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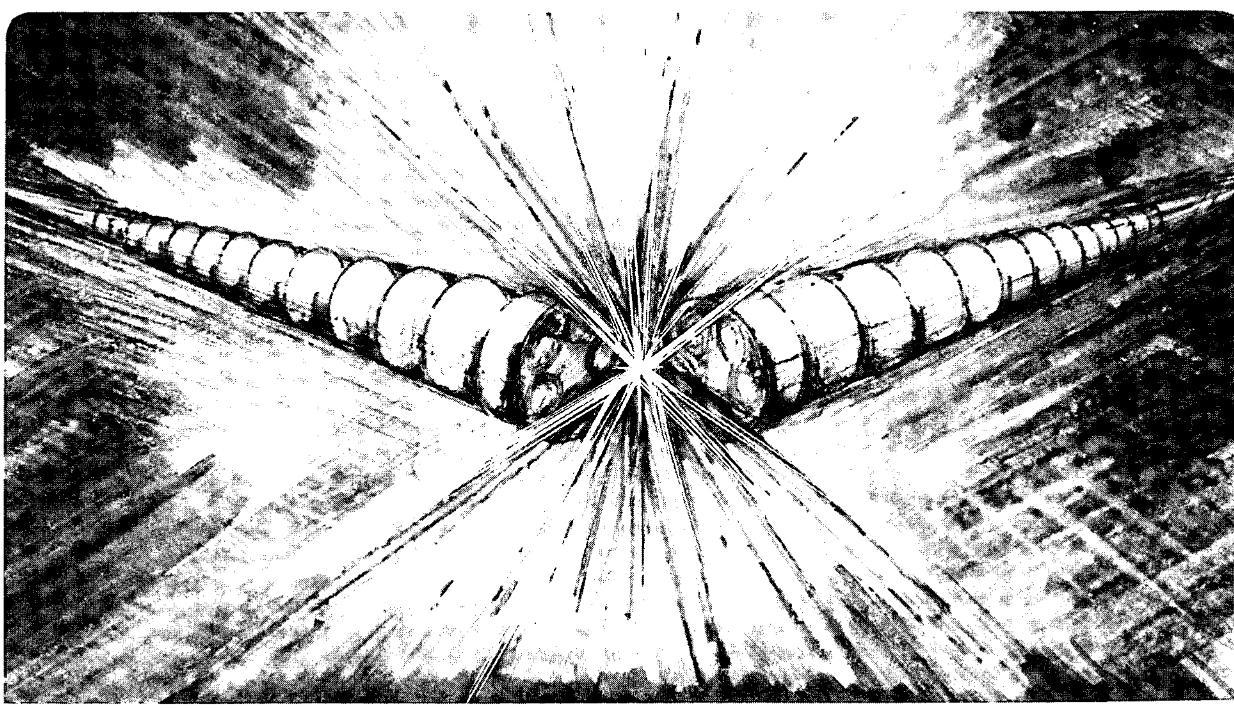

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Electron-hose instability in the ion-focussed regime

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A relativistic electron beam propagating through an unmagnetized, underdense, collisionless plasma exhibits a transverse instability due to the coupling of the beam centroid to plasma electrons at the ion-channel edge. The transverse wakefield corresponding to this "electron-hose" effect is calculated in the "frozen-field" approximation, for a low current, cylindrical beam. The asymptotic growth of beam centroid oscillations is computed and possible damping and saturation mechanisms are discussed.

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In recent years, the demands of the TeV-energy electron-positron collider,[1] have spurred considerable theoretical interest in the application of the "ion-focussed regime" (IFR), for the transport in plasma of intense relativistic electron beams. Proposed applications include the plasma lens,[2] the continuous plasma focus,[3] and the plasma emittance damper.[4] At the same time, the IFR has been considered theoretically for the production of coherent radiation from intense beams,[5,6,7,8] and experimental work in this area is in progress.[9] These novel concepts for transport, acceleration, and radiation, draw on a considerable body of work in beam-plasma physics,[10,11,12] and extensive practical application of the IFR in accelerator and radiation research.[13,14,15]

In this Letter, we show that IFR devices relying on an unneutralized ion-channel, surrounded by a quasineutral plasma, suffer from a previously unrecognized hose instability. This instability is similar to the ion-hose instability,[16] and other varieties of two-species transverse coupling instability,[17] except that here the coupling is between the beam and plasma electrons at the boundary between the ion-channel and the surrounding quasineutral plasma.

