-2x} = \int_{-\infty}^{\infty} dk_y \left[E_{lin} e^{-ik_{0z}[z+2(d_0-d_{-1})]} \right] e^{ik_y y} \tag{161}$$

$$H_{-2y} = \int_{-\infty}^{\infty} dk_y \frac{k_{-2z}}{\omega\mu_{-2}} \left[-E_{lin} e^{-ik_{0z}[z+2(d_0-d_{-1})]} \right] e^{ik_y y}$$

$$H_{-2z} = \int_{-\infty}^{\infty} dk_y \frac{-k_y}{\omega\mu_{-2}} \left[E_{lin} e^{-ik_{0z}[z+2(d_0-d_{-1})]} \right] e^{ik_y y} \tag{162}$$

The field in Region -1 is

$$E_{-1x} = \int_{-\infty}^{\infty} dk_y \left[E_{lin} e^{ik_{0z}[z-2d_0]} \right] e^{ik_y y} \tag{163}$$

$$H_{-1y} = \int_{-\infty}^{\infty} dk_y \frac{k_{-1z}}{\omega\mu_{-1}} \left[-E_{lin} e^{ik_{0z}[z-2d_0]} \right] e^{ik_y y}$$

$$H_{-1z} = \int_{-\infty}^{\infty} dk_y \frac{-k_y}{\omega\mu_{-1}} \left[E_{lin} e^{ik_{0z}[z-2d_0]} \right] e^{ik_y y} \tag{164}$$

In Region 0 for $z \leq 0$

$$E_{0x} = \int_{-\infty}^{\infty} dk_y \left[E_{lin} e^{-ik_{lz}z} \right] e^{ik_y y} \tag{165}$$

$$H_{0y} = \int_{-\infty}^{\infty} dk_y \frac{k_{0z}}{\omega\mu_0} \left[-E_{lin} e^{-ik_{lz}z} \right] e^{ik_y y}$$

$$H_{0z} = \int_{-\infty}^{\infty} dk_y \frac{-k_y}{\omega\mu_0} \left[E_{lin} e^{-ik_{lz}z} \right] e^{ik_y y} \tag{166}$$

In Region 0 for $z \geq 0$

$$E_{0x} = \int_{-\infty}^{\infty} dk_y \left[E_{lin} e^{ik_{lz}z} \right] e^{ik_y y} \tag{167}$$

$$H_{0y} = \int_{-\infty}^{\infty} dk_y \frac{k_{0z}}{\omega\mu_0} \left[E_{lin} e^{ik_{lz}z} \right] e^{ik_y y}$$

$$H_{0z} = \int_{-\infty}^{\infty} dk_y \frac{-k_y}{\omega\mu_0} \left[E_{lin} e^{ik_{lz}z} \right] e^{ik_y y} \tag{168}$$

In region 1

$$E_{1x} = \int_{-\infty}^{\infty} dk_y \left[E_{lin} e^{-ik_{0z}[z-2d_1]} \right] e^{ik_y y} \tag{169}$$

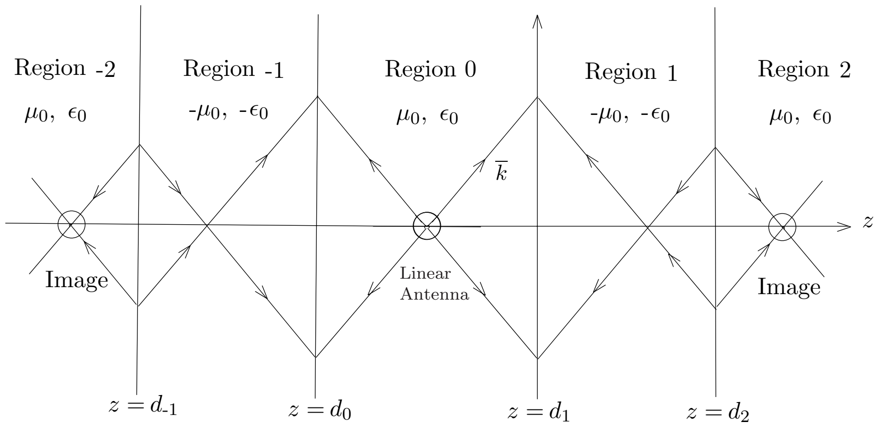

Figure 8. Images of a linear antenna in Region 0.

$$H_{1y} = \int_{-\infty}^{\infty} dk_y \frac{k_{1z}}{\omega\mu_1} \left[E_{lin} e^{-ik_{0z}[z-2d_1]}\right] e^{ik_y y}$$
$$H_{1z} = \int_{-\infty}^{\infty} dk_y \frac{-k_y}{\omega\mu_1} \left[E_{lin} e^{-ik_{0z}[z-2d_1]}\right] e^{ik_y y} \tag{170}$$

In region 2

$$E_{2x} = \int_{-\infty}^{\infty} dk_y \left[E_{lin} e^{ik_{0z}[z-2(d_2-d_1)]}\right] e^{ik_y y} \tag{171}$$
$$H_{2y} = \int_{-\infty}^{\infty} dk_y \frac{k_{2z}}{\omega\mu_2} \left[E_{lin} e^{ik_{0z}[z-2(d_2-d_1)]}\right] e^{ik_y y}$$
$$H_{2z} = \int_{-\infty}^{\infty} dk_y \frac{-k_y}{\omega\mu_2} \left[E_{lin} e^{ik_{0z}[z-2(d_2-d_1)]}\right] e^{ik_y y} \tag{172}$$

It is thus seen that the field in Region 2 is due to a linear antenna situated at $z = 2(d_2 - d_1)$, which is a perfect image of the original line source. Likewise a perfect image is formed in Region -2 and located at $z = 2(d_{-1} - d_0)$.

6.3. Linear Antenna in Front of a Negative Isotropic Slab

For linear antennas in front of stratified media, all regions -1 to $-s$ are absent, we have $E_{0-}^{+} = 0$ and $R_{0-} = 0$. It follows that

$$\left.\begin{array}{ll} E_{0-}^{+} = 0 & E_{0+}^{+} = E_{lin} \\ E_{0-}^{-} = (1 + R_{0+})E_{lin} & E_{0+}^{-} = R0{+}E_{lin} \end{array}\right\} \tag{173}$$

For a linear antenna in front of a slab medium with boundary surfaces at $z = d_1$ and $z = d_2$, we find the reflection coefficient

$$R_{0+} = \frac{R_{01} + R_{12}e^{i2k_{1z}(d_2-d_1)}}{1 + R_{01}R_{12}e^{i2k_{1z}(d_2-d_1)}}e^{i2k_{0z}d_1} \tag{174}$$

the amplitudes inside the slab medium

$$E_1^+ = \frac{2E_{lin}e^{-i(k_{1z}-k_{0z})d_1}}{(1+p_{01})(1+R_{01}R_{1t}e^{i2k_{1z}(d_2-d_1)})} \tag{175}$$

$$E_1^- = \frac{2R_{12}E_{lin}e^{-i(k_{1z}-k_{0z})d_1}e^{i2k_{1z}d_2}}{(1+p_{01})(1+R_{01}R_{1t}e^{i2k_{1z}(d_2-d_1)})} \tag{176}$$

and the transmission coefficient

$$T = \frac{4e^{ik_{0z}d_1}e^{ik_{1z}(d_2-d_1)}e^{-ik_{2z}d_2}}{(1+p_{01})(1+p_{1t})(1+R_{01}R_{1t}e^{i2k_{1z}(d_2-d_1)})} \tag{177}$$

Consider a linear antenna in front of the slab medium,

(A) In region 0; $z \leq 0$

$$E_{0x} = \int_{-\infty}^{\infty} dk_y \left[(1+R_{0+})E_{lin}e^{-ik_{lz}z}\right]e^{ik_y y} \tag{178}$$

$$H_{0y} = \int_{-\infty}^{\infty} dk_y \frac{k_{0z}}{\omega\mu_0}\left[-(1+R_{0+})E_{lin}e^{-ik_{lz}z}\right]e^{ik_y y} \tag{179}$$

$$H_{0z} = \int_{-\infty}^{\infty} dk_y \frac{-k_y}{\omega\mu_0}\left[(1+R_{0+})E_{lin}e^{-ik_{lz}z}\right]e^{ik_y y} \tag{180}$$

(B) In region 0; $z \geq 0$

$$E_{0x} = \int_{-\infty}^{\infty} dk_y E_{lin}\left[e^{ik_{lz}z} + R_{0+}e^{-ik_{lz}z}\right]e^{ik_y y} \tag{181}$$

$$H_{0y} = \int_{-\infty}^{\infty} dk_y \frac{k_{0z}}{\omega\mu_0}E_{lin}\left[e^{ik_{lz}z} - R_{0+}e^{-ik_{lz}z}\right]e^{ik_y y} \tag{182}$$

$$H_{0z} = \int_{-\infty}^{\infty} dk_y \frac{-k_y}{\omega\mu_0}E_{lin}\left[e^{ik_{lz}z} + R_{0+}e^{-ik_{lz}z}\right]e^{ik_y y} \tag{183}$$

(C) In region 1

$$E_{1x} = \int_{-\infty}^{\infty} dk_y \left[E_1^+e^{ik_{lz}z} + E_1^-e^{-ik_{lz}z}\right]e^{ik_y y} \tag{184}$$

$$H_{1y} = \int_{-\infty}^{\infty} dk_y \frac{k_{1z}}{\omega\mu_1} \left[E_1^+ e^{ik_{lz}z} - E_1^- e^{-ik_{lz}z}\right] e^{ik_y y} \tag{185}$$

$$H_{1z} = \int_{-\infty}^{\infty} dk_y \frac{-k_y}{\omega\mu_1} \left[E_1^+ e^{ik_{1z}z} + E_1^- e^{-ik_{1z}z}\right] e^{ik_y y} \tag{186}$$

(D) In region 2, $k_{2z} = k_{0z}$

$$E_{2x} = \int_{-\infty}^{\infty} dk_y \left[TE_{lin} e^{ik_{2z}z}\right] e^{ik_y y} \tag{187}$$

$$H_{2y} = \int_{-\infty}^{\infty} dk_y \frac{k_{2z}}{\omega\mu_2} \left[TE_{lin} e^{ik_z z}\right] e^{ik_y y} \tag{188}$$

$$H_{2z} = \int_{-\infty}^{\infty} dk_y \frac{-k_y}{\omega\mu_2} \left[TE_{lin} e^{ik_{2z}z}\right] e^{ik_y y} \tag{189}$$

For a linear antenna in front of a negative isotropic slab in free space, we let $\mu_1 = -\mu_0$, $\epsilon_1 = -\epsilon_0$, and $\mu_t = \mu_0$, $\epsilon_t = \epsilon_0$. We find $k_{1z} = -k_{0z}$, $k_{2z} = k_{0z}$, $p_{01} = p_{12} = 1$, $R_{01} = R_{12} = R_{0+} = 0$, $E_1^- = 0$, $E_{lin} = -\omega\mu_0 I/4\pi k_{0z}$, $E_1^+ = E_{lin} e^{i2k_{0z}d_1}$, $T = e^{-i2k_{0z}(d_2-d_1)}$.

