Mon. Not. R. astr. Soc. (1970) **148**, 367–381.

ELECTRON DENSITIES IN PLANETARY NEBULAE

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(Received 1970 January 29)

SUMMARY

Emissivities are tabulated for lines of [O II] (configuration $2p^3$) and [S II], [Cl III], [Ar IV] and [K V] (configuration $3p^3$), using the most accurate atomic data available. The intensity ratios

$$R = I(^2D_{3/2} \rightarrow {}^4S)/I(^2D_{5/2} \rightarrow {}^4S)$$

are sensitive to electron density. Densities are obtained from observed ratios in eleven planetary nebulae. The results obtained from [O II], [S II] and [Cl III] are in good agreement. Larger densities are obtained from [Ar IV] and [K V] in high excitation planetaries; this may be evidence for large-scale density variations.

1. INTRODUCTION

Observed intensity ratios

$$R = I(^2D_{3/2} \rightarrow {}^4S)/I(^2D_{5/2} \rightarrow {}^4S) \qquad (1.1)$$

for [O II], configuration $2p^3$, have been used by Seaton & Osterbrock (1957) to obtain electron densities in gaseous nebulae. The possibility of obtaining densities from ratios R for $3p^3$ ions, [S II], [Cl III], [Ar IV] and [K V], has been discussed in a number of recent papers (Weedman 1968; Khromov 1969; Czyzak, Walker & Aller 1969; Krueger, Aller & Czyzak 1970). In the present paper we tabulate emissivities, as functions of electron temperature T_e and electron density N_e, for transitions between the np^3 levels $^2P_{3/2}$, $^2P_{1/2}$, $^2D_{5/2}$, $^2D_{3/2}$ and ^4S in [O II], [S II], [Cl III], [Ar IV] and [K V].

The intensity ratios R are sensitive to the density parameter $x = 10^{-2}N_e/T_e^{1/2}$ (N_e in cm^{-3}, T_e in °K). We use observed values of R to deduce densities in eleven planetary nebulae. The tabulated emissivities may also be used to calculate the intensity ratios $I(^2P \rightarrow {}^2D)/I(^2D \rightarrow {}^4S)$ and $I(^2P \rightarrow {}^4S)/I(^2D \rightarrow {}^4S)$ as functions of N_e and T_e; the interpretation of these ratios is not discussed in the present paper.

2. CALCULATION OF EMISSIVITIES

The level populations are determined by electron collisions and spontaneous emission of radiation.

2.1 *Collisional transition rates*

Let $q(j \rightarrow i)$ be the probability per unit time that an ion in level j undergoes a collisional transition to level i. For super-elastic collisions

$$q(j \rightarrow i) = \frac{8.63 \times 10^{-6}}{\omega_j} \frac{N_e}{T_e^{1/2}} \Upsilon(j \rightarrow i)\ \mathrm{s}^{-1} \qquad (E_j > E_i) \qquad (2.1)$$

where ω_j is the statistical weight of level j, T_e is in °K, and N_e in cm^{-3}. The quantity $\Upsilon(j \rightarrow i)$ is given by

$$\Upsilon(j \rightarrow i) = \int_0^\infty \Omega(j, i) e^{-\xi_j} d\xi_j \tag{2.2}$$

where $\Omega(j, i)$ is the collision strength,

$$\xi_j = \frac{\frac{1}{2} m v_j^2}{kT_e} = \frac{157890\, k_j^2}{T_e} \tag{2.3}$$

and v_j is the velocity of the colliding electron and k_j^2 is its energy in Rydberg units. If $\Omega(j, i)$ is a constant, independent of energy, one obtains $\Upsilon(j \rightarrow i) = \Omega(j, i)$; in general $\Upsilon(j \rightarrow i)$ is a slowly-varying function of T_e.

For collisional excitation,

$$q(i \rightarrow j) = \frac{\omega_j}{\omega_i} q(j \rightarrow i) e^{-E_{ji}/kT_e} \qquad (E_j > E_i) \qquad (2.4))$$

where

$$E_{ji} = E_j - E_i. \tag{2.5}$$

In (2.4), (2.5) the observed ion energies E_j, E_i must be used (in the calculation of collision strengths some use is made of calculated ion energies).

The quantities Υ have been calculated using the collision strengths of Martins & Seaton (1969) for [O II] and the collision strengths of Czyzak *et al.* (1970) for [S II], [Cl III], [Ar IV] and [K V]. For the transitions ^4S–$^2D_{3/2}$, ^4S–$^2D_{5/2}$ and $^2D_{3/2}$–$^2D_{5/2}$ the collision strengths are calculated allowing for resonance structures at energies below the threshold for excitation of the ^2P term. The resonances are narrow and one may therefore average over resonance structures before integrating over Maxwell distributions.

The most accurate collision strengths currently available are obtained from solutions of coupled integro-differential equations, taking account of all states in the np^3 configurations (the 'close-coupling' approximation). This method has been used for energies above the ^2P threshold, and without consideration of transitions between fine structure levels. The collision strengths of Martins and Seaton for [O II] agree with results of close-coupling calculations to within about 10 per cent (Henry, Burke & Sinfailam 1969). Close-coupling calculations for [S II], [Cl III] and [Ar IV] have been made by Conneely, Smith & Lipsky (1970), at energies some way above the ^2P threshold. The calculations of Czyzak *et al.* (1970) for these ions are made using the distorted wave approximation. A comparison of close-coupling and distorted-wave results is given by Czyzak *et al.* (1970). The largest discrepancy occurs for $\Omega(^4S, {}^2D)$ in [S II]; for this transition the close-coupling result is larger than the distorted-wave result by a factor of 1·8. The agreement between the two calculations improves as the ion charge increases. For [Ar IV] the distorted wave results are in very good agreement with the close-coupling results.