To characterize the IFR and the equilibrium to be perturbed, we consider propagation of a relativistic electron beam in an unmagnetized, preformed plasma. We assume an unperturbed beam charge density of the form $\rho_{b0}=-en_bH(a-r)$, where H is the step function, $-e$ is the electron charge, r is the radial coordinate in the x-y plane and a is the beam radius (Fig. 1). The beam density on axis is n_b and is a function of $s=t-z/c$, where t is time, z is axial displacement and c is the speed of light.

As the beam head propagates through the plasma, it expels plasma electrons from the beam volume, on the time scale of a plasma electron period. Typically, $\omega_e \tau_r >> 1$, where τ_r is the current rise time, and ω_e is the plasma electron frequency, $\omega_e^2 = 4\pi n_e e^2/m$. The electron mass is m and n_e is the initial plasma density. When the plasma is "underdense" ($n_e < n_b$) plasma electrons are adiabatically expelled from a cylindrical volume or "ion-channel" of radius $b \sim a(n_b/n_e)^{1/2}$. This nonneutral ion-channel is maintained for a time of order ω_i^{-1}, where ω_i is the ion plasma frequency, $\omega_i^2 = 4\pi n_e e^2/m_i$, with m_i the ion mass. We assume $2\omega_i \tau < 1$, where τ is the pulse length, so that ion-neutralization of the beam can be neglected.

For effective focussing of the beam the transverse pinch force on the beam due to the ion charge should be much larger than the transverse force due to self-fields. This imposes the Budker condition[10] on the plasma density, $n_e >> n_b/\gamma^2$, where γ is the Lorentz factor for the beam. In this limit, the Lorentz force equation shows that electrons undergo transverse oscillations at the "betatron frequency"[18] $\omega_\beta \sim \omega_e/(2\gamma)^{1/2}$. The main attraction of the IFR and the primary motivation for using a plasma in beam transport, is that ω_β is much larger than that achievable with conventional magnets.

For this work, we will assume that the collisionless plasma skin-depth, c/ω_e, is much larger than the channel radius. Thus $\omega_e b/c = 2(I/I_0)^{1/2} << 1$, where $I_0 = mc^3/e \sim 17$ kA, and I is the beam current. Since, collisionless plasmas characteristically neutralize magnetic fields on the scale of a skin depth, we expect the axial plasma electron current, and drift velocity, to be negligible in this limit. In this case, the equilibrium plasma electron charge density is $\rho_{e0} = -en_e H(r-b)$.

We consider next the effect of a perturbation to the beam centroid (Fig. 2) in the form of a small displacement, ξ, of the beam centroid, in the x direction. The perturbation to the beam charge density is then $\rho_{b1} = -en_b \xi \delta(a-r)\cos\theta$, where θ is the azimuthal angle in the x-y plane. We proceed to compute the perturbed scalar and axial vector potentials, ϕ_1 and A_{z1}, and the perturbed plasma electron density ρ_{e1}, due to ξ.

Maxwell's equations in the Lorentz gauge are

$$\left\{\nabla_\perp^2 + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right\}a_1 = 4\pi\rho_{e1} \; ,$$
$$\left\{\nabla_\perp^2 + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right\}\phi_1 = -4\pi(\rho_{b1} + \rho_{e1}) \, . \tag{1}$$

where $a_1=A_{z1}-\phi_1$. The transverse gradient is $\nabla_\perp$. We will change variables from z, t to z, s and simplify Eq. (1) with the "frozen-field" approximation, in which the D'Alembertian operators are approximated by $\nabla_\perp^2$ and radiative effects are neglected. We shall find that this amounts to a neglect of $\omega_e b/c\gamma << 1$.