(A) In region 0; $z \leq 0$

$$E_{0x} = \int_{-\infty}^{\infty} dk_y E_{lin} \left[e^{-ik_{0z}z}\right] e^{ik_y y} \tag{190}$$

$$H_{0y} = \int_{-\infty}^{\infty} dk_y \frac{k_{0z}}{\omega\mu_0} E_{lin} \left[-e^{-ik_{0z}z}\right] e^{ik_y y} \tag{191}$$

$$H_{0z} = \int_{-\infty}^{\infty} dk_y \frac{-k_y}{\omega\mu_0} E_{lin} \left[e^{-ik_{0z}z}\right] e^{ik_y y} \tag{192}$$

(B) In region 0; $z \geq 0$

$$E_{0x} = \int_{-\infty}^{\infty} dk_y E_{lin} \left[e^{ik_{0z}z}\right] e^{ik_y y} \tag{193}$$

$$H_{0y} = \int_{-\infty}^{\infty} dk_y \frac{k_{0z}}{\omega\mu_0} E_{lin} \left[e^{ik_{0z}z}\right] e^{ik_y y} \tag{194}$$

$$H_{0z} = \int_{-\infty}^{\infty} dk_y \frac{-k_y}{\omega\mu_0} E_{lin} \left[e^{ik_{0z}z}\right] e^{ik_y y} \tag{195}$$

$$\begin{aligned} H_0^{(1)}(k\rho) &= \frac{1}{\pi} \int_{-\infty}^{\infty} dk_y \, \frac{1}{k_{0z}} e^{ik_y y + ik_{0z}|z|} \\ &= \frac{1}{\pi} \int_{-\infty}^{\infty} dk_y \, \frac{1}{k_{0z}} e^{ik_y y} \left\{ \begin{array}{c} e^{ik_{0z}z} \\ e^{-ik_{0z}z} \end{array} \right\} \qquad \begin{array}{c} z \geq 0 \\ z \leq 0 \end{array} \end{aligned} \tag{196}$$

(C) In region 1, $d_1 < z < 2d_1$, $k_{0z} \to ik_y$

$$E_{1x} = \int_{-\infty}^{\infty} dk_y E_{lin} e^{-ik_{0z}(z-2d_1)} e^{ik_y y}$$
$$= H_0^{(1)} \left(k\sqrt{y^2 + |z - 2d_1|^2} \right) \tag{197}$$

$$\overline{H}_1 = \frac{-1}{i\omega\mu_0}(\hat{y}\frac{\partial}{\partial z} - \hat{z}\frac{\partial}{\partial y})E_{1x} \tag{198}$$

$$H_{1y} = \int_{-\infty}^{\infty} dk_y \frac{k_{0z}}{\omega\mu_0} E_{lin} e^{-ik_{0z}(z-2d_1)} e^{ik_y y} \tag{199}$$

$$H_{1z} = \int_{-\infty}^{\infty} dk_y \frac{k_y}{\omega\mu_0} E_{lin} e^{-ik_{0z}(z-2d_1)} e^{ik_y y} \tag{200}$$

(D) In region 1, $2d_1 < z < d_2$, $k_y \to -k_y$, $k_{0z} \to -ik_y$

$$E_{1x} = \int_{-\infty}^{\infty} dk_y E_{lin} e^{-ik_{0z}(z-2d_1)} e^{-ik_y y} \tag{201}$$

$$H_{1y} = \int_{-\infty}^{\infty} dk_y \frac{k_{0z}}{\omega\mu_0} E_{lin} e^{-ik_{0z}(z-2d_1)} e^{-ik_y y} \tag{202}$$

$$H_{1z} = -\int_{-\infty}^{\infty} dk_y \frac{k_y}{\omega\mu_0} E_{lin} e^{-ik_{0z}(z-2d_1)} e^{-ik_y y} \tag{203}$$

(E) In region 2, $d_2 < z < 2(d_2 - d_1)$, $k_y \to -k_y$, , $k_{0z} \to -ik_y$

$$E_{2x} = \int_{-\infty}^{\infty} dk_y E_{lin} e^{ik_{0z}(z-2(d_2-d_1))} e^{-ik_y y} \tag{204}$$

$$H_{2y} = \int_{-\infty}^{\infty} dk_y \frac{k_{0z}}{\omega\mu_0} E_{lin} e^{ik_{0z}(z-2(d_2-d_1))} e^{-ik_y y} \tag{205}$$

$$H_{2z} = \int_{-\infty}^{\infty} dk_y \frac{k_y}{\omega\mu_0} E_{lin} e^{ik_{0z}(z-2(d_2-d_1)} e^{-ik_y y} \tag{206}$$

(F) In region 2, $2(d_2 - d_1) < z$, $k_{0z} \to ik_y$

$$E_{2x} = \int_{-\infty}^{\infty} dk_y E_{lin} e^{ik_{0z}(z-2(d_2-d_1))} e^{ik_y y} \tag{207}$$

$$H_{2y} = \int_{-\infty}^{\infty} dk_y \frac{k_{0z}}{\omega\mu_0} E_{lin} e^{ik_{0z}(z-2(d_2-d_1))} e^{ik_y y} \tag{208}$$

$$H_{2z} = \int_{-\infty}^{\infty} dk_y \frac{-k_y}{\omega\mu_0} E_{lin} e^{ik_{0z}(z-2(d_2-d_1)} e^{ik_y y} \tag{209}$$

It is seen that in the transmitted region, the field originates from a linear antenna located at $z = 2(d_2 - d_1)$, which is a perfect image of the original antenna.

7. DIPOLE ANTENNAS IN STRATIFIED MEDIA

7.1. Hertzian Electric and Magnetic Dipoles

The most fundamental model for radiating structures is a Hertzian electric dipole which consists of a current-carrying element with an infinitesimal length l. Denoting the current dipole moment with Il, the current density $\overline{J}(\overline{r})$ of a Hertzian dipole pointing in the $\hat{z}$ direction and located at the origin is

$$\overline{J}(\overline{r}') = \hat{z} Il\, \delta(\overline{r}') \tag{210}$$

The exact expressions for the electric field vector $\overline{E}(\overline{r})$ for the Hertzian dipole is calculated to be

$$\begin{aligned}
\overline{E}(\overline{r}) &= i\omega\mu \left[\overline{\overline{I}} + \frac{1}{k^2}\nabla\nabla \right] \cdot \iiint d\overline{r}' \frac{e^{ik|\overline{r}-\overline{r}'|}}{4\pi|\overline{r}-\overline{r}'|} \hat{z} Il\delta(\overline{r}') \\
&= i\omega\mu Il \left[\hat{z} + \frac{1}{k^2}\nabla\frac{\partial}{\partial z} \right] \frac{e^{ikr}}{4\pi r} \\
&= \frac{i\omega\mu Il}{4\pi} \left\{ \hat{z}\frac{1}{r}e^{ikr} + \frac{1}{k^2}(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}) \left[(\frac{ikz}{r^2} + \frac{-z}{r^3})e^{ikr} \right] \right\} \\
&= \frac{i\omega\mu Il}{4\pi} \left\{ \hat{z}\frac{1}{r}\left[1 + \frac{i}{kr} - \frac{1}{k^2r^2} \right] - \hat{x}\frac{xz}{r^3}\left[1 + \frac{3i}{kr} - \frac{3}{k^2r^2} \right] \right. \\
&\qquad \left. -\hat{y}\frac{yz}{r^3}\left[1 + \frac{3i}{kr} - \frac{3}{k^2r^2} \right] - \hat{z}\frac{zz}{r^3}\left[1 + \frac{3i}{kr} - \frac{3}{k^2r^2} \right] \right\} e^{ikr} \\
&= \frac{i\omega\mu Il}{4\pi r} \left\{ \hat{z}\left[1 + \frac{i}{kr} - \frac{1}{k^2r^2} \right] - \hat{z}\frac{zz}{r^2}\left[1 + \frac{3i}{kr} - \frac{3}{k^2r^2} \right] \right. \\
&\qquad \left. - \hat{\rho}\frac{z\rho}{r^2}\left[1 + \frac{3i}{kr} - \frac{3}{k^2r^2} \right] \right\} e^{ikr} \\
&= \frac{i\omega\mu e^{ikr}}{4\pi r} Il \left\{ \hat{z}\left[1 + \frac{i}{kr} - \frac{1}{k^2r^2} \right] - \hat{r}\frac{z}{r}\left[1 + \frac{3i}{kr} - \frac{3}{k^2r^2} \right] \right\}
\end{aligned} \tag{211}$$

Notice that with $g(r) = e^{ikr}/4\pi r$ and $\partial g(r)/\partial z = (ik - 1/r)\cos\theta\, g(r)$. To cast in spherical coordinates, note that $\hat{z} = \hat{r}\cos\theta - \hat{\theta}\sin\theta$ and $z = r\cos\theta$. We find from (211)

$$\overline{E}(\overline{r}) = -\frac{i\omega\mu e^{ikr}}{4\pi r} Il \left\{ \hat{r}\left[\frac{i}{kr} + \left(\frac{i}{kr}\right)^2 \right] 2\cos\theta + \hat{\theta}\left[1 + \frac{i}{kr} + \left(\frac{i}{kr}\right)^2 \right] \sin\theta \right\} \tag{212}$$

The magnetic field follows from Faraday's law

$$\overline{H}(\overline{r}) = \frac{1}{i\omega\mu}\nabla\times\overline{E} = -\hat{\phi}\, ikIl\frac{e^{ikr}}{4\pi r}\left[1 + \frac{i}{kr} \right] \sin\theta \tag{213}$$