The effects of collisional coupling with higher configurations are neglected in both the distorted-wave and close-coupling calculations. It is shown by Eissner, *et al.* (1969) that these effects will be most important for the more highly ionized systems.

The accuracy of the calculations is discussed further in Section 4.1.

2.2 *Radiative transition probabilities*

It was first pointed out by Aller, Ufford & Van Vleck (1949) that, in calculating the ^2D–^4S transition probabilities, it is necessary to allow for spin–spin interactions in addition to second-order spin–orbit interactions. The [O II] transition probabilities are sensitive to the spin–spin parameter η, the spin–orbit parameter ζ and the quadrupole integral s_q. The intensity ratio R in the limit of high densities, denoted by $R(\infty)$, is proportional to the ratio of transition probabilities, $A(^2D_{3/2} \rightarrow {}^4S)/A(^2D_{5/2} \rightarrow {}^4S)$. Using calculated transition probabilities, Aller, *et al.* (1949) obtained $R(\infty) = 1{\cdot}7$, Garstang (1952) obtained $T(\infty) = 2{\cdot}1$ and Seaton and Osterbrock obtained $R(\infty) = 2{\cdot}3$. From the observed ratio in the high density planetary IC 4997, Seaton and Osterbrock deduced that $R(\infty) = 2{\cdot}9 \pm 0{\cdot}3$. They concluded that the discrepancy between the observed and calculated values of $R(\infty)$ was consistent with the probable error in the calculations, and therefore adjusted the ratio of transition probabilities, $A(^2D_{3/2} \rightarrow {}^4S)/A(^2D_{5/2} \rightarrow {}^4S)$, so as to obtain $R(\infty) = 2{\cdot}9$. We use the [O II] transition probabilities of Seaton and Osterbrock (which should be more accurate than the calculated values quoted by Garstang (1968)), but return to a discussion of their accuracy in Section 4.1 below.

The transition probabilities for [S II], [Cl III] and [Ar IV] have been calculated by Czyzak & Krueger (1963 and Corrigendum 1965), and those for [K V] by Garstang (1968). The results of Czyzak and Krueger for [S II] are quoted incorrectly by Garstang. The transition probabilities for $^2D_{3/2} \rightarrow {}^4S$ are dominated by the magnetic dipole contributions A_m, whereas those for $^2D_{5/2} \rightarrow {}^4S$ are dominated by the electric quadrupole contribution A_q. The ratios $R(\infty)$ increase rather rapidly as the ion charge increases, due to an increase in A_m/A_q. For the $3p^3$ ions the calculated values of $R(\infty)$ are sensitive to the quadrupole integrals s_q but, compared with [O II], less sensitive to the spin–spin parameter η.

2.3 *The equilibrium equations*

The equilibrium equations are

$$N(i) \sum_j P(i \rightarrow j) = \sum_j N(j) P(j \rightarrow i) \tag{2.6}$$

where $N(i)$ is the number of ions per cm^3 in level i, and $P(i \rightarrow j) = q(i \rightarrow j) + A(i \rightarrow j)$ ($A(i \rightarrow j)$ is, of course, zero for $E_i < E_j$). We solve these equations for the ratios $N(i)/N$ where

$$N = \sum_i N(i). \tag{2.7}$$

2.4 *Line emissivities*

Let $4\pi J(j \rightarrow i)$ be the emission in the line $j \rightarrow i$, in erg cm^{-3} s^{-1}. Then

$$4\pi J(j \rightarrow i) = N(j) A(j \rightarrow i) E_{ji}. \tag{2.8}$$

It is convenient to consider the total emission in all lines from level j,

$$4\pi J(j) = 4\pi \sum_i J(j \rightarrow i). \tag{2.9}$$

The line emissivities are then given in terms of $J(j)$ and branching ratios $B(j \rightarrow i)$ which are independent of T_e and N_e:

$$J(j \rightarrow i) = J(j) B(j \rightarrow i) \tag{2.10}$$

where

$$B(j \to i) = \frac{A(j \to i)E_{ji}}{\sum_k A(j \to k)E_{jk}}. \tag{2.11}$$

Branching ratios for the transitions from levels ^2P_J are given in Table I; for the levels ^2D_J only one transition has to be considered, $^2D_J \to {}^4S$.

TABLE I

Branching ratios, $B(j \to i)$

Ion	j	i: $^2D_{3/2}$	$^2D_{5/2}$	^4S
S II	$^2P_{1/2}$	0·319	0·138	0·542
	$^2P_{3/2}$	0·137	0·169	0·694
Cl III	$^2P_{1/2}$	0·251	0·077	0·672
	$^2P_{3/2}$	0·123	0·113	0·764
Ar IV	$^2P_{1/2}$	0·209	0·037	0·754
	$^2P_{3/2}$	0·114	0·083	0·803
K V	$^2P_{1/2}$	0·194	0·024	0·782
	$^2P_{3/2}$	0·115	0·074	0·811
O II	$^2P_{1/2}$	0·432	0·263	0·305
	$^2P_{3/2}$	0·172	0·325	0·502

The main temperature dependence of $J(j)$ is through an exponential factor, $\exp(-E_{j1}/kT_e)$, where E_{j1} is the excitation energy of level j. We therefore put

$$4\pi J(j) = NK(j)e^{-E_{ji}/kT_e} \tag{2.12}$$

and tabulate $\log_{10}K(j)$, in Table II, as a function of the variables

$$t = 10^{-4}T_e,\ x = 10^{-4}N_e/t^{1/2}. \tag{2.13}$$

It should be noted that, for the $3p^3$ ions, the values of $K(j)$ given in Table II are calculated using distorted-wave collision strengths. In Section 4.1 we shall recommend that certain corrections should be made. These corrections are deduced from an examination of the observational results and from comparisons of collision strengths calculated in the distorted-wave and close-coupling approximations.