The perturbed plasma electron charge density is determined from the potentials through the electron cold-fluid equations,

$$\frac{\partial\rho_{e1}}{\partial s} + \vec{\nabla}\bullet\left(\rho_{e0}\vec{v}_{e1}\right) \approx 0 \quad ,$$
$$\frac{\partial\vec{v}_{e1}}{\partial s} \approx \frac{e}{m}\vec{\nabla}\phi_1 \quad . \tag{2}$$

Inspection of Eq. (2) shows that ρ_{e1} consists entirely of a surface charge layer at $r=b$. Thus it is convenient to define,

$$P(z,s)\cos\theta = \int_{b^-}^{b^+} dr \; \rho_{e1}(r,\theta,z,s) \quad , \tag{3}$$

the dipole moment density induced on the channel wall by the beam charge.

In terms of ξ and P the potentials from Eq. (1) are

$$\phi_1 = 2\pi \cos(\theta) \begin{cases} (P - n_b e \xi) r & ; r < a \\ Pr - \dfrac{n_b e \xi a^2}{r} & ; a < r < b \\ (Pb^2 - n_b e \xi a^2)\dfrac{1}{r} & ; b < r \end{cases} , \tag{4}$$

and

$$a_1 = -2\pi \cos(\theta) \begin{cases} Pr & ; r < b \\ \dfrac{Pb^2}{r} & ; b < r \end{cases} . \tag{5}$$

The polarization is determined from ξ, through Eq. (2),

$$\left(\frac{\partial^2}{\partial s^2} + \omega_0^2\right) P(z,s) = \frac{a^2}{b^2} \omega_0^2 n_b e \xi(z,s) , \tag{6}$$

where $\omega_0 = \omega_e / 2^{1/2}$. Thus P responds as a simple harmonic oscillator with characteristic angular frequency ω_0. This frequency differs from ω_e because the surface at $r=b$ is the boundary between a region of electron density n_e, and a region of zero density.

The Lorentz force law for the displacement of the beam centroid is

$$\left(\frac{\partial}{\partial z} \gamma \frac{\partial}{\partial z} + \gamma k_\beta^2\right) \xi = - \frac{e}{mc^2} \frac{\partial}{\partial x} a_1 , \tag{7}$$

where $k_\beta \sim \omega_\beta / c$ is the betatron wavenumber. This describes the deflection of the beam by the image polarization on the ion-channel wall.

For an infinite beam and beamline, we may combine Eqs. (6) and (7), taking a perturbation varying as $\xi \propto exp(ikz-i\omega s)$, to obtain the dispersion relation,

$$\left(1-\frac{k^2}{k_\beta^2}\right)\left(1-\frac{\omega^2}{\omega_0^2}\right)=1 \qquad (8)$$

Equation (8) predicts instability for real $\omega^2<\omega_0^2$ or real $k^2<k_\beta^2$ with growth rates diverging as $\omega^2\to\omega_0^2$, or $k^2\to k_\beta^2$ from below. As in the "rigid beam" model of the resistive-hose instability,[19] we expect these singularities to result in an instability which is absolute in both the beam and lab frames.

To obtain a more quantitative result, we solve the initial value problem for a semi-infinite beam and beamline. We invert Eq. (6) to obtain P, and using Eqs. (5) and (7), we obtain a "beam break-up equation"[20] for ξ,

$$\left(\frac{\partial}{\partial z}\gamma\frac{\partial}{\partial z}+\gamma k_\beta^2\right)\xi(z,s)=\int_0^s ds'\left(\frac{I(s')}{I_0}\right)W(s-s')\,\xi(z,s') \quad , \qquad (9)$$

where we identify the electron-hose "dipole wake potential"[21] as $W(s)=W_0 sin(\omega_0 s)$, with $W_0=2\omega_0/b^2$. This wake is formally identical to that of an undamped microwave cavity,[22] with a shunt impedance per unit length[23] of $2/\omega_0 b^2$, and a resonant frequency ω_0. Interestingly, $(I/I_0)W_0=\omega_0^3/c^2$, independent of current, so that the details of the variation $I(s')$ of current along the beam are irrelevant to growth. Indeed, inspection of Eq. (9) shows that with the scaling of z by k_β, and s by ω_0, for γ, ω_0 independent of s and z, no free parameters remain. There is only one, universal solution for prescribed initial conditions.