The complex Poynting power density is calculated by taking the cross product of $\overline{E}$ and the complex conjugate of $\overline{H}$

$$\overline{S} = \overline{E}\times\overline{H}^{*}=\eta \left[\frac{kIl}{4\pi r}\right]^2 \left\{\hat{r}\left[1-\left(\frac{i}{kr}\right)^3\right]\sin^2\theta-\hat{\theta}\left[\left(\frac{i}{kr}\right)-\left(\frac{i}{kr}\right)^3\right]\sin 2\theta\right\} \tag{214}$$

The time-average Poynting power density is

$$<\overline{S}>= \frac{1}{2}\mathrm{Re}\{\overline{S}\} = \hat{r}\frac{\eta}{2}\left[\frac{kIl}{4\pi r}\right]^2 \sin^2\theta \tag{215}$$

Let the electric current moment be in a general direction, we write $\overline{I}l = \hat{x}I_x l + \hat{y}I_y l + \hat{z}I_z l$. From (211), (212), and (213) we find

$$\overline{E}(\overline{r}) = \frac{i\omega\mu e^{ikr}}{4\pi r}\left\{\overline{I}l\left(1+\frac{i}{kr}-\frac{1}{k^2r^2}\right)-\hat{r}(\hat{r}\cdot\overline{I}l)\left[1+\frac{3i}{kr}-\frac{3}{k^2r^2}\right]\right\} \tag{216}$$

$$\overline{H}(\overline{r}) = \hat{r}\times\overline{I}l\frac{ike^{ikr}}{4\pi r}\left[1+\frac{i}{kr}\right] \tag{217}$$

Notice that $\hat{r} = \hat{z}z/r + \hat{\rho}\rho/r = \hat{x}x/r + \hat{y}y/r + \hat{z}z/r$. The Poynting vector is

$$\overline{S} = \overline{E}\times\overline{H}^{*} = \eta\left[\frac{k}{4\pi r}\right]^2$$
$$\left\{\hat{r}(Il)^2(1+\frac{i}{k^3r^3})-\hat{r}(\hat{r}\cdot\overline{I}l)^2\left[1+\frac{2i}{kr}+\frac{3i}{k^3r^3}\right]+(\hat{r}\cdot\overline{I}l)\overline{I}l\left[\frac{2i}{kr}+\frac{2i}{k^3r^3}\right]\right\} \tag{218}$$

The time-average Poynting power density is

$$<\overline{S}>= \frac{1}{2}\mathrm{Re}\{\overline{S}\} = \hat{r}\ \eta k^2\frac{[(Il)^2-(\hat{r}\cdot\overline{I}l)^2]}{2(4\pi r)^2} \tag{219}$$

In a negative isotropic medium when $k = -|k|$ the phase velocity of the radiated wave points towards the dipole while the Poynting power is propagating in the direction of increasing $\overline{r}$.

The dual of a Hertzian electric dipole is a magnetic dipole. A Hertzian magnetic dipole can be realized with the model of a small current loop with area A and carrying current I. The correspondence between the electric and magnetic dipoles can be quantified by letting the Hertzian dipole moment Il to be [7]

$$(Il)_e = (ikIA)_m \tag{220}$$

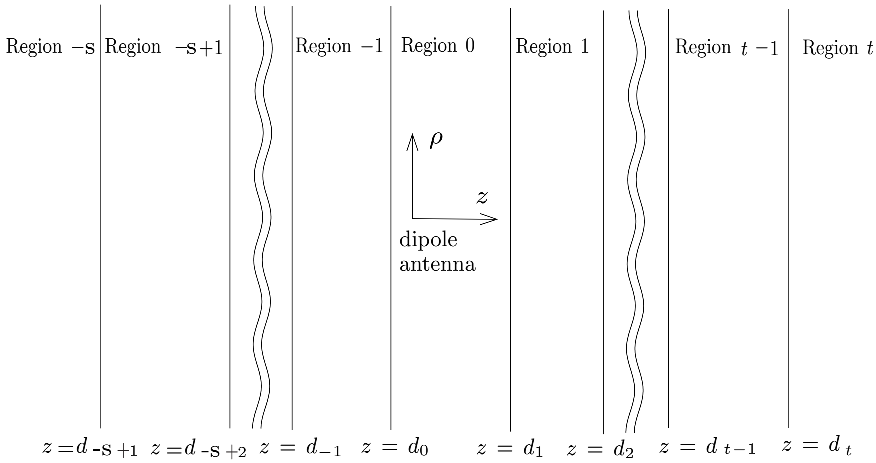

Figure 9. Hertzian dipole source in stratified media.

$$\overline{E}_m = \eta \overline{H}_e \tag{221}$$

$$\overline{H}_m = -\frac{\overline{E}_e}{\eta} \tag{222}$$

where the subscripts e and m are used to denote the electric dipole and the current loop, respectively. The solutions for a small current loop thus follow from the solutions for electric dipoles.

7.2. Integral Formulation for Dipoles in Stratified Media

The geometrical configuration of the problem is shown in Figure 9. The origin of the coordinate system is placed in the location of the dipole which can be a z-directed electric dipole (ZED), a z-directed magnetic dipole (ZMD), an x-directed electric dipole (XED), an y-directed electric dipole (YED), an x-directed magnetic dipole (XMD). or an y-directed magnetic dipole (YMD). There are t layers at $z = d_1, d_2, \ldots d_t$ and $s+1$ layers at $z = d_0, d_{-1}, \ldots, d_{-s+1}$. We shall first assume that all regions contain isotropic media. In region l, we denote the permittivity and permeability by ϵ_l and μ_l. Notice that in region $0, \epsilon_0$, and μ_0 are not necessarily equal to the free space permittivity and permeability which we denote by ϵ_o and μ_o .

Making use of cylindrical coordinate system, the integrands of transverse field components $\overline{E}_s = \hat{\rho} E_\rho + \hat{\phi} E_\phi$ and $\overline{H}_s = \hat{\rho} H_\rho + \hat{\phi} H_\phi$

are derived from those of the longitudinal components E_z and H_z. Let

$$E_z = \int_{-\infty}^{\infty} dk_\rho \, E_z(k_\rho) \tag{223}$$

$$H_z = \int_{-\infty}^{\infty} dk_\rho \, H_z(k_\rho) \tag{224}$$

We have from the Maxwell equations in source-free regions

$$\overline{E}_s(k_\rho) = \frac{1}{k_\rho^2}\left[\nabla_s \frac{\partial}{\partial z} E_z(k_\rho) + i\omega\mu_l \nabla_s \times \overline{H}_z(k_\rho)\right] \tag{225}$$

$$\overline{H}_s(k_\rho) = \frac{1}{k_\rho^2}\left[\nabla_s \frac{\partial}{\partial z} H_z(k_\rho) - i\omega\epsilon_l \nabla_s \times \overline{E}_z(k_\rho)\right] \tag{226}$$

The fields of a dipole radiating in unbounded space with permittivity ϵ_0 and μ_0 can be transformed from spherical coordinates to cylindrical coordinates by using the Sommerfeld identity

$$\frac{e^{ik_0 r}}{r} = \frac{i}{2}\int_{-\infty}^{\infty} dk_\rho \, \frac{k_\rho}{k_{0z}} H_0^{(1)}(k_\rho \rho) e^{ik_{0z}|z|} \tag{227}$$

From the previsous section, we find for a Hertzian electric dipole $Il\delta(\overline{r})$ situated at the origin,

$$\begin{aligned} \overline{E}(\overline{r}) &= \frac{i\omega\mu}{4\pi}\left[\overline{\overline{I}} + \frac{1}{k^2}\nabla\nabla\right] \cdot \overline{I}l \, \frac{e^{ikr}}{r} \\ &= \frac{-\omega\mu}{8\pi}\left[\overline{\overline{I}} + \frac{1}{k^2}\nabla\nabla\right] \cdot \overline{I}l \int_{-\infty}^{\infty} dk_\rho \, \frac{k_\rho}{k_{0z}} H_0^{(1)}(k_\rho \rho) e^{ik_{0z}|z|} \end{aligned} \tag{228}$$

$$\overline{H}(\overline{r}) = \frac{1}{i\omega\mu}\nabla \times \overline{E} = \frac{i}{8\pi}\nabla \times \overline{I}l \int_{-\infty}^{\infty} dk_\rho \, \frac{k_\rho}{k_{0z}} H_0^{(1)}(k_\rho \rho) e^{ik_{0z}|z|} \tag{229}$$

Noticing that $H_0^{(1)\prime}(k_\rho \rho) = -H_1^{(1)}(k_\rho \rho)$, we find

(A) z-directed electric dipole (ZED): $\overline{I}l = \hat{z}Il$

$$H_z = 0; \qquad E_z = \int_{-\infty}^{\infty} dk_\rho \, E_{zed} \begin{Bmatrix} e^{ik_{0z}z} \\ e^{-ik_{0z}z} \end{Bmatrix} H_0^{(1)}(k_\rho \rho) \qquad \begin{matrix} z \geq 0 \\ z \leq 0 \end{matrix} \tag{230}$$

with

$$E_{zed} = -\frac{Il k_\rho^3}{8\pi\omega\epsilon_0 k_{0z}} \tag{231}$$

where Il is the electric dipole moment.

(B) x-directed electric dipole (XED): $\overline{I}l = \hat{x}Il$

$$E_z = \int_{-\infty}^{\infty} dk_\rho \, E_{xed} \left\{ \begin{array}{c} e^{ik_{0z}z} \\ -e^{-ik_{0z}z} \end{array} \right\} H_1^{(1)}(k_\rho \rho) \cos\phi \qquad \begin{array}{c} z \geq 0 \\ z \leq 0 \end{array} \tag{232}$$

$$H_z = \int_{-\infty}^{\infty} dk_\rho \, H_{xed} \left\{ \begin{array}{c} e^{ik_{0z}z} \\ e^{-ik_{0z}z} \end{array} \right\} H_1^{(1)}(k_\rho \rho) \sin\phi \qquad \begin{array}{c} z \geq 0 \\ z \leq 0 \end{array} \tag{233}$$

with

$$E_{xed} = i\frac{Ilk_\rho^2}{8\pi\omega\epsilon_0}; \quad H_{xed} = i\frac{Ilk_\rho^2}{8\pi k_{0z}} \tag{234}$$

(C) y-directed electric dipole (YED): $\overline{I}l = \hat{y}Il$

$$E_z = \int_{-\infty}^{\infty} dk_\rho \left\{ \begin{array}{c} E_{yed} \, e^{ik_{0z}z} \\ -E_{yed} \, e^{-ik_{0z}z} \end{array} \right\} H_1^{(1)}(k_\rho \rho) \sin\phi \qquad \begin{array}{c} z \geq 0 \\ z \leq 0 \end{array} \tag{235}$$

$$H_z = -\int_{-\infty}^{\infty} dk_\rho \, H_{yed} \left\{ \begin{array}{c} e^{ik_{0z}z} \\ e^{-ik_{0z}z} \end{array} \right\} H_1^{(1)}(k_\rho \rho) \cos\phi \qquad \begin{array}{c} z \geq 0 \\ z \leq 0 \end{array} \tag{236}$$

with

$$E_{yed} = E_{xed} = i\frac{Ilk_\rho^2}{8\pi\omega\epsilon_0}; \quad H_{yed} = H_{xed} = i\frac{Ilk_\rho^2}{8\pi k_{0z}} \tag{237}$$

The results of the y-directed electric dipole (YED) can be obtained from that for XED by replacing ϕ with $-\pi/2 + \phi$.