2.5 *Limiting forms for N_e small and N_e large*

From (2.8) and (2.12),

$$K(j) = \frac{N(j)}{N} e^{E_{ji}/kT_e} \sum_i A(j \to i)E_{ji}. \tag{2.14}$$

In the limit of N_e small, $N(j)$ for $j \neq 1$ is small compared with $N(1)$, the number of ground-state ions. We may therefore neglect transitions due to collisions of electrons with ions in excited states. If T_e is not too large, we may also neglect the rate of population of the ^2D levels due to cascade from the ^2P levels. We then obtain

$$\frac{N(j)}{N} = \frac{q(1 \to j)}{A(j)} \quad (N_e \text{ small}) \tag{2.15}$$

TABLE II(a)

Values of $\log_{10}K(j)$ for [S II]

$\log_{10}x$	$j =$ $^2D_{3/2}$			$^2D_{5/2}$			$^2P_{1/2}$			$^2P_{3/2}$		
$t =$	0·5	1·0	2·0	0·5	1·0	2·0	0·5	1·0	2·0	0·5	1·0	2·0
−4·0	−19·13	−19·08	−19·02	−18·96	−18·92	−18·87	−19·55	−19·52	−19·50	−19·19	−19·16	−19·14
−3·0	−18·13	−18·08	−18·02	−17·96	−17·92	−17·88	−18·55	−18·52	−18·50	−18·19	−18·16	−18·14
−2·0	−17·13	−17·07	−17·02	−16·96	−16·92	−16·88	−17·55	−17·52	−17·50	−17·18	−17·15	−17·13
−1·5	−16·62	−16·57	−16·51	−16·47	−16·44	−16·40	−17·03	−17·00	−16·98	−16·66	−16·64	−16·61
−1·0	−16·10	−16·05	−16·01	−16·01	−15·98	−15·95	−16·49	−16·47	−16·45	−16·12	−16·09	−16·08
−0·5	−15·58	−15·54	−15·51	−15·59	−15·57	−15·58	−15·91	−15·88	−15·88	−15·52	−15·50	−15·50
0·0	−15·08	−15·07	−15·09	−15·25	−15·27	−15·31	−15·26	−15·25	−15·28	−14·87	−14·87	−14·90
+0·5	−14·70	−14·73	−14·80	−15·00	−15·05	−15·15	−14·58	−14·61	−14·68	−14·20	−14·23	−14·31
1·0	−14·46	−14·54	−14·66	−14·84	−14·93	−15·07	−13·95	−14·02	−14·13	−13·58	−13·64	−13·76
2·0	−14·32	−14·43	−14·59	−14·73	−14·85	−15·03	−12·95	−13·05	−13·20	−12·54	−12·63	−12·79
3·0	−14·30	−14·41	−14·59	−14·71	−14·83	−15·01	−12·40	−12·51	−12·69	−11·86	−11·97	−12·15

Note: The results in this table are obtained using collision strengths calculated in the distorted wave approximation. In Section 4.1.2 it is *recommended that* 0·26 *should be subtracted from the values of log x.*

TABLE II(b)

Values of $\log_{10}K(j)$ for [Cl III]

$\log_{10}x$	$j =$ $^2D_{3/2}$			$^2D_{5/2}$			$^2P_{1/2}$			$^2P_{3/2}$		
$t =$	0·5	1·0	2·0	0·5	1·0	2·0	0·5	1·0	2·0	0·5	1·0	2·0
−3·0	−17·88	−17·88	−17·90	−17·71	−17·72	−17·75	−18·30	−18·32	−18·37	−17·95	−17·97	−18·02
−2·0	−16·88	−16·88	−16·89	−16·71	−16·72	−16·76	−17·29	−17·32	−17·37	−16·95	−16·97	−17·02
−1·0	−15·85	−15·85	−15·88	−15·75	−15·76	−15·80	−16·27	−16·29	−16·34	−15·92	−15·94	−15·99
−0·5	−15·31	−15·32	−15·35	−15·32	−15·34	−15·38	−15·72	−15·74	−15·80	−15·36	−15·39	−15·45
0·0	−14·76	−14·78	−14·84	−14·98	−15·00	−15·06	−15·13	−15·16	−15·23	−14·77	−14·80	−14·87
0·5	−14·29	−14·32	−14·40	−14·71	−14·75	−14·84	−14·49	−14·54	−14·63	−14·14	−14·18	−14·28
1·0	−13·94	−13·99	−14·11	−14·51	−14·57	−14·70	−13·85	−13·91	−14·04	−13·49	−13·55	−13·69
1·5	−13·75	−13·82	−13·97	−14·38	−14·46	−14·62	−13·26	−13·34	−13·50	−12·89	−12·97	−13·13
2·0	−13·67	−13·75	−13·91	−14·32	−14·41	−14·58	−12·75	−12·84	−13·01	−12·36	−12·45	−12·62
3·0	−13·62	−13·70	−13·88	−14·29	−14·37	−14·55	−12·07	−12·15	−12·33	−11·53	−11·61	−11·79

Note: The results in this table are obtained using collision strengths calculated in the distorted wave approximation. In Section 4.1.3 it is *recommended that* 0·18 *should be subtracted from the values of log x.*

TABLE II(c)

Values of $\log_{10}K(j)$ for [Ar IV]