We obtain this solution up to quadrature by Laplace transforming Eq. (9), solving the simple harmonic oscillator equation, and inverting the Laplace transform, to find

$$\xi(z,s) = \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} dp\, \frac{1}{p} \exp(ps) \cos\left\{ z\left[k_\beta^2 - \left(\frac{I}{\gamma I_0}\right) w(p)\right]^{1/2}\right\}, \tag{10}$$

where $w(p)$ is the Laplace transform of the wake and the initial condition $\xi(0,s)=H(s)$ is assumed.

This solution has been studied extensively, in connection with microwave cavity BBU,[22,23] and the distinction between the "weak" and "strong" focussing limits is worth noting.[24,25] Focussing is "weak" when $\lambda_\beta > L_g$, where L_g is the instability growth length, and $\lambda_\beta = 2\pi/k_\beta$ is the betatron period. In the ion-channel, k_β and W_0 are not free parameters, and when $\omega_0\tau >> 1$, focussing is always weak with respect to electron-hose growth. This is seen by the asymptotic form of Eq. (10), obtained by the method of steepest descents,

$$\xi(z,s) \approx \frac{2^{3/2}}{3^{5/4}\pi^{1/2}} \frac{A^{1/2}}{\omega_0 s} e^A \sin\left\{\omega_0 s - 3^{-1/2}A - \frac{\pi}{12}\right\}. \tag{11}$$

The term in the exponent is $A=(z/L_g)^{2/3}$, where the growth length is

$$L_g = \frac{2^3}{3^{9/4}}\left(\frac{\gamma I_0}{I}\right)^{1/2} \frac{1}{\sqrt{W_0 s}} = \frac{2^2}{3^{9/4}\pi} \frac{\lambda_\beta}{\sqrt{\omega_0 s}}, \tag{12}$$

and $\omega_0 s >> A >> 1$ is assumed. Inspection of Eqs. (11) and (12) confirms the frozen field approximation and the weak-focussing approximation. This result is remarkable in that it shows ion-focussing is rendered ineffective by the presence of free plasma electrons at the channel wall.

We turn next to consider mechanisms which will tend to reduce growth. We observe from the dispersion relation of Eq. (8), that there are in principle two methods of "curing" the electron-hose. We may diminish the resonance at $\omega^2 \to \omega_0^2$, or at $k^2 \to k_\beta^2$. On the other hand, since focussing is typically weak, damping mechanisms relying on a spread or sweep in betatron wavenumber, $\Delta k_\beta \sim 1/L_g$, are ineffective, as they require an impractically large spread, $\Delta k_\beta / k_\beta \sim 1/k_\beta L_g > 1$. This rules out Landau damping due to a spread in energy within a beam slice,[24] and "BNS damping" due to a sweep in energy from head to tail.[26] This also rules out "phase-mix damping" of BBU growth due to nonlinear focussing, arising from a radially non-uniform plasma.[13] (These conclusions contrast with those for resistive-hose growth,[27] where focussing is typically strong.)

Thus, to diminish electron-hose growth, we must look to the resonant coupling at $\omega^2 \to \omega_0^2$, and a number of mitigating factors suggest themselves. First, the electron-hose could be eliminated entirely by ionizing a channel of radius less than b. In this case, all plasma electrons are ejected to the beam-pipe wall, leaving no free plasma electrons at the channel edge. Alternatively, an axial variation in plasma density, as in the continuous plasma focus,[3] may produce phase-mix damping. In this case, the plasma density should vary appreciably over a length $L_g < \lambda_\beta$.

Growth could also be reduced by varying the resonant frequency of plasma oscillations, through the external geometry. For example, if we add a conducting pipe of radius R to the problem, we find a resonant frequency

$\omega_0' \sim \omega_0(1+b^2/2R^2)$, i.e., the image dipole, P oscillates at a slightly higher frequency, dependent on R. Thus a variation of the pipe radius on the scale of a growth length could in principle produce the effect of "stagger tuning" used in conventional microwave beam break-up, where successive cavities are slightly detuned to produce phase-mixing in the driving term on the right side of Eq. (9).