Notice that the magnetic dipoles produce fields which are duals of those produced by the corresponding electric dipoles. The results for the magnetic dipoles ZMD, XMD, and YMD, can be obtained by the replacement $\overline{E} \rightarrow \overline{H}$, $\overline{H} \rightarrow -\overline{E}$, $\mu_0 \rightleftarrows \epsilon_0$, and $Il \rightarrow i\omega\mu_0 IA$. We note in particular that at $z = 0$, the following field components vanish:

(A) z-directed electric dipole (ZED)

$$E_\rho = 0 \tag{238}$$

(B) x-directed electric dipole (XED)

$$E_z = H_\rho = H_\phi = 0 \tag{239}$$

This is seen from (225)–(226) and by noting from (227) that

$$\frac{\partial}{\partial z}\frac{e^{ik_0 r}}{r} = 0 \qquad \text{at} \quad z = 0 \tag{240}$$

We now consider dipole sources placed in Region 0 of the stratified isotropic medium (Fig. 9). We assume that all regions contain isotropic media. In region l, we denote the permittivity and permeability by ϵ_l and μ_l. The solutions to the wave equations can be written as superpositions of TE and TM wave components. Let E_l^+ and E_l^- denote amplitudes for the TM waves and H_l^+ and H_l^- denote amplitudes for the TE waves. We find in region l the following solutions:

(A) z-directed electric dipole (ZED): $\overline{I}l = \hat{z}Il$

$$E_{lz} = \int_{-\infty}^{\infty} dk_\rho \left[E_l^+ e^{ik_{lz}z} + E_l^- e^{-ik_{lz}z}\right] H_0^{(1)}(k_\rho \rho) \quad (241)$$

$$E_{l\rho} = \int_{-\infty}^{\infty} dk_\rho \frac{ik_{lz}}{k_\rho} \left[E_l^+ e^{ik_{lz}z} - E_l^- e^{-ik_{lz}z}\right] H_0^{(1)\prime}(k_\rho \rho) \quad (242)$$

$$H_{l\phi} = \int_{-\infty}^{\infty} dk_\rho \frac{i\omega\epsilon_l}{k_\rho} \left[E_l^+ e^{ik_{lz}z} + E_l^- e^{-ik_{lz}z}\right] H_0^{(1)\prime}(k_\rho \rho) \quad (243)$$

(B) x-directed electric dipole (XED): $\overline{I}l = \hat{x}Il$

$$E_{lz} = \int_{-\infty}^{\infty} dk_\rho \left[E_l^+ e^{ik_{lz}z} + E_l^- e^{-ik_{lz}z}\right] H_1^{(1)}(k_\rho \rho) \cos\phi \quad (244)$$

$$\begin{aligned} E_{l\rho} &= \int_{-\infty}^{\infty} dk_\rho \frac{ik_{lz}}{k_\rho} \left[E_l^+ e^{ik_{lz}z} - E_l^- e^{-ik_{lz}z}\right] H_1^{(1)\prime}(k_\rho \rho) \cos\phi \\ &\quad + \int_{-\infty}^{\infty} dk_\rho \frac{i\omega\mu_l}{k_\rho^2 \rho} \left[H_l^+ e^{ik_{lz}z} + H_l^- e^{-ik_{lz}z}\right] H_1^{(1)}(k_\rho \rho)[\cos\phi] \quad (245) \end{aligned}$$

$$\begin{aligned} E_{l\phi} &= \int_{-\infty}^{\infty} dk_\rho \frac{ik_{lz}}{k_\rho^2 \rho} \left[E_l^+ e^{ik_{lz}z} - E_l^- e^{-ik_{lz}z}\right] H_1^{(1)}(k_\rho \rho)[-\sin\phi] \\ &\quad + \int_{-\infty}^{\infty} dk_\rho \frac{-i\omega\mu_l}{k_\rho} \left[H_l^+ e^{ik_{lz}z} + H_l^- e^{-ik_{lz}z}\right] H_1^{(1)\prime}(k_\rho \rho) \sin\phi \quad (246) \end{aligned}$$

$$H_{lz} = \int_{-\infty}^{\infty} dk_\rho \left[H_l^+ e^{ik_{lz}z} + H_l^- e^{-ik_{lz}z}\right] H_1^{(1)}(k_\rho \rho) \sin(\phi) \quad (247)$$

$$\begin{aligned} H_{l\rho} &= \int_{-\infty}^{\infty} dk_\rho \frac{ik_{lz}}{k_\rho} \left[H_l^+ e^{ik_{lz}z} - H_l^- e^{-ik_{lz}z}\right] H_1^{(1)\prime}(k_\rho \rho) \sin\phi \\ &\quad + \int_{-\infty}^{\infty} dk_\rho \frac{-i\omega\epsilon_l}{k_\rho^2 \rho} \left[E_l^+ e^{ik_{lz}z} + E_l^- e^{-ik_{lz}z}\right] H_1^{(1)}(k_\rho \rho)[-\sin\phi] \quad (248) \end{aligned}$$

$$\begin{aligned} H_{l\phi} &= \int_{-\infty}^{\infty} dk_\rho \frac{ik_{lz}}{k_\rho^2 \rho} \left[H_l^+ e^{ik_{lz}z} - H_l^- e^{-ik_{lz}z}\right] H_1^{(1)}(k_\rho \rho)[\cos\phi] \\ &\quad + \int_{-\infty}^{\infty} dk_\rho \frac{i\omega\epsilon_l}{k_\rho} \left[E_l^+ e^{ik_{lz}z} + E_l^- e^{-ik_{lz}z}\right] H_1^{(1)\prime}(k_\rho \rho) \cos\phi \quad (249) \end{aligned}$$

where $H_n^{(1)}(k_\rho \rho)$ is the nth order Hankel function of the first kind and $H_n^{(1)\prime}(k_\rho \rho)$ denotes the derivative of $H_n^{(1)}(\xi)$ with respect to its argument ξ. The integrands of transverse field components $\overline{E}_s = \hat{\rho}E_\rho + \hat{\phi}E_\phi$ and $\overline{H}_s = \hat{\rho}H_\rho + \hat{\phi}H_\phi$ are derived from those of the longitudinal components E_z and H_z.

In region 0 it becomes necessary that we distinguish the wave amplitudes in region 0 for $z \geq 0$ from those in region 0 for $z < 0$. For $z > 0$ we use E_{0+}^{+}, E_{0+}^{-}, H_{0+}^{+}, and H_{0+}^{-}; and for $z < 0$ we use E_{0-}^{+}, E_{0-}^{-}, H_{0-}^{+}, and H_{0-}^{-}.

(A) z-directed electric dipole (ZED)

$$\left.\begin{array}{ll} E_{0-}^{+} = E_0^{+z} & E_{0+}^{+} = E_0^{+z} + E_{zed} \\ E_{0-}^{-} = E_0^{-z} + E_{zed} & E_{0+}^{-} = E_0^{-z} \\ H_{0-}^{+} = H_{0-}^{-} = 0 & H_{0+}^{+} = H_{0+}^{-} = 0 \end{array}\right\} \tag{250}$$

where E_0^{-z} and E_0^{+z} characterize contributions due to the stratified medium. Let

$$R_{0+}^{TM} = \frac{E_{0+}^{-}}{E_{0+}^{+}} = \frac{E_0^{-z}}{E_0^{+z} + E_{zed}} \tag{251}$$

$$R_{0-}^{TM} = \frac{E_{0-}^{+}}{E_{0-}^{-}} = \frac{E_0^{+z}}{E_0^{-z} + E_{zed}} \tag{252}$$

we find

$$E_0^{+z} = \frac{R_{0-}^{TM}(1 + R_{0+}^{TM})}{1 - R_{0-}^{TM} R_{0+}^{TM}} E_{zed} \tag{253}$$

$$E_0^{-z} = \frac{R_{0+}^{TM}(1 + R_{0-}^{TM})}{1 - R_{0-}^{TM} R_{0+}^{TM}} E_{zed} \tag{254}$$

$$\left.\begin{array}{ll} E_{0-}^{+} = \dfrac{R_{0-}^{TM}(1 + R_{0+}^{TM})}{1 - R_{0-}^{TM} R_{0+}^{TM}} E_{zed} & E_{0+}^{+} = \dfrac{1 + R_{0-}^{TM}}{1 - R_{0-}^{TM} R_{0+}^{TM}} E_{zed} \\ E_{0-}^{-} = \dfrac{1 + R_{0+}^{TM}}{1 - R_{0-}^{TM} R_{0+}^{TM}} E_{zed} & E_{0+}^{-} = \dfrac{R_{0+}^{TM}(1 + R_{0-}^{TM})}{1 - R_{0-}^{TM} R_{0+}^{TM}} E_{zed} \\ H_{0-}^{+} = H_{0-}^{-} = 0 & H_{0+}^{+} = H_{0+}^{-} = 0 \end{array}\right\} \tag{255}$$