$\log_{10}x$ $j=$	$^2D_{3/2}$			$^2D_{5/2}$			$^2P_{1/2}$			$^2P_{3/2}$		
$t=$	0·5	1·0	2·0	0·5	1·0	2·0	0·5	1·0	2·0	0·5	1·0	2·0
−2·0	−17·21	−17·20	−17·20	−17·05	−17·05	−17·06	−17·66	−17·66	−17·68	−17·33	−17·34	−17·35
−1·0	−16·20	−16·20	−16·19	−16·05	−16·06	−16·07	−16·65	−16·65	−16·67	−16·32	−16·32	−16·34
−0·5	−15·69	−15·68	−15·68	−15·57	−15·58	−15·59	−16·13	−16·13	−16·15	−15·79	−15·79	−15·81
0·0	−15·15	−15·14	−15·15	−15·13	−15·13	−15·16	−15·58	−15·58	−15·61	−15·22	−15·22	−15·25
0·5	−14·59	−14·59	−14·61	−14·75	−14·77	−14·81	−14·97	−14·98	−15·02	−14·60	−14·60	−14·64
1·0	−14·05	−14·06	−14·11	−14·48	−14·51	−14·58	−14·33	−14·34	−14·40	−13·94	−13·96	−14·02
1·5	−13·59	−13·62	−13·70	−14·27	−14·31	−14·42	−13·65	−13·68	−13·76	−13·27	−13·31	−13·40
2·0	−13·26	−13·31	−13·44	−14·09	−14·15	−14·30	−12·98	−13·03	−13·16	−12·61	−12·66	−12·79
2·5	−13·07	−13·15	−13·31	−13·97	−14·05	−14·23	−12·39	−12·46	−12·62	−12·01	−12·08	−12·24
3·0	−12·99	−13·07	−13·25	−13·91	−14·00	−14·19	−11·95	−12·02	−12·20	−11·50	−11·58	−11·75

TABLE II(d)

Values of $\log_{10}K(j)$ for [K V]

$\log_{10}x$ $j=$	$^2D_{3/2}$			$^2D_{5/2}$			$^2P_{1/2}$			$^2P_{3/2}$		
$t=$	0·5	1·0	2·0	0·5	1·0	2·0	0·5	1·0	2·0	0·5	1·0	2·0
−2·0	−17·41	−17·40	−17·40	−17·25	−17·25	−17·26	−17·99	−17·98	−17·97	−17·67	−17·65	−17·65
−1·0	−16·41	−16·40	−16·40	−16·26	−16·25	−16·26	−16·98	−16·97	−16·97	−16·66	−16·65	−16·65
0·0	−15·40	−15·39	−15·39	−15·27	−15·27	−15·28	−15·95	−15·94	−15·94	−15·61	−15·60	−15·60
0·5	−14·87	−14·86	−14·87	−14·80	−14·81	−14·83	−15·39	−15·38	−15·38	−15·03	−15·02	−15·02
1·0	−14·32	−14·31	−14·32	−14·39	−14·40	−14·44	−14·76	−14·76	−14·77	−14·37	−14·36	−14·38
1·5	−13·76	−13·76	−13·79	−14·08	−14·10	−14·16	−14·08	−14·08	−14·12	−13·67	−13·68	−13·72
2·0	−13·24	−13·26	−13·32	−13·85	−13·89	−13·98	−13·38	−13·39	−13·46	−12·98	−13·00	−13·07
2·5	−12·82	−12·86	−12·96	−13·66	−13·71	−13·85	−12·66	−12·70	−12·80	−12·29	−12·32	−12·44
3·0	−12·54	−12·60	−12·75	−13·49	−13·57	−13·75	−12·01	−12·07	−12·22	−11·63	−11·69	−11·84

TABLE II(e)

Values of $\log_{10} K(j)$ for [O II]

$\log_{10} x$ / $j =$	$^2D_{5/2}$			$^2D_{3/2}$			$^2P_{3/2}$			$^2P_{1/2}$		
$t =$	0·5	1·0	2·0	0·5	1·0	2·0	0·5	1·0	2·0	0·5	1·0	2·0
−4·0	−19·04	−19·01	−18·98	−19·21	−19·19	−19·15	−19·64	−19·62	−19·61	−20·02	−20·00	−19·99
−3·0	−18·04	−18·01	−17·99	−18·21	−18·18	−18·15	−18·64	−18·62	−18·60	−19·02	−19·00	−18·98
−2·0	−17·06	−17·04	−17·01	−17·20	−17·17	−17·14	−17·60	−17·58	−17·57	−17·98	−17·96	−17·95
−1·5	−16·60	−16·58	−16·57	−16·68	−16·65	−16·63	−17·03	−17·01	−17·00	−17·42	−17·40	−17·39
−1·0	−16·21	−16·20	−16·20	−16·15	−16·13	−16·12	−16·40	−16·38	−16·38	−16·80	−16·78	−16·78
−0·5	−15·91	−15·90	−15·93	−15·68	−15·67	−15·68	−15·72	−15·71	−15·73	−16·12	−16·10	−16·13
0·0	−15·70	−15·70	−15·77	−15·34	−15·34	−15·39	−15·05	−15·04	−15·10	−15·44	−15·43	−15·49
0·5	−15·57	−15·59	−15·68	−15·15	−15·16	−15·24	−14·44	−14·44	−14·52	−14·83	−14·83	−14·91
1·0	−15·52	−15·54	−15·64	−15·07	−15·09	−15·19	−13·90	−13·90	−14·00	−14·28	−14·28	−14·38
2·0	−15·49	−15·51	−15·63	−15·04	−15·05	−15·17	−12·93	−12·93	−13·04	−13·33	−13·33	−13·44
3·0	−15·49	−15·51	−15·63	−15·03	−15·05	−15·17	−12·26	−12·27	−12·39	−12·71	−12·72	−12·84

where $A(j) = \sum_i A(j \to i)$. Substituting (2.15) in (2.14) and using (2.1) and (2.4) we obtain

$$K(j) = \frac{8{\cdot}63 \times 10^{-4}}{A(j)\omega_1} x\Upsilon(j, 1) \sum_i A(j \to i) E_{ji} \qquad (N_e \text{ small}). \tag{2.16}$$

In this approximation $K(j)$ is proportional to x and varies slowly with t.