In addition, growth will be mitigated somewhat by the plasma return current, neglected in the approximation $\omega_e b/c<<1$. In the low current limit we have considered, the electron-hose is formally analagous to the image-displacement effect in conventional accelerators.[28] Were a conducting boundary or a sufficiently dense plasma nearby, it would carry a dipole image current, and the combined Lorentz force on the beam due to the image fields would be a factor of $1/\gamma^2$ less than for the electric field term alone. On the other hand, to achieve even $\omega_e b/c \sim 2$ requires $I \sim I_0$, a current larger than typically envisioned.

Ultimately, as a result of hosing, plasma electrons will be heated, and the instability will saturate. The simplest estimate would give saturation when $\xi \sim b$, corresponding to substantial growth in the beam emittance, and a significant electron temperature$\sim mc^2(I/I_0)$. In fact, this omits the subtler feature that, at lesser temperatures, the channel wall will take on the character of a Debye sheath, with a radial variation in the plasma frequency and a phase-mix reduction of the wake driving term. A simliar effect may obtain due to beam "halo". Numerical studies are in progress to assess these effects.

Finally, it is interesting to note that for an infinitely wide planar beam, the electron-hose dipole wake vanishes. This is because a one-dimensional dipole field vanishes outside the source. However, in this case we have found a flute-like analog of the electron-hose, where ripples develop in the beam density and provide a coupling of the beam and the channel wall. Nevertheless, it may be possible that for ellipsoidal beams, electron-hose growth could be reduced with a sufficiently large aspect ratio.

In conclusion, we have set down a simple, first theory for a rapidly growing cumulative instability of a relativistic beam-plasma system. Its consequences could be severe for a number of recently proposed beam-plasma devices, and its role in plasma-heating, radiation, and stable beam-plasma equilibria is likely of fundamental importance. Further analytic work and numerical simulation is in progress to assess damping and saturation mechanisms.

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[1]R.B. Palmer, *New Developments in Particle Acceleration Techniques*, edited by S. Turner, CERN 87-11, Vol.1, p.80.

[2]J. J. Su, T. Katsouleas, J. M. Dawson, and R. Fedele, Phys. Rev. A **41**, 3321 (1990).

[3]P. Chen, K. Oide, A.M. Sessler, S. S. Yu, Phys. Rev. Lett. **64**, 1231 (1990); D.H. Whittum, *Proceedings of the Topical Conference on Research Trends in Nonlinear and Relativistic Effects in Plasmas*, (LaJolla Institute, 1990).

[4]W. A. Barletta, in *Proceedings of the Workshop on New Developments in Particle Acceleration Techniques*, (Orsay, 1987).

[5]K. Takayama and S. Hiramatsu, Phys. Rev. A **37**, 173 (1988).

[6]W. A. Barletta and A. M. Sessler, *Proceedings of the INFN International School on Electromagnetic Radiation and Particle Beam Acceleration* (Varenna, 1988).

[7]D. H. Whittum, A. M. Sessler and J. M. Dawson, Phys. Rev. Lett. **64**, 2511 (1990).

[8]K. R. Chen, T. Katsouleas, and J. M. Dawson, "On the Amplification Mechanism of the Ion-Channel Laser", IEEE Trans. Plasma Sci. (accepted for publication).

[9]S. Hiramatsu, K. Ebihara, Y. Kimura, J. Kishiro, M. Kumada, H. Kurino, Y. Mizumachi, T. Ozaki and K. Takayama, Nucl. Instrum. Methods **A285**, 83 (1989).

[10]G.J. Budker, *Proceedings of the CERN Symposium on High Energy Accelerators and Pion Physics*, (Geneva, 1956) pp. 68-75.

[11]R. Okamura, Y. Nakmura, and N. Kawashima, Plasma Phys. **19**, 997 (1977).