(B) x-directed electric dipole (XED)

$$\left.\begin{array}{ll} E_{0-}^{+} = E_0^{+x} & E_{0+}^{+} = E_0^{+x} + E_{xed} \\ E_{0-}^{-} = E_0^{-x} - E_{xed} & E_{0+}^{-} = E_0^{-x} \\ H_{0-}^{+} = H_0^{+x} & H_{0+}^{+} = H_0^{+x} + H_{xed} \\ H_{0-}^{-} = H_0^{-x} + H_{xed} & H_{0+}^{-} = H_0^{-x} \end{array}\right\} \quad (256)$$

where E_0^{-z}, E_0^{+z}, H_0^{-z}, and H_0^{+z} characterize contributions due to the stratified medium. Let

$$R_{0+}^{TM} = \frac{E_{0+}^{-}}{E_{0+}^{+}} = \frac{E_0^{-x}}{E_0^{+z} + E_{xed}} \quad (257)$$

$$R_{0-}^{TM} = \frac{E_{0-}^{+}}{E_{0-}^{-}} = \frac{E_0^{+x}}{E_0^{-x} - E_{xed}} \quad (258)$$

$$R_{0+}^{TE} = \frac{H_{0+}^{-}}{H_{0+}^{+}} = \frac{H_0^{-x}}{H_0^{+x} + H_{xed}} \quad (259)$$

$$R_{0-}^{TE} = \frac{H_{0-}^{+}}{H_{0-}^{-}} = \frac{H_0^{+x}}{H_0^{-x} + H_{xed}} \quad (260)$$

we find

$$E_0^{+x} = -\frac{R_{0-}^{TM}(1 - R_{0+}^{TM})}{1 - R_{0-}^{TM} R_{0+}^{TM}} E_{xed} \quad (261)$$

$$E_0^{-x} = \frac{R_{0+}^{TM}(1 - R_{0-}^{TM})}{1 - R_{0-}^{TM} R_{0+}^{TM}} E_{xed} \quad (262)$$

$$H_0^{+x} = \frac{R_{0-}^{TE}(1 + R_{0+}^{TE})}{1 - R_{0-}^{TE} R_{0+}^{TE}} H_{xed} \quad (263)$$

$$H_0^{-x} = \frac{R_{0+}^{TE}(1 + R_{0-}^{TE})}{1 - R_{0-}^{TE} R_{0+}^{TE}} H_{xed} \quad (264)$$

$$\left.\begin{array}{ll} E_{0-}^{+} = -\dfrac{R_{0-}^{TM}(1 - R_{0+}^{TM})}{1 - R_{0-}^{TM} R_{0+}^{TM}} E_{xed} & E_{0+}^{+} = \dfrac{1 - R_{0-}^{TM}}{1 - R_{0-}^{TM} R_{0+}^{TM}} E_{xed} \\ E_{0-}^{-} = -\dfrac{1 - R_{0+}^{TM}}{1 - R_{0-}^{TM} R_{0+}^{TM}} E_{xed} & E_{0+}^{-} = \dfrac{R_{0+}^{TM}(1 - R_{0-}^{TM})}{1 - R_{0-}^{TM} R_{0+}^{TM}} E_{xed} \\ H_{0-}^{+} = \dfrac{R_{0-}^{TE}(1 + R_{0+}^{TE})}{1 - R_{0-}^{TE} R_{0+}^{TE}} H_{xed} & H_{0+}^{+} = \dfrac{1 + R_{0-}^{TE}}{1 - R_{0-}^{TE} R_{0+}^{TE}} H_{xed} \\ H_{0-}^{-} = \dfrac{1 + R_{0+}^{TE}}{1 - R_{0-}^{TE} R_{0+}^{TE}} H_{xed} & H_{0+}^{-} = \dfrac{R_{0+}^{TE}(1 + R_{0-}^{TE})}{1 - R_{0-}^{TE} R_{0+}^{TE}} H_{xed} \end{array}\right\} \quad (265)$$

We have expressed the solution in region 0 where the dipoles are located in terms of superpositions of the primary excitations in the absence of the stratified medium and the homogeneous solutions of the stratified medium in the absence of the source. It is easily shown that they satisfy the boundary conditions at $z = 0$ by remembering the vanishing field components as listed in (238)–(239) for the primary excitations.

The boundary conditions of the interfaces require that tangential electric and magnetic field components be continuous for all ρ and ϕ. At $z = d_{l+1}$, we obtain

$$k_{lz}\left(E_l^+ e^{ik_{lz}d_{l+1}} - E_l^- e^{-ik_{lz}d_{l+1}}\right) = k_{(l+1)\,z}\left(E_{l+1}^+ e^{ik_{(l+1)\,z}d_{l+1}} - E_{l+1}^- e^{-ik_{(l+1)\,z}d_{l+1}}\right) \quad (266)$$

$$\epsilon_l\left(E_l^+ e^{ik_{lz}d_l} + E_l^- e^{-ik_{lz}d_{l+1}}\right) = \epsilon_{(l+1)}\left(E_{l+1}^+ e^{ik_{(l+1)\,z}d_{l+1}} + E_{l+1}^- e^{-ik_{(l+1)\,z}d_{l+1}}\right) \quad (267)$$

$$k_{lz}\left(H_l^+ e^{ik_{lz}d_{l+1}} - H_l^- e^{-ik_{lz}d_{l+1}}\right) = k_{(l+1)\,z}\left(H_{l+1}^+ e^{ik_{(l+1)\,z}d_{l+1}} - H_{l+1}^- e^{-ik_{(l+1)\,z}d_{l+1}}\right) \quad (268)$$

$$\mu_l\left(H_l^+ e^{ik_{lz}d_{l+1}} + H_l^- e^{-ik_{lz}d_{l+1}}\right) = \mu_{(l+1)}\left(H_{l+1}^+ e^{ik_{(l+1)\,z}d_{l+1}} + H_{l+1}^- e^{-ik_{(l+1)\,z}d_{l+1}}\right) \quad (269)$$

We now determine the wave amplitudes in region l. For TM waves, (266)–(267) can be solved to express E_l^+ and E_l^- in terms of E_{l-1}^+ and E_{l-1}^- or to express E_{l-1}^+ and E_{l-1}^- in terms of E_l^+ and E_l^- . We find

$$E_l^+ e^{ik_{lz}d_{l+1}} = \frac{1}{2}\left(\frac{\epsilon_{l+1}}{\epsilon_l} + \frac{k_{(l+1)\,z}}{k_{lz}}\right)\left[E_{l+1}^+ e^{ik_{(l+1)\,z}d_{l+1}} + R_{l(l+1)}^{TM} E_{l+1}^- e^{-ik_{(l+1)\,z}d_{l+1}}\right] \quad (270a)$$

$$E_l^- e^{-ik_{lz}d_{l+1}} = \frac{1}{2}\left(\frac{\epsilon_{l+1}}{\epsilon_l} + \frac{k_{(l+1)\,z}}{k_{lz}}\right)\left[R_{l(l+1)}^{TM} E_{l+1}^+ e^{ik_{(l+1)\,z}d_{l+1}} + E_{l+1}^- e^{-ik_{(l+1)\,z}d_{l+1}}\right] \quad (270b)$$

for E_l^+ and E_l^- in terms of E_{l+1}^+ and E_{l+1}^-, and

$$E_{l+1}^+ e^{ik_{(l+1)\,z}d_{l+1}} = \frac{1}{2}\left(\frac{\epsilon_l}{\epsilon_{l+1}} + \frac{k_{lz}}{k_{(l+1)\,z}}\right)\left[E_l^+ e^{ik_{lz}d_{l+1}} + R_{(l+1)\,l}^{TM} E_l^- e^{-ik_{lz}d_{l+1}}\right] \quad (271a)$$

$$E_{l+1}^{-}e^{-ik_{(l+1)z}d_{l+1}} = \frac{1}{2}\left(\frac{\epsilon_l}{\epsilon_{l+1}} + \frac{k_{lz}}{k_{(l+1)z}}\right)$$
$$\left[R_{(l+1)l}^{TM}E_l^{+}e^{ik_{lz}d_{l+1}} + E_l^{-}e^{-ik_{lz}d_{l+1}}\right] \quad (271b)$$

for E_{l+1}^{+} and E_{l+1}^{-} in terms of E_l^{+} and E_l^{-}. The Fresnel reflection coefficient

$$R_{(l+1)l}^{TM} = \frac{1-\epsilon_{l+1}k_{lz}/\epsilon_l k_{(l+1)\,z}}{1+\epsilon_{l+1}k_{lz}/\epsilon_l k_{(l+1)\,z}} = -R_{l(l+1)}^{TM} \quad (272)$$

A similar procedure applies to the case of TE waves. The results are duals of those of (270a)–(272) with the replacements of E^{+} by H^{+}, E^{-} by H^{-}, and ϵ by μ.

There are altogether $s+t$ boundaries which give rise to $4(s+t)$ equations as shown above. There are altogether $s+t+1$ regions. In regions t and $-s$ we have $E_t^{-} = H_t^{-} = 0$ and $E_{-s}^{+} = H_{-s}^{+} = 0$ because there are no waves originating from infinity. Thus we have a total of $4(s+t+1)-4 = 4(s+t)$ unknowns to be solved from the $4(s+t)$ equations. The wave amplitudes are related to the configurations and the excitation amplitudes of the dipole antenna in region 0. Thus field amplitudes in Region 0 require special attention.

For $z \geq 0$, we notice that $E_t^{-} = E_t^{-} = 0$. Letting $l = 0$, we obtain the reflection coefficients $R_{0+}^{TM} = E_{0+}^{-}/E_{0+}^{+}$ and $R_{0+}^{TE} = H_{0+}^{-}/H_{0+}^{+}$ in the form of continued fractions. We find

$$R_{0+}^{TM} = \frac{E_{0+}^{-}}{E_{0+}^{+}} = \frac{e^{i2k_{0z}d_1}}{R_{01}^{TM}} + \frac{\left[1-\left(1/R_{01}^{TM}\right)^2\right]e^{i2(k_{0z}+k_{1z})d_1}}{(1/R_{01}^{TM})e^{i2k_{1z}d_1} + (E_1^{-}/E_1^{+})} \quad (273)$$

$$R_{0+}^{TE} = \frac{H_{0+}^{-}}{H_{0+}^{+}} = \frac{e^{i2k_{0z}d_1}}{R_{01}^{TE}} + \frac{\left[1-\left(1/R_{01}^{TE}\right)^2\right]e^{i2(k_{0z}+k_{1z})d_1}}{(1/R_{01}^{TE})e^{i2k_{1z}d_1} + (H_1^{-}/H_1^{+})} \quad (274)$$

where E_1^{-}/E_1^{+} and H_1^{-}/H_1^{+} can be expressed in terms of E_2^{-}/E_2^{+} and H_2^{-}/H_2^{+} and so on until region t where $E_t^{-}/E_t^{+} = 0 = H_t^{-}/H_t^{+}$.