In the limit of N_e large, the rates of collisional transitions are large compared with the rates of radiative transitions and the level populations are given by the Boltzmann equation,

$$\frac{N(j)}{N} = \frac{\omega_j e^{-E_{j1}/kT_e}}{\mathscr{P}} \qquad (N_e \text{ large}) \tag{2.17}$$

where

$$\mathscr{P} = \sum_i \omega_i e^{-E_{i1}/kT_e}. \tag{2.18}$$

For T_e not too large, $\mathscr{P} \simeq \omega_1$ and

$$K(j) \simeq \frac{\omega_j}{\omega_1} \sum_i A(j \to i) E_{ji} \qquad (N_e \text{ large}). \tag{2.19}$$

In this limit $K(j)$ is independent of x and t.

To summarize, we may say that, for all values of x, $K(j)$ varies slowly with t; and that $K(j)$ is proportional to x for small x and independent of x for large x.

The slow variation with t is seen in the results given in Table II, for $t = 0{\cdot}5$, $1{\cdot}0$ and $2{\cdot}0$; the results for other values of t are easily obtained by interpolation. The variation with x, or N_e, is illustrated in Fig. 1, which shows $\log_{10}K(j)$ for [Cl III] as functions of $\log_{10}N_e$, for $T_e = 10^4$ °K. It is seen that the low density approximation (2.16), $K(j) \propto N_e$, is valid for $N_e \leqslant 10^2$. For $j = {}^2D_{3/2}$ and ${}^2D_{5/2}$ the high density approximation (2.16), K independent of N_e, is valid for $N_e > 10^5$.

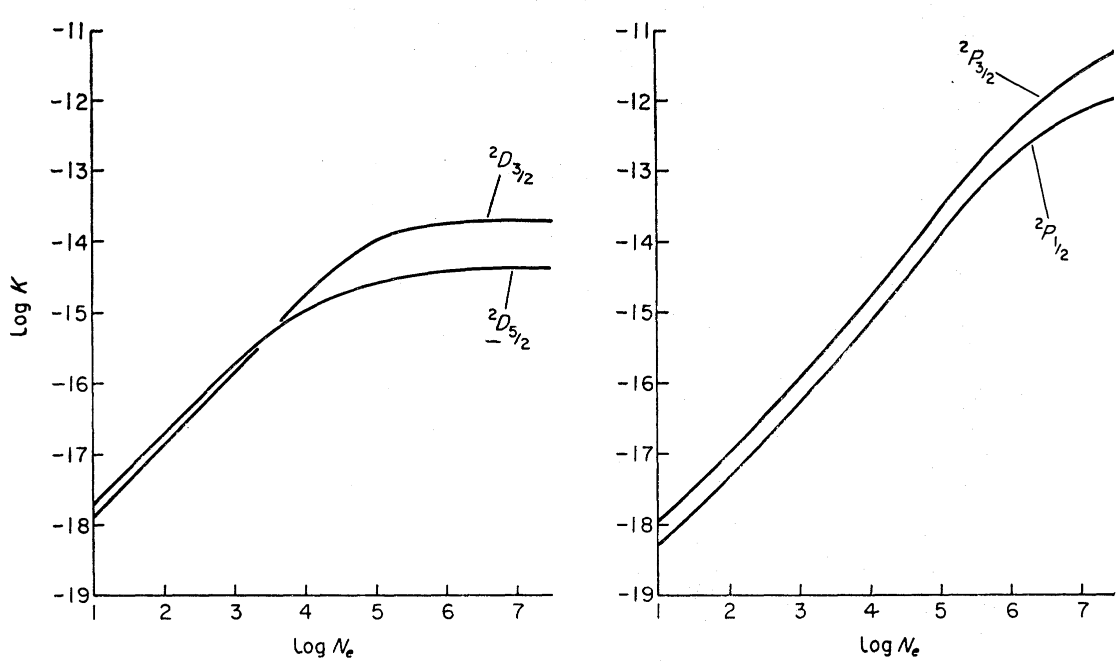

FIG. 1. *$\log_{10}K(j)$ for [Cl III] as functions of N_e for $T_e = 10^4$ °K.*

For the ^2P states the radiative transition probabilities are larger than those for the ^2D states, and K approaches the high density limit at larger values of N_e. It is of interest to note that for N_e in the range $10^2 < N_e < 10^4$, $K(^2P_{1/2})$ and $K(^2P_{3/2})$ increase somewhat faster than linear functions of N_e; at these densities excitation of ^2P from ^2D begins to be of importance.

It is seen from Fig. 1 that $K(^2D_{3/2})$ is smaller than $K(^2D_{5/2})$ at low N_e but larger at high N_e. Over a considerable range of densities the intensity ratio R is therefore sensitive to N_e. For a given value of $x = 10^{-2}N_e/T_e^{1/2}$, R is insensitive to T_e. Table III gives R for [S II], [Cl III], [Ar IV] and [K V], as functions of x for $T_e = 1 \times 10^4$ °K. Values of $1/R$ for [O II] have been tabulated by Eissner *et al.* (1969).