[12]P. C. de Jagher, F. W. Sluijter, and H. J. Hopman, Physics Reports **167**, 177 (1988).

[13]G. J. Caporaso, F. Rainer, W. E. Martin, D. S. Prono, and A. G. Cole, Phys. Rev. Lett **57**, 13 (1986).

[14]R. B. Miller, *Physics of Particle Accelerators,* edited by Melvin Month and Margaret Dienes, AIP Conf. Proc. **184**, Rita G. Lerner, series editor, (AIP, New York, 1989), Vol. 2, p. 1730.

[15]R. F Lucey, Jr., R. M. Gilgenbach, J. D. Miller, J. E. Tucker, and R. A. Bosch, Phys. Fluids B **1**, 430 (1989).

[16]H. L. Buchanan, Phys. Fluids, **30**, 221 (1987).

[17]L. J. Laslett, D. Mohl, and A. M. Sessler, *Proceedings of the Third All-Union National Conference on Particle Accelerators,* (Moscow, 1972).

[18]Stanley Humphries, *Principles of Charged Particle Acceleration,* (John Wiley & Sons, New York, 1986).

[19]E.P. Lee, Phys. Fluids **21**, 1327 (1978).

[20]K. L. F. Bane, *Physics of Particle Accelerators,* AIP Conf. Proc. **153**, edited by Melvin Month and Margaret Dienes (AIP, New York, 1987), pp. 972-1012.

[21]Alexander W. Chao, *Physics of High Energy Particle Accelerators,* AIP Conf. Proc. **105**, edited by Melvin Month (AIP, New York, 1983), pp. 353-523.

[22]W. K. H. Panofsky and M. Bander, Rev. Sci. Instrum. **39**, 206 (1968).

[23]V. K. Neil, L. S. Hall, and R. K. Cooper, Particle Accelerators **9**, 213 (1979).

[24]Y. Y. Lau, Phys. Rev. Lett. **63**, 1141 (1989).

[25]D. H. Whittum, G. A. Travish, A. M. Sessler, G. D. Craig, and J. F. DeFord, *Proceedings of the 1989 IEEE Particle Accelerator Conference,* edited by Floyd Bennett and Joyce Kopta, (IEEE, New York, 1989) pp. 1190-1192 .

[26]V.E. Balakin, A.V. Novokhatsky, and V.P. Smirnov, *Proceedings of the 12th International Conference on High-Energy Accelerators*, edited by F.T Cole and R. Donaldson (Fermi National Accelerator Laboratory, Batavia, Illinois, 11-16 August 1984), pp. 119-120.

[27]M. Lampe, W. Sharp, R. F. Hubbard, E. P. Lee, R. J. Briggs, Phys. Fluids **27**, 2921 (1984).

[28]R. J. Adler, B. B. Godfrey, M. M. Campbell, D. J. Sullivan, and T. C. Genoni, Particle Accelerators **13**, 25 (1983).

FIG. 1. In equilibrium, a relativistic electron beam of radius a propagates through a channel of unneutralized ions. Plasma electrons have been expelled to a radius b.

FIG. 2. A beam slice in the ion-channel is displaced by an amount ξ in the x-direction, inducing a polarization $P\cos\theta$ on the channel wall. The polarization responds to the beam dipole field as a simple harmonic oscillator with characteristic angular frequency ω_0. This image dipole then deflects follow-on portions of the beam.