For $z \leq 0$, we notice that $E_{-s}^{+} = H_{-s}^{+} = 0$. Letting $l = 0$, we obtain the reflection coefficients $R_{0-}^{TM} = E_{0-}^{+}/E_{0-}^{-}$ and $R_{0-}^{TE} = H_{0-}^{+}/H_{0-}^{-}$ in the form of continued fractions. We find

$$R_{0-}^{TM} = \frac{E_{0-}^{+}}{E_{0-}^{-}} = \frac{e^{-i2k_{0z}d_0}}{R_{0(-1)}^{TM}} + \frac{\left[1-\left(1/R_{0(-1)}^{TM}\right)^2\right]e^{-i2(k_{0z}+k_{-1z})d_0}}{(1/R_{0(-1)}^{TM})e^{-i2k_{-1z}d_0} + (E_{-1}^{+}/E_{-1}^{-})} \quad (275)$$

$$R_{0-}^{TE} = \frac{H_{0-}^{+}}{H_{0-}^{-}} = \frac{e^{-i2k_{0z}d_0}}{R_{0(-1)}^{TE}} + \frac{\left[1 - \left(1/R_{0(-1)}^{TE}\right)^2\right] e^{-i2(k_{0z}+k_{-1z})d_0}}{\left(1/R_{0(-1)}^{TE}\right) e^{-i2k_{-1z}d_0} + (H_{-1}^{+}/H_{-1}^{-})} \quad (276)$$

where E_{-1}^{+}/E_{-1}^{-} and H_{-1}^{+}/H_{-1}^{-} are expressible in terms of E_{-2}^{+}/E_{-2}^{-} and H_{-2}^{+}/H_{-2}^{-} and so on until region $-s$, where $E_{-s}^{+}/E_{-s}^{-} = 0 = H_{-s}^{+}/H_{-s}^{-} = 0$.

Once the wave amplitudes in region 0 are found, wave amplitudes in other regions can be determined by the use of propagation matrices, and from a set of dual equations for TE waves.

7.3. Dipoles Between Two Negative Isotropic Slabs

For a z-directed electric dipole (ZED) antenna situated inbetween two negative isotropic slabs, we let $\mu_{-2} = \mu_0$, $\epsilon_{-2} = \epsilon_0$; $\mu_{-1} = -\mu_0$, $\epsilon_{-1} = -\epsilon_0$; $\mu_1 = -\mu_0$, $\epsilon_1 = -\epsilon_0$, $\mu_t = \mu_0$, $\epsilon_t = \epsilon_0$. We find $k_{-2z} = k_{2z} = k_{0z}$, $k_{-1z} = k_{1z} = -k_{0z}$, $p_{(-2)(-1)} = p_{(-1)0} = p_{01} = p_{12} = 1$, $R_{(-2)(-1)}^{TM} = R_{(-1)0}^{TM} = R_{01}^{TM} = R_{12}^{TM} = R_{0-}^{TM} = R_{0+}^{TM} = 0$.

$$\left.\begin{array}{ll} E_{0-}^{+} = 0 & E_{0+}^{+} = E_{zed} \\ E_{0-}^{-} = E_{zed} & E_{0+}^{-} = 0 \end{array}\right\} \quad (277)$$

From (270a) to (270b), we find

$$E_l^{+} = -E_{l+1}^{+} e^{i(k_{(l+1)z} - k_{lz})d_{l+1}} \quad (278)$$

$$E_l^{-} = -E_{l+1}^{-} e^{-i(k_{(l+1)z} - k_{lz})d_{l+1}} \quad (279)$$

It follows that

$$\left.\begin{array}{l} E_{-1}^{+} = 0 \\ E_{-1}^{-} = -E_{zed} e^{-i(k_{0z} - k_{-1z})d_0} = -E_{zed} e^{-i2k_{0z}d_0} \end{array}\right\} \quad (280)$$

$$\left.\begin{array}{l} E_{-2}^{+} = 0 \\ E_{-2}^{-} = E_{zed} e^{-i(k_{-1z} - k_{-2z})d_{-1}} e^{-i2k_{0z}d_0} = E_{zed} e^{i2k_{0z}(d_{-1} - d_0)} \end{array}\right\} \quad (281)$$

$$\left.\begin{array}{l} E_1^{+} = -E_{zed} e^{-i(k_{1z} - k_{0z})d_1} = -E_{zed} e^{i2k_{0z}d_1} \\ E_1^{-} = 0 \end{array}\right\} \quad (282)$$

$$\left.\begin{array}{l} E_2^{+} = E_{zed} e^{-i(k_{2z} - k_{1z})d_2} e^{i2k_{0z}d_1} = E_{zed} e^{-i2k_{0z}(d_2 - d_1)} \\ E_2^{-} = 0 \end{array}\right\} \quad (283)$$

The field in Region −2 is

$$E_{-2z} = \int_{-\infty}^{\infty} dk_\rho \left[E_{zed} e^{-ik_{0z}[z-2(d_{-1}-d_0)]} \right] H_0^{(1)}(k_\rho \rho) \tag{284}$$

$$E_{-2\rho} = \int_{-\infty}^{\infty} dk_\rho \frac{ik_{-2z}}{k_\rho} \left[-E_{zed} e^{-ik_{0z}[z-2(d_{-1}-d_0)]} \right] H_0^{(1)\prime}(k_\rho \rho) \tag{285}$$

$$H_{-2\phi} = \int_{-\infty}^{\infty} dk_\rho \frac{i\omega\epsilon_{-2}}{k_\rho} \left[E_{zed} e^{-ik_{0z}[z-2(d_{-1}-d_0)]} \right] H_0^{(1)\prime}(k_\rho \rho) \tag{286}$$

The field in Region −1 is

$$E_{-1z} = \int_{-\infty}^{\infty} dk_\rho \left[-E_{zed} e^{ik_{0z}[z-2d_0]} \right] H_0^{(1)}(k_\rho \rho) \tag{287}$$

$$E_{-1\rho} = \int_{-\infty}^{\infty} dk_\rho \frac{ik_{-1z}}{k_\rho} \left[E_{zed} e^{ik_{0z}[z-2d_0]} \right] H_0^{(1)\prime}(k_\rho \rho)$$

$$H_{-1\phi} = \int_{-\infty}^{\infty} dk_\rho \frac{i\omega\epsilon_{-1}}{k_\rho} \left[-E_{zed} e^{ik_{0z}[z-2d_0]} \right] H_0^{(1)\prime}(k_\rho \rho) \tag{288}$$

In Region 0 for $z \leq 0$

$$E_{0z} = \int_{-\infty}^{\infty} dk_\rho \left[E_{zed} e^{-ik_{0z}z} \right] H_0^{(1)}(k_\rho \rho) \tag{289}$$

$$E_{0\rho} = \int_{-\infty}^{\infty} dk_\rho \frac{ik_{0z}}{k_\rho} \left[-E_{zed} e^{-ik_{0z}z} \right] H_0^{(1)\prime}(k_\rho \rho)$$

$$H_{0\phi} = \int_{-\infty}^{\infty} dk_\rho \frac{i\omega\epsilon_0}{k_\rho} \left[E_{zed} e^{-ik_{0z}z} \right] H_0^{(1)\prime}(k_\rho \rho) \tag{290}$$

In Region 0 for $z \geq 0$

$$E_{0z} = \int_{-\infty}^{\infty} dk_\rho \left[E_{zed} e^{ik_{0z}z} \right] H_0^{(1)}(k_\rho \rho) \tag{291}$$

$$E_{0\rho} = \int_{-\infty}^{\infty} dk_\rho \frac{ik_{0z}}{k_\rho} \left[E_{zed} e^{ik_{0z}z} \right] H_0^{(1)\prime}(k_\rho \rho)$$

$$H_{0\phi} = \int_{-\infty}^{\infty} dk_\rho \frac{i\omega\epsilon_0}{k_\rho} \left[E_{zed} e^{ik_{0z}z} \right] H_0^{(1)\prime}(k_\rho \rho) \tag{292}$$

In region 1

$$E_{1z} = -\int_{-\infty}^{\infty} dk_\rho \left[E_{zed} e^{-ik_{0z}[z-2d_1]} \right] H_0^{(1)}(k_\rho \rho) \tag{293}$$

$$E_{1\rho} = -\int_{-\infty}^{\infty} dk_\rho \frac{ik_{1z}}{k_\rho} \left[E_{zed} e^{-ik_{0z}[z-2d_1]} \right] H_0^{(1)\prime}(k_\rho \rho)$$

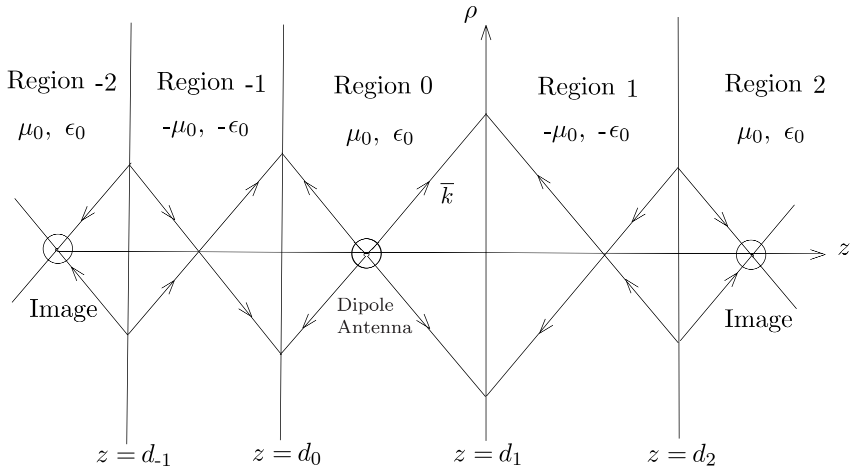

Figure 10. Images of a dipole antenna in Region 0.

$$H_{1\phi} = -\int_{-\infty}^{\infty} dk_\rho \frac{i\omega\epsilon_1}{k_\rho} \left[E_{zed} e^{-ik_{0z}[z-2d_1]}\right] H_0^{(1)\prime}(k_\rho \rho) \tag{294}$$

In region 2

$$E_{2z} = \int_{-\infty}^{\infty} dk_\rho \left[E_{zed} e^{ik_{0z}[z-2(d_2-d_1)]}\right] H_0^{(1)}(k_\rho \rho) \tag{295}$$

$$E_{2\rho} = \int_{-\infty}^{\infty} dk_\rho \frac{ik_{2z}}{k_\rho} \left[E_{zed} e^{ik_{0z}[z-2(d_2-d_1)]}\right] H_0^{(1)\prime}(k_\rho \rho)$$

$$H_{2\phi} = \int_{-\infty}^{\infty} dk_\rho \frac{i\omega\epsilon_2}{k_\rho} \left[E_{zed} e^{ik_{0z}[z-2(d_2-d_1)]}\right] H_0^{(1)\prime}(k_\rho \rho) \tag{296}$$

It is thus seen that the field in Region 2 is due to a dipole antenna situated at $z = 2(d_2 - d_1)$, which is a perfect image of the original line source. Similarly, a perfect image of the original dipole antenna is formed in Region -2 and located at $z = 2(d_{-1} - d_0)$.