Table III

Intensity ratio $R = I(^2D_{3/2} \rightarrow {}^4S)/I(^2D_{5/2} \rightarrow {}^4S)$ *for* $T_e = 10^4$ °*K*

log x	R [S II]	[Cl III]	[Ar IV]	[K V]
−4·0	0·695	0·697	0·709	0·726
−3·0	0·697	0·698	0·709	0·726
−2·0	0·711	0·710	0·712	0·727
−1·5	0·743	0·737	0·718	0·728
−1·0	0·839	0·821	0·738	0·732
−0·5	1·09	1·06	0·800	0·744
0·0	1·57	1·67	0·990	0·783
0·5	2·13	2·72	1·54	0·90
1·0	2·48	3·78	2·85	1·27
1·5	2·63	4·39	5·03	2·27
2·0	2·67	4·63	7·07	4·42
2·5	2·66	4·70	8·18	7·37
3·0	2·63	4·69	8·58	9·60

Note: The results in this table are obtained using collision strengths calculated in the distorted wave approximation. In Section 4.1 *the following changes are recommended: for S II subtract* 0·26 *from log x; for Cl III subtract* 0·18 *from log x.*

Fig. 2 shows R for the various ions, as functions of N_e for $T_e = 10^4$ °K. To a good approximation at low densities, R is equal to the ratio of collision strengths, $\Omega(^4S, {}^2D_{3/2})/\Omega(^4S, {}^2D_{5/2})$, and these ratios are equal to 4/6, the ratio of statistical weights in the upper state. All p^3 ions therefore have similar ratios R for small N_e. For large N_e, R is determined by the ratio of transition probabilities, and these ratios depend much more on the ion considered.

3. Determination of electron densities in gaseous nebulae

There has been much recent interest in the construction of models for the structures of nebulae and in considering inhomogeneous models, that is to say models for which T_e and N_e may vary from point to point. We have presented our main results in the form of emissivities, rather than intensity ratios, since the former are required if one wishes to compute the spectrum line intensities for inhomogeneous nebular models.

In the remainder of the present paper we consider the interpretation of observed intensity ratios R for homogeneous models. Table IV gives the wavelengths of the

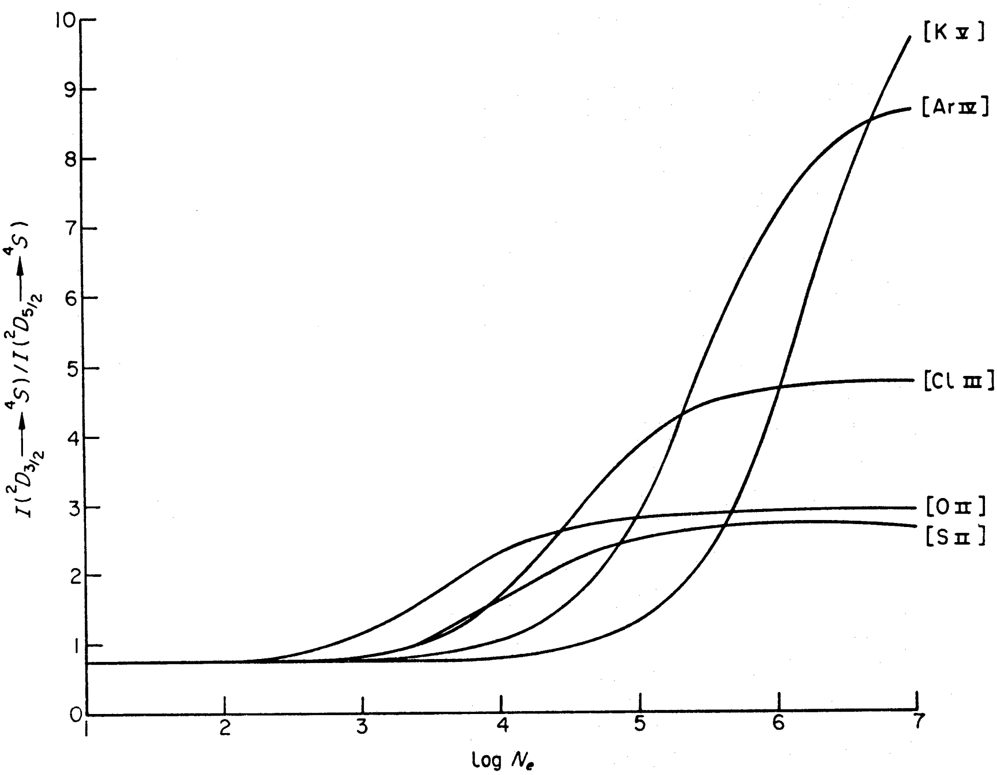

FIG. 2. *Intensity ratios* $R = I(^2D_{3/2} - {}^4S)/I(^2D_{5/2} - {}^4S)$ *as functions of* $\log N_e$ *for* $T_e = 10^4$ °K. *The uncorrected data of Table III is used.*

TABLE IV

Wavelengths for transitions $p^3\ {}^2D_J \rightarrow {}^3S$, *and ionization potentials*

Ion	λ, Ångström $^2D_{3/2} \rightarrow {}^4S$	λ, Ångström $^2D_{5/2} \rightarrow {}^4S$	I_1 eV	I_2 eV
[S II]	6731	6717	10·4	23·4
[O II]	3726	3729	13·6	35·1
[Cl III]	5538	5518	23·8	39·9
[Ar IV]	4740	4711	40·9	59·8
[K V]	4163	4123	60·9	82·6

observed ^2D_J–^4S transitions (from Aller, Bowen & Minowski 1955) and, for each ion, two ionization potentials: I_1 for production from the next lower stage of ionization and I_2 for ionization to the next higher stage. The ions are arranged in order of increasing ionization potentials.

Table V gives results for eleven planetary nebulae for which at least two intensity ratios R have been measured. The Table gives ratios obtained by different observers, adopted ratios, and values of N_e calculated for $T_e = 10^4$ °K. If any other value of T_e is assumed, these densities should be multiplied by $(T_e/10^4)^{1/2}$.