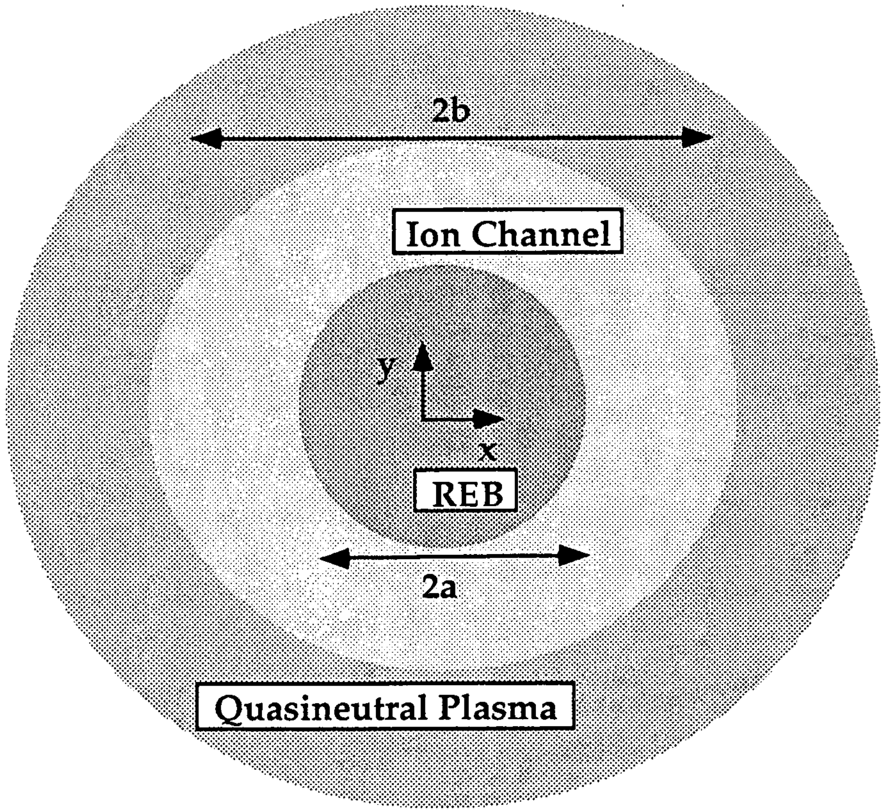

FIG. 1

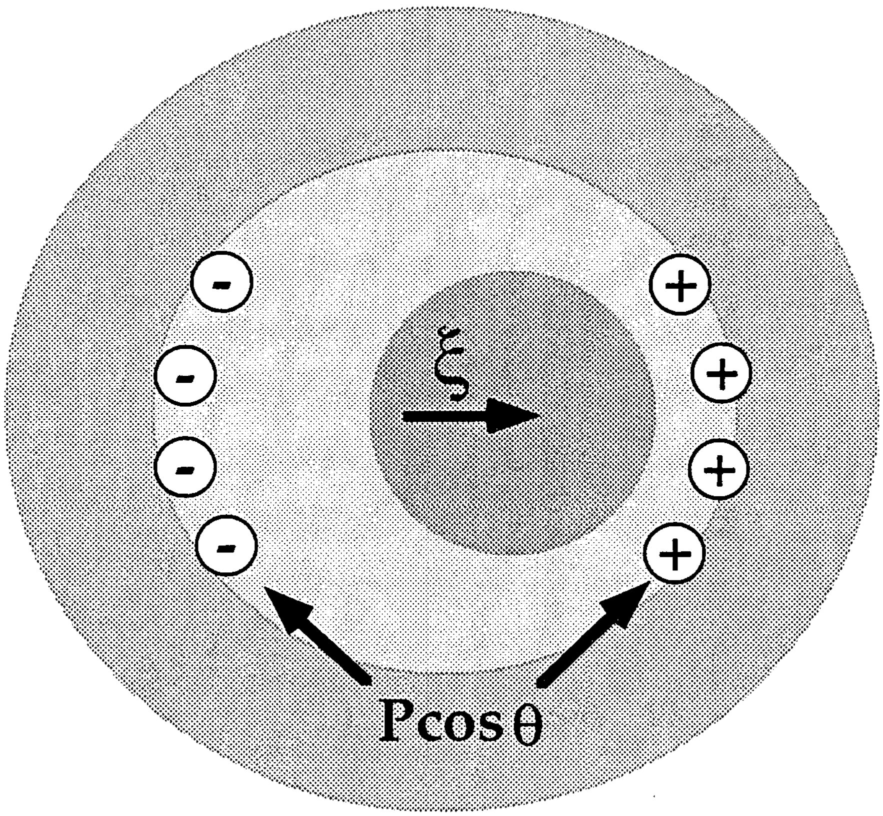

FIG. 2

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