7.4. Dipoles in Front of Stratified Isotropic Media

For dipoles in front of stratified media, all regions -1 to $-s$ are absent, we have $E_{0-}^{+} = H_{0-}^{+} = 0$ and $R_{0-}^{TM} = R_{0-}^{TE} = 0$. It follows that for
(1) z-directed electric dipole (ZED)

$$\left.\begin{array}{ll} E_{0-}^{+} = 0 & E_{0+}^{+} = E_{zed} \\ E_{0-}^{-} = (1 + R_{0+}^{TM}) E_{zed} & E_{0+}^{-} = R_{0+}^{TM} E_{zed} \\ H_{0-}^{+} = H_{0-}^{-} = 0 & H_{0+}^{+} = H_{0+}^{-} = 0 \end{array}\right\} \tag{297}$$

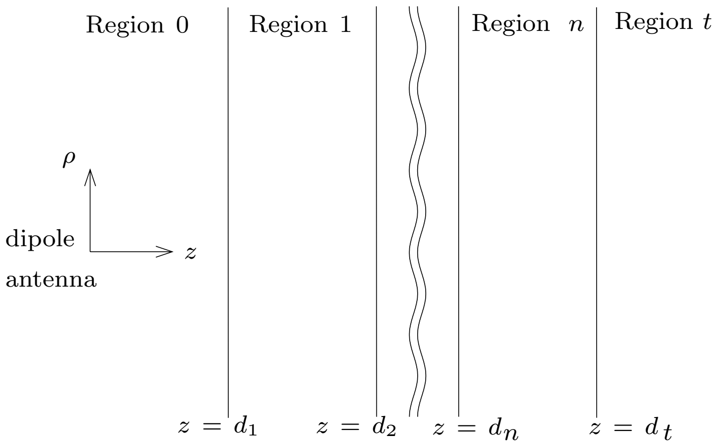

Figure 11. Dipole in layered medium.

(2) x-directed electric dipole (XED)

$$\left.\begin{array}{ll} E_{0-}^{+} = 0 & E_{0+}^{+} = E_{xed} \\ E_{0-}^{-} = -(1 - R_{0+}^{TM})E_{xed} & E_{0+}^{-} = R_{0+}^{TM} E_{xed} \\ H_{0-}^{+} = 0 & H_{0+}^{+} = H_{xed} \\ H_{0-}^{-} = (1 + R_{0+}^{TE})H_{xed} & H_{0+}^{-} = R_{0+}^{TE} H_{xed} \end{array}\right\} \tag{298}$$

The results of the y-directed electric dipole (YED) can be obtained from that for XED by replacing ϕ with $\phi - \pi/2$. The magnetic dipoles produce fields which are duals of those produced by the corresponding electric dipoles. The results for the magnetic dipoles ZMD, XMD, and YMD, can be obtained by the replacement $\overline{E} \to \overline{H}$, $\overline{H} \to -\overline{E}$, $\mu_0 \rightleftarrows \epsilon_0$, $Il \to i\omega\mu_0 IA$ and TE $\rightleftarrows$ TM.

7.5. Dipoles in Front of a Negative Isotropic Slab

For a z-directed electric dipole (ZED) in front of a slab medium with boundary surfaces at $z = d_1$ and $z = d_2$, we find the reflection coefficient

$$R^{TM} = \frac{R_{01}^{TM} + R_{12}^{TM} e^{i2k_{1z}(d_2 - d_1)}}{1 + R_{01}^{TM} R_{12}^{TM} e^{i2k_{1z}(d_2 - d_1)}} e^{i2k_{0z}d_1} \tag{299}$$

the amplitudes inside the slab medium

$$E_1^+ = \frac{2E_{zed}e^{-i(k_{1z}-k_{0z})d_1}}{(\frac{\epsilon_1}{\epsilon_0}+\frac{k_{1z}}{k_{0z}})(1+R_{01}^{TM}R_{12}^{TM}e^{i2k_{1z}(d_2-d_1)})} \quad (300)$$

$$E_1^- = \frac{2R_{12}^{TM}E_{zed}e^{-i(k_{1z}-k_{0z})d_1}e^{i2k_{1z}d_2}}{(\frac{\epsilon_1}{\epsilon_0}+\frac{k_{1z}}{k_{0z}})(1+R_{01}^{TM}R_{12}^{TM}e^{i2k_{1z}(d_2-d_1)})} \quad (301)$$

and the transmission coefficient

$$T^{TM} = \frac{4e^{ik_{0z}d_1}e^{ik_{1z}(d_2-d_1)}e^{-ik_{2z}d_2}}{(\frac{\epsilon_1}{\epsilon_0}+\frac{k_{1z}}{k_{0z}})(\frac{\epsilon_2}{\epsilon_1}+\frac{k_{2z}}{k_{1z}})(1+R_{01}^{TM}R_{12}^{TM}e^{i2k_{1z}(d_2-d_1)})} \quad (302)$$

where for TM waves, $p_{l(l+1)}^{TM} = \epsilon_l k_{(l+1)z}/\epsilon_{l+1}k_{lz}$, and

$$R_{l(l+1)}^{TM} = \frac{1-p_{l(l+1)}^{TM}}{1+p_{l(l+1)}^{TM}}$$

is the reflection coefficient at the boundary between regions l and $l+1$.

Consider a z-directed electric dipole (ZED) in front of the slab medium, we find

(A) In region 0; $z \le 0$

$$E_{0z} = \int_{-\infty}^{\infty} dk_\rho \left[(1+R_{0+}^{TM})E_{zed}e^{-ik_{0z}z}\right] H_0^{(1)}(k_\rho\rho) \quad (303)$$

$$E_{0\rho} = \int_{-\infty}^{\infty} dk_\rho \frac{ik_{0z}}{k_\rho}\left[-(1+R_{0+}^{TM})E_{zed}e^{-ik_{0z}z}\right] H_0^{(1)\prime}(k_\rho\rho) \quad (304)$$

$$H_{0\phi} = \int_{-\infty}^{\infty} dk_\rho \frac{i\omega\epsilon_0}{k_\rho}\left[(1+R_{0+}^{TM})E_{zed}e^{-ik_{0z}z}\right] H_0^{(1)\prime}(k_\rho\rho) \quad (305)$$

(B) In region 0; $z \ge 0$

$$E_{0z} = \int_{-\infty}^{\infty} dk_\rho E_{zed}\left[e^{ik_{0z}z}+R_{0+}^{TM}e^{-ik_{0z}z}\right] H_0^{(1)}(k_\rho\rho) \quad (306)$$

$$E_{0\rho} = \int_{-\infty}^{\infty} dk_\rho \frac{ik_{0z}}{k_\rho}E_{zed}\left[e^{ik_{0z}z}-R_{0+}^{TM}e^{-ik_{0z}z}\right] H_0^{(1)\prime}(k_\rho\rho) \quad (307)$$

$$H_{0\phi} = \int_{-\infty}^{\infty} dk_\rho \frac{i\omega\epsilon_0}{k_\rho}E_{zed}\left[e^{ik_{0z}z}+R_{0+}^{TM}e^{-ik_{0z}z}\right] H_0^{(1)\prime}(k_\rho\rho) \quad (308)$$

(C) In region 1

$$E_{1z} = \int_{-\infty}^{\infty} dk_\rho \left[E_1^+e^{ik_{1z}z}+E_1^-e^{-ik_{1z}z}\right] H_0^{(1)}(k_\rho\rho) \quad (309)$$

$$E_{1\rho} = \int_{-\infty}^{\infty} dk_\rho \frac{ik_{1z}}{k_\rho} \left[E_1^+ e^{ik_{1z}z} - E_1^- e^{-ik_{1z}z}\right] H_0^{(1)\prime}(k_\rho\rho) \quad (310)$$

$$H_{1\phi} = \int_{-\infty}^{\infty} dk_\rho \frac{i\omega\epsilon_1}{k_\rho} \left[E_1^+ e^{ik_{1z}z} + E_1^- e^{-ik_{1z}z}\right] H_0^{(1)\prime}(k_\rho\rho) \quad (311)$$

(D) In region 2, $k_{2z} = k_{0z}$

$$E_{2z} = \int_{-\infty}^{\infty} dk_\rho \left[TE_{zed}e^{-ik_{2z}z}\right] H_0^{(1)}(k_\rho\rho) \quad (312)$$

$$E_{2\rho} = \int_{-\infty}^{\infty} dk_\rho \frac{ik_{2z}}{k_\rho} \left[TE_{zed}e^{-ik_{2z}z}\right] H_0^{(1)\prime}(k_\rho\rho) \quad (313)$$

$$H_{2\phi} = \int_{-\infty}^{\infty} dk_\rho \frac{i\omega\epsilon_2}{k_\rho} \left[TE_{zed}e^{-ik_{2z}z}\right] H_0^{(1)\prime}(k_\rho\rho) \quad (314)$$

For a z-directed electric dipole (ZED) in front of a negative isotropic slab, we let $\mu_1 = -\mu_0$, $\epsilon_1 = -\epsilon_0$, and $\mu_t = \mu_0$, $\epsilon_t = \epsilon_0$. We find $k_{1z} = -k_{0z}$, $k_{2z} = k_{0z}$, $p_{01} = p_{12} = 1$, $R_{01} = R_{12} = R_{0+}^{TM} = 0$, $E_1^- = 0$, $E_1^+ = -E_{zed}e^{i2k_{0z}d_1}$, $T = e^{-i2k_{0z}(d_2-d_1)}$.