4. DISCUSSION OF RESULTS

4.1 *Results from different ions*

4.1.1 *The* [O II] *ratio.* We have used transition probabilities for [O II] calculated on the assumption that the observed value of R for IC 4997 is equal to $R(\infty)$. From the observed ratio we are therefore not able to deduce a value of N_e for this object.

TABLE V

Electron densities from observed intensity ratios R

NGC, etc.	Ion	R	Reference	Adopted R	log N_e From Table III, uncorrected	log N_e Recommended
IC 418	S II	1·2	4			
		1·98	12	1·8	4·2	3·9
		1·75	13			
	O II	2·5	4	2·6	4·4	⩽4·4
		2·7±0·03	11			
	Cl III	2·24	7	2·2	4·25	4·1
IC 2149	S II	1·7–1·85	13	1·8	4·2	3·9
	O II	1·45	10	1·45	3·3	3·3
2392	O II	2·1	10	2·1	3·8	3·8
	Cl III	0·6–1·3	7	⩽1·3	⩽3·7	⩽3·6
2440	O II	1·6	10	1·6	3·4	3·4
	Cl III	1·1	7	1·1	3·6	3·4
3242	O II	1·4	10	1·4	3·2	3·2
	Cl III	1·0	7	1·0	3·4	3·2
6543	S II	1·7	13	1·7	4·1	3·85
	O II	2·0	10	2·0	3·7	3·7
	Cl III	1·7	7	1·7	4·0	3·85
6572	S II	1·5	4			
		2·6	12	1·5	3·9	3·65
		1·3–1·6	13			
	O II	2·0	4			
		2·2	9	2·2	3·9	3·9
		2·2	10			
	Cl III	1·5	4	1·6	4·0	3·8
		1·9	7			
	Ar IV	2·0	8	2·0	4·7	4·7
4997	S II	1·6	12	1·6	4·0	3·75
	O II	2·9	8	2·9	—	—
		2·9	11			
	Cl III	2·66	7	2·7	4·5	4·3
	Ar IV	⩽2·4	8	⩽2·4	⩽4·9	⩽4·9
7009	S II	1·5–2·0	13	1·7	4·1	3·85
	O II	2·0	10	2·0	3·7	3·7
	Cl III	1·2	7	1·2	3·7	3·5
7027	S II	1·39	1			
		1·56–1·85	13	1·7	4·1	3·85
		1·9	4			
	O II	1·96	1			
		2·0	4	2·0	3·7	3·7
		1·93	6			
		2·1	10			
	Cl III	2·7	1			
		3·3	4	2·7	4·5	4·3
		2·7	7			

TABLE V—*continued*

NGC, etc.	Ion	R	Reference	Adopted R	log N_e From Table III, uncorrected	log N_e Recommended
7027	Ar IV	3·9	1	3·9	5·3	5·3
		⩾2·3	4			
		⩾1·3	5			
	K V	1·5	1	1·5	5·1	5·1
		1·4	6			
7662	S II	1·51–1·75	13	1·6	4·0	3·75
	O II	1·8	2	1·8	3·55	3·55
		1·8	3			
		1·6	4			
	Cl III	1·5	3	1·3	3·75	3·6
		1·0	4			
		0·89	7			
	Ar IV	0·95	2	1·0	4·1	4·1
		1·04	3			
		0·8	4			

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2. Aller, Kaler & Bowen 1966.
3. Minkowski & Aller 1956*.
4. Wyse 1942.
5. O'Dell 1963.
6. Aller, Bowen & Wilson 1963.
7. Czyzak, Walker & Aller 1969.
8. Aller & Kaler 1964.
9. Aller, Ufford & Van Vleck 1949.
10. Osterbrock 1960.
11. Seaton & Osterbrock 1957.
12. Andrillat & Houziaux 1968.
13. Weedman 1968.

* We quote results for the centre of the bright ring in NGC 7662.

Consideration of the observed [O III] ratio

$$R' = \{I(\lambda\, 4959)+I(\lambda\, 5007)\}/I(\lambda\, 4363) \qquad (4.1)$$

for IC 4997 led Seaton & Osterbrock (1957) to believe that the electron density was very high. Thus from $R' = 13{\cdot}1$ observed by Aller in 1938 (Aller 1941), Seaton and Osterbrock deduced that $N_e = 1{\cdot}3\times 10^6\ \mathrm{cm}^{-3}$ assuming $T_e = 2\times 10^4\,{}^\circ\mathrm{K}$ and $N_e = 7\times 10^6\ \mathrm{cm}^{-3}$ assuming $T_e = 1\times 10^4\,{}^\circ\mathrm{K}$. The IC 4997 results for [S II], [Cl III] and [Ar IV] give much smaller values of N_e. These results suggest that $R(\infty)$ for [O II] may be slightly larger than the value which we have adopted (say $R(\infty) = 3{\cdot}1$ in place of 2·9).

4.1.2. *The* [S II] *ratio*. The ionization potential for formation of S^+ (10·4 eV) is less than that for formation of H^+ (13·6 eV). Sulphur will therefore remain ionized throughout the transition region between H^+ and H^0. The ionization potential for formation of O^+ is equal to that for formation of H^+. Assuming that a transition region exists, the outermost parts of the region will give [S II] emission to be stronger than [O II] emission. One might therefore expect that, if there are any systematic differences between the densities deduced from [S II] and [O II], they should be such that the densities from [S II] are slightly smaller. This is not borne out by the results in Table V; for the seven nebulae for which N_e can be obtained from both [S II] and [O II] we obtain mean values of log N_e of 4·1 from [S II] and

3·8 from [O II]. This result is probably a consequence of errors in the distorted-wave collision strengths used for [S II]. The ratio of the close-coupling collision strengths of Conneely *et al.* (1970) to the distorted-wave collision strengths is 1·8 for $\Omega(^4S, {}^2D)$ and 1·4 for $\Omega(^4S, {}^2P)$. This is for an energy of 0·0762 Rydbergs above the ^2P threshold (Conneely 1969). The ratio R depends mainly on the collision strengths $\Omega(^4S, {}^2D_{3/2})$ and $\Omega(^4S, {}^2D_{5/2})$ (which are proportional to $\Omega(^4S, {}^2D)$) and $\Omega(^2D_{3/2}, {}^2D_{5/2})$. We expect that the fractional error in $\Omega(^2D_{3/2}, {}^2D_{5/2})$ will be similar to the error in $\Omega(^4S, {}^2D)$, and that the fractional errors will vary fairly slowly with energy. Since the electron density enters the expressions for the emissivities as the product of electron density times collision strength, collision strengths which are too small will give densities which are too large. It appears, both from the close-coupling calculations and from the observational results, that the adopted [S II] collision strengths are too small, by a factor of about 1·8.

We recommend that electron densities deduced from the data of Table III for [S II] should be divided by 1·8 (*subtract* 0·26 *from log x, in Table II*(*a*) *and in Table III for* [*S II*]).

With this correction there is no systematic difference between the densities obtained from [O II] and from [S II].

4.1.3. *The* [Cl III] *ratio.* Comparison of ionization potentials suggests that the [O II] and [Cl III] emissions come from similar regions in the nebulae, although [Cl III] emission will be less important in the transition region. There is good general agreement between the densities from [O II] and [Cl III], but the latter tend to be a little larger. The close-coupling calculations suggest that the distorted-wave collision strengths are too small, by factors of about 1·5.

We recommend that densities deduced from the data of Table III for [*Cl III*] *should be divided by* 1·5 (*subtract* 0·18 *from log x, in Table II*(*b*) *and from Table III for* [*Cl III*]).

4.1.4. *The* [Ar IV] *and* [K V] *ratios.* Although the distorted-wave and close-coupling results are in good agreement for [Ar IV], one cannot rule out the possibility that the collision strengths for [Ar IV] and [K V] are in error due to effects neglected in both calculations (collisional coupling between $3s^23p^3$ and $3s3p^4$).

4.2. *Results for individual nebulae*

In the following discussion we consider the electron densities given in the last column of Table V, calculated allowing for the corrections recommended in Section 4.1.

IC 418. The [O II] intensity ratio R observed for this nebula is rather close to the adopted value of $R(\infty)$. The deduced electron density is therefore sensitive to the exact value of R, and to the value adopted for $R(\infty)$. IC 418 appears to be a particularly uniform nebula, and considering the results for [S II], [O II] and [Cl III] we adopt a mean density $\log N_e = 4{\cdot}1 \pm 0{\cdot}1$.

IC 2149. We obtain $\log N_e = 4{\cdot}0$ from [S II] and $\log N_e = 3{\cdot}5$ from [O II].

NGC 2392. Czyzak, Walker and Aller obtain different [Cl III] ratios R on making observations for different points in this nebula. A mean density would have little significance.

NGC 2440 *and* 3242. Both [O II] and [Cl III] give rather low values of N_e for these two nebulae. We adopt $\log N_e = 3{\cdot}5 \pm 0{\cdot}1$ for NGC 2440 and $\log N_e = 3{\cdot}3 \pm 0{\cdot}1$ for NGC 3242.

NGC 6543. From [S II], [O II] and [Cl III], $\log N_e = 3{\cdot}8 \pm 0{\cdot}1$.

NGC 6572. From [S II], [O II] and [Cl III], $\log N_e = 3{\cdot}8 \pm 0{\cdot}2$. A higher density is indicated by [Ar IV].

IC 4997. This nebula has been discussed in Section 4.1.1. The outstanding problem is to reconcile the high density indicated by the [O III] ratio R' with the lower densities indicated by [S II] and [Cl III]. It has been shown by Aller & Liller (1966) that R' varies with time; from their results we deduce that a measurement in 1968 would have given $R' = 23$, compared with $R' = 13$ in 1938. The 1968 value of R' would give densities smaller than those deduced from the 1938 values, considered by Seaton and Osterbrock, but still larger than those obtained from the ratios R. In order to explain all of the observations it may be necessary to adopt an inhomogeneous model. Further observations would be of value, and would enable a more accurate estimate of $R(\infty)$ for [O II] to be obtained.

NGC 7009. From [S II], [O II] and [Cl III], $\log N_e = 3{\cdot}8 \pm 0{\cdot}2$.

NGC 7027. The results for this nebula indicate large scale density variations. The density increases with increasing ionization potential, from $\log N_e = 3{\cdot}8 \pm 0{\cdot}1$ for [S II] and [O II] to $\log N_e = 5{\cdot}2 \pm 0{\cdot}1$ for [Ar IV] and [K V].

NGC 7662. From [S II], [O II] and [Cl III], $\log N_e = 3{\cdot}6 \pm 0{\cdot}2$. A somewhat higher density is indicated by [Ar IV].

CONCLUSIONS

The present data, with the corrections recommended in Section 4.1, is the most accurate currently available for the determination of densities in nebulae from the intensity ratios of ions with p^3 configurations.

There is good general agreement in the densities deduced from [S II], [O II] and [Cl III]. Higher densities are obtained from [Ar IV] and [K V] observed in high excitation planetaries. This could be evidence for large-scale variations in density, the highest density occurring in those parts of the nebulae closest to the central stars.

ACKNOWLEDGMENTS

Our work on these problems has been supported by a grant from the Science Research Council. A part of the work was done during a visit by M. J. Seaton to the Joint Institute for Laboratory Astrophysics, Boulder, Colorado.

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