$$\left.\begin{array}{ll} E_{0-}^+ = 0 & E_{0+}^+ = E_{zed} \\ E_{0-}^- = E_{zed} & E_{0+}^- = 0 \\ H_{0-}^+ = H_{0-}^- = 0 & H_{0+}^+ = H_{0+}^- = 0 \end{array}\right\} \quad (315)$$

$$\begin{aligned} \overline{E}(\overline{r}) &= \hat{\rho}E_\rho + \hat{z}E_z \\ &= \frac{-i\omega\mu Ile^{ikr}}{4\pi r} \left\{\hat{z}\frac{zz}{r^2}\left[1 + 3\frac{i}{kr} + 3(\frac{i}{kr})^2\right] - \hat{z}\left[1 + \frac{i}{kr} + (\frac{i}{kr})^2\right] \right. \\ &\quad \left. + \hat{\rho}\frac{z\rho}{r^2}\left[1 + 3\frac{i}{kr} + 3(\frac{i}{kr})^2\right]\right\} \quad (316) \end{aligned}$$

The magnetic field follows from Faraday's law

$$\overline{H}(\overline{r}) = \hat{\phi}H_\phi = \hat{\phi}\frac{-ikIle^{ikr}}{4\pi r}\frac{\rho}{r}\left[1 + \frac{i}{kr}\right] \quad (317)$$

The complex Poynting power density is calculated by taking the cross product of $\overline{E}$ and the complex conjugate of $\overline{H}$

$$\begin{aligned} \overline{S} &= \frac{\omega\mu k(Il)^2}{(4\pi r)^2} \left\{-\hat{\rho}\frac{zz}{r^2}\left[1 + 3\frac{i}{kr} + 3(\frac{i}{kr})^2\right] + \hat{\rho}\left[1 + \frac{i}{kr} + (\frac{i}{kr})^2\right] \right. \\ &\quad \left. + \hat{z}\frac{z\rho}{r^2}\left[1 + 3\frac{i}{kr} + 3(\frac{i}{kr})^2\right]\right\}\frac{\rho}{r}\left[1 - \frac{i}{kr}\right] \quad (318) \end{aligned}$$

The time-average Poynting power density is

$$< \overline{S} > = \frac{\omega\mu k(Il)^2}{2(4\pi r)^2}\left\{\hat{\rho}\frac{\rho}{r} + \hat{z}\frac{z}{r}\right\}(\frac{\rho}{r})^2 \tag{319}$$

(A) In region 0; $z \leq 0$

$$E_{0z} = \int_{-\infty}^{\infty} dk_\rho \left[E_{zed}e^{-ik_{0z}z}\right] H_0^{(1)}(k_\rho\rho) = E_z \tag{320}$$

$$E_{0\rho} = \int_{-\infty}^{\infty} dk_\rho \frac{ik_{0z}}{k_\rho}\left[-E_{zed}e^{-ik_{0z}z}\right] H_0^{(1)\prime}(k_\rho\rho) = E_\rho \tag{321}$$

$$H_{0\phi} = \int_{-\infty}^{\infty} dk_\rho \frac{i\omega\epsilon_0}{k_\rho}\left[E_{zed}e^{-ik_{0z}z}\right] H_0^{(1)\prime}(k_\rho\rho) = H_\phi \tag{322}$$

(B) In region 0; $z \geq 0$

$$E_{0z} = \int_{-\infty}^{\infty} dk_\rho \left[E_{zed}e^{ik_{0z}z}\right] H_0^{(1)}(k_\rho\rho) = E_z \tag{323}$$

$$E_{0\rho} = \int_{-\infty}^{\infty} dk_\rho \frac{ik_{0z}}{k_\rho}\left[E_{zed}e^{ik_{0z}z}\right] H_0^{(1)\prime}(k_\rho\rho) = E_\rho \tag{324}$$

$$H_{0\phi} = \int_{-\infty}^{\infty} dk_\rho \frac{i\omega\epsilon_0}{k_\rho}\left[E_{zed}e^{ik_{0z}z}\right] H_0^{(1)\prime}(k_\rho\rho) = H_\phi \tag{325}$$

(C) In region 1, $k_{1z} = -k_{0z}$

$$E_{1z} = \int_{-\infty}^{\infty} dk_\rho \left[-E_{zed}e^{-ik_{0z}(z-2d_1)}\right] H_0^{(1)}(k_\rho\rho) = -E_z \tag{326}$$

$$E_{1\rho} = \int_{-\infty}^{\infty} dk_\rho \frac{ik_{0z}}{k_\rho}\left[E_{zed}e^{-ik_{0z}(z-2d_1)}\right] H_0^{(1)\prime}(k_\rho\rho) = -E_\rho \tag{327}$$

$$H_{1\phi} = \int_{-\infty}^{\infty} dk_\rho \frac{i\omega\epsilon_0}{k_\rho}\left[E_{zed}e^{-ik_{0z}(z-2d_1)}\right] H_0^{(1)\prime}(k_\rho\rho) = H_\phi \tag{328}$$

(D) In region 2, $k_{2z} = k_{0z}$

$$E_{2z} = \int_{-\infty}^{\infty} dk_\rho \left[E_{zed}e^{ik_{0z}[z-2(d_2-d_1)]}\right] H_0^{(1)}(k_\rho\rho) = E_z \tag{329}$$

$$E_{2\rho} = \int_{-\infty}^{\infty} dk_\rho \frac{ik_{2z}}{k_\rho}\left[E_{zed}e^{ik_{0z}[z-2(d_2-d_1)]}\right] H_0^{(1)\prime}(k_\rho\rho) = E_\rho \tag{330}$$

$$H_{2\phi} = \int_{-\infty}^{\infty} dk_\rho \frac{i\omega\epsilon_2}{k_\rho}\left[E_{zed}e^{ik_{0z}[z-2(d_2-d_1)]}\right] H_0^{(1)\prime}(k_\rho\rho) = H_\phi \tag{331}$$

It is seen that in the transmitted region, the field originates from a ZED located at $z = 2(d_2 - d_1)$.

7.6. Dipoles In Front of Negative Isotropic Slab Backed by a Perfect Conductor

Region t is assumed to be a perfect conductor. With $\mu_1 = -\mu_0$, $\epsilon_1 = -\epsilon_0$, $k_{1z} = -k_{0z}$, we find $p_{01} = \epsilon_0 k_{1z}/\epsilon_1 k_{0z} = 1$, $\epsilon_t \to \infty$, $R_{12}^{TM} = 1$, $R_{01}^{TM} = 0$, and the following solutions:

$$T^{TM} = 0 \tag{332}$$

$$E_1^+ = -E_{zed} e^{i2k_{0z}d_1} \tag{333}$$

$$E_1^- = -E_{zed} e^{-i2k_{0z}(d_2-d_1)} \tag{334}$$

$$R_{0+}^{TM} = e^{-i2k_{0z}(d_2-d_1)} e^{i2k_{0z}d_1} \tag{335}$$

(A) In region 1

$$E_{1z} = \int_{-\infty}^{\infty} dk_\rho \left[E_1^+ e^{-ik_{0z}z} + E_1^- e^{ik_{0z}z} \right] H_0^{(1)}(k_\rho \rho) \tag{336}$$

$$E_{1\rho} = \int_{-\infty}^{\infty} dk_\rho \frac{-ik_{0z}}{k_\rho} \left[E_1^+ e^{-ik_{0z}z} - E_1^- e^{ik_{0z}z} \right] H_0^{(1)\prime}(k_\rho \rho) \tag{337}$$

$$H_{1\phi} = \int_{-\infty}^{\infty} dk_\rho \frac{-i\omega\epsilon_0}{k_\rho} \left[E_1^+ e^{-ik_{0z}z} + E_1^- e^{ik_{0z}z} \right] H_0^{(1)\prime}(k_\rho \rho) \tag{338}$$

(B) In region 0; $z \geq 0$

$$E_{0z} = \int_{-\infty}^{\infty} dk_\rho E_{zed} \left[e^{ik_{0z}z} + R_{0+}^{TM} e^{-ik_{0z}z} \right] H_0^{(1)}(k_\rho \rho) \tag{339}$$

$$E_{0\rho} = \int_{-\infty}^{\infty} dk_\rho \frac{ik_{0z}}{k_\rho} E_{zed} \left[e^{ik_{0z}z} - R_{0+}^{TM} e^{-ik_{0z}z} \right] H_0^{(1)\prime}(k_\rho \rho) \tag{340}$$

$$H_{0\phi} = \int_{-\infty}^{\infty} dk_\rho \frac{i\omega\epsilon_0}{k_\rho} E_{zed} \left[e^{ik_{0z}z} + R_{0+}^{TM} e^{-ik_{0z}z} \right] H_0^{(1)\prime}(k_\rho \rho) \tag{341}$$

(C) In region 0; $z \leq 0$

$$E_{0z} = \int_{-\infty}^{\infty} dk_\rho \left[(1 + R_{0+}^{TM}) E_{zed} e^{-ik_{0z}z} \right] H_0^{(1)}(k_\rho \rho) \tag{342}$$

$$E_{0\rho} = \int_{-\infty}^{\infty} dk_\rho \frac{-ik_{0z}}{k_\rho} \left[(1 + R_{0+}^{TM}) E_{zed} e^{-ik_{0z}z} \right] H_0^{(1)\prime}(k_\rho \rho) \tag{343}$$

$$H_{0\phi} = \int_{-\infty}^{\infty} dk_\rho \frac{i\omega\epsilon_0}{k_\rho} \left[(1 + R_{0+}^{TM}) E_{zed} e^{-ik_{0z}z} \right] H_0^{(1)\prime}(k_\rho \rho) \tag{344}$$

From solutions in Region 0 for $z \leq 0$, we notice the remarkable result that an image dipoe has been generated with the amplitude equal to that of the source with $|R_{0+}^{TM}| = 1$ and positioned at $z = -2d_2 + 4d_1$!

As a concluding remark, note all previous illustrations of the negative isotropic material have assumed the constitutive parameters $\mu = -\mu_o$ and $\epsilon = -\epsilon_o$. The realization of such material may never be attainable. For practical consideration of material properties, two issues require extensive study are dispersion of tne material and their loss, aside from many interesting mathematical and physical conceptual issues.